CHAPTER 1

UNDERSTANDING SCHOOL MATHEMATICS

KEY TERMS

Communication patterns
Conceptual understanding
Curriculum
Evaluated curriculum
Implemented curriculum
Intended curriculum
Knowledge for teaching mathematics
Mathematical problem solving
Mathematical reasoning
Open-ended questions
Pedagogical content knowledge
Pedagogy
Problem solving
Procedural fluency
Representations
Rich tasks
Targeted teaching

CHAPTER OBJECTIVES

This chapter will enable the reader to:

- Understand the contribution of mathematics to society and the role of school mathematics in developing creative, informed, responsible citizens
- Understand the important contribution effective teachers of school mathematics make to the lives of their students both at and beyond school
- Develop a broad understanding of what is involved in the teaching and learning of mathematics in Foundation to Year 9 classrooms
- Recognise the important role of life-long learning, reflection and research in mathematics education.

BIG IDEAS

- We have never needed mathematics more.
- The depth and connectedness of teachers' knowledge for teaching mathematics makes a difference.

- Knowledge of students and students' learning of mathematics is essential.
- Teaching is a highly rewarding but complex and demanding task that requires thoughtful planning.
- Learning mathematics is both an individual and social enterprise—beliefs and social norms matter.
- School mathematics needs to change and is changing to meet the needs of all learners.

EVERYONE CAN DO MATHS

When teachers of mathematics are asked to say what they do in a social context, the questioner often responds with something like, 'I was never any good at maths ... I dropped it as soon as I could.' Such responses are incredible when you consider the person concerned is highly unlikely to make the same claims about his or her capacity to read or write. It seems that it is socially acceptable to admit to disliking or not being 'very good at' mathematics. It is a sad irony that those who profess such views frequently demonstrate sophisticated uses of mathematics in their everyday activities. For instance, I recall a taxi driver who, having confessed that he had 'failed mathematics in Year 9', expertly gauged the flow of the traffic, consulted a global positioning system, decided to take an alternative route to ensure we arrived in time, and at the end of the journey mentally added the airport tax to the fare and calculated the change. Everyone can do mathematics—there is no gene that determines who is 'good' at school maths and who is not—it just takes a caring, knowledgeable teacher of mathematics, who takes the time to identify where students are in their learning journey, and knows where to go to next, and how to get them there (Wiliam, 2011).

1. When was the last time you truly, madly, deeply, really enjoyed doing some mathematics? What did you do and how did it make you feel?
2. Can you recall a teacher who made a significant impact on your learning of mathematics or an experience when you were 'turned off' mathematics? If so, describe the circumstances in terms of who, what, when and where. How did it make you feel?

Introduction

In the preface to this book we talked about teacher quality and what it means to be an effective teacher of mathematics. While it is relatively easy to list generic characteristics, what matters is what students experience, individually and collectively in mathematics classrooms. Teacher quality is clearly related to teacher knowledge and confidence, but what actually happens in classrooms is also affected by what teachers feel they have to teach, how they go about teaching it, and the social contexts in which teaching takes place (Siemon, 2019).

Teachers choose to teach for a variety of reasons—and it is rarely for the money! The most commonly cited reason for choosing to teach is 'to make a difference'. In *A Sense of Calling: Who Teaches and Why*, Farkas, Johnson and Foleno (2000) report that 96 per cent of new teachers surveyed reported that they chose teaching because it 'involves work they love doing' and gave them 'a sense of contributing to society and helping others' (p. 11). 'Understanding what matters to people, what motivates them and why they do what they do can make the difference between a conversation that moves forward and one that goes nowhere' (p. 8). To this end, it is important to regularly reflect on what we do and why we do it.

TRY IT YOURSELF

→ Job, vocation or career

For me, in Year 12, teaching was the obvious choice. Where else do you get an opportunity to do all the things you like doing, such as learning mathematics, studying history, discussing literature, playing sport, singing and performing? But most of all, I became a teacher because I somehow felt it was the right thing to do. Helping others learn is not only very satisfying, but it also increases people's opportunities in life. Education is the key to making the world a better, fairer, more environmentally responsible place.

1. What were your reasons for choosing to become a teacher? Was it because of particular strengths and interests, role models or opportunities?
2. What do you hope to achieve as a teacher of mathematics?
3. What do you see yourself doing in ten years' time?

→ **LINKAGE:** The 'Try it yourself' sections throughout this book are designed to assist you to build your professional knowledge for teaching mathematics.

Given that most teachers are motivated by a desire to support the learning and well-being of others, and given what we know about effective mathematics teaching, why is it that students' experiences of learning mathematics are not that much different from what they were in the past, particularly in the middle years of schooling (Bodin & Capponi, 1996; Yates, 2005)? It appears that teachers tend to teach mathematics in the way that they were taught (e.g. Brady, 2007; Stigler & Hiebert, 1999) and that their decisions about content and **pedagogy** are very much related to what they know and believe about the nature of mathematics and the teaching and learning of mathematics (e.g. Brady, 2007; Fennema, Carpenter & Peterson, 1989; Handal & Herrington, 2003; Siemon, 2019).

In what follows we will look at what we mean by mathematics, consider the goals of contemporary school mathematics, make a case that despite our access to sophisticated technological tools we have never needed mathematics more, and explore what it takes to become a great teacher of mathematics.

What is mathematics?

We all have views on what constitutes mathematics, and these views shape our decisions about how we teach and learn mathematics. For instance, if mathematics is viewed as a set of universal truths, teachers are more likely to see their task as transferring a given set of facts and skills to students and to view student learning as the capacity to reproduce these facts and skills as instructed. If, on the other hand, mathematics is viewed as a socio-cultural practice, a product of reflective human activity, then it is more likely that teachers will see their task as engaging students in meaningful mathematical practices and view student learning in terms of conceptual change. While the two views may co-exist to some extent, what teachers believe about the fundamental nature of mathematics has important implications for practice as we shall see below.

Many views have been advanced to describe the nature of mathematics:

> Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality. (Courant & Robbins, 1941, p. xv)

> Mathematics reveals hidden patterns that help us understand the world around us … Mathematics is a science of pattern and order. (Mathematical Sciences Education Board, 1989, p. 31)

> [A]ll cultural traditions … find connecting patterns and apparent symmetries throughout nature. Individually and collectively, we accept these patterns as part of the background of our lives: beyond question or doubt, all Australians agree we live in an orderly, knowable universe. Yet the patterns which organise, and the laws which govern, European knowledge and perception apparently have little in common with the patterns which make sense of the Aboriginal world … both kinds of patterning form complex and mathematically sophisticated systems, both have powerful ideological underpinnings, both are social constructions emerging from historically identifiable contexts, both attempt to account for natural as well as social phenomena, both are rational in principle and practical in application. (Watson, 1989, p. 31)

These views all propose that mathematics offers a way of understanding the world we live in. Mathematics provides a consistent framework, a symbolic technology, by which we can model 'reality', solve problems and support predictions, but how this is described and communicated depends very much on our cultural values and traditions. For instance, our taken-for-granted view of mathematics—the mathematics of our schooling that Bishop (1991) refers to as 'Mathematics with a capital "M"'—is governed by number patterns and relations. There are other cultures whose ways of understanding the world are not governed by 'how much and how many' but by complex kinship patterns that connect all things to each other without the need of numbers (e.g. Bishop, 1991; Watson, 1989). In other words, mathematics is a 'pan-cultural phenomenon: that is, it exists in all cultures', and 'Mathematics' is a 'particular variant of mathematics, developed through the ages by various societies' (Bishop, 1991, p. 19).

This is a difficult notion to grapple with, as we can scarcely imagine a world without mathematics, and yet we happily accept that different languages are developed by different cultural groups in order to communicate. This prompted Bishop to ask: if language develops from the need for, and activity of, *communicating*, what are 'the activities and processes which lead to the development of mathematics?' (1991, p. 22). According to Bishop, there are six fundamental mathematical activities that occur in some form across all cultures, which he describes as follows.

1 *Counting*—the use of a systematic way to compare and order discrete phenomena, which may involve tallying, using objects or string to record, or special number words or names
2 *Locating*—exploring one's spatial environment and conceptualising and symbolising that environment, with models, diagrams, drawings, words or other means
3 *Measuring*—quantifying qualities for the purposes of comparison and ordering, using objects or tokens as measuring devices with associated units or 'measure-words'
4 *Designing*—creating a shape or design for an object or for any part of one's spatial environment, which may involve making the object as a 'mental template', or symbolising it in some conventional way
5 *Playing*—devising, and engaging in, games and pastimes, with more or less formalised rules of play that all players abide by
6 *Explaining*—finding ways to account for the existence of phenomena, be they religious, animistic or scientific (Bishop, 1988, pp. 182–3).

In looking at these activities and processes, we can see how it might be possible for there to be many mathematics underpinned by different cultural norms and values. For Bishop, mathematics can be viewed as a 'way of knowing'.

MAKING CONNECTIONS

→ A way of knowing

The cultured nature of mathematics is illustrated by an apocryphal story that describes a group of tourists who were being escorted through the central Australian desert by a local Indigenous man. When they had been travelling for some time and were feeling overwhelmed by the seemingly unchangeable landscape, one of the tourists asked the guide, 'What do you do when you get lost out here?' The man turned to the tourist somewhat bemused, and said, 'I go home.' In other words, for him, there was no notion of 'lost'. He knew the landscape intimately, and he could discern its patterns and variations. By this means he knew exactly where he was and how to return home, without the need of GPS, compass or paper map.

Bishop's (1988) set of six fundamental mathematical activities can be seen in the structure of the *Australian Curriculum: Mathematics* (Australian Curriculum Assessment & Reporting Authority [ACARA], 2015) which is discussed further below.

Goals of school mathematics

Niss (1996) suggests that there are three fundamental reasons for the existence of school mathematics as we know it today. They are that the study of mathematics contributes to:

> the technological and socio-economic development of society; the political, ideological and cultural maintenance and development of society; [and] the provision of individuals with prerequisites which may assist them in coping with private and social life, whether in education, occupation, or as citizens. (p. 22)

In ancient times, the study of mathematics was restricted to a select few, generally young men with the time to converse with more learned others. For some, this was simply an enjoyable pastime, an intellectual pursuit for its own sake, while for others such as the Pythagoreans, in Greece, this activity was shrouded in secrecy and mysticism. However, as production became more specialised and trade expanded, mathematical know-how was acquired as a matter of necessity. Merchants, builders, navigators, tax collectors, artisans and religious leaders all needed some mathematics, and they inducted those that followed them into their particular mathematical practices.

Before the nineteenth century, there was little offered in the way of formal education beyond the opportunity to learn a trade. For the very few who did receive some sort of formal education, this was largely justified on the grounds that it contributed to 'the political, ideological and cultural maintenance of society' (Niss, 1996, p. 22). This situation changed with the advent of the industrial age and the introduction of compulsory schooling. Elementary mathematics was included as a core component of the curriculum, presumably 'to contribute to the technological and socio-economic development of society, while at the same time placing some emphasis on equipping individuals with tools for mastering their vocational and everyday private lives' (Niss, 1996, p. 23).

Today, all three of Niss's reasons for including mathematics in school curricula can be found in the *Australian Curriculum: Mathematics* (ACARA, 2019), which aims to ensure that students:

- are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens
- develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in number and algebra, measurement and geometry, and statistics and probability
- recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.

Source: https://australiancurriculum.edu.au/f-10-curriculum/mathematics/aims/

Problem solving is a one of the most important reasons for studying mathematics. But, as indicated above, the problems have changed significantly from the days of our great-great-grandparents. Today's problems can be found in popular culture more than in everyday calculations, and we need to be able to reason mathematically in order to make sense of claims that are made in increasingly sophisticated ways. For example, mobile phone companies advertise their phone plans in ways that make it very difficult to decide which plan is the most appropriate. Solutions to these types of problems require a capacity to model and investigate complex situations, seek and process data, and communicate this effectively in a much wider range of ways than was the case in the past.

TRY IT YOURSELF

→ From an 1875 book of arithmetical examples

A wine merchant buys 3 hhds. (hogsheads) of wine at Bordeaux, at £15 per hhd., pays duty 1 s. (shilling) per gallon, and carriage £3. What must he sell the wine at per gallon, to clear £10 in the transaction? (Davis, 1875, p. 47)

1. What does this say about the assumptions underpinning the teaching and learning of mathematics at the time? How different are these from the assumptions underpinning your experience of school mathematics? Discuss in terms of the three fundamental reasons for studying mathematics described by Niss above.
2. What knowledge and skills are needed to solve this problem? (Hint: you may need to rewrite this in a more familiar form first.)

   In the same book, a 'simple multiplication' problem was given as 856 439 082 × 7 008 001 (p. 13). These days, it is difficult to understand why this was regarded as 'simple' or why such a problem would be set, but these examples nicely illustrate that school mathematics is a human construction reflecting societal values and priorities at particular points in time. The fact that all students, not just a select few, were expected to solve these problems is indicative of the goals of school mathematics at the time and the assumptions made about the teaching and learning of mathematics. Contrast this with how an Aboriginal child living in a remote part of Australia in 1875 might have been expected to learn to find his or her way home in the desert.

TEACHING TIP

Use a cyclical framework such as ASK–THINK–DO to explicitly model the problem-solving process. Questions that focus on problem type, what information is needed, and link to past experience (e.g. have I seen something like this before?) can be listed under the ASK and THINK headings. Strategies such as 'draw a diagram', 'use a table', and 'if then reasoning' can be listed under the DO heading. Questions and strategies can be added to as they arise in practice.

In recent years, the decline in the relative performance of Australian students on international assessments of school mathematics, such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme of International Student Assessment (PISA), and the significant decline in the proportion of Year 12 students undertaking the more advanced mathematics courses have dramatically increased public scrutiny of school mathematics not least because its critical role in supporting careers in science, technology, engineering and mathematics (STEM) fields (Siemon, Banks & Prasad, 2018). Parents, governments and industry groups all have a heightened awareness of the importance of school mathematics. Teachers of mathematics from the early to the middle years of schooling have a crucial role to play in helping students build the knowledge and confidence they need to successfully engage with school mathematics in the senior years and thereby increase their options for meaningful participation in our increasingly high-tech society.

We have never needed mathematics more

While technology has replaced many of the routine procedures traditionally taught in school mathematics, the advent of sophisticated information and communication technologies has also drastically changed the way we conduct our everyday lives. Mathematics underpins much of this technology, and mathematics is used in increasingly powerful and subtle ways to persuade voters and consumers to act in certain ways. This together with the fact that the fastest growing employment opportunities require some form of STEM qualification suggest that we have never needed mathematics more. But the mathematics we need is not the school mathematics of the past with its emphasis on isolated skills and rote learning. Students at all levels of schooling now need to develop a deep understanding of the concepts and principles underpinning school mathematics in order to apply the mathematics they know to solve unfamiliar problems, justify their reasoning to others, and equip them to understand the ways in which mathematics is being used in our modern, technological world to monitor and shape our lives (Gellert & Jablonka, 2008).

Numeracy

Although not everyone needs to achieve a highly sophisticated level of mathematics, school mathematics has a particular (but not the sole) responsibility for developing student *numeracy*.

> In the Australian Curriculum, students become numerate as they develop the knowledge and skills to use mathematics confidently across other learning areas at school and in their lives more broadly. Numeracy encompasses the knowledge, skills, behaviours and dispositions that students need to use mathematics in a wide range of situations. It involves students recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.
>
> *Source: https://australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy/*

LINKAGE: ←
Chapter 9 examines what we mean by numeracy and explores its relationship to school mathematics in Years F–9. In particular, this chapter focuses on how mathematics can be applied across the curriculum and in everyday contexts to solve problems and inform decision making.

ACTIVITY 1.1 — 7 TO 14 YEARS

Images of mathematics in use

1. Collect a range of newspapers, magazines, food containers and screen displays from a variety of websites.
2. Ask students to identify where mathematics is being used and how in each of the items.
3. Encourage younger students to look beyond numbers to measures. Encourage older students to look for instances of proportion and other, more subtle, uses of mathematics such as location of visual displays and headings.
4. Represent the collection as a collage of annotated images for classroom display.

Today, individuals need to be able to make sense of vast amounts of quantitative and spatial information presented in increasingly sophisticated multimedia formats; make decisions on the basis of that understanding; and communicate their reasons for doing so if challenged. According to Becker and Selter (1996), the 'ultimate objective of student learning at all levels is the acquisition of a *mathematical disposition* rather than the absorption of a set of isolated concepts or skills' (p. 542). In their opinion, students should learn to:

LINKAGE: Chapter 4 considers what is involved in building conceptual understanding and procedural fluency. Chapter 5 explores the mathematical proficiencies of problem solving and mathematical reasoning.

> *be creative*: to look for patterns, make conjectures, generate new problems ... to *reason*: to give arguments, uncover contradictions, distinguish between facts and assertions ... to *mathematize*: to collect data, process information, interpret data and solutions ... and to *communicate*: to express their own thoughts, accept the ideas of others, establish forms of cooperation. (p. 542)

This is consistent with Bishop's view of mathematics as 'a way of knowing' rather than 'a way of doing'. The values they represent are reflected in the *Australian Curriculum: Mathematics* (ACARA, 2019) in the form of proficiencies, that is, understanding, fluency, problem solving and reasoning.

Becoming a teacher of mathematics

Graduating from university with a teaching qualification is a major milestone that recognises the learning that has occurred in the preceding years and acknowledges the graduates' readiness to join the teaching profession. It is both a celebration of achievement and the beginning of the next stage in the ongoing journey of becoming a teacher. Many beginning teachers find the first year(s) of teaching demanding, not only in terms of taking responsibility for their students but also in establishing themselves as professionals separate from the university context (Patkin & Gesser, 2009). However, support is at hand. For a start, there will be knowledgeable school leaders and experienced teachers at your school who can provide local advice and support, and there are professional associations, such as the Australian Association of Mathematics Teachers (AAMT) and its affiliates in each State and Territory, that provide access to on-going professional learning, professional networks and quality teaching resources.

The Australian Institute for School Teaching and Leadership (AITSL) and the associated teacher registration bodies in each State/Territory provide professional advice and support of a more general nature, but AITSL is responsible for setting the non-discipline-specific Professional Standards which serve as a framework to promote excellence in teaching and school leadership. Standards are provided for each of four career stages: Graduate, Proficient, Highly Accomplished and Lead Teacher. At each stage, the standards are organised in three overarching domains: Professional knowledge, Professional practice and Professional engagement. At whatever stage of your career you may be, the standards are intended to provide a means of self-assessment and a framework for planning career development and professional learning. The higher stages can be regarded as standards to aspire to, while the Graduate standards recognise the considerable knowledge and skills with which beginning teachers are expected to enter the profession.

So, what does it take to become a great teacher of mathematics?

Since 2002, AAMT has promoted standards for excellence in teaching mathematics in Australian schools organised in terms of: Professional Knowledge, Professional Attributes and Professional Practice. Unlike the AITSL standards, the AAMT standards are concerned only with excellence and relate most closely to the Highly accomplished and Lead stages of the AITSL standards. They are also concerned only with mathematics, and hence they can provide more specific guidance for teachers wanting to develop their expertise in teaching that discipline. To this end, the AAMT standards have been used in a number of professional learning projects involving teachers of Years F–12 (e.g. Bishop, Clarke & Morony, 2006). Because our concern is with mathematics teaching, the AAMT standards have been used to structure the remainder of this chapter.

Professional knowledge

Teaching mathematics is a complex, demanding, but rewarding task that requires a knowledge of students, knowledge of content and knowledge of how that content might best be represented to engage and support learners. Referred to by Shulman (1986) as **pedagogical content knowledge** (PCK) and, more recently and with respect to mathematics teaching and learning, as **knowledge for teaching mathematics** or KTM (Ball, Thames & Phelps, 2008) it clearly involves much more than a knowledge of mathematics at the level taught.

> Teaching is about building relationships—between students and the teacher and among students themselves around mathematics—and engaging together in constructing mathematical meaning. Teaching involves orchestrating the content, the representation, and the people in relation to one another. It is about making decisions in the moment that serve the individuals and collective. It is about understanding the students … It is about working together to negotiate meaning. (Franke, Kazemi & Battey, 2007, p. 228)

To do this, teachers need to have a deep understanding of what makes particular mathematics content difficult to learn, what **representations** and instructional strategies are best suited to the needs of individual students, and how best to manage the relationships in which the teaching and learning takes place. Although indistinguishable in practice, these key aspects of teacher knowledge are considered briefly below, that is, knowledge of students, knowledge of mathematics (for teaching) and knowledge of how students learn mathematics.

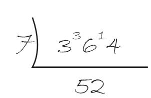
Knowledge of students

> Excellent teachers of mathematics have a thorough knowledge of the students they teach. This includes knowledge of students' social and cultural contexts, the mathematics they know and use, their preferred ways of learning, and how confident they feel about learning mathematics. (AAMT, 2006, p. 2)

Students come to school from an extremely diverse range of social, cultural and linguistic backgrounds. They come with particular experiences, personal characteristics, beliefs and attitudes that impact how they engage with school and with learning mathematics in particular. Access to the knowledge, skills and dispositions needed to participate effectively in society is an obligation of all teachers, but teachers of mathematics have a particular responsibility in relation to numeracy. No matter what their personal circumstances, mathematics achievement can be expected for all learners with good teaching and the right support. Identifying and addressing the learning needs of all students in ways that are appropriate for them is fundamental to the task of teaching mathematics.

LINKAGE: Chapter 8 explores what we mean by diversity and the implications of this for the teaching and learning of mathematics.

FIGURE 1.1 Nick's 'doing division'

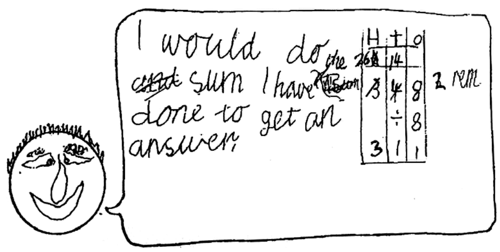


Source: Siemon, 1993

Teachers also need to be aware of how students view mathematics and what they see as their role in learning mathematics. Student responses to mathematical tasks can reveal a lot about their perceptions of school mathematics. For example, Nick was a Year 4 student who knew his number facts and was 'good at' 'doing division' using the traditional 'goes into' algorithm. For example, he could confidently divide 364 by 7 by saying '7 goes into 3? No, carry 3 … 7 goes into 36? … yes, 5 times and 1 left over' [writes 5 underneath and 1 beside the 4] … '7 goes into 14, yes 2 times' [writes 2] (see Figure 1.1). After some time in class exploring an alternative approach involving sharing and base 10 materials, which Nick clearly understood and could justify to his peers, he was given the problem 'Eight families shared a prize of $348. How much did each family receive?'

In responding to this problem, Nick provided the 'talking head' response shown in Figure 1.2. This format requires students to draw a face to show how he or she felt about a particular task and write a brief explanation—framed in terms of a 'how and why' story—to explain what they had done. Despite his demonstrated understanding of both forms of division, Nick chose to create his own algorithm based on what he knew about subtraction and renaming numbers. Nick had obviously not been taught to do this—it was something he constructed in the moment based on his beliefs and experience. Both his drawing (happy face) and his comments suggest that he believes school mathematics is about doing 'sums' (i.e. algorithms) to get answers. His response also suggests he is applying a learnt procedure without consideration for what the numbers actually refer to (i.e. 4 tens, 3 hundred).

FIGURE 1.2 Nick's response to a division word problem



Source: Siemon, 1993

Reading from the top down and starting with the ones Nick appears to have reasoned, '8 how many 8s? ... 1'. He records 1 in the answer line for ones and proceeds, '4 how many 8s?'. Realising he can't do that he trades 1 hundred for 10 tens. Proceeding he then thinks, '14 how many 8s? ... 1 and 6 over' recording 1 ten in the answer line and literally recording the 6 'over' the 2 that had been recorded in the hundreds trading box. Realising '2 how many 8s?' is not going to work, he crosses the 6 out and rewrites the 2 and the 6 as 26 and proceeds, '26 how many 8s? ... 3 and 2 remainder', which he records. When asked about his answer, Nick said, 'Oh if it was real money, I wouldn't do it like that.' Prompted to explain how he would do it, Nick replied, 'Well 8 families, $40 each that'd be $320, $50 each would be $400, I reckon it's about $43.' Nick's problem was not with division, but with the values and beliefs he held about the nature and purpose of school mathematics. Asked why he did this, Nick said that he knew his 'old way of doing it would work, but Mrs ... didn't like that' and he could do it the new way 'but that was too long'.

STUDENT DISPOSITION TO LEARN

Issues related to agency (available means) and identity are often ignored in attempts to understand why some students are indifferent or resistant to mathematics (Sfard & Prusak, 2005). Sagor and Cox (2004) have identified five essential feelings they believe are crucial to a young person's well-being and success at school: 'the need to feel competent, the need to feel they belong, the need to feel useful, the need to feel potent, and the need to feel optimistic' (p. 4). They explain why working only on the behaviours and attitudes of discouraged learners is insufficient, and suggest including an additional dimension, the need to feel they have a valued role to play in the social context.

Knowledge of mathematics

> Excellent teachers of mathematics have a sound, coherent knowledge of the mathematics appropriate to the student level they teach, … which is situated in their knowledge and understanding of the broader mathematics curriculum. They understand how mathematics is represented and communicated, and why mathematics is taught. They are confident and competent users of mathematics who understand connections within mathematics, between mathematics and other subject areas, and how mathematics is related to society. (AAMT, 2006, p. 2)

In Australia, decisions about what mathematics to teach are largely guided by the *Australian Curriculum: Mathematics* (ACARA, 2019). While this is implemented in slightly different ways by the educational authorities in each state and territory, the structure and content are essentially the same. The mathematics **curriculum** is organised by Year level (F–12), content strands and proficiencies as shown in Table 1.1.

TABLE 1.1 Structure of the *Australian Curriculum: Mathematics*

CONTENT STRANDS	PROFICIENCIES
Number and Algebra	**[Conceptual] Understanding**
Measurement and Geometry	**[Procedural] Fluency**
Statistics and Probability	**Mathematical Problem Solving**
	Mathematical Reasoning

The bracketed terms in the list of proficiencies in Table 1.1 refer to the terms used by Kilpatrick, Swafford, & Findell (2001), who first articulated these mathematical proficiencies. In their original list, they included a fifth proficiency *productive disposition* to describe an 'habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy' (p. 138). It is unclear why this was not included in the *Australian Curriculum: Mathematics*, but it is an important one to remember as we consider what it takes to be an effective teacher of mathematics.

The curriculum produced by education systems is often referred to as the **intended curriculum**. It sets the standard for what is valued and what will be assessed at a system or national level. However, schools and teachers need to interpret the intended curriculum in the light of their own knowledge and experience and what they know about their particular student population. This can lead to variations in the curriculum that is translated into practice (the **implemented curriculum**) and assessed in classrooms (the **evaluated curriculum**). These variations highlight the competing tensions faced by teachers as they try to balance the learning needs of the students with system and community expectations. This is a non-trivial task that can result in underachievement or learners being left behind if it is not managed well. This requires accurate information about what each student knows and is able to do, a deep knowledge of the key ideas and strategies needed to progress student learning, and a sound knowledge of how this can be achieved.

TEACHERS IN PRACTICE

Kathy Arnold

Becoming a teacher of primary mathematics has been one of the hardest things I've ever experienced. I've always hated maths, so my confidence was always an issue. Preparation for this role began in my school years with memories from this time suggesting that primary mathematics teaching mainly involves chanting times tables and weekly timed tests of number facts.

Shortly after deciding to apply to university to become a teacher, panic ensued about my adequacy to teach mathematics. As an adult my mental arithmetic skills had always been below par, so based on the assessment that I 'needed to learn more maths', I decided to complete a Year 11 maths course, before teacher training. As part of my course, I received extra tutoring, and consequently, I built up my repertoire of procedural knowledge.

At university, I developed strategies for teaching mathematics—but with hindsight, this was like a drop in the ocean and barely adequate to begin a teaching career. At the outset, I based my lessons on a prescribed mathematics textbook and careful mimicry of the model I observed on practicum.

My teaching model consisted of teacher explanation followed by independent practice supplemented by repetition 'on the mat' for those needing extra support. Over time I learned to 'differentiate' my practice although this generally equated to three groups deemed 'high, middle and low', with the groups remaining the same throughout the year and never quite meeting my students' needs.

Eventually, with the realisation that I was short-changing my students, I sought further professional development. This began an ongoing journey of reflection and learning based on the Big Ideas in Number and developing confidence to put this into practice in my classroom. The big ideas proved exactly what I needed to cater for all of my students' needs in an achievable way. Using the *Assessment for Common Misunderstandings* and *Scaffolding Numeracy in the Middle Years* materials, I have learned to use formative assessment to inform my teaching. This has deepened my own knowledge and confidence to teach mathematics and resulted in a passion for teaching mathematics that I now share with other teachers.

My journey towards strong mathematical knowledge for teaching seems never-ending, and I suspect this applies to many other teachers. Nowadays, I look at every student as an opportunity to learn more.