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The knowledge and skills gap in Australian primary mathematics classrooms

By Lee Walker, Oxford University Press

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Foreword

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Understanding the place of mathematics in the modern world has never been more important. The future of work is becoming more STEM focused, and mathematical knowledge and skills have been defined as critical to success in life (Tout, 2020). Primary school educators know that engaging students in learning to be numerate is as important as teaching children to read to prepare them for their futures. However, research by Oxford University Press reveals the challenge of teaching a classroom of students whose mathematical knowledge and skills are at varying levels. Teachers are concerned with ensuring all children are adequately progressing.

How do teachers deliver best teaching for all their students no matter their levels of ability, so they can all make good progress? How do teachers ensure that all their students are ready to transition from primary to secondary school? How can we best highlight the challenging circumstances that teachers face?

> *'For those who have strong skills, it is very hard to give them the time and attention they deserve to complete quality tasks at their level. This is similar for students operating at grade level, who deserve to have their skills extended. Unfortunately, when there are a significant number of students with major gaps in the prerequisite skills required to teach 'grade level' curriculum topics, you need to spend time with them 'filling in the gaps'. Those who are already behind can easily have their confidence further destroyed, those who are waiting to get your attention (or quality feedback) can become disengaged. Often you feel like you are not doing a good job at teaching any of the students.'*
> **(Primary teacher, 2022 survey respondent)**

The research in this report encourages educators and school leaders to consider:

- Reviewing assessment techniques and timing of assessment, including assessing students at the beginning of a new school year to inform teaching programs
- Choosing the right and best resources for assessment and differentiated teaching
- Implementing a sustained program of professional development and training for individual teachers and whole staff to develop and share knowledge and expertise
- How to integrate content area literacy and disciplinary literacy to mathematics teaching and learning
- The need for greater access to teaching support to attend to the diverse learning needs of students whose ability levels span 1–5+ years in any one classroom.

We are grateful to all the Australian primary teachers who informed this latest research; collectively, the contributions that make up this report reveal strong themes about the challenges of teaching mathematics to young students to prepare them for their futures. We hope that you find the information useful, that it provides you with insights to inform your teaching, and it recognises the heroic efforts from you and other teachers to ensure students are engaged in their learning and are making progress. We also hope that this research supports school leaders, the wider education industry, and government policy makers in their plans to train and retain our most valued teaching professionals – we can't do without them, we must value them, and we must invest in them.

The knowledge and skills gap in Australian primary mathematics classrooms

Oxford University Press Australia & New Zealand (OUP) conducts thought leadership in partnership with Australian and international experts, universities, subject associations, and classroom educators. This paper presents research about the knowledge and skills gap in Australian primary mathematics classrooms. Specifically, the paper reveals:

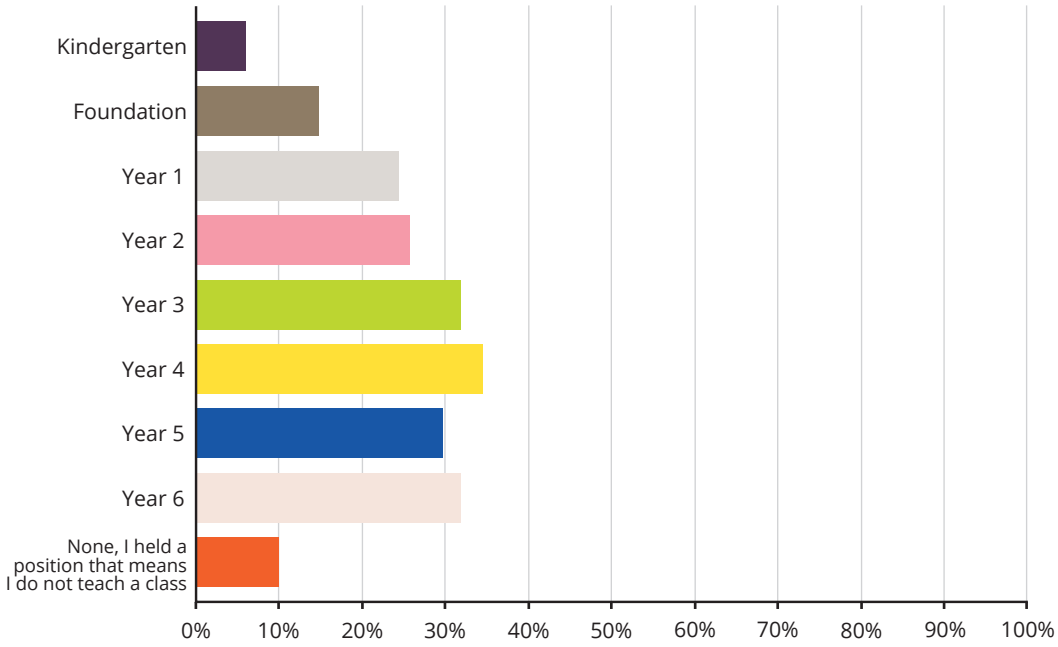
- Australian student achievement trends in mathematics
- The importance of assessing students all along their learning journeys – past and present
- The challenge of transitioning from primary to secondary school
- Impacts on engagement, confidence, and anxiety levels when students aren't making adequate progress
- The importance of reading comprehension to ensure progress in mathematics
- Why teachers don't have enough time to teach mathematics to all their students.

Complementing this paper are articles written by some of Australia's leading mathematics educators, who provide practical implications for teaching mathematics to primary students:

- On taking time with mathematics (Janine Sprakel)
- Making a difference with differentiation (Annie Facchinetti)
- Demonstrating the relevance of mathematics to real-world situations – the key to engagement and motivation (Peter Maher)
- Including all students in whole-class mathematically rich learning opportunities (Peter Sullivan).

The Oxford University Press research

In early 2022, OUP surveyed 228 teachers of primary Foundation to Year 6 students from around Australia, the majority (94%) of whom teach mathematics and interact directly and daily with students. There was representation of teachers for each year level, although more teachers represented years 3–6 (64%) than F–2 (36%). Only 13 respondents (6%) teach Foundation.

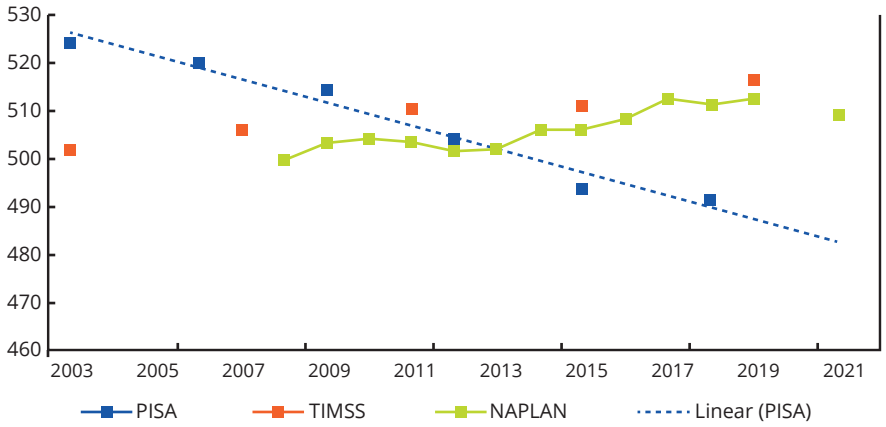


Survey respondents by year level/s taught

Australian student achievement trends in mathematics

Australian student achievement trends in national and international testing are mixed:

- The Trends in International Maths and Science Study (TIMMS) and National Assessment Program – Literacy and Numeracy (NAPLAN) reveal relatively no improvement over time.
- Australia's results in the Programme for International Student Assessment (PISA) have declined more steeply and consistently than in any other country (other than Finland) and decline has been greatest in mathematics (Fahey, 2021).



Australian student achievement trends in mathematics

Source: Programme for International Student Assessment (PISA) 2018 (15-year-olds), Trends in International Maths and Science Study (TIMSS) 2019 (average year 4, 8), National Assessment Program – Literacy and Numeracy (NAPLAN) 2021 (average year 3, 5, 7, 9), retrieved from Fahey, 2021.

Assessing students all along their learning journeys – past and present

The OUP research tells us that primary teachers place a good focus on assessment to understand how their students are progressing at different stages of their learning journey. Assessment methods used to measure progress and identify gaps in students' mathematics knowledge and skills include (in order of prevalence):

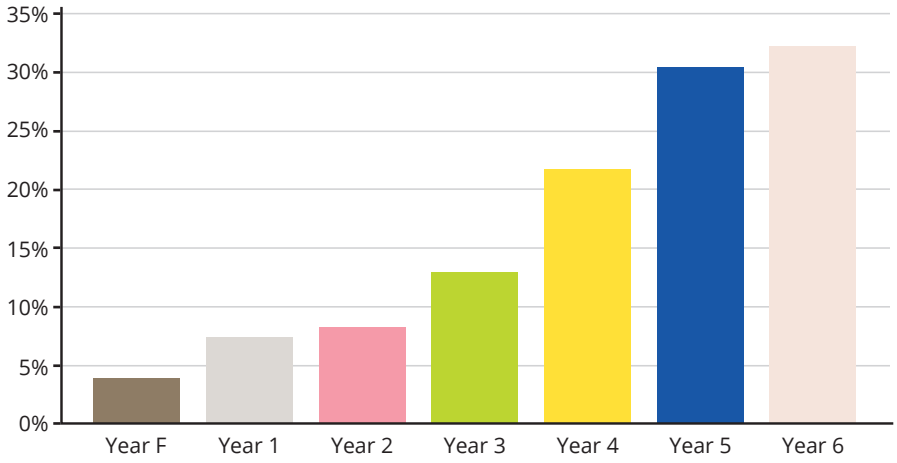
- Summative assessment at the completion of a topic (80%)
- Observing and monitoring learning (79%)
- Testing prior to the commencement of a topic (72%)
- Formative assessment (during learning of a topic) (64%)
- Placement assessment at the beginning of the year (56%)
- Student reports or assessment results from previous years (44%).

Just over half of teachers surveyed assess their students at the beginning of the year. Additionally, less than half of teachers review student achievement from previous years. For those teachers who don't know the knowledge and skills students are (and aren't) bringing with them from the previous year, this might contribute to the challenge of teaching a classroom of students whose ability levels and gaps in learning vary from the first day of the new school year.

Mathematics knowledge and skills gaps exist in every year of primary school

Teachers tell us that, in any one classroom, primary students' levels of mathematical knowledge and skills range from 1–5+ years, depending on the year of school, and as children progress through primary school the gap continues to widen:

- In the first year of school, more than 70% of teachers believe that students' gaps in knowledge and skills range 1–2 years.
- By Year 3, the range increases to 1–4 years, with almost 30% of teachers claiming a 3-year gap and 24% a 4-year gap.
- In years 4–6, most teachers claim that the knowledge and skills gap ranges from 3–5 years or more, with 33% of teachers asserting that the gap in Year 6 is 5 or more years.



Teacher respondents claiming a 5+ year gap in mathematical skills and knowledge

Similarly, research conducted by OUP in 2021 revealed that the size of the knowledge and skills gap in Australian secondary schools also ranged from 1–5+ years, with the widest skills gap in years 7–9. One secondary teacher said:

> *'The spread of student levels has increased hugely over the past 15 years to a point that it is virtually impossible for a single teacher in a classroom to successfully engage [all] students…'.* **(Secondary teacher, 2021 survey respondent)**

Are knowledge and skills gaps in mathematics unique to students, or does teacher instruction have gaps, too?

A recent analysis by the Centre of Independent Studies of initial teacher education in mathematics suggests an unsatisfactory integration of evidence-based practice in pre-service training, which might lead to a lack of effective teacher-led teaching approaches in Australian schools. The review suggests that teachers might not be equipped with a full range of practices to teach effectively to all learners. According to Fahey, O'Sullivan & Bussell (2021), teaching practices that are proven to be among the most effective for all learners include:

- Clear teacher demonstrations that recognise the implications of cognitive load
- Guided, scaffolded practice opportunities that allow students to verbalise
- Immediate corrective feedback to clarify and confirm students' progress
- Spaced and interleaved practice to facilitate the cumulative review of content.

As a result of this research, in 2021 the Federal Government implemented a Quality Initial Teaching Review to support better preparation of graduates to be more successful teachers (ibid).

Almost 60% of teachers who responded to the 2022 OUP survey say professional development needs to continue in-service, that one of their biggest teaching challenges is attending to the diverse learning needs of their students, and that they need greater support in choosing the right approaches for instruction:

> *'Professional development and training to accompany resources is always incredibly helpful. Online portals and community teaching hubs are also incredibly helpful as teachers can share successful ways they have implemented intervention strategies, differentiated learning tasks, etc.'* **(Primary teacher, 2022 survey respondent)**

Primary teachers surveyed also say that they need greater access to support staff to successfully attend to the diverse learning needs of their students:

> *'Appropriate support staff to accompany the classroom teacher.'*
> *'Extra staff.'*
> *'Support to complete... 1:1 assessment tasks.'*
> *'Human resources... teacher aide help.'*
> *'More human resources support.'*
> *'More teachers!'* **(Primary teachers, 2022 survey respondents)**

The challenge of transitioning from Year 6 primary to Year 7 secondary school

The research conducted by OUP in 2021 revealed that 94% of secondary school teachers observed that students' ability levels varied, and many were attempting Year 7-level mathematics with inadequate knowledge and skills. The research conducted by OUP in 2022 reveals that most (70%) primary teachers agree.

> *'Students are coming from primary school without fundamentals, such as knowing their multiplication tables. They have no concept of number and reasonableness of results. They do not predict answers through estimation to understand the reasonableness of 'calculator' answers.'* **(Secondary teacher, 2021 survey respondent)**

Almost all secondary teachers surveyed in 2021 also believed that gaps identified in Year 7 students' ability levels would have consequences for their longer-term learning, and that hindering individual progress is one difficulty of teaching to a wide, differentiated classroom of students:

> *'A huge divide forms between those students who come in with good skills already, and those who don't. It can make it difficult to teach all students with such a vast difference of abilities.'* **(Secondary teacher, 2021 survey respondent)**

One secondary teacher expressed concern for students' futures:

> *'Reduced outcomes for their futures and behavioural issues, puts them off maths and getting certain jobs in the future.'* **(Secondary teachers, 2021 survey respondent)**

Impacts on student engagement, confidence, and anxiety levels

When reviewing the results from both the 2021 and 2022 OUP research with years F–10 educators, there is overwhelming consensus that lack of student progress in mathematics results in confidence continuing to decline, and anxiety levels continuing to increase.

Most (64%) primary teachers surveyed in 2022 believe that lack of progress in acquiring the right level of knowledge and skills in mathematics reduces student engagement with the subject, and most (75%) agree that it reduces confidence in their ability to learn and increases anxiety levels. Similarly, the research conducted by OUP in 2021 revealed that the effects of declining progress in mathematics at the secondary level were reduced student engagement (77%), reduced student confidence and increased anxiety (86%).

These views include students who need more support to progress their learning as well as students who are excelling in mathematics:

> *'For those who have strong skills, it is very hard to give them the time and attention they deserve to complete quality tasks at their level. This is similar for students operating at grade level, who deserve to have their skills extended.'* **(Primary teacher, 2022 survey respondent)**

The importance of reading comprehension to ensure progress in mathematics

Most 2022 survey respondents agree that that it is likely that gaps in reading comprehension impact students' progress in mathematics, and that the impact continues to worsen as students move through school. Research confirms that reading performance can explain a considerable proportion of the variance in mathematics performance in the Programme for International Student Assessment (PISA; Huiming & Homer, 2020).

Students need content area literacy – the literacy knowledge and skills students use across learning areas – and disciplinary literacy – the literacy knowledge and skills students use to achieve the purpose of learning (Merga, 2022). For example, working through a mathematics problem requires the student to be able to read and comprehend language specific to mathematics, including understanding instructional text. If reading comprehension is not sufficient, students will struggle to understand how to apply their learning.

> *'The Australian Curriculum positions literacy as a general capability to be taught in every learning area. Despite this, few Australian schools have whole-literacy policies that include practical plans for building literacy across learning areas.'* **(Merga, 2022)**

Why don't teachers have enough time to teach mathematics to all their students?

Most primary teachers (87%) claim that not having enough time to address gaps in all students' understanding of mathematics is the key issue to helping all students progress. Comparatively, the 2021 research revealed that 97% of secondary teachers also cited time constraints as a key issue. Stated reasons for not having enough time to teach all students include:

- Class sizes and different ability levels
- Difficulty catering to different learning levels, needs and abilities
- Overcrowded curriculum – too many concepts and not enough depth at each year level
- Lack of focus on professional development and teacher capacity to teach numeracy
- Lack of support staff.

Overwhelmingly, primary teachers cite time constraints with the challenge of differentiating teaching for varying abilities:

- *'Having such a spread of skills…'*
- *'Too diverse a range of needs…'*
- *'Spread of student ability.'*
- *'Too many students at different levels in one class – years apart.'*
- *'Large class sizes and differentiating to student needs…'*
- *'Ability to extend whilst supporting and teaching mainstream.'*
- *'Try to address gaps but they are huge.'* **(Primary teachers, 2022 survey respondents)**

When asked what they need to help them differentiate their teaching, 87% of primary educators said they needed the right resources to address knowledge and skills gaps. In the 2021 OUP research, a secondary teacher confirmed that the right resources are required because:

> *'…learning is enhanced when learning opportunities are matched to individuals' current levels of knowledge, skill and understanding, so classroom activities are likely to be most effective in raising achievement levels if they are differentiated'.* **(Secondary teacher, 2021 survey respondent)**

It could be suggested that the 'right resources' to support assessment and differentiated teaching also includes the right instruction techniques to implement the best evidence-based, teacher-led approaches to engage and motivate all students in their learning, and that ongoing professional development to hone teaching knowledge and expertise is key.

Conclusion

This paper presents potential reasons for the mathematics knowledge and skills gap that exists in Australian primary classrooms, that continues to widen from year to year. In the 2022 OUP survey, teachers highlight the challenges they face when teaching groups of students with a range of abilities. Importantly, our Australian primary educators reveal their passion for teaching young learners and their desire to continue to build repertoires of knowledge and expertise to address gaps in students' learning. They cite the need for ongoing professional development and in-class support, the right assessment and teaching resources, and the best instructional approaches for classrooms of students with diverse learning needs.

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On taking time in mathematics

Janine Sprakel

AUSTRALIAN MATHS TRUST

Take some time to do some mathematics today. I mean really take the time. Do a problem that you are about to set for your students. Stare out the window and ponder. Scratch around with paper and pencil. Draw a sketch, scribble out a table. Look for a problem that really engages you, for it will most likely engage your students. Get to know the problem. Think about how your students will approach it. What will they need to be successful? How can you help them along if they need it? How can you extend the learning for all students? What will the hurdles be? Take the time.

I have been thinking about the importance of time in the mathematics classroom, and outside of it, a lot recently. There is a new curriculum coming and the usual issues persist. Engagement and differentiation, explicit teaching, and just-in-time skills, thinking deeply, finding just the right resources and assessment and reporting – where will you find time to do it all?

As a trained primary teacher with many years of classroom experience, I know how hard it is to find the time to plan for engagement and student growth as well as curriculum and jurisdictional requirements. Problem solving can help teachers address this by combining concepts and contexts in the one problem or series of interconnected problems.

I'd like to propose a challenge: do one problem with your class this week, and really spend some time on it.

Before the session

Finding a good problem is easier said than done, especially when you want to find just the right problem, but there are lots of resources available to help you. Look in your textbook, search online, go through your school library, attend your local mathematics association conferences, and read their teacher journals. The Australian Maths Trust's (AMT's) Problemo (problemo.edu.au) is our platform for teachers, with over 800 problems, and we are now working on a program of teacher support to accompany it. You are looking for problems that cater to a range of student abilities and interests – a low floor, high ceiling and wide wall problem that will really engage. As you do more problem solving you will begin to recognise a good problem and may even author some yourself.

Lean on good maths people... seek them out. Yes, do join Facebook groups, get some just-in-time professional learning or a good idea for tomorrow from Twitter, look at Pinterest, read the latest pedagogy in maths resource, the one everyone is talking about. By developing a personal mathematical identity and doing some mathematics we begin to understand what students will do with a problem, and only then can we consider how to enable their access into a problem and beyond it. Build a network of people you can go to when the problem is out of your reach, or when you need a colleague to bounce ideas around with. This adds to your professional learning network (PLN) in an informal way.

Before you deliver your problem-solving session, do the problem. Teachers doing the problem is the single most important step in problem-solving implementation. Plan and prepare what you will say and do in the mathematics session, which may go over a number of lessons. How will you get students back on track? How will you extend all students and ensure every one of them has some success? How will you know when students are thinking?

During the session

Go deeply on one problem – at least once a week. Allow students time to go deeply into solving it and collaborating towards new understanding.

Structure helps you go deeply into the mathematics of a single problem with your class, and your students will appreciate and come to expect the comfort that comes from knowing what is coming next. We use the structure:

Launch – Understand the problem. Students read the question and the class discusses the key concepts and clarify terms they are unsure of.

Solve – Students devise a plan and carry it out. The teacher lets students get to work and is there to circulate, prompt, but not solve the problem. The teacher notices interesting student strategies and solutions.

Reflect – Look back and check. Student strategies and solutions are carefully managed by the teacher in a class conversation, stringing the previously noticed student responses together as the conversation progresses.

Extend – Build on what all students have learned. Make use of appropriate extending prompts. This is a summary point in your lesson. Allow time for consolidation where a similar task is posed, allowing students to apply newly acquired skills.

Connect – Students make connections to self, family, art and symbols, nature, and previously solved problems. Teachers connect the behaviours, strategies and dispositions to the curriculum and assessment protocols for their school and class.

Setting up your classroom so that communication and collaboration are a focus is key to helping students develop a deep understanding of mathematics. It also assists them to grapple with problem-solving strategies and heuristics. To encourage your students to have a voice, never say anything one of them could say. When one of their peers explains some thinking or elaborates on a strategy, students are more likely to recognise where they could have done something new or different and the learning 'sticks'.

After the session

Making some time to reflect on how your problem-solving session went is important. This may be as simple as a conversation with a colleague over a cup of tea in the staffroom or as formal as a meeting with your PLN jotting down some notes for the next time you do this problem with students. Reflect on it, prepare to collaborate on the next one. Perhaps you could have a WOW day where each teacher is given the gift of Watching Others Work, for one problem-solving session. We learn so much from each other and seeing how one of your colleagues implements problem solving can be valuable.

And then – come back and do it all again next week. Good luck!

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Article

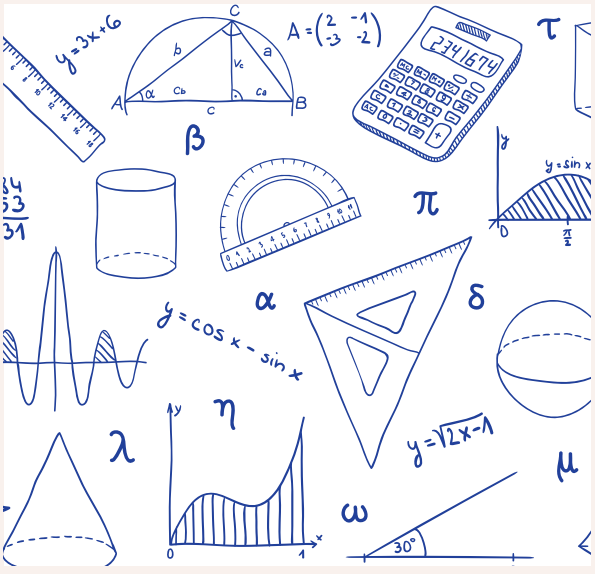
Making a difference with differentiation

Annie Facchinetti

Any generalist primary teacher working in a school today knows that the demands on their time, in and outside the classroom, have increased dramatically over the last 10 years. While it is widely acknowledged that the span of abilities in a maths class could be anywhere from one to five years or even more, the thought of having to formulate differentiated plans to meet the needs of all learners on top of all the extra responsibilities teachers now have can be overwhelming. Add to this, the fact that for many students, maths is a stressful activity. According to data from the 2012 Programme for International Student Assessment (PISA) studies, 59% of the 15- to 16-year-old students who sat the tests worldwide indicated that they worry about the difficulty of maths classes (OECD, 2013). The already complex considerations of planning for effective maths instruction therefore also need to encompass student wellbeing and engagement.

Laying the foundation for differentiated learning

One way to lay the foundation for differentiated learning that builds confidence and certainty for students is to begin with setting up regular class routines and structures, with a view to establishing students as independent learners of maths. You can start with something as simple as co-constructing an anchor chart detailing 'What good mathematicians do'. This opens the way for discussions about problem-solving strategies and fundamental proficiencies for learning and applying mathematical procedures and reasoning, but on a more basic level, it can also embed organisational expectations during maths lessons. When students know where to find the materials they need for a lesson and more importantly, what they can do if they get stuck, they automatically begin to be less reliant on the teacher and to feel in control of their learning.



Instructional routines to support all learners

Once classroom routines are established, instructional routines can be implemented that support learners of any level to think more deeply and to articulate their questions and their learning (McCallum & Nowak, 2018). 'Think-Pair-Share' is an example of an instructional routine that teachers are probably already familiar with. Another routine useful for maths classes is 'I Notice, I Wonder'. Used when presenting a problem, this provides a structure for students to share their thinking and ideas without pressure to find a solution (National Council of Teachers of Mathematics, n.d.). Regularly using routines such as these gives students a sense of familiarity and clear expectations, even when new content is being explored. While not exclusively a differentiation strategy, cultivating student independence frees the classroom teacher up to oversee investigations or give targeted instruction to particular individuals or groups that supports their learning at point of need.

Using open-ended questions

It can be a fine balancing act to formulate a maths activity that suits, or can be adapted to, all students in the class, so that it is not too hard for some students to attempt and not too easy for other students to complete without some level of challenge. Open-ended questions offer a way to do this that doesn't require the teacher to design separate activities or content to differentiate learning. Problems with several correct answers, some relatively simple and some that involve more sophisticated responses that reflect a deeper understanding of mathematical concepts, are particularly valuable. They have the added advantage of 'foster[ing] higher level thinking because they

encourage students to develop their problem-solving expertise at the same time as they are acquiring mathematical skills' (Sullivan & Lilburn, 2004, p. 4).

Using extending or enabling prompts

Incorporating the use of extending or enabling prompts with open-ended questions, or with other activities during a maths class, is another strategy that can allow students with a range of abilities to access a single task. A recent study into how these prompts are used in maths lessons found that teachers were able to think about ways to engage less able learners and consider students' prior learning when they formulated enabling prompts (Cheeseman, Downton & Livy, 2017). Teachers also reported feeling that student confidence and success was nurtured when the cognitive demands for these students were considered. At the other end, teachers used enabling prompts to challenge students to extend their thinking and apply their knowledge to new problems. The study concluded that the enabling and extending prompts, '... allowed teachers to differentiate their teaching and they stimulated teachers' noticing of students' reasoning, strategic thinking, and mathematical communication'.

Sharing thinking and strategies

Supporting students to share their thinking and strategies not just with the teacher but with each other can also lead to increased learning across different ability levels. This does, however, take a bit of management on the teacher's part, at least initially. Rather than just asking who has learned something at the end of a session, reviewing and communicating how problems were approached and solved can be more meaningful if a few students are prepped to share something of note that the teacher saw or heard during the lesson to make for a more 'orchestrated' productive maths discussion (Stein & Smith, 2018). Students of any level can be supported to participate, not just the most able mathematicians who are often keen to share their answers. This approach values the thinking of every student and exposes all class members to higher-level mathematical problem solving. After all, if you only ever group the lower students together, they don't get the benefit of hearing the thinking and strategies of other students. And sometimes it is those who struggle the most who come up with creative answers or methods that surprise their classmates!

Every child should experience success, every maths lesson

When I speak about maths teaching, I often quote the well-known mathematics educator Michael Ymer, who at a professional learning day I attended said, 'Every child should experience success, every maths lesson'. It's a thought that has stuck with me for many years, summing up the essence of differentiation. While it takes some forethought, if we can use strategies that support students to feel successful in maths, rather than anxious, and that meet their learning needs, value their contributions, and encourage critical thinking at the same time, we are a long way down the path to nurturing both the everyday and the extraordinary mathematicians of the future.

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Demonstrating the relevance of mathematics to real-world situations - the key to engagement and motivation

Peter Maher

I can clearly remember asking my Year 9 mathematics teacher why we were studying a particular topic, that appeared to me, at the time, to be rather obscure and boring. The topic itself evades my memory, but I can clearly remember his response: 'In case you become a Maths teacher'. This comment, upon reflection, is ironic on a number of levels but does highlight a major issue in the teaching of the subject that persists across all year levels to this very day. Students who cannot identify with what they are learning will lack engagement and motivation.

This raises the question: when are our students engaged and what do lessons that are full of student engagement look like? As teachers, we know when a lesson has gone well. Our students are motivated, the lesson aims have been met and you derive satisfaction and often joy from the progress made. But what has caused this success? What has made the students motivated and eager to learn? Perhaps the activity planned was fun. Perhaps the social chemistry in the room was conducive to learning. Perhaps the low threshold–high ceiling structure of the lesson allowed all students to achieve. However, I bet the one thing the lesson did do was capture the imagination of the students through its perceived relevance to them. In other words, the students saw the context of the topic in a real-world setting.

Students learn mathematics at school so they can use it outside of school

A mantra that all teachers should adopt is: 'We learn maths at school so we can use it outside of school'. As educators our fundamental role is to prepare our students to cope with the demands of life beyond the classroom, hopefully becoming empathetic, resilient, creative thinkers in the process. In our mathematics classrooms it is incumbent on us all to constantly demonstrate the nexus between school, home, and the community. Why teach a topic if it cannot be used to benefit of the learner in both intrinsic and extrinsic ways?

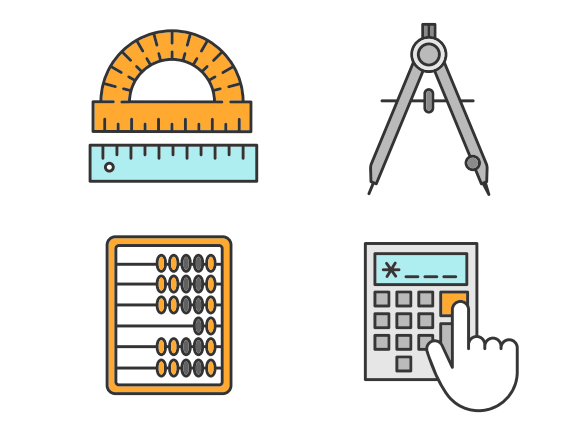
The mathematics that we teach in the primary classroom lends itself wonderfully well to the real world. Every time we begin a new topic, time should be devoted to the students' experiences of that theme. For example:

- When studying place value, ask the students where they have heard 'big' numbers being used. Crowd sizes, populations of towns, cities and countries, personal fortunes and distances lend themselves to contexts related to the topic, such as number pronunciation, digit values and rounding off.

- When studying probability, talk about the games of chance that the students play that use dice, cards, or spinners. Discuss the way a sporting event usually begins by the tossing of a coin. Show the likelihood of rain today and discuss the risk/reward nature of insurances.
- A study of fractions, a topic that can appear dry and rather esoteric to children, can come alive when applied to recipes and the time divisions in games.

Parents' and carers' interest in their children's educational experiences

Without doubt, the best learning occurs in a triangular arrangement, where motivated students are taught by caring professionals, supported by parents who are interested in the educational experiences of their children. It is an excellent idea to bring in parents at the start of each academic year to advise them of the way key learning areas are taught at each year level. This is a wonderful opportunity to show parents how they can best assist with their child's mathematical understandings. This can best be achieved by parents demonstrating the way mathematics affects their everyday lives and the ubiquitous nature of the subject. A quick glance around the home and a walk in the community will reveal rich mathematical examples in tessellated tiles, digital numbers on appliance displays, on street signs and letterboxes, on news services, in newspapers both physical and online, on utility bills, on packets, cans and bottles, at the petrol station and when shopping. The list is as long as one's imagination can conjure. And each example will reinforce the purpose of learning the skills, concepts and knowledge that are imparted and developed at school.



To be engaged with learning mathematics, students need to be motivated

There is little doubt that having a solid background in mathematics is becoming more important in the workplace. Therefore, we need to encourage our students to pursue studies in mathematics for as long as possible. To be engaged with learning mathematics, they need to be motivated. To be motivated, they need to be engaged in the classroom. To be engaged in the classroom, they need to be able to appreciate the relevance of what they are learning to real-world contexts. I believe that this, for teachers, is one of our fundamental obligations.

To all educators and policy makers, further proof of the continuing decline of Australian students' results in PISA tests should be both disturbing and seen as justification for policy change. Essentially, the questions found on PISA tests ask students to apply their knowledge, skills, and concepts to real life situations. Surely this should be an integral part of every mathematics classroom from Foundation to Year 12.

Peter Maher

Peter Maher is the author of *Making Maths Count - Exploring Maths Concepts in Real-world Contexts*. He has been a Foundation–Year 6 mathematics coordinator for over 40 years and has conducted professional development sessions for both teachers and parents across the country. Author of 34 texts designed to improve the teaching of mathematics in the primary school, Peter was awarded an OAM in 2021 for his services to mathematics education.

Including all students in whole-class mathematically rich learning opportunities

Peter Sullivan

Perhaps the greatest challenge for teachers of mathematics is catering effectively for differences in student readiness. Students vary in the nature of their prior mathematics learning experiences, motivation, willingness to persist, aspirations and confidence. Most importantly, there are differences in the amount of time students need to think and engage with mathematical ideas. Student achievement and readiness vary from domain to domain, from time to time, and in their preferred styles for interpreting and representing mathematical ideas.

Some schools seek to address these differences through grouping students by their achievement levels. This has the disadvantage that students placed in low achieving groups effectively have future opportunities limited, partly due to the impact this grouping has on their self-concept and partly because they may not, as a result, be exposed to the broad range of mathematical thinking needed for their subsequent learning.

Over various research projects, at different levels of schooling, my colleagues and I have explored approaches to addressing this diversity of readiness in heterogeneous classrooms. We have consistently identified some key elements of inclusive teaching.

Elements of inclusive teaching

First, teachers can structure lessons so that students have time to think and freedom to choose their own intuitive approaches, strategies, and representations. Predominantly, this means that students explore productively challenging problems prior to any active instruction.

Second, teachers can select or create tasks that are open-ended (meaning those that have multiple correct possible responses) or open-middled (meaning those that can be solved or represented in different ways), and that are accessible at different levels thereby allowing all students to engage. Some students might find one solution or strategy; others might find many.

Third, teachers can develop variations to the initial learning experiences, which are consistent with the essence of the original problem. For students who may require additional support, teachers can prepare tasks supplementary to the original, termed *enabling prompts*. Three key ways in which such prompts can be created are by changing the:

- representation (rather than representing a problem with symbols, students might be prompted to imagine or draw a diagram or picture, or use materials);

- numbers (by reducing the size of the numbers – for example, if the original task involves numbers in the hundreds, numbers up to 100 might be suggested); and
- number of steps (if the problem involves two steps, a one-step version might be posed).

The intention is that students work on the prompt after which they return to the original task. Basically, the goal is for all students to have engaged sufficiently with the prompt and the original tasks to participate in class discussions.

There are also students who might complete the original task quickly. The intention is for teachers to create and pose *extending prompts* that encourage these students to think more deeply and perhaps generalise their ideas. Such extending prompts engage students in further thinking which also has the advantage of allowing all other students more time to solve the original task.

Strategies for creating such tasks are the same as for enabling prompts except in the opposite direction. That is, making the representations, increasing the size or nature of the numbers (or shapes, or measures, or data, etc.) and increasing the number of steps. Some other generic extending prompts teachers can use are:

- What patterns in your answers can you describe?
- Work out your answer a different way. Which way is better and why?
- Convince me you have found all possibilities.
- What are you wondering about? How would you explore this?
- What if the numbers (shapes, measures, data) were different? Would your method still work? What are the characteristics of the numbers (shapes, measures, data)?
- Explain your reasoning to someone from "marketing".
- Write a question to prove that you know what the big mathematical idea is?
- How are these questions the same and how are they different?

Some teachers pose these prompts separately to individuals as needed, and some have them available for all students to choose when needed.

Fourth, teachers orchestrate classroom discussions that draw on the thinking of students, but which explicitly connect student generated ideas with the mathematical intention of the original learning experience.

While students are working, teachers select student solutions for sharing and then orchestrate discussions based on student solutions. Where necessary, teachers make the intended learning explicit.

Fifth, teachers pose further tasks that are "a bit the same and a bit different" to consolidate the learning activated by student engagement with the original task. Students who may have experienced difficulty with the original task can learn from the class discussions and apply this new learning to the subsequent task.

Putting it all together

Note that it is not the individual elements that are critical but the overall approach. By using such a lesson structure consistently all students, especially those who may lack confidence and be unwilling to persist, know the structure and become aware that there are multiple opportunities to engage with tasks and to learn. A key feature of the overall approach is that this structure of lessons, consistently applied, gives students confidence of what is to come and reduces anxiety. The tasks which are readily accessible, the additional time, the enabling prompts, the class discussions, and the consolidating tasks are all opportunities for students to engage.

Peter Sullivan

Peter's career combines research into task design with the development of teacher support resources and classroom trialling. He is the author of the Shape of the Australian Mathematics Curriculum, a paper commissioned by the National Curriculum Board to lay the foundation of the Australian Curriculum. Subsequently, he was appointed the lead writer for the development phases of the Australian Curriculum and has supported ongoing development of the documents.

Peter is the author of the Australian Education Review publication, *Teaching mathematics: using research-informed strategies* (Australian Council for Educational Research) that has been downloaded more than 240,000 times. He was editor for seven years, and for three years chief editor, of the prestigious *Journal for Mathematics Education Research Journal* (Springer Science+Business Media on behalf of the Mathematics Education Research Group of Australasia). Peter's extensive list of publications for teachers and researchers includes books, book chapters, journal articles and conference publications. He is the author of *Open-ended Activities*, *Challenging Mathematical Tasks* and *Building Engagement in the Middle Years*.

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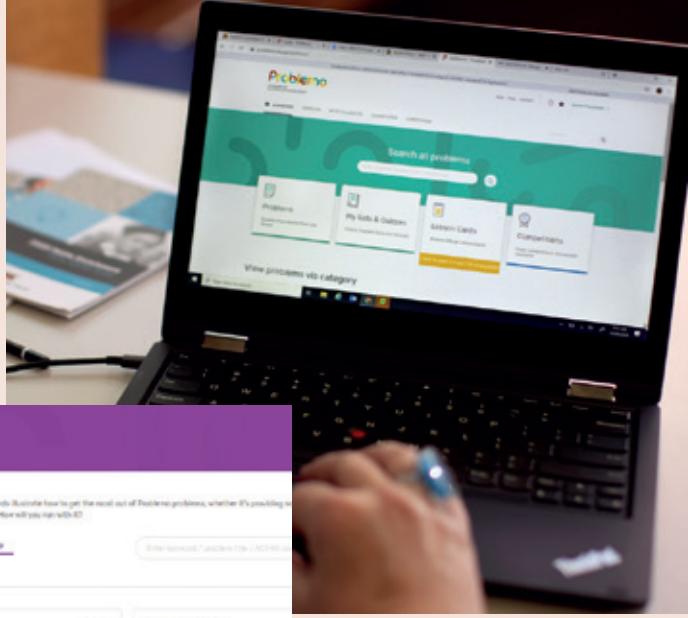

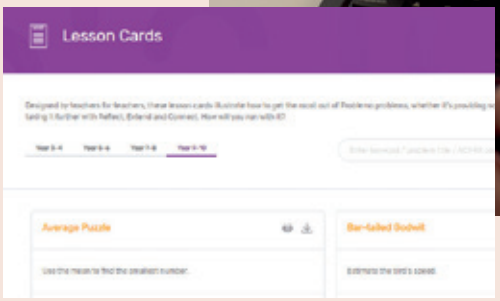

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