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IT'S NOT AS SCARY AS YOU THINK!

Welcome to *Maths Skills for Success at University*. We have written this book for students just like you – who need to do some maths as part of their course but are concerned about doing so. For some students this is because they may not have done much maths since leaving school (which could have been quite a while ago). For other students maths might have always been a bit of a struggle and so they may be lacking in confidence or even feeling quite anxious.

We would like to start this book by emphasising that maths is not as scary as you think and that by using this book you will be prepared to conquer the maths requirements in your course. In this introductory chapter, we will look at a few strategies you can use on your road to success. We will give you a chance to record how you feel about maths because doing so is a great way to identify what your hurdles might be. We have found that when we encourage students to reflectively record their attitudes towards maths they develop a more positive outlook.

So why might university students think that maths is scary? Well, there are many reasons that may include:

- Only those that are good at 'it' can do maths.
- I haven't done 'it' in years.
- I was never good at 'it' at school.
- I just haven't got a maths brain.
- I can't use a calculator.
- There's always too much information that is taught too fast.
- All of those numbers and symbols are plain confusing.

▶ REFLECTIVE ACTIVITY 1

So ... is maths a bit scary for you and what makes it so? Record your thoughts in your notebook.

LEARNING MATHS IS LIKE LEARNING A NEW LANGUAGE

There are some things that are peculiar to learning and using maths that can make it a challenge. To begin with, learning maths is like learning a new language. Maths is full of words that can be foreign, including terms such as:

- quotient
- exponent
- factor
- reciprocal
- hypotenuse
- histogram.

A glossary is included in this book for you to use for maths words that may be unfamiliar.

And maths is not just about new or unfamiliar words and terms; it also involves the use of many symbols and other forms of notation that replace words. Knowing that learning maths is like learning a new language means that some of the strategies that work well when learning a language will also work well when learning maths such as:

- creating your own glossary with new terms and their definitions
- continually reviewing the meaning of new terms
- making a conscious effort to use the language of maths correctly.

REFLECTIVE ACTIVITY 2

Use the index at the back of this book to locate the terms that have been listed above, and then try to write in your own words a basic definition for each term. Alternatively, locate the meaning of these terms in an easy to use web-based resource (such as www.mathsisfun.com). Hopefully, you can see that this process is both easy and useful.

MATHS IS SOMETHING THAT YOU DO, NOT JUST READ OR VIEW

Another aspect that is specific to learning and using maths is that it simply must be done, and not just read or viewed. We have heard students say many times that they understood everything perfectly well in our lectures when watching us complete examples on the board, and then when they sat down to do exercises themselves they went blank. What these students do not realise is that they do not really get 'it' when watching the lecture, even if they think they do. You can only get 'it' by doing 'it' with repeated practice. Think of learning and using maths

in the same way as learning and using a sport skill, such as playing golf. If only learning to play golf was as easy as watching the great champions on television. Anyone who has only partially mastered the game of golf will tell you that it only results from constant, repeated practice.

The message here is not to get lulled into a false sense of security, thinking that you understand something new without having done it repeatedly by yourself. You can also optimise your learning by applying these study strategies:

- Do not complete all of your out-of-class maths study in one great big block of time – 3 or 4 hours – on just one day.
- Do a little bit of maths each day – 20 to 30 minutes is all that is required.

This will keep the maths concepts and processes fresh in your mind.

STUDYING MATHS AT UNIVERSITY

Studying anything at university is quite different to being at school, and maths is no exception. For students who are already thinking that maths is a bit scary, these differences can provide further challenges. Some of the differences are:

- At school attendance is mandatory; at university attendance is optional and your lecturer is not going to follow-up if you do not turn up!
- At school teachers constantly monitor student progress and achievements; at university students receive their grades at the end of the semester and need to monitor their own progress.
- At school students see their teacher every day; at university students normally see their lecturer once or twice a week.
- At school teachers prepare students for tests that occur frequently; at university assessment activities occur less frequently and students are required to manage their own preparation.

Given these differences, the key to success at university is to manage your time well and to take responsibility for your progress through good study habits such as:

- doing some maths every day
- attending all your classes, unless it is unavoidable
- ensuring you catch up using resources that your lecturer puts online, if you miss a lecture
- keeping an organised notebook for maths
- finding or organising a study group with other students to support each other.

MATHS ANXIETY

Maths anxiety, which is a feeling of tension, apprehension or fear that interferes with learning or using maths, has been well documented as a key reason for people to have an aversion to studying maths and using even simple maths in their daily lives. Research has

found that the brain areas that are active when maths-anxious people prepare to do maths overlap with the same brain areas that register the threat of bodily harm. So the main problem is that fear stops people from using maths, and not a lack of maths skills.

There are a number of factors that cause maths anxiety. The first is that some people have less than positive maths learning experiences at school especially at a young age. These experiences can trigger a fear of not being good at maths, or even not being able to do maths at all. A second factor is that, unfortunately, there exists a general societal view that maths is hard and that only smart people can do maths. This can create unreasonable expectations and pressure, especially during the school years. Once the seeds of maths anxiety have been sown, many negative consequences can arise. For example, maths-anxious people might:

- lose confidence in themselves and in their academic abilities
- trust blindly any bills they receive, because they do not want to engage with figures and numbers
- not be able to help their children with their homework
- avoid enrolling in courses in case they contain maths
- leave courses when they encounter the ‘maths part’.

So, what can be done about maths anxiety? As we have already noted, reflectively articulating your attitudes towards maths is one strategy that can develop a more positive outlook. Specifically, we have found that encouraging our students to create a personal maths metaphor is a useful reflective activity that can highlight deep-seated emotions that might exist regarding maths.

PERSONAL MATHS METAPHOR

When creating a personal maths metaphor, you compare maths to specific things or objects as a way of focusing how you feel about learning or using maths. We have found that in creating a personal maths metaphor our students have been helped to recognise the value of mathematics, which has enhanced their chance of success in their studies. So why not try it yourself?

REFLECTIVE ACTIVITY 3

Let’s start creating your personal maths metaphor by firstly imagining that you are describing what maths is to someone. In your notebook, write a list of the words or phrases you might use.

REFLECTIVE ACTIVITY 4

Now imagine that you are using maths either at university or in your everyday life. In your notebook, write a list of words or phrases to describe what doing or using maths feels like to you.

REFLECTIVE ACTIVITY 5

Next think about things or objects that reflect what maths is like for you. For example:

- If mathematics was weather, what kind of weather would it be?
- If mathematics was a food, what food would it be?
- If mathematics was a colour, what colour would it be?
- If mathematics was an animal, what animal would it be?
- If mathematics was a type of music, what type of music would it be?

In your notebook, write a list of things or objects that you think mathematics is like.

REFLECTIVE ACTIVITY 6

Read over the list of words and phrases that best describes mathematics for you and the list that describes how you feel about learning or using mathematics. Now from the list of things or objects that you think mathematics is like, select the item on your list that *best* describes what mathematics is like for you. In your notebook, note all the ways that mathematics and this thing or object are alike.

REFLECTIVE ACTIVITY 7

Now you are ready to create your personal maths metaphor. Start your metaphor with the phrase: 'For me, maths is like ...' Complete your metaphor by adding a short paragraph to describe the ways the thing or object you have selected and maths are similar. Think in particular about how this metaphor describes how you feel about using or doing mathematics. Here are some metaphors our students have written as an example:

- For me maths is like a roller coaster because you put a lot of courage into getting on it and doing it. And after a crazy ride, you get off the roller coaster that is maths, and you feel pride for having done it.
- For me maths is like a snowstorm. Firstly, it looks cold and terrible but once you have the right warm clothes you can see the beauty of the snow.
- For me maths is like an electrical circuit. When all the components are in place, the light bulb comes on. When a component breaks down, the circuit is cut and you're left in the dark.

In your notebook, create your personal maths metaphor.

So, we hope that we have convinced you that maths is not as scary as you think. If you apply any or all the strategies that we have suggested in this chapter, we hope that you will continue to develop your Maths Skills for Success at University.

2

WHOLE NUMBER FUNDAMENTALS

CHAPTER CONTENT

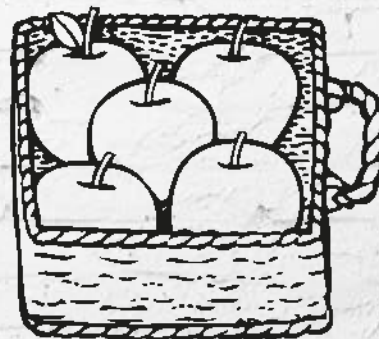
- » Introduction to whole numbers
- » Place value for whole numbers
- » Adding and subtracting whole numbers
- » Multiplying whole numbers
- » Dividing whole numbers
- » Order of operations
- » Rounding whole numbers
- » Significant figures

CHAPTER OBJECTIVES

- » Determine the place value of digits in whole numbers
- » Add and subtract whole numbers
- » Multiply and divide whole numbers
- » Apply the correct order of operations to whole number calculations
- » Round whole numbers to a specified number of decimal places or significant figures

USING WHOLE NUMBERS FOR ESTIMATION IN EVERYDAY LIFE

Whether we realise it or not, we use whole numbers extensively in our everyday life, and estimation is one of the most important skills involving whole numbers. When you shop for groceries, you might need to stick to a budget. So you probably keep a running total of the cost of items in your trolley, discarding the cents and just using whole dollar values—whole numbers. This ensures that you have enough money when you get to the checkout. For example, if you had \$25 left until payday and bought milk for \$3.50, cheese for \$6.99, bread for \$3.90, apples for \$5.50 and bananas for \$2.50, you could add up $\$4 + \$7 + \$4 + \$6 + \$3 = \24 and know that you would come in under budget.



Another shopping situation where you might estimate with whole numbers could be in calculating a discount. If you wanted a pair of jeans that were \$59 but they were on sale for 30% off, a quick estimation would tell you that 10% of \$60 is \$6, three lots of \$6 is \$18, and $\$60 - \$18 = \$42$, which would approximately be the cost of the jeans when on sale.

What about when you go out to dinner with friends? Rather than add up exactly who has spent what, there are probably times when you have simply divided the total cost of the bill by the number of people at dinner and obtained a rough, whole number estimate of each individual cost. For example, if the cost of dinner for eight people was \$241.45, you could divide \$241 by 8 to get approximately \$30 per person as the individual cost. The reality is that whole numbers are used extensively in many everyday life situations for estimation purposes.



Making connections
Decimal numbers are further discussed in Chapter 6.

2.1 INTRODUCTION TO WHOLE NUMBERS

In ancient times, there were no numbers to represent quantities like two, three and four. Fingers, rocks and sticks came in handy for communicating quantities. Words like ‘flock’, such as a flock of sheep, or ‘swarm’, such as a swarm of bees, indicated groups of similar animals (flock) or a large group of insects (swarm); the exact number of animals or insects represented was non-specific. Symbols for numbers came into being with the advent of villages and settlements, where bartering occurred between neighbouring groups.

Early numerical systems used symbols instead of the numbers we use now. For example, the early Egyptians used symbols such as those shown top right and the Chinese used sticks laid on tables to represent numbers, as shown bottom right. Roman numerals are still used in some contexts today; for example, the year that a movie is created is usually given in Roman numerals. A movie made in 2014 would have MMXIV listed for its year, with M being the symbol for 1000, X the symbol for 10, and the I (symbol for 1) being positioned to the left of the V (symbol for 5) together indicating 4 ($IV = 5 - 1 = 4$). Over many years, counting systems have evolved to give us the numbers from 0 to 9 that we use today.

1	10	100	1000
Stroke	Arch	Coiled Rope	Lotus Flower

10000	100000	1000000
Pointed Finger	Tadpole	Surprised Man

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90

Whole number

A positive number with no fraction or decimal part that is comprised of at least one of the digits from 0 to 9

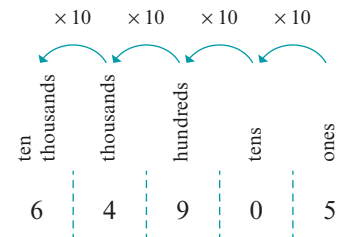
2.2 PLACE VALUE FOR WHOLE NUMBERS

Place value refers to the numerical value that a digit has according to its position in a number, as shown below. Each place has a value of 10 times the place to its right. The rightmost digit in a **whole number** is in the ‘ones place’—because the rightmost digit has a value of itself multiplied by one. The next digit to the left is in the ‘tens place’, because it has a value of itself multiplied by ten. Moving to the left digit by digit, each digit has a place value of ten times the previous one. For example, based on the position of the digits in the number 64905, each digit has the following value:

- 5 means $5 \times 1 = 5$
- 0 means $0 \times 10 = 0$
- 9 means $9 \times 100 = 900$
- 4 means $4 \times 1000 = 4000$
- 6 means $6 \times 10000 = 60000$

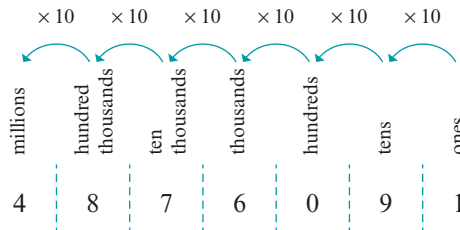
In other words, 64905 can be written as:

$$\begin{aligned} & 6 \times 10000 + 4 \times 1000 + 9 \times 100 + 0 \times 10 + 5 \times 1 \\ &= 60000 + 4000 + 900 + 0 + 5 \\ &= 64905 \end{aligned}$$



As another example, the digits in the number 4876091 have the place value shown below and the number can be written as:

$$\begin{aligned} & 4 \times 1000000 + 8 \times 100000 + 7 \times 10000 + 6 \times 1000 + 0 \times 100 + 9 \times 10 + 1 \times 1 \\ &= 4000000 + 800000 + 70000 + 6000 + 0 + 90 + 1 \\ &= 4876091 \end{aligned}$$



EXAMPLE 1

Write in words the place value of the underlined digit in each of the given numbers.

a $1\text{2}36$

b 5596084

c $\text{9}214695288$

Solutions

a The digit 2 is in the hundreds place in $1\text{2}36$. (Counting left from the rightmost digit: ones, tens, hundreds)

b The digit 4 is in the ones place in 5596084 . (The ones place is the rightmost digit of a whole number.)

c The digit 9 is in the billions place in $\text{9}214695288$. (Counting left from the rightmost digit: ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions, ten millions, hundred millions, billions)

PRACTICE 1

Write in words the place value of the underlined digit in each of the given numbers.

a $412\text{53}6$

b $1\text{0}559608467$

c $\text{9}7042$

EXAMPLE 2

Underline the place value of the digit in the named place in the following numbers.

a ten thousands place in 345903

b ones place in 12384

c hundred millions place in 9760954321

Solutions

a $3\text{4}5903$

b 12384

c $\text{9}760954321$

PRACTICE 2

Underline the place value of the digit in the named place in the following numbers.

a hundreds place in 345903

b ten thousands place in 12384

c billions place in 97609546321

EXAMPLE 3

Write the following numbers in place value form.

a 2906

b 1083920

c 40067

Solutions

$$\begin{aligned} \text{a } 2906 &= 2 \times 1000 + 9 \times 100 + 0 \times 10 + 6 \times 1 \\ &= 2000 + 900 + 0 + 6 \end{aligned}$$

$$\begin{aligned} \text{b } 1083920 &= 1 \times 1\,000\,000 + 0 \times 100\,000 + 8 \times 10\,000 \\ &\quad + 3 \times 1\,000 + 9 \times 100 + 2 \times 10 + 0 \times 1 \\ &= 1\,000\,000 + 0 + 80\,000 + 3\,000 + 900 \\ &\quad + 20 + 0 \end{aligned}$$

$$\begin{aligned} \text{c } 40067 &= 4 \times 10\,000 + 0 \times 1\,000 + 0 \times 100 + 6 \times 10 \\ &\quad + 7 \times 1 \\ &= 40\,000 + 0 + 0 + 60 + 7 \end{aligned}$$

PRACTICE 3

Write the following numbers in place value form.

a 12086

b 291

c 4670068

2.3 ADDING AND SUBTRACTING WHOLE NUMBERS

Adding or subtracting whole numbers involves two steps:

- 1 lining up two numbers according to their place value
- 2 adding or subtracting corresponding digits in each number, working from right to left.

Addition

The first step in addition is lining up numbers according to place value. This means lining up the numbers so that digits with the same place value are directly above or below each other, as shown on the left below. When adding 782 and 7, the 2 in 782 and the 7 in 7 are both in the ones place, so these digits need to be directly above and below each other. On the right below, crossed out in green, are two incorrect ways of lining up the two numbers 782 and 7.

H means hundreds
T means tens
O means ones

H	T	O
7	8	2
+		7

Here numbers are lined up correctly according to place value.

Here numbers are *not* lined up according to place value.

Here numbers are *not* lined up according to place value.

The second step in addition—adding digits in the same column, working from right to left—is shown below for the sum $782 + 7$. The three substeps involved in the second addition step are shown as (a), (b) and (c). First (a), the digits in the rightmost column (the ones column) are added ($2 + 7$), then (b) the digits in the tens column ($8 + 0$), and (c) finally the digits in the hundreds column ($7 + 0$). This gives the result $782 + 7 = 789$.

(a): $2 + 7 = 9$ (b): $8 + 0 = 8$ Note that 7 is the same as 07 and 007, so the blank spaces in front of the 7 are counted as zeros. (c): $7 + 0 = 7$

Th	H	T	O
	7	8	2
+			7
<hr/>			
			9

Th	H	T	O
	7	8	2
+			7
<hr/>			
		8	9

Th	H	T	O
	7	8	2
+			7
<hr/>			
	7	8	9

Th means thousands

The second step in addition is shown below for the sum $433 + 52$. The three substeps involved in the second addition step are shown as (a), (b) and (c). First (a), the digits in the rightmost column (the ones column) are added ($3 + 2$), then (b) the digits in the tens column ($3 + 5$), and finally (c) the digits in the hundreds column ($4 + 0$). This gives the result $433 + 52 = 485$.

(a): $3 + 2 = 5$ (b): $3 + 5 = 8$ (c): $4 + 0 = 4$ Note that 52 is the same as 052, so the blank space in front of 52 is counted as zero.

Th	H	T	O
	4	3	3
+		5	2
<hr/>			
			5

Th	H	T	O
	4	3	3
+		5	2
<hr/>			
		8	5

Th	H	T	O
	4	3	3
+		5	2
<hr/>			
	4	8	5

Th means thousands
H means hundreds
T means tens
O means ones

EXAMPLE 4

Add 40672 and 7213.

Solution

Line up the numbers according to place value, and then add individual digits one by one, moving from right to left, as shown below.

(a): Add rightmost column (ones column): $2 + 3 = 5$ (b): Add next column to the left (tens column): $7 + 1 = 8$ (c): Add next column to the left (hundreds column): $6 + 2 = 8$ (d): Add next column to the left (thousands column): $0 + 7 = 7$ (e): Add next column to the left (ten thousands column): $4 + 0 = 4$

40672	40672	40672	40672	40672
+ 7213	+ 7213	+ 7213	+ 7213	+ 7213
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
5	85	885	7885	47885

The result is $40672 + 7213 = 47885$.

PRACTICE 4

Add the following numbers.

a 451 032 and 64

b 528 and 9431

c 45274 and 4312

Addition with carrying

When adding two numbers together, if two digits are added and the result is more than 9, then this number needs to be carried and added onto the digits in the next column to the left. An example of addition with carrying is shown below. Note that the numbers have been lined up correctly according to place value before the digit-by-digit addition is completed.

The three substeps involved in the second addition step are shown below as (a), (b) and (c). First (a), the digits in the rightmost column (the ones column) are added ($6 + 7$). Since $6 + 7 = 13$ and this is greater than 9, the 3 from 13 is written as the result in the ones column and the 1 from 13 is carried to the tens column. Then (b), the digits in the tens column are added ($5 + 4$), and the carry (1) is added to them, giving $5 + 4 + 1 = 10$. Since 10 is greater than 9, the 0 from 10 is written as the result in the tens column and the 1 from 10 is carried to the hundreds column. Finally (c), the digits in the hundreds column are added ($7 + 0$), and the carry (1) is added to them, giving $7 + 0 + 1 = 8$. This result is written in the hundreds column. Since there are no more digits, the addition is complete.

(a): $6 + 7 = 13$; since 13 is more than one digit (more than 9), we put the 3 in the ones position and carry the 1

Th	H	T	O
		1	5
		4	6
		+	7
			3

(b): $5 + 4 = 9$, then add 1 (carry) is 10; since 10 is more than one digit we put the 0 in the tens position and carry the 1

Th	H	T	O
		1	5
		4	6
		+	7
		0	3

(c): $7 + 0 = 7$, then add 1 (carry) is 8

Th	H	T	O
	1	7	5
		4	6
		+	7
	8	0	3

Th means thousands
H means hundreds
T means tens
O means ones

EXAMPLE 5

What is the sum of 93 672 and 72 53?

Solution

The numbers are lined up according to place value and then added, digit by digit, as shown below.

(a): Add rightmost column (ones column): $2 + 3 = 5$. Since 5 is less than 10, the 5 is simply written in the ones column of the result.

(b): Add next column to the left (tens column): $7 + 5 = 12$. Since 12 is greater than 9, write the 2 in the result and carry the 1.

(c): Add next column to the left (hundreds column) and add the carry (1): $6 + 2 + 1 = 9$. Since 9 is less than 10, there is no carry.

(d): Add next column to the left (thousands column): $3 + 7 = 10$. Since 10 is greater than 9, write the 0 in the result and carry the 1.

(e): Add next column to the left (ten thousands column) and add the carry: $9 + 0 + 1 = 10$. Since this is the leftmost column, the 10 can be written directly in the result instead of carrying the 1.

9	3	6	7	2
+	7	2	5	3
				5

9	3	6	7	2
+	7	2	5	3
			2	5

9	3	6	7	2
+	7	2	5	3
		9	2	5

1	9	3	6	7	2
+	7	2	5	3	
		0	9	2	5

1	9	3	6	7	2
+	7	2	5	3	
	1	0	9	2	5

The sum of 93 672 and 72 53 is 100 925.

PRACTICE 5

Determine the sum of the following numbers.

- a 761 992 and 99
- c 45658 and 4808
- b 528 and 9723 and 10 198
- d 0909 and 444

EXAMPLE 6: APPLICATION

Jim's coffee shop sold eight hundred and thirty-three cups of coffee on Monday, six hundred and forty on Tuesday, five hundred and ninety-two on Wednesday, six hundred and twelve on Thursday, and eight hundred and twenty-nine on Friday. How many cups of coffee did they sell for the week?

Solution

The total number of cups of coffee sold is calculated by adding all the cups sold on individual days, as shown below.

2

8

3

3

6

4

0

5

9

2

6

1

2

+

8

2

9

3

5

0

6

The shop sold a total of 3506 cups of coffee during the week.

PRACTICE 6: APPLICATION

Hospital admissions for two hospitals are shown in the table below. Which hospital had more admissions for the week shown?

	SUN	MON	TUE	WED	THU	FRI	SAT
Hospital A	22	33	18	19	21	30	42
Hospital B	31	22	14	23	46	18	30

Subtraction

Subtraction is lined up the same way as addition but the individual digits are subtracted, and instead of carrying, if necessary, the technique of borrowing is used (see next subsection on subtraction with borrowing). The first step in subtraction is lining up the digits according to their place value, as shown overpage. The second and final step is to subtract the corresponding digits in each of the two numbers, going from right to left, and taking the bottom digit away from the top digit—just the same as for addition except that the bottom digit is subtracted from the top digit instead of being added.

An example of subtraction is shown below. The three substeps involved in the second subtraction step are shown as (a), (b) and (c). First (a), the digits in the rightmost column (the ones column) are subtracted ($3 - 2 = 1$) and the result (1) is written in the ones column. Then (b), the digits in the tens column—the next column to the left—are subtracted ($8 - 5 = 3$) and the result (3) is written in the tens column. Finally (c), the digits in the hundreds column are subtracted ($4 - 0 = 4$) and the result (4) is written in the hundreds column. For example:

(a): $3 - 2 = 1$

(b): $8 - 5 = 3$

(c): $4 - 0 = 4$

Th thousands
H hundreds
T tens
O ones

Th	H	T	O
	4	8	3
—		5	2
			1

Th	H	T	O
	4	8	3
—		5	2
		3	1

Th	H	T	O
	4	8	3
—		5	2
	4	3	1

Note that 52 is the same as 052, so the blank space in front of 52 is counted as zero.

EXAMPLE 7

Subtract 8443 from 998576.

Solution

Line up the numbers vertically according to place value and then proceed as shown below.

(a): Subtract rightmost column (ones column):
 $6 - 3 = 3$

(b): Subtract next column to the left (tens column):
 $7 - 4 = 3$

(c): Subtract next column to the left (hundreds column):
 $5 - 4 = 1$

(d): Subtract next column to the left (thousands column):
 $8 - 8 = 0$

(e): Subtract next column to the left (ten thousands column):
 $9 - 0 = 9$

(f): Subtract next column to the left (hundred thousands column):
 $9 - 0 = 9$

9	9	8	5	7	6
—	8	4	4	3	
				3	

9	9	8	5	7	6
—	8	4	4	3	
				3	3

9	9	8	5	7	6	
—	8	4	4	3		
				1	3	3

9	9	8	5	7	6		
—	8	4	4	3			
				0	1	3	3

9	9	8	5	7	6			
—	8	4	4	3				
				9	0	1	3	3

9	9	8	5	7	6			
—	8	4	4	3				
				9	0	1	3	3

The result is 990133.

PRACTICE 7

Subtract the following numbers.

- a 2605 from 163798
- b 417 from 9839
- c 2027 from 5278

Subtraction with borrowing

When subtracting, if the bottom digit is larger than the corresponding top digit, the bottom digit cannot be subtracted from the top one without getting a negative number as a result. In this case, to avoid a negative number result, the strategy is to borrow from the next column to the left. This is done by reducing the digit in the next column to the left by one, and then adding ten to the current digit. An example of subtraction with borrowing is shown overleaf.

Note that the two numbers have been lined up according to place value before the digit-by-digit subtraction is completed.

The three substeps involved in the second subtraction step are shown below as (a), (b) and (c). First (a), the digits in the rightmost column are considered. Since the top digit (3) is less than the bottom digit (4), ten is borrowed from the next column to the left; the 8 is reduced to 7 and a 1 is placed in front of the 3 in the rightmost column so that it becomes 13. Subtraction for the digits in the rightmost column is then performed ($13 - 4 = 9$) and the result (9) is written in the rightmost column of the answer. The digits in the tens column are then considered (b) and since the top digit (7) is greater than or equal to the bottom digit (5), subtraction is performed ($7 - 5 = 2$) and the result (2) is written in the corresponding column in the answer. Similarly, for the final column (c), the subtraction is performed ($4 - 0 = 4$) and the result (4) is written in the answer. Overall, this subtraction demonstrates that $483 - 54 = 429$.

(a): 4 is larger than 3, so borrow from the tens column: $13 - 4 = 9$

Th	H	T	O
		4	8 ¹ 3
+		5	4
			9

(b): Subtract next column to left (tens column): 7 is larger than 5, so just subtract to get $7 - 5 = 2$

Th	H	T	O
		4	8 ¹ 3
+		5	4
			2 9

(c): Subtract next column to left (hundreds column): 4 is larger than 0, so just subtract to get $4 - 0 = 4$

Th	H	T	O
	4	8 ¹ 3	
+		5	4
			4 2 9

Note that 54 is the same as 054, so the blank space in front of 54 is counted as zero.

EXAMPLE 8

Subtract 54 from 402.

Solution

The three substeps involved in the second subtraction step are shown overpage as (a), (b) and (c). First, in part (a), the digits in the rightmost column are considered. Since the top digit (2) is less than the bottom digit (4), we look to borrow ten from the next column to the left (the tens column). However, the tens column has a zero in it so cannot be borrowed from—so we look to the next column to the left (the hundreds column) and borrow from the 4 in the hundreds column, making it 3. This means we can add 10 to the tens column, making that now 10 instead of 0. Now we can borrow from the tens column, making that 9, and adding 10 to the ones column, making the 2 now 12. Subtraction for the digits in the rightmost column is then performed ($12 - 4 = 8$) and the result (8) is written in the rightmost column of the answer. The digits in the tens column are then considered and since the top digit (9) is greater than or equal to the bottom digit (5), subtraction is performed ($9 - 5 = 4$) and the result (4) is written in the corresponding column in the answer. Similarly, for the final column ($3 - 0 = 3$) and the result (3) is written in the answer. Overall, this subtraction demonstrates that $402 - 54 = 348$.

(a): 2 is smaller than 4, so look to borrow from tens column. But tens column is zero, so borrow from hundreds column, reducing hundreds column from 4 to 3 and adding ten to tens column (step not shown). Then borrow from tens column, reducing it from 10 to 9 and adding 10 to ones column to give 12. Then perform subtraction: $12 - 4 = 8$

Th	H	T	O
3	4	0	2
—	5	4	8
8			

(b): Now subtract digits in the next column to the left (tens column): $9 - 5 = 4$

Th	H	T	O
3	4	0	2
—	5	4	8
4 8			

(c): Now subtract digits in the next column to the left (hundreds column): $3 - 0 = 3$

Th	H	T	O
3	4	0	2
—	5	4	8
3 4 8			

The result is 348.

PRACTICE 8

Subtract the following numbers.

a 2605 from 163284

c 36297 from 43006

b 487 from 9013

d $804 - 56 - 49$

EXAMPLE 9: APPLICATION

Viv's pay is \$1204 but she owes a friend \$709. How much does Viv have left to spend?

Solution

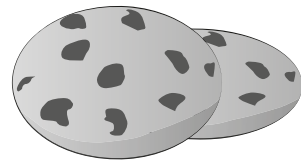
The amount left to spend is the result when 709 is subtracted from 1204:

Th	H	T	O
1	2	0	4
—	7	0	9
4 9 5			

So, the amount left for Viv to spend is \$495.

PRACTICE 9: APPLICATION

When baking recently, Tiffany made 12 dozen cookies for a party. Before they had cooled down, 25 of them had been eaten by the family. How many cookies were left for the party?



2.4 MULTIPLYING WHOLE NUMBERS

Multiplication is the process of adding a number to itself a certain number of times. For example, '5 times 2' means five lots of 2, or '2 plus 2 plus 2 plus 2 plus 2'. In other words, $5 \times 2 = 10$.

When two numbers are multiplied together, each of the numbers being multiplied is called a **factor** and the result of the multiplication is called a **product**.

$$\begin{array}{c} 6 \times 2 = 12 \\ \swarrow \quad \uparrow \quad \searrow \\ \text{Factor} \quad \text{Factor} \quad \text{Product} \end{array}$$

The multiplication of two, single-digit numbers is usually done simply by using the times tables of those digits. When one or more of the numbers has more than one digit, long multiplication may be useful. The first step in doing long multiplication is lining up numbers according to place value. The second step in doing long multiplication is multiplying the top number by each of the digits in the bottom number, in turn, going from right to left. Each answer is lined up according to the place value of the digit in the bottom row. Note that when you multiply individual digits and the result of the multiplication has more than one digit, you may need to carry.

Consider the example of multiplying 49 by 63:

Step 1: line up numbers according to place value

H	T	O
4	9	
\times	6	3

Step 2a: multiply 49 by 3

first: 9×3

Th	H	T	O
		24	9
		\times	6
			3
			7

Note that instead of writing 27 as the result, we write the 7 in the ones column and carry the 2.

second: 40×3 (think of this as 4×3 , put result in tens place), but you must add the carry to the result

Th	H	T	O
		24	9
		\times	6
			3
		1	4
			7

We write the 4 in the tens column and since 4 is the leftmost digit of the top number, we have now finished multiplying and can write 14 straight in as the result.

Step 2b: multiply 49 by 60 (think of this as 49×6 , put the zero in the ones place, and then start writing the result from the tens place)

first: 9×60 (think of this as 9×6 , put 0 in ones place and start result in tens place)

Th	H	T	O
		54	9
		\times	6
			3
		1	4
			7

Note that instead of writing 54 as the result, we put the 4 in the tens place and carry the 5.

second: 40×60 (think of this as 40×6 , put result in hundreds place)

Th	H	T	O
		54	9
		\times	6
			3
		1	4
			7
		2	9
			4
			0

Note that our result is 29 ($4 \times 6 + 5$, where 5 is the carry from the first part of Step 2b). Since 4 is the leftmost digit of the top number, we have now finished multiplying and can write 29 straight in as the result.

Factor

A whole number that multiplies with another whole number with the result being the product. A factor can also be defined as a whole number that divides into another whole number without a remainder.

Product

The result of the multiplication of two or more numbers

Step 3: add each answer obtained from the digit-by-digit multiplication

Th

H

T

O

4

9

×

6

3

+

1

4

7

1

2

9

4

0

3

0

8

7

Note that when we add 9 and 1 we get 10, so we put the 0 as part of our result and carry the 1.

Overall, the result of this multiplication is that $49 \times 63 = 3087$.

EXAMPLE 10

Multiply 4903×812 .

Solution

The two numbers are lined up according to place value (Step 1). For Step 2, going from right to left, each digit in the bottom number is multiplied by the top number. Finally, for Step 3 the results of each substep of the multiplication are added together. Steps 2 and 3 are shown below.

Step 2a: Multiply 4903 by 2.

1

4

9

0

3

×

8

1

2

9

8

0

6

Step 2b: Multiply 4903 by 1; since 1 is in the tens column put 0 in the ones column.

4

9

0

3

×

8

1

2

9

8

0

6

4

9

0

3

0

Step 2c: Multiply 4903 by 8; since 8 is in the hundreds column put zeros in the ones and tens columns.

7

4

9

2

0

3

×

8

1

2

9

8

0

6

4

9

0

3

0

3

9

2

2

4

0

0

Step 3: Add the result of each step of multiplication to get the final answer.

4

9

0

3

×

8

1

2

1

9

8

0

6

2

4

9

0

3

0

+

3

9

2

2

4

0

0

3

9

8

1

2

3

6

The result is 3981 236.

PRACTICE 10

Multiply the following numbers.

a 493×89

b 96074×318

c 4152×5934

EXAMPLE 11

Multiply 576×804 .

Solution

Step 1 is to line up numbers according to place value. Steps 2 and 3 are shown below.

Step 2a: Multiply 576 by 4.

$$\begin{array}{r} 3 \overline{) 576} \\ \times 804 \\ \hline 2304 \end{array}$$

Step 2b: Put 0 in ones place and then multiply 576 by 0. Note that any number multiplied by 0 is 0.

$$\begin{array}{r} 576 \\ \times 804 \\ \hline 2304 \\ 0000 \end{array}$$

Step 2c: Put 0 in ones and tens place. Then multiply 576 by 8.

$$\begin{array}{r} 6 \overline{) 576} \\ \times 804 \\ \hline 2304 \\ 0000 \\ 460800 \end{array}$$

Step 3: Add up the results of Steps 2a, 2b and 2c.

$$\begin{array}{r} 576 \\ \times 804 \\ \hline 1 \overline{) 2304} \\ 2304 \\ 0000 \\ + 460800 \\ \hline 463104 \end{array}$$

NOTE: When two numbers are arranged one above the other for multiplication, according to place value, if one of the digits in the bottom number is zero, then the step that involves that multiplication (Step 2b in the diagram above) can be expedited as shown below.

Step 2a: Multiply 576 by 4.

$$\begin{array}{r} 3 \overline{) 576} \\ \times 804 \\ \hline 2304 \end{array}$$

Step 2b: Put 0 in ones place. The next step is to multiply 576 by 0, but since 576×0 will result in a row of zeros, as shown in Step 2b above, it is sufficient to put a zero in the tens place and then continue with Step 2c on the same row, starting from the hundreds place.

$$\begin{array}{r} 576 \\ \times 804 \\ \hline 2304 \\ 00 \end{array}$$

Step 2c: Multiply 576 by 8. (Extra zeros are already in place in ones and tens places.)

$$\begin{array}{r} 6 \overline{) 576} \\ \times 804 \\ \hline 2304 \\ 460800 \end{array}$$

Step 3: Add up the results of Steps 2a, 2b and 2c.

$$\begin{array}{r} 576 \\ \times 804 \\ \hline 1 \overline{) 2304} \\ 2304 \\ 0000 \\ + 460800 \\ \hline 463104 \end{array}$$

The result is 463 104.

PRACTICE 11

Multiply the following numbers.

a 9879×809

b 847×900

c 1603×8042

EXAMPLE 12: APPLICATION

Apples are packed in boxes of 128. A particular truck can carry 236 boxes. How many apples can be taken in one trip on the truck?

Solution

Step 1 is to line up the numbers according to place value. Steps 2 and 3 are shown below.

Step 2a:

Multiply
236 by 8.

$$\begin{array}{r} ^2^4^6 \\ \times 128 \\ \hline 1888 \end{array}$$

Step 2b: Put 0 in

ones place and
then multiply
236 by 2.

$$\begin{array}{r} ^2^3^6 \\ \times 128 \\ \hline 1888 \\ 4720 \end{array}$$

Step 2c: Put 0 in

ones and tens place.
Then multiply
236 by 1.

$$\begin{array}{r} ^2^3^6 \\ \times 128 \\ \hline 1888 \\ 4720 \\ 23600 \end{array}$$

Step 3: Add up

the results of
Steps 2a, 2b
and 2c.

$$\begin{array}{r} ^2^3^6 \\ \times 128 \\ \hline ^2^3^6 1888 \\ 4720 \\ + ^1^3^6 23600 \\ \hline 30208 \end{array}$$

So, 30208 apples can be taken in one trip on the truck.

PRACTICE 12: APPLICATION

A group of thirty-six people is planning a trip to London together. The adult fare from Sydney to London is \$2236. What will be the total airfare cost for the group?

2.5 DIVIDING WHOLE NUMBERS

Dividing one number into another means working out how many times one number fits into another one. For example, when we say 6 divided by 2 is 3, what we mean is that 2 fits into 6, 3 times. This can be written in many ways, as shown below.

$6 \div 2 = 3$	$\frac{6}{2} = 3$	$6/2 = 3$	$\begin{array}{r} 3 \\ 2 \overline{)6} \end{array}$
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All of these statements are different ways of saying that 2 fits into 6, 3 times.

As shown, the 6 is called the **dividend** (that's the number we are dividing 'into'), the 2 is called the **divisor** (that's the number of 'groups' we have), and 3 is called the **quotient** (that's our answer). The quotient represents how many items are in each of the groups that is formed.

$$\begin{array}{c} 6 \div 2 = 12 \\ \swarrow \quad \uparrow \quad \nwarrow \\ \text{Dividend} \quad \text{Divisor} \quad \text{Quotient} \end{array}$$

Dividend

The number
being divided
into

Divisor

The number
of groups that
the dividend is
being split into

Quotient

The result of a
division

Division method 1: Short division

Short division involves working out how many times one number fits into each digit of another number, digit by digit, going from left to right. For example, the question ‘What is 230 divided by 5?’ or ‘What is $230 \div 5$?’ is answered using short division below.

Step 1: Does 5 go into 2?
No it does not. 5 is bigger than 2.

$$\begin{array}{r} 5 \overline{) 230} \end{array}$$

Step 2: Does 5 go into 23?
Note that since 5 did not go into 2, we now look at 23 rather than just 3.

$$\begin{array}{r} 4 \\ 5 \overline{) 230} \\ \underline{20} \\ 3 \end{array}$$

5 goes into 23, 4 times, with 3 left over.

Step 3: Does 5 go into 30? Note that the 30 is made up of the carry (3) and the next digit in the dividend (0).

$$\begin{array}{r} 46 \\ 5 \overline{) 230} \\ \underline{20} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

5 goes into 30, 6 times, with nothing left over.

Overall, the result of this division is $230 \div 5 = 46$.

EXAMPLE 13

What is $356 \div 2$?

Solution

Step 1: Does 2 go into 3?
Yes it does. It goes in once, with 1 left over.

$$\begin{array}{r} 1 \\ 2 \overline{) 356} \\ \underline{2} \\ 1 \end{array}$$

Step 2: Does 2 go into 15?
Note that the 15 is made up of the carry (1) and the next digit in the dividend (5).

$$\begin{array}{r} 17 \\ 2 \overline{) 356} \\ \underline{2} \\ 15 \\ \underline{14} \\ 1 \end{array}$$

2 goes into 15, 7 times, with 1 left over.

Step 3: Does 2 go into 16?
It goes in exactly 8 times, with no remainder.

$$\begin{array}{r} 178 \\ 2 \overline{) 356} \\ \underline{2} \\ 15 \\ \underline{14} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

The result is 178.

PRACTICE 13

What is $7092 \div 4$?

EXAMPLE 14

What is $93\,168 \div 3$?

Solution

As shown below, the answer (the quotient) when 93 168 is divided by 3 is 31 056. First, 3 goes into 9 three times exactly. Second, 3 goes into 3 one time exactly. Third, 3 does not fit into 1, so the 1 is carried to the fourth step. Fourth, 3 goes into 16 five times with 1 left over. Fifth, three goes into 18 six times exactly.

$$\begin{array}{r} 31056 \\ 3 \overline{)93168} \\ \underline{9} \\ 3 \\ \underline{3} \\ 1 \\ \underline{1} \\ 6 \\ \underline{6} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

The result is 31 056.

PRACTICE 14

What is $23\,176 \div 8$?

Short division with remainders

Sometimes, one number does not fit exactly into another number. A simple example is if we ask ‘How many times does 2 fit into 7?’ our answer would be that 2 fits into 7, 3 times—but with 1 left over. This leftover number is called the remainder, as shown below.

$$7 \div 2 = 3 \text{ R } 1$$

Dividend
Divisor
Quotient (3 with remainder 1 is the quotient)
Remainder (just the 1)
(Note: the R stands for Remainder)

As another example of division with remainders, the question ‘What is 125 divided by 2?’ or ‘What is $125 \div 2$?’ is shown below.

Step 1: Does 2 go into 1?
No it does not. 2 is bigger than 1.

$$2 \overline{)125}$$

Step 2: Does 2 go into 12?
Note that since 2 did not go into 1, we now look at 12 rather than just 2.

$$2 \overline{)125} \quad 6$$

2 goes into 12, 6 times exactly. Note there is nothing left over, so no carry.

Step 3: 2 goes into 5, 2 times, with 1 left over. Since we still have a number left over after dividing into the last digit of 125, we write Remainder 1 (or R1 for short) at the end of our answer.

$$2 \overline{)125} \quad 62 \text{ Remainder } 1$$

Our answer (or quotient) is 62 remainder 1.

EXAMPLE 15What is $5387 \div 6$?**Solution**

Step 1: Does 6 go into 5? No it does not — it's too big. So in the next step, we look not just at the next digit (3) but at 53 (the number comprised of this digit and the next digit).

$$\begin{array}{r} 6 \overline{) 5387} \end{array}$$

Step 2: Does 6 go into 53? It goes in 8 times ($6 \times 8 = 48$), with 5 left over. Note the 8 is written above the 3 in 53.

$$\begin{array}{r} 8 \\ 6 \overline{) 5387} \\ \underline{48} \end{array}$$

Step 3: Does 6 go into 58? It goes in 9 times ($6 \times 9 = 54$), with 4 left over. Note the 58 is made up of the carry from the previous step (5) and the current digit (8).

$$\begin{array}{r} 89 \\ 6 \overline{) 5387} \\ \underline{48} \end{array}$$

Step 4: Does 6 go into 47? It goes in 7 times ($6 \times 7 = 42$), with 5 left over. Since we are at the last digit the 5 left over is our remainder.

$$\begin{array}{r} 897 \text{ R } 5 \\ 6 \overline{) 5387} \\ \underline{42} \end{array}$$

The result is 897 remainder 5.

PRACTICE 15What is $4236 \div 9$?**EXAMPLE 16**What is $70216 \div 7$?**Solution**

Step 1: Does 7 go into 7? Yes it does, exactly 1 time ($7 \times 1 = 7$) with nothing left over.

$$\begin{array}{r} 1 \\ 7 \overline{) 70216} \end{array}$$

Step 2: Does 7 go into 0? No it does not — it's too big. But since the 0 is not the first (leftmost) digit in the dividend, we need to write in that 7 does not fit into 0 (or that 7 fits into 0, zero times), so we write a 0. Since 7 does not fit into 0, in the next step we look not just at the next digit (2) but at 02 (the number comprised of this digit and the next digit) — but as it happens, 02 is the same as 2, so we can just focus on the 2.

$$\begin{array}{r} 10 \\ 7 \overline{) 70216} \end{array}$$

Step 3: Does 7 fit into 2? No it does not. But since the 2 is not the first (leftmost) digit in the dividend, we need to write in that 7 does not fit into 2 (or that 7 fits into 2, zero times), so we write a 0. Since 7 does not fit into 2, in the next step we look not just at the next digit (1) but at 21 (the number comprised of this digit and the next digit).

$$\begin{array}{r} 100 \\ 7 \overline{) 70216} \end{array}$$

Step 4: Does 7 go into 21? It goes in 3 times exactly ($7 \times 3 = 21$)

$$\begin{array}{r} 1003 \\ 7 \overline{) 70216} \end{array}$$

Step 5: Does 7 go into 6? It does not (it is too big). Since the 6 is not the first (leftmost) digit in the dividend, we need to write in that 7 does not fit into 6, so we write a 0. Since 6 is our last digit and there was 6 left over when we tried to fit 7 into 6 unsuccessfully, our remainder (R) is 6.

$$\begin{array}{r} 10030R6 \\ 7 \overline{) 70216} \end{array}$$

The result for $70216 \div 7$ is 10030 remainder 6.

PRACTICE 16

What is $48063 \div 6$?

EXAMPLE 17: APPLICATION

Grandma has \$1300 allocated for buying birthday presents for her grandchildren this year. She likes to spend the same amount on each child and she has fifteen grandchildren. How much money will she be able to spend on each grandchild? Ignore cents in your answer and state how many dollars she will be able to spend on each grandchild.

Solution

The amount grandma has to spend on each child will be the result when the total amount of money (\$1300) is divided by the number of grandchildren (15). This division is shown below.

Step 1: Does 15 go into 1? No it does not – it's too big. So in the next step, we look not just at the next digit (3) but at 13 (the number comprised of this digit and the next digit).

$$\begin{array}{r} 15 \overline{) 1300} \end{array}$$

Step 2: Does 15 go into 13? No, it's still too big. So in the next step, we look not just at the next digit (5) but at 130 (the number comprised of 13 and the next digit).

$$\begin{array}{r} 15 \overline{) 1300} \end{array}$$

Step 3: Does 15 go into 130? Yes. It goes in 8 times ($15 \times 8 = 120$), with 10 left over.

$$\begin{array}{r} 8 \\ 15 \overline{) 1300} \end{array}$$

Step 4: Does 15 go into 100? Yes. It goes in 6 times ($15 \times 6 = 90$), with 10 left over. (Note the 100 is made up of the carry from the previous step (10) and the current digit (0)). Since we are at the last digit, the 10 now left over is our remainder.

$$\begin{array}{r} 86 \text{ R } 10 \\ 15 \overline{) 1300} \end{array}$$

The result of the division is 86 remainder 10. This means that ignoring cents, Grandma has \$86 to spend on each grandchild.

PRACTICE 17: APPLICATION

Mr Collings is buying pizza for his class of 16 students. He is buying 13 pizzas, each of which have 8 pieces. In order to share the pizzas fairly, how many pieces should he advise each student that they can have and still leave enough for everyone?

Making connections

The remainder of ten can be used to work out how many cents Grandma has. The techniques for division with decimals are described in Chapter 6.

Division method 2: Long division

Long division involves the same underlying process as short division: it involves working out how many times one number fits into each digit of another number, digit by digit, going from left to right. However, with long division, there is more writing down of calculations and less making calculations in your head. Either method can be used for division according to personal preference. The question ‘What is 230 divided by 5?’ or ‘What is $230 \div 5$?’ is answered using long division below.

Step 1: Does 5 go into 2? No it does not – it’s too big. So in the next step, we look not just at the next digit (3) but at 23 (the number comprised of this digit and the next digit).

$$\begin{array}{r} 5 \overline{) 230} \end{array}$$

Step 2: Does 5 go into 23? Yes, it fits in 4 times ($5 \times 4 = 20$) with 3 left over. In long division, the calculation of what is left over (3 in this case) is shown explicitly as $23 - 20$.

$$\begin{array}{r} 4 \\ 5 \overline{) 230} \\ \underline{- 20} \\ 3 \end{array}$$

Step 3: Bring the next digit (0) down and put it on the end of the remainder (3) from the previous step to get 30. Does 5 go into 30? Yes. It goes in 6 times exactly ($5 \times 6 = 30$), so the remainder is zero (there is no remainder).

$$\begin{array}{r} 16 \\ 5 \overline{) 230} \\ \underline{- 20} \\ 30 \\ \underline{- 30} \\ 0 \end{array}$$

The result when 230 is divided by 5 is 16.

EXAMPLE 18

Use long division to divide 2754 by 17.

Solution

The long division process to solve this problem is shown below.

Step 1: Does 17 go into 2? No it does not – it's too big. So in the next step, we look not just at the next step we look but at 27 (the number comprised of this digit and the next digit).

$$\begin{array}{r} 17 \overline{) 2754} \end{array}$$

Step 2: Does 17 go into 27? Yes, it fits in once ($17 \times 1 = 17$) with 10 left over. In long division, the calculation of what is left over (10 in this case) is shown explicitly as $27 - 17$.

$$\begin{array}{r} 1 \\ 17 \overline{) 2754} \\ \underline{- 17} \\ 10 \end{array}$$

Step 3: Bring the next digit (5) down and put it on the end of the remainder (10) from the previous step to get 105. Does 17 go into 105? Yes, it goes in 6 times ($17 \times 6 = 102$), with 3 left over.

$$\begin{array}{r} 16 \\ 17 \overline{) 2754} \\ \underline{- 17} \\ 105 \\ \underline{- 102} \\ 3 \end{array}$$

Step 4: Bring the next digit (4) down and put it on the end of the remainder (3) from the previous step to get 34. Does 17 go into 34? Yes. It goes in 2 times exactly ($17 \times 2 = 34$), so the remainder is zero (there is no remainder).

$$\begin{array}{r} 162 \\ 17 \overline{) 2754} \\ \underline{- 17} \\ 105 \\ \underline{- 102} \\ 34 \\ \underline{- 34} \\ 0 \end{array}$$

So, 2754 divided by 17 is 162.

PRACTICE 18

Use long division to calculate $1646 \div 18$.

Long division with remainders

Remainders when using long division are dealt with in the same way as for short division. The following example uses long division and demonstrates how the remainder is found and recorded as part of the answer.

EXAMPLE 19

Use long division to calculate $32035 \div 13$.

Solution

Step 1: Does 13 go into 3? No it does not – it's too big. So in the next step, we look not just at the next digit (2) but at 32 (the number comprised of this digit and the next digit).

$$\begin{array}{r} 13 \overline{) 32035} \end{array}$$

Step 2: Does 13 go into 32? Yes, it fits in 2 times ($13 \times 2 = 26$) with 6 left over. In long division, the calculation of what is left over (6 in this case) is shown explicitly as $32 - 26$.

$$\begin{array}{r} 2 \\ 13 \overline{) 32035} \\ \underline{- 26} \\ 6 \end{array}$$

Step 3: Bring the next digit (0) down and put it on the end of the remainder (6) from the previous step to get 60. Does 13 go into 60? Yes. It goes in 4 times ($13 \times 4 = 52$), with 8 left over.

$$\begin{array}{r} 24 \\ 13 \overline{) 32035} \\ \underline{- 26} \\ 60 \\ \underline{- 52} \\ 8 \end{array}$$

Step 4: Bring the next digit (3) down and put it on the end of the remainder (8) from the previous step to get 83. Does 13 go into 83? Yes. It goes in 6 times ($13 \times 6 = 78$), with 5 left over.

$$\begin{array}{r} 246 \\ 13 \overline{) 32035} \\ \underline{- 26} \\ 60 \\ \underline{- 52} \\ 83 \\ \underline{- 78} \\ 5 \end{array}$$

Step 5: Bring the next digit (5) down and put it on the end of the remainder (5) from the previous step to get 55. Does 13 go into 55? Yes. It goes in 4 times ($13 \times 4 = 52$), with 3 left over. Since we are at the last (rightmost) digit, the 3 left over is our remainder.

$$\begin{array}{r} 2464 \text{ R } 3 \\ 13 \overline{) 32035} \\ \underline{- 26} \\ 60 \\ \underline{- 52} \\ 83 \\ \underline{- 78} \\ 55 \\ \underline{- 52} \\ 3 \end{array}$$

The result (quotient) when 32035 is divided by 13 is 2464 with a remainder of 3.

PRACTICE 19

Use long division to calculate $8216 \div 17$.

EXAMPLE 20: APPLICATION

A group of 1520 Adelaide Crows fans are travelling to Melbourne to watch the Crows play in the AFL final. Available buses have 46 seats each. How many buses are needed to transport the entire group?

Solution

The number of buses needed is obtained by dividing the total number of fans (1520) by the number of seats per bus (46), as shown below.

$$\begin{array}{r}
 \overline{) 1520} \\
 \underline{- 138} \\
 140 \\
 \underline{- 138} \\
 2
 \end{array}$$

3 3 R 2

So, all except two of the fans will fit in 33 buses ... but that means we need 34 buses to fit in everyone.

PRACTICE 20: APPLICATION

The dinner for a large conference anticipates 3408 guests. Tables seat 16 people each. How many tables are needed?

2.6 ORDER OF OPERATIONS

Consider the following mathematics problem: $3 + 4 \times 5$. This problem might be tackled in two ways. One way could be to do the $3 + 4$ first, with the result being 7, and then to do 7×5 to get 35. Another way could be to do the 4×5 first, with the result being 20, and then to do $3 + 20$ to get 23. Which way is correct?

Now consider another mathematics problem: $3 \times 10 \div 5$. This problem might also be tackled in two ways. One way could be to do the 3×10 first, with the result being 30, and then to do $30 \div 5$ to get 6. Another way could be to do the $10 \div 5$ first, with the result being 2, and then to do 3×2 to get 6. Which way is correct?

The rule that tells us the correct order in which to carry out mathematics operations is **BEDMAS** (or sometimes known as **BODMAS**). The letters in **BEDMAS** stand for the following:

- B: brackets
 - E: exponents (also known as O operators)
 - D: division
 - M: multiplication
 - A: addition
 - S: subtraction
- } Division and multiplication have *equal* precedence but must be done in order *from left to right*.
 } Addition and subtraction have *equal* precedence but must be done in order *from left to right*.

So in the first example above, the second method—doing the 4×5 first, then $3 + 20$ to get 23—is the correct method and gives the correct answer. In the second example above, the first method—doing the 3×10 first and then $30 \div 5$ to get 6—is the correct method and gives the correct answer, because when two operations have equal precedence (multiplication and division have equal precedence), the order in which they should be calculated is from left to right.

In general, BEDMAS says that calculations within brackets in a mathematics problem need to be done first. Then, any numbers raised to a power (operators/exponents) are worked out next. Division and multiplication then have equal precedence, as do addition and subtraction. However, division and multiplication must be done going from left to right, and the same with addition and subtraction, because in some problems, not going from left to right will give a different answer. The following examples highlight how to use BEDMAS to guide the order in which you tackle individual calculations within a larger mathematics problem. Note that the following examples do not include exponents. Exponents are covered in Chapter 8 and examples of BEDMAS with exponents will be provided there.

EXAMPLE 21

Solve the following problems.

a $30 - 8 \times 2$

b $4 + 7 \times 3 - 6 \div 2$

c $20 \div 4 + 6 - 2 \times 5 + 1$

d $4 + 7 \times (14 - 6) \div 2$

e $20 \div 4 + (40 - 2 \times 5) \div 3 \times 5 + 1$

Solutions

- a BEDMAS tells us that multiplication has higher precedence than subtraction, so multiplication is done first in the equation $30 - 8 \times 2$. Calculations are as follows:

$$\begin{aligned} 30 - 8 \times 2 &= 30 - 16 \\ &= 14 \end{aligned}$$

- b BEDMAS tells us that multiplication and division are calculated before addition and subtraction, and that multiplication and division have equal precedence and should be carried out from left to right. Calculations are as follows:

$$\begin{aligned} 4 + 7 \times 3 - 6 \div 2 \\ &= 4 + 21 - 6 \div 2 \\ &= 4 + 21 - 3 \\ &= 25 - 3 \\ &= 22 \end{aligned}$$

Multiplication is on the left so done before division

Division is done next

Addition is on the left so done before subtraction

Subtraction is done next

- c BEDMAS tells us that multiplication and division are calculated before addition and subtraction, and that multiplication and division have equal precedence and should be carried out from left to right. Calculations are as follows:

$$20 \div 4 + 6 - 2 \times 5 + 1$$

Division is on the left so done before multiplication

$$= 5 + 6 - 2 \times 5 + 1$$

Multiplication is done next

$$= 5 + 6 - 10 + 1$$

Addition of 5 and 6 is leftmost so is next

$$= 11 - 10 + 1$$

Subtraction is leftmost so done next

$$= 1 + 1$$

Addition is the only remaining operation

$$= 2$$

- d BEDMAS tells us that the first calculation to be done is any calculation inside brackets. After brackets, comes exponents, but there are no exponents in this problem. Next, BEDMAS tells us that multiplication and division are calculated before addition and subtraction and that multiplication and division have equal precedence and should be carried out from left to right. Calculations are as follows:

$$4 + 7 \times (14 - 6) \div 2$$

Brackets first

$$= 4 + 7 \times 8 \div 2$$

Multiplication is further to the left than division so is done next

$$= 4 + 56 \div 2$$

Division is done next

$$= 4 + 28$$

Addition is done next

$$= 32$$

- e BEDMAS tells us that the first calculation to be done is any calculation inside brackets. After brackets, comes exponents, but there are no exponents in this problem. Next, BEDMAS tells us that multiplication and division are calculated before addition and subtraction and that multiplication and division have equal precedence and should be carried out from left to right. Calculations are as follows:

$$20 \div 4 + (40 - 2 \times 5) \div 3 \times 5 + 1$$

Brackets first, within brackets multiplication first

$$= 20 \div 4 + (40 - 10) \div 3 \times 5 + 1$$

Brackets first, within brackets subtraction next

$$= 20 \div 4 + 30 \div 3 \times 5 + 1$$

Division of 20 by 4 is leftmost division/multiplication so next

$$= 5 + 30 \div 3 \times 5 + 1$$

Division of 30 by 3 is further left than multiplication so is done next

$$= 5 + 10 \times 5 + 1$$

Multiplication is done next

$$= 5 + 50 + 1$$

Addition of 5 and 50 is leftmost so done next

$$= 55 + 1$$

Addition of 55 and 1 is all that is left to do

$$= 56$$

PRACTICE 21

Solve the following problems.

a $12 + 18 \div 9$

b $24 - 20 \div 5 + 8 \times 4$

c $6 + 20 \times 4 \div 2 - 3 \times 6 + 2$

d $4 + 7 \times ((14 - 6) \div 2)$

e $40 \div (20 - 2 \times 6) \times 4 \div 2 \times 5 + 1$

2.7 ROUNDING WHOLE NUMBERS

Sometimes an approximation of a number is sufficient. For example, if three houses are sold for \$540 510, \$423 200 and \$681 300, respectively, then when comparing the sale prices, hundreds of dollars are unlikely to be relevant. It is sufficient to consider an approximation of each sale price to, say, the nearest ten thousand dollars: \$540 000, \$420 000 and \$680 000.

To **round** a whole number to a given place value, the process is as follows:

- 1 Underline the digit in the place to which the number is being rounded.
- 2 Note that the digit to the right of the underlined digit is called the critical digit.
 - a If the critical digit is 5 or more, then add one to the underlined digit.
 - b If the critical digit is less than 5, then the underlined digit remains unchanged.

Rounding

The process of approximating a number to a given place value

EXAMPLE 22

Round the number 123 405 to the following places.

- a thousands b tens c hundred thousands

Solutions

- a Underline the digit in the thousands place: 123405. The digit to the right of the thousands place is a 4. Since this is less than 5, the 3 remains unchanged and the rounded number is 123 000.
- b Underline the digit in the tens place: 123405. The digit to the right of the tens place is 5. Since this is in the category '5 or more', add one to the digit in the tens place (0 becomes 1). The rounded number is 123 410.
- c Underline the digit in the hundred thousands place: 123 405. The digit to the right of the hundred thousands place is 2. Since this is in the category 'less than 5', it remains unchanged. The rounded number is 100 000.

PRACTICE 22

Round the number 9 026 913 to the following places.

- a thousands b tens c millions

When rounding a number, if the underlined digit is a 9 and the critical digit is 5 or more, then the underlined digit is rounded up to 10. What this means in practice is that the underlined digit is changed to a zero (0) and the next digit to the left is rounded up by one.

EXAMPLE 23

- a Round the number 3089 695 to the thousands place.
- b Round the number 3997 421 to the ten thousands place.
- c Round the number 3997 421 to the hundred thousands place.

Solutions

- a Underline the digit in the thousands place: 3089 695. The digit to the right of the thousands place is a 6. Since this is in the '5 or more' category, the underlined 9 is rounded up to a 10. This means that the underlined 9 is changed to a zero (0) and the next digit to the left (8) is rounded up to a 9. The rounded number is 3090000.
- b Underline the digit in the ten thousands place: 3997 421. The digit to the right of the ten thousands place is 7 and since this is in the category '5 or more', the underlined 9 is rounded up to a 10. This means that the underlined 9 is changed to a 0 and the next digit to the left (the 9 in the hundred thousands place) is rounded up to a 10. This means in turn that the 9 in the hundred thousands place is replaced by a zero (0) and the next digit to the left (the 3 in the millions place) is rounded up to a 4. The rounded number is 4000000.
- c Underline the digit in the hundred thousands place: 3997 421. The digit to the right of the hundred thousands place is 9 and since this is in the category '5 or more', the underlined 9 is replaced by a zero (0) and the next digit to the left (the 3 in the millions place) is rounded up to a 4. The rounded number is 4000000. Note that for this particular number (but not for every number), rounding to the nearest ten thousand gives the same result as rounding to the nearest hundred thousand.

PRACTICE 23

- a Round the number 249 526 to the thousands place.
- b Round the number 3999 951 to the hundreds place.
- c Round the number 3909 991 to the tens place.

2.8 SIGNIFICANT FIGURES

The accuracy required when using numbers depends upon the context in which the number is used. For example, if we were describing someone's wealth in terms of millions of dollars, a couple of hundred dollars more, or less, would not make any substantial difference. While it would be possible to specify that we want to round someone's wealth to the nearest hundred thousand dollars, this is only effective if we know just how wealthy someone is. If we decided to round to the nearest hundred thousand dollars but the wealth being measured is in the billions of dollars, rounding to the nearest hundred thousand dollars would be almost irrelevant. In some circumstances, particularly where we are unsure of the magnitude of the number being approximated, it is useful to have a rounding technique that works in a different way. We need a technique that uses for approximation only the figures (or digits) that provide the important information about the size of a number. These digits are known as **significant figures**.

Suppose we want to write the number that is closest to 46 892 using, at most, two non-zero digits. This number would be 47 000. (The third digit, 8, is greater than 4, so the 6 is rounded up.) Therefore, we can say that 46 892 to two significant figures (s. f.) is 47 000. The number 1 349 029 to two significant figures would be 1 300 000. (The third digit, 4, is less than 5, so the 3 remains as is and the number is rounded down.)

If we need to write a number to two significant figures, we look at the first three digits, with the third digit being the critical digit that determines whether the second digit is rounded up or remains the same. Similarly, to write a number to three significant figures we look at the first four digits with the fourth digit being the critical digit, and so on. In general, we always look at one more digit than the number of significant figures required.

EXAMPLE 24

Write 654319 to:

a 2 s. f.

b 4 s. f.

c 1 s. f.

Solutions

- a To write the number to 2 s. f. we need to look at the first 3 digits. The 3rd digit is a 4, so the 2nd digit, the 5, remains a 5, and the number is rounded to 650 000.
- b To write the number to 4 s. f. we need to look at the first 5 digits. The 5th digit is a 1, so the 4th digit, the 3, remains a 3, and the number is rounded to 654 300.
- c To write the number to 1 s. f. we need to look at the first 2 digits. The 2nd digit is a 5, so the 1st digit, the 6, is rounded up to 7, and the number is rounded to 700 000.

Significant figure

An approximation to a specified number of digits, starting from the highest place value digits in the number

PRACTICE 24

Write 36 482 to:

a 3 s. f.**b** 4 s. f.**c** 2 s. f.**EXAMPLE 25**

Write 4990013 to:

a 2 s. f.**b** 1 s. f.**c** 4 s. f.**Solutions**

- a** To write the number to 2 s. f. we need to look at the first 3 digits. The 3rd digit is a 9, so the 2nd digit, the 9, is rounded up to 10, which means the 2nd digit becomes zero and the 1st digit is rounded up to a 5. The number is rounded to 5000000.
- b** To write the number to 1 s. f. we need to look at the first 2 digits. The 2nd digit is a 9, so the 1st digit, the 4, is rounded up to 5, and the number is rounded to 5000000. Note that because the 2nd and 3rd digits are both 9, rounding to 1 s. f. produces the same result as rounding to 2 s. f.
- c** To write the number to 4 s. f. we need to look at the first 5 digits. The 5th digit is a zero, so the 4th digit, also a zero, remains a zero, and the number is rounded to 4990000.

PRACTICE 25

Write 4003997 to:

a 2 s. f.**b** 1 s. f.**c** 5 s. f.**d** 6 s. f.

2.9 CHAPTER SUMMARY

SKILL OR CONCEPT	DEFINITION OR DESCRIPTION	EXAMPLES
Whole number	A number without a fractional or decimal part, made up of one or more digits.	24, 0, 987 645
Place value	The rightmost digit in a whole number is in the ones place, the next digit to the left is in the tens place, the next digit to the left is in the hundreds place, and so on, with the place value of each digit being ten times the place value of the digit next to it on the right.	$14352 = 1 \times 10000 + 4 \times 1000 + 3 \times 100 + 5 \times 10 + 2 \times 1$
Adding or subtracting whole numbers	<ul style="list-style-type: none">• Write numbers vertically lining up according to place value.• Add or subtract the digits working from right to left; subtract bottom digit from top digit.• Carry if the sum of two digits is greater than 9; borrow if the difference when subtracting the bottom digit is negative.	$8641 + 237 =$ $\begin{array}{r} 8641 \\ + 237 \\ \hline 9228 \end{array}$ $2307 - 259 =$ $\begin{array}{r} 2307 \\ - 259 \\ \hline 2046 \end{array}$
Multiplying whole numbers	<ul style="list-style-type: none">• Write numbers vertically lining up according to place value.• Working from right to left, multiply each digit in the top number by the rightmost digit in the bottom number.• Repeat the above step for each digit in the bottom number, working from right to left.• Add up the results of all the multiplication steps to get the final answer.	$1681 \times 356 =$ $\begin{array}{r} 1681 \\ \times 356 \\ \hline 10086 \\ 84050 \\ 504300 \\ \hline 598436 \end{array}$
Dividing whole numbers	<ul style="list-style-type: none">• Divide the dividend by the divisor.• Carry out the division digit by digit working from left to right.• If you use short division, just note the carry as you go.• If you use long division, write out the multiplication and subtraction needed to calculate the carry before moving on to the next digit.	Short division: $1520 \div 46 = 33 \text{ remainder } 2$ $\begin{array}{r} 33 \text{ R } 2 \\ 46 \overline{) 1520} \end{array}$ Long division: $1520 \div 46 = 33 \text{ remainder } 2$ $\begin{array}{r} 33 \text{ R } 2 \\ 46 \overline{) 1520} \\ \underline{- 138} \\ 140 \\ \underline{- 138} \\ 2 \end{array}$

SKILL OR CONCEPT	DEFINITION OR DESCRIPTION	EXAMPLES
Order of operations	BEDMAS (Brackets, Exponents, Division and Multiplication, Addition and Subtraction) specifies the order in which operations should be carried out.	$(4 + 5) \times 9 - 21 \div 3$ (brackets first) $= 9 \times 9 - 21 \div 3$ (then multiplication) $= 81 - 21 \div 3$ (then division) $= 81 - 7$ (then subtraction) $= 74$
Rounding whole numbers	<ul style="list-style-type: none">Underline the digit in the place value to which the number is being rounded.The digit to the right of the underlined digit is the critical digit.If the critical digit is 5 or more, add one to the underlined digit.If the critical digit is less than 5, the underlined digit remains unchanged.All other digits to the right of the underlined digit become zero.	Round 852567 to the nearest thousand. 85 <u>2</u> 567 = 853000 (nearest thousand) Round 34969 to the nearest hundred. 34 <u>9</u> 69 = 35000 (nearest hundred; 9 rounds up to 10, so 4 becomes 5 and 9 becomes 0)
Significant figures (s. f.)	<ul style="list-style-type: none">Starting from the leftmost digit, underline the <i>n</i>th digit where you are rounding to <i>n</i> significant figures.The digit to the right of the underlined digit is the critical digit.If the critical digit is 5 or more, add one to the underlined digit.If the critical digit is less than 5, the underlined digit remains unchanged.All other digits to the right of the underlined digit become zero.	Write 9853468 to 4 s. f. 985 <u>3</u> 468 = 9853000 (4 s. f.) Write 239969 to 3 s. f. 23 <u>9</u> 969 = 240000 (9 rounds up to 10, so 3 becomes 4 and 9 becomes 0)

2.10 REVIEW QUESTIONS

A SKILLS

- 1 Write in words the place value of the underlined digit in the following numbers.

a 1 <u>3</u> 6 5 4 1 9	b 2 5 7 6 <u>3</u> 4 1
c <u>2</u> 3 9 4	d 4 1 6 0 8 1 <u>8</u> 2
- 2 Underline the digit with the specified place value in the given number.

a thousands in 135419	b tens in 3411728
c millions in 32964176	
- 3 Carry out the following additions or subtractions.

a $2254 - 1032$	b $813 + 164$	c $4305 + 15296$
d $2731 - 889$	e $30048 - 12567$	f $49991 + 202436$
- 4 Carry out the following multiplications or divisions.

a 81×63	b $486 \div 3$	c 7×891
d $9076 \div 12$	e 486×316	f $1302 \div 21$
g 48×79	h $1351 \div 7$	i 42×4015
j $6238 \div 9$	k 5179×742	l $2104 \div 24$
- 5 Carry out the following calculations.

a $5 + (22 - 6) \times 2$	b $7 + 84 \div 2 \times (14 - 8)$
c $(45 - 3 \times 9) \div 3 \times 5 + 60 \div 4$	d $45 - 3 \times 9 \div 3 \times 5 + 60 \div 4$
- 6 Round the following numbers to the required place value.

a 19863 (tens)	b 5823789 (hundreds)
c 26767456 (hundred thousands)	d 98996 (tens)
- 7 Write the following numbers to the required number of significant figures.

a 36072 (1 s. f.)	b 2041005 (2 s. f.)
c 124881 (3 s. f.)	d 9009965 (4 s. f.)

B APPLICATIONS

1 In a cricket match, the number of runs made by each of the eleven players was as follows: 45, 13, 68, 143, 14, 28, 35, 44, 32, 3 and 5. What was the total number of runs made by the team?

2 Approximate data for the total rainfall for Adelaide during 2014 is shown in the table (Source: www.bom.gov.au).



MONTH	RAINFALL (MM)
Jan	10
Feb	98
Mar	19
Apr	51
May	64
Jun	104
Jul	100
Aug	21
Sep	31
Oct	5
Nov	24
Dec	6

- a What was the total rainfall for each season (spring, summer, autumn, winter)?
- b Round the rainfall amounts in part (a) to the nearest ten.
- c What was the total rainfall for the year?
- d The average of a set of numbers is the sum of the numbers divided by how many numbers there are in the set. Use your calculations from part (a) to find the average monthly rainfall.
- e Express the average rainfall for spring and winter to one significant figure.

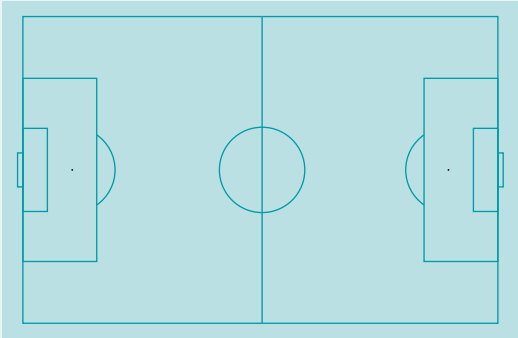
3 The price of a stock opened at \$70 and dropped by \$3 per hour for the next four hours, before rising by \$7 during the next hour, then dropping by \$4 per hour for the next two hours, and finally rising by \$12 before the day's trading closed. What was the closing price of the stock?

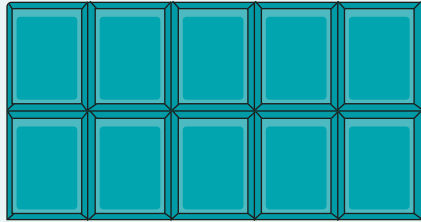
4 Tom had \$248 in his bank account. After buying eight DVDs, each at the same price, he had \$120 left. How much did each DVD cost?

5 The population of Springfield was 24 950. It decreased by 326 each year for five years. What was its population after five years?

6 The formula for the area of a rectangle is $length \times width$. If a soccer field has a length of 105 metres and a width of 68 metres, what is its area?

7 On a recent trip, James drove 2496 kilometres. The fuel consumption for the trip was 192 litres. How many kilometres per litre did the car average?



- 8 Macy's interest-free student loan is \$1350. If she pays off \$75 per month, how long will it take her to pay it off completely?
- 9 An irrigation subcontractor is installing water pipes. If she is installing 744 metres of pipe and each piece of pipe is 3 metres long, how many pieces of pipe will she need?
- 10 To supply food for a party, Jack needs to buy four pizzas at \$26 each, six bottles of drink at \$2 each, a packet of paper plates for \$6 and a packet of napkins at \$4. What is the total cost of the party? Express your answer to two significant figures to obtain a rough budget for the party.
- 11 Dr Dianati accidentally misread his travel itinerary and as a result his plane fare will cost \$709 instead of the original \$488. How much extra does Dr Dianati have to pay?
- 12 How many seconds are there in one day?
- 13 A customer has asked for 850 pencils. The store has 63 boxes of pencils in stock. There are 14 pencils in each box. Does the store have enough pencils to supply to the customer?
- 14 A store chain is placing an order for chocolate bars. They estimate the chain will sell 5500 of this chocolate bar in a month.
The chocolate bars come in boxes of 24.
How many boxes does the store need to buy to meet expected sales?
- 
- 15 A group of West Coast Eagles supporters is going to Melbourne by bus to see the Eagles in a football final. There are 531 supporters making the trip. Each bus seats 48 people. How many buses are needed to transport all the supporters?
- 16 There are 324 students studying Biology, 499 students studying Chemistry and 250 students studying Physics. Estimate how many students are studying these sciences in total by rounding your answer to the nearest hundred.
- 17 There are 849 students studying Law, 4099 students studying Education, 1256 students studying Business and 467 students studying Engineering. Estimate how many students in total are studying in these areas by expressing each number to two significant figures and then calculating a total based on your estimated numbers.
- 18 Five houses in the Sunnyvale area sold for the following prices: \$524 000, \$306 500, \$290 000, \$610 100 and \$738 000. Work out a rough average for these house prices by rounding each price to the nearest ten thousand, adding the rounded numbers, and dividing by the number of houses sold.