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Study Buddy

Revision and Exam Guide

QCE SPECIALIST MATHEMATICS

UNITS 3 & 4

VOLUME 1

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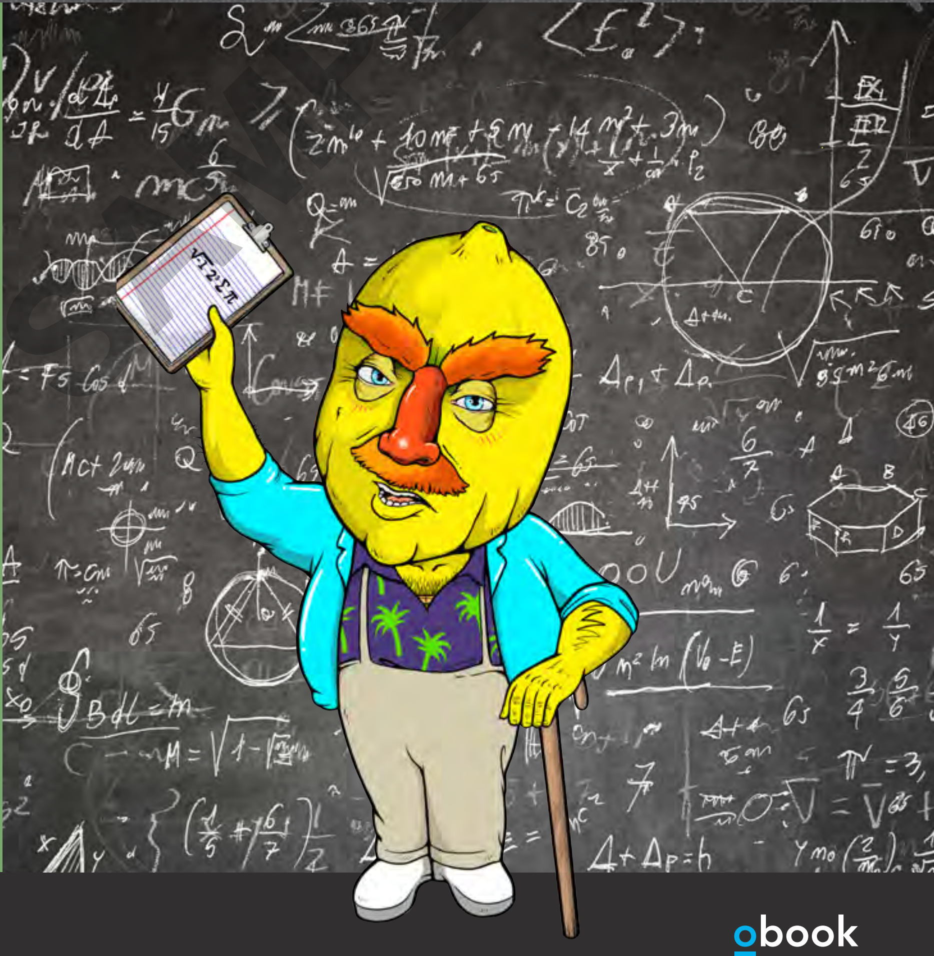

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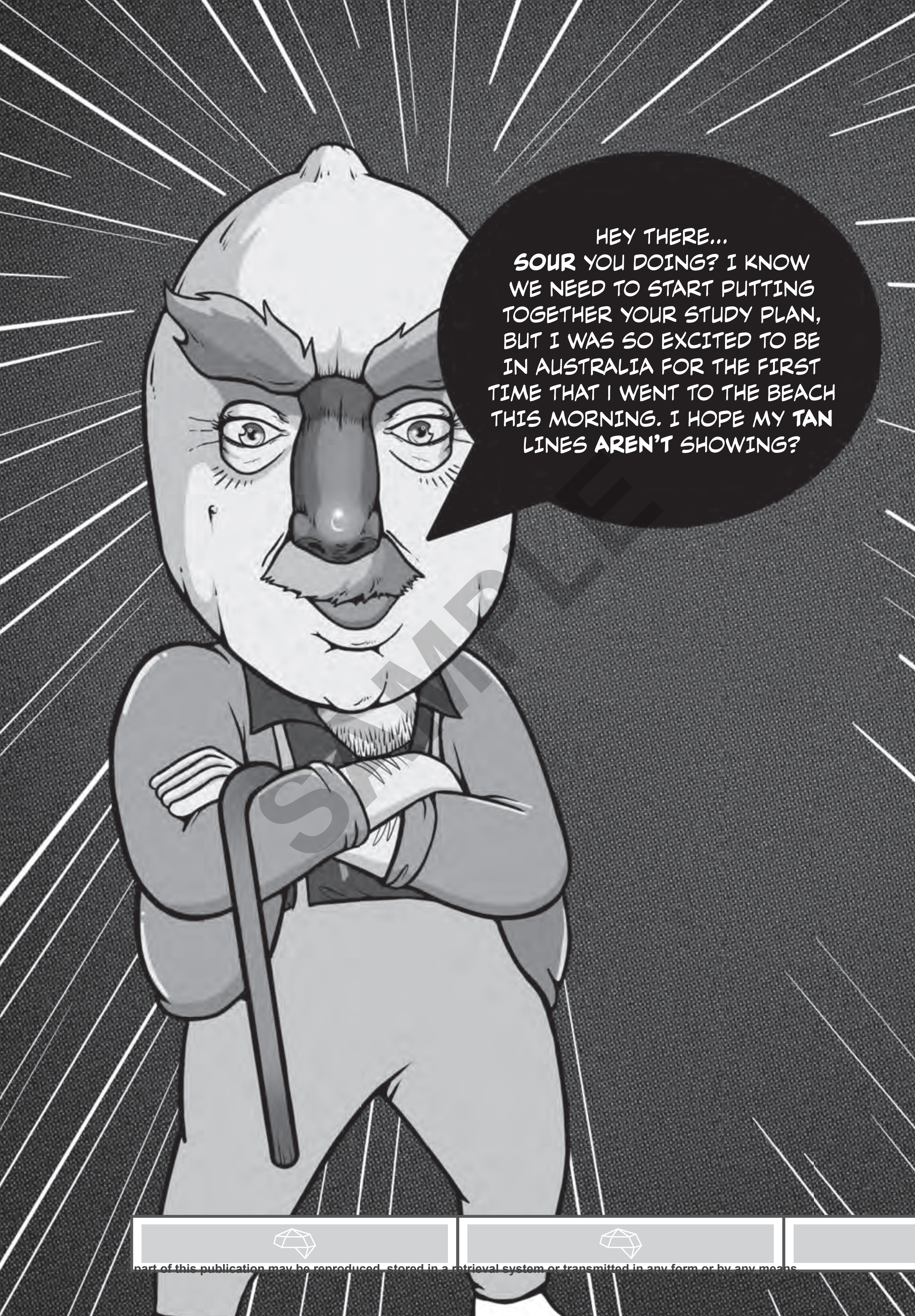
Unit 3: Mathematical induction, and further vectors, matrices and complex numbers

Unit 4: Further calculus and statistical inference


HEY THERE...
SOUR YOU DOING? I KNOW WE NEED TO START PUTTING TOGETHER YOUR STUDY PLAN, BUT I WAS SO EXCITED TO BE IN AUSTRALIA FOR THE FIRST TIME THAT I WENT TO THE BEACH THIS MORNING. I HOPE MY **TAN** LINES **AREN'T** SHOWING?

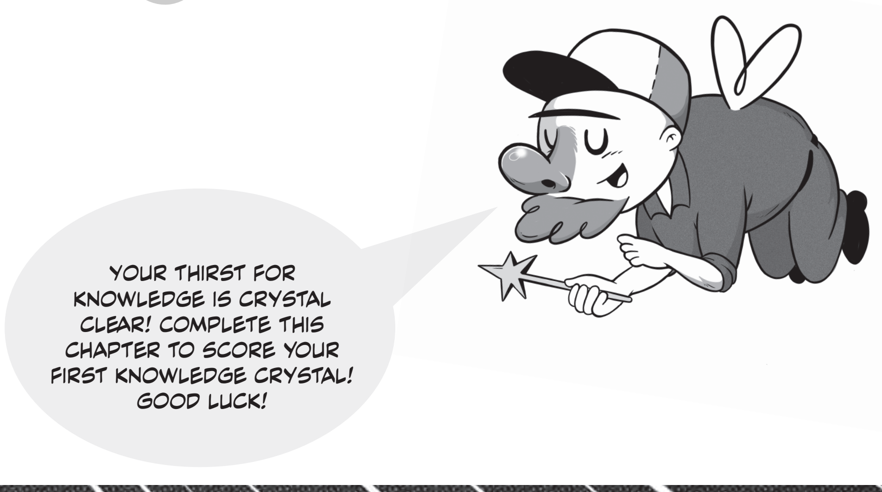
CHAPTER

1 STUDYING FOR SUCCESS

Before you start studying for your QCE Specialist Mathematics exams, it's important to set yourself up for success. That's exactly what this chapter is designed to do, so thanks for stopping by!

As soon as it's time to start studying for the external assessment, we recommend that you work through this whole chapter before doing anything else.

You might do this at the start of the school year or at the start of your exam study period, but whatever you do, don't skip this chapter; it contains a bunch of really important information and tips that might just give you the edge you're looking for.



1.1 OVERVIEW OF QCE SPECIALIST MATHEMATICS UNITS 3 & 4

In this section we will:

- provide a brief overview of how the QCE Specialist Mathematics Units 3 & 4 course is structured
- list all of the concepts and topics that you will need to learn and understand
- explain how you will be assessed.

1.1.1
Resource: Specialist Mathematics General Senior Syllabus

Study tip

The QCE Specialist Mathematics General Senior Syllabus sets out all of the information you are expected to learn and also provides important information on how you will be assessed.

In this chapter, we have summarised the key information relating to external assessment but the QCAA may update the syllabus from time to time, so it's important that you check the most up-to-date version.

Make sure you visit the QCAA website and download a copy of the Specialist Mathematics General Senior Syllabus and read the key information carefully before you sit your external assessment. To save you time, we've also included a link to it on your obook assess!

UNDERSTANDING THE QCE SPECIALIST MATHEMATICS UNITS 3 & 4 COURSE STRUCTURE

The QCE Specialist Mathematics General Senior Syllabus is the most important document supporting the QCE Specialist Mathematics course. It sets out all the content – known as subject matter – that you will be expected to learn and provides important information about how you will be assessed.

QCE Specialist Mathematics is a course of study consisting of four units (i.e. Units 1 & 2 and Units 3 & 4) taught over 2 years, but in this revision and exam guide, we will focus on information relating to Units 3 & 4 of the course. The topics you will learn about in Units 3 & 4 are summarised in Table 1.

Study tip

The notional hours shown in Table 1 are provided by the QCAA to help teachers with their planning and to give them an estimate of how long to spend teaching the subject matter in each topic.

Notional hours can be a handy way to help you structure and allocate your revision and preparation time for the external assessment because – as a general rule – there are likely to be more questions on subject matter with higher notional hours.

Course structure for QCE Specialist Mathematics Units 3 & 4

Unit 3 Mathematical induction, and further vectors, matrices and complex numbers	Unit 4 Further calculus and statistical inference
Topic 1: Proof by mathematical induction *Subject matter:* • Mathematical induction [7 hours] **Topic 2: Vectors and matrices** *Subject matter:* • The algebra of vectors in three dimensions [4 hours] • Vector and Cartesian equations [10 hours] • Systems of linear equations [6 hours] • Applications of matrices [7 hours] • Vector calculus [5 hours] **Topic 3: Complex numbers 2** *Subject matter:* • Cartesian forms [4 hours] • Complex arithmetic using polar form [3 hours] • The complex plane (the Argand plane) [2 hours] • Roots of complex numbers [3 hours] • Factorisation of polynomials [4 hours]	**Topic 1: Integration and applications of integration** *Subject matter:* • Integration techniques [10 hours] • Applications of integral calculus [9 hours] **Topic 2: Rates of change and differential equations** *Subject matter:* • Rates of change [10 hours] • Modelling motion [10 hours] **Topic 3: Statistical inference** *Subject matter:* • Sample means [8 hours] • Confidence intervals for means [8 hours]

Table 1 Each unit is developed to a notional (i.e. estimated) time of 55 hours of teaching and learning, including assessment. Notional times for each sub-topic are also provided.

Modified from *Specialist Mathematics General Senior Syllabus 2019 v1.2*, © State of Queensland (QCAA) 2018, licensed under CC BY 4.0

UNDERSTANDING THE QCE SPECIALIST MATHEMATICS UNITS 3 & 4 ASSESSMENT STRUCTURE

You will be expected to complete a total of ***four* summative assessments** in QCE Specialist Mathematics Units 3 & 4. Summative assessments are designed to evaluate your understanding of the subject matter and compare your performance against the understanding of students from across the state.

Three of these assessments are **internal** and one is **external**, but all will contribute to your Australian Tertiary Admission Rank (ATAR) calculation and to your Queensland Certificate of Education (QCE).

INTERNAL ASSESSMENTS

- Schools will develop ***three* internal assessments** for QCE Specialist Mathematics, based on the subject matter described in Units 3 & 4 of the syllabus.


I'D NEVER CALL YOU **AVERAGE**... THAT WOULD BE **MEAN**! ANYWAY, THERE'S NO TIME FOR FIGHTING, YOU'VE GOT SOME REVISION TO GET ON WITH! LET'S GO!

CHAPTER

2 REVISION

In this chapter, we provide a clear, concise summary of all examinable content from QCE Specialist Mathematics Units 3 & 4 to help you revise and prepare for the external assessment.

Everything has been organised by Unit, Topic and Sub-topic in the General Senior Syllabus to help you focus your time and attention where it is needed most.

The revision notes are not designed to replace your teacher or your textbook. Instead, they have been designed to help you gauge your level of understanding and confidence of the subject matter before the exam. You can use them to identify those topics you know inside out and those that still require some extra attention.

The revision notes are also supported by a bunch of handy features, tips and icons designed to help you get the very best result on the day. Here's an overview of what's covered:

2.1 UNIT 3 TOPIC 1 – PROOF BY MATHEMATICAL INDUCTION

MATHEMATICAL INDUCTION

Questions on pages 106–110

SUBJECT MATTER

Before the external assessment, you should be able to:

- understand the nature of inductive proof including the 'initial statement' and inductive step
- prove results for sums for any positive integer n.
- prove divisibility results for any positive integer n.

INDUCTION PROOFS

KEY CONCEPT

- A **mathematical statement** is a sentence or equation that must be either true or false.
- Sentences or equations that are quantified 'for all positive integers' are typically represented with capital letters, e.g. $P(n)$
- Some **quantifiers** commonly used in proofs are:
  - 'for all positive integers': $\forall n \in Z^+$
  - 'for all negative integers': $\forall n \in Z^-$
  - 'for all integers': $\forall n \in Z$
  - 'for all real numbers': $\forall n \in R$

Study tip

All proofs that use the principle of mathematical induction have the same key steps. Learn these steps and ensure that every time you practise a question involving mathematical induction, set out your response in the same way. This will make the process feel more automatic. Write out the structure and then fill it in.

The principal of mathematical induction is similar to 'climbing a ladder' and 'the domino effect'. When we push the first domino, all consecutive dominos will also fall. Similarly, for proofs by induction, when the initial case, $P(1)$, is true, it can be shown that $P(2)$, $P(3)$, $P(4)$... $P(n)$ will also be true.

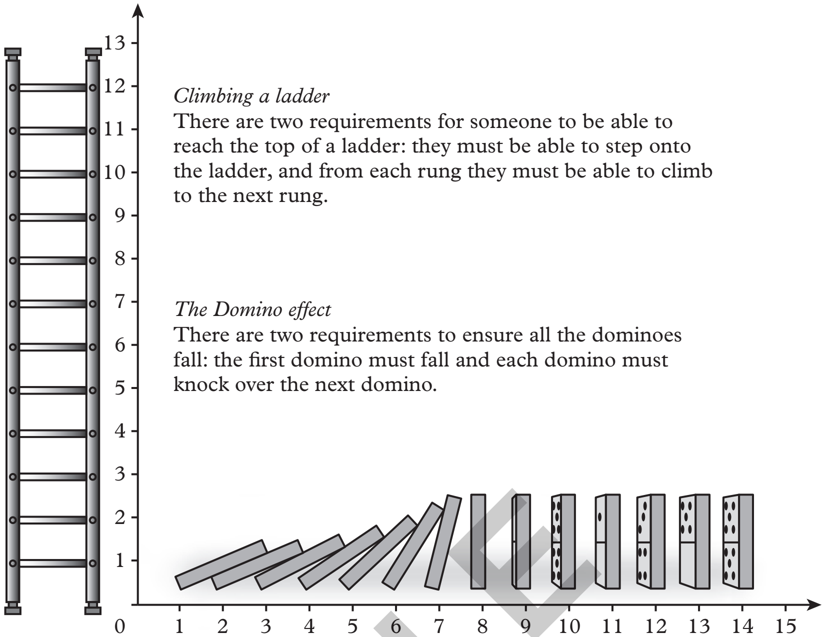


Figure 1 Mathematical induction can be illustrated by imagining climbing a ladder or knocking over a set of dominos.

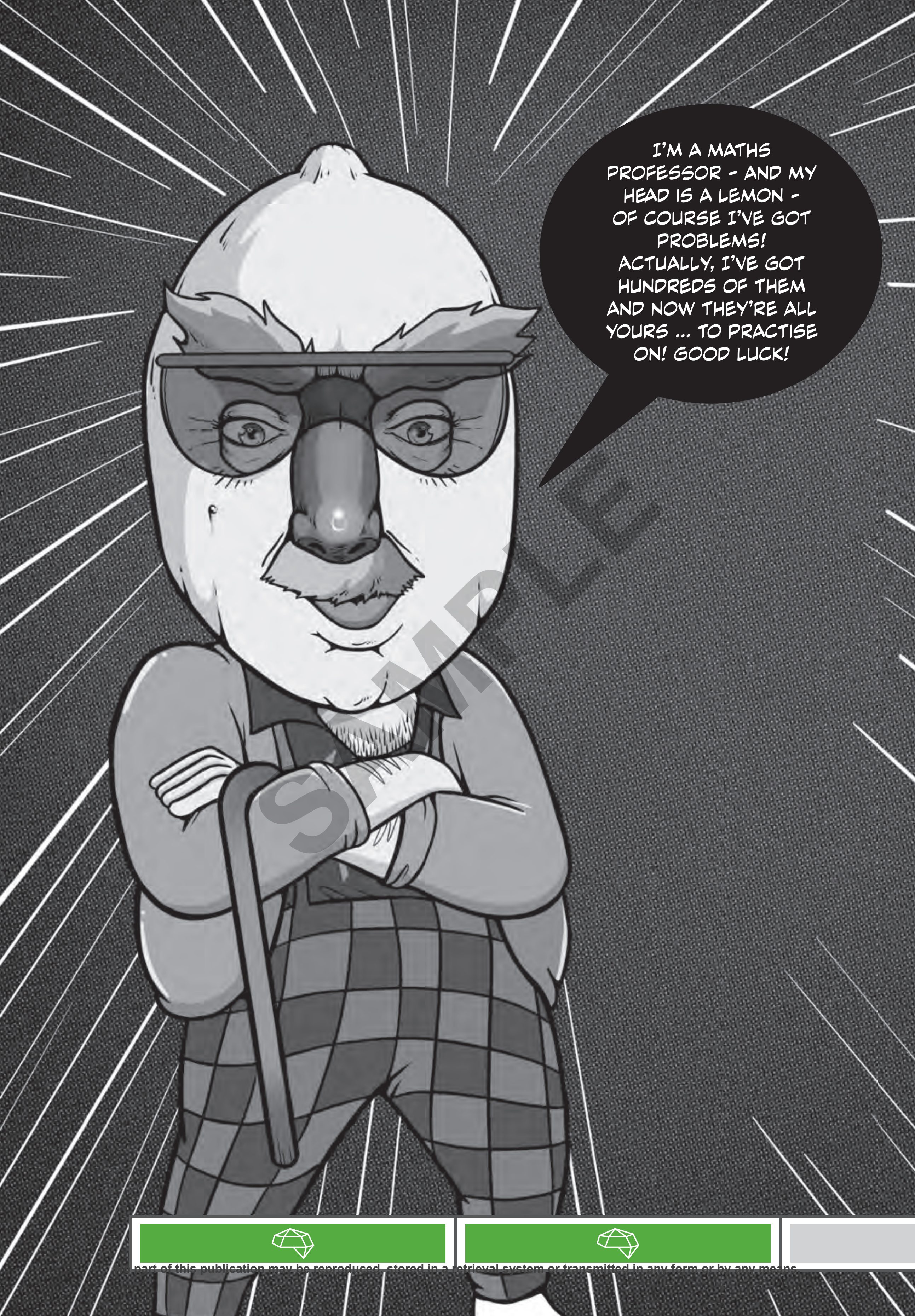
THE NATURE OF INDUCTIVE PROOF

1 INTRODUCTION

Remember that it is important to read every exam question carefully. For a question involving mathematical proof, you need to identify precisely what statement you are required to prove – and to clarify both its logical structure and quantification.

Let $P(n)$ be the given statement: $\underbrace{n(n+1) = 4m}_{\text{structure}}, \underbrace{\forall n \in N}_{\text{quantifier}}$

Start by writing out the identified statement, $P(n)$ or $S(n)$, and declare that you are proving the statement by mathematical induction. But don't just blindly copy from the question stem, make sure you understand what it is asking before moving on.


I'M A MATHS PROFESSOR - AND MY HEAD IS A LEMON - OF COURSE I'VE GOT PROBLEMS! ACTUALLY, I'VE GOT HUNDREDS OF THEM AND NOW THEY'RE ALL YOURS ... TO PRACTISE ON! GOOD LUCK!

CHAPTER

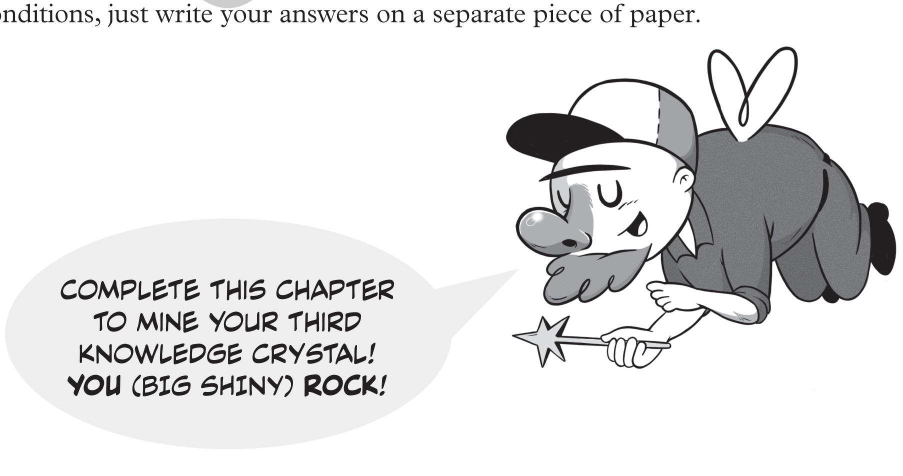
3 PRACTICE QUESTIONS

In this chapter, we provide a range of practice questions for all of the examinable content from QCE Specialist Mathematics Units 3 & 4. What a surprise… not! I have a sneaking suspicion the title gave it away!

No fancy tricks here, we just provide over 300 questions organised by Unit and Topic so you can move between revision and practice as you study. This will help you get the practice you need and build up your confidence!

To help you direct your time and effort where it's needed most, we've grouped the questions by type. Multiple choice and short response questions are also clearly labelled **Technology free** or **Technology active** so that you can prepare for Paper 1 and Paper 2 effectively.

You'll notice that we've provided a small amount of space under each question for you to jot down your answers or do some working out. In most cases it won't be as much space as you'll be given on the exam itself, but we know you wouldn't want to waste your money on a book full of empty pages. You're here for the questions, so that's what we've given you. If you want to practise under exam conditions, just write your answers on a separate piece of paper.



3.1 UNIT 3 TOPIC 1 – PROOF BY MATHEMATICAL INDUCTION

TECHNOLOGY FREE

MULTIPLE CHOICE QUESTIONS

Answers on page 240

QUESTION 1

If the sum $\sum_{r=1}^{n} n^{r}$ appears in the statement $P(n)$, what sum would appear in the inductive step of a proof by induction?

(A) $k + k^2 + \ldots + k^k + k^{k+1}$

(B) $1 + k + k^2 + \ldots + k^k + k^{k+1}$

(C) $(k+1) + (k+1)^2 + \ldots + (k+1)^{k+1}$

(D) $1 + (k+1) + (k+1)^2 + \ldots + (k+1)^{k+1}$

QUESTION 2

If you were to prove that a statement $P(n)$ is true for all positive even values of n, which of these would be a valid process?

(A) Establish $P(1)$ then show that $P(k) \Rightarrow P(k+1)$

(B) Establish $P(2)$ then show that $P(k) \Rightarrow P(k+2)$

(C) Establish $P(1)$ then show that $P(k) \Rightarrow P(2k)$

(D) Establish $P(2)$ then show that $P(2k) \Rightarrow P(2k+1)$

QUESTION 3

For which of the following propositions can the intital statement be proven, given $n \in Z^+$.

(A) $n^3 + (n+1)^2 + (n+1)^3$ is divisible by 6

(B) $\sum_{1}^{n} n^2 = n^2(n+1)$

(C) $4^{2n+1} \times 5^{n+1} - 1$ is divisible by 13

(D) $\frac{1}{1+2+3} + \frac{1}{2+3+4} + \frac{1}{3+4+5} \ldots + \frac{1}{n+(n+1)+(n+2)} = \frac{n}{3(n+3)}$

QUESTION 4

To prove by induction that $P(n) = 7^n - 4$ is divisible by 3 for all positive integers $n \geq k$, what is the minimum value of k?

(A) 0

(B) 1

(C) 2

(D) 4

QUESTION 5

When proving the sum below, what is the value of the statement in the initial case?

$$\sum_{k=1}^{n} k(k+4) = \frac{1}{6}\left(k^{2}+k\right)(2k+13)$$

(A) 1
(B) 4
(C) 5
(D) 6

SHORT RESPONSE QUESTIONS

QUESTION 6 (6 marks)

Answers on pages 240–242

Use mathematical induction to prove $4^n + 5$ is divisible by 3, for all $n \in N$.

MY MARK /6

QUESTION 7 (6 marks)

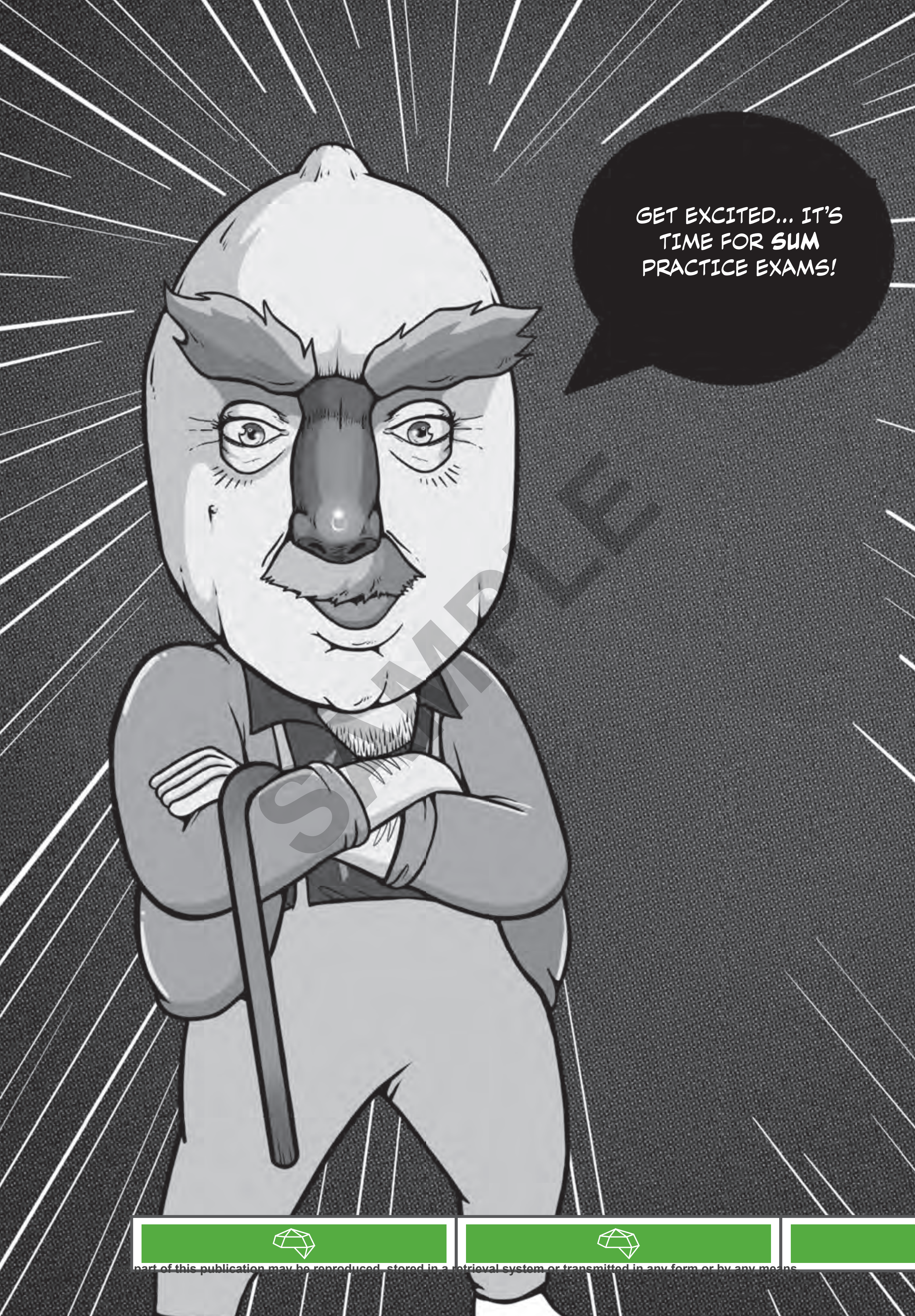
Prove by mathematical induction that
$-1 + 1 + 3 + 5 + \ldots + (2n - 3) = n^2 - 2n$, for all $n \in Z^+$.

MY MARK /6

QUESTION 8 (6 marks)

Use mathematical induction to prove $\sum_{r=1}^{n}(r-1) = \frac{n^2 - n}{2}$, for all $n \in Z^+$.

MY MARK /6


GET EXCITED... IT'S TIME FOR SUM PRACTICE EXAMS!

CHAPTER

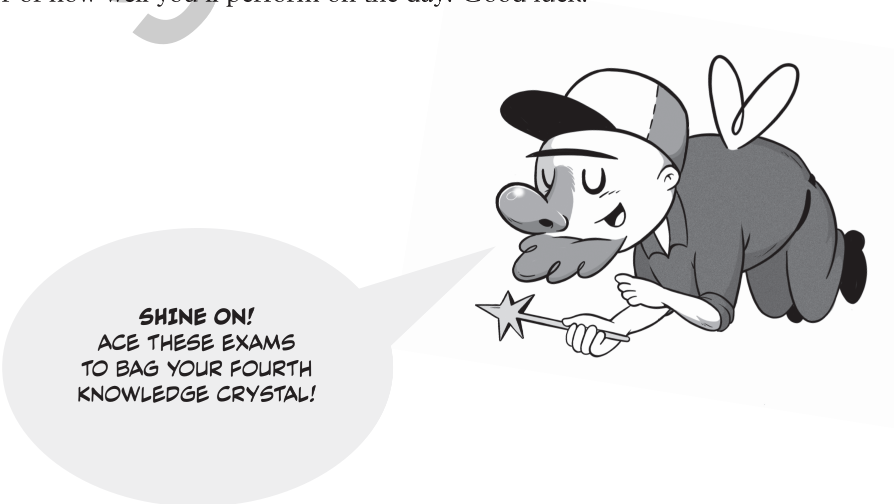
4 OFFICIAL PAST PAPERS

In this chapter, things get serious! It's now time for you to put your revision and practice to the test – literally – by completing the official QCE Specialist Mathematics External assessment from 2020!

We recommend you:

- don't look at this chapter until you've finished your revision and completed all of the practice questions in Chapter 3.
- complete these papers under exam conditions (i.e. follow the instructions regarding perusal time and working time, and don't refer to any notes or other materials that will not be allowed during the real exams)
- refer to the answers in Chapter 5 and use the marking advice to self-assess your responses once you've finished.

Remember… these are the QCE Specialist Mathematics papers from 2020, so – if you complete them under exam conditions – they are arguably the best indicator of how well you'll perform on the day! Good luck!



4.1 EXTERNAL ASSESSMENT 2020: SPECIALIST MATHEMATICS PAPER 1 (TECHNOLOGY FREE)

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are **not** permitted.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

Section 1 (10 marks)

Answers
on pages 306–310

QUESTION 1

The indefinite integral $\int \frac{3x - A}{1 - x^2}\,dx$ can be determined using the partial fractions $\frac{-1}{1 + x} + \frac{2}{1 - x}$.

The value of A is

(A) –3
(B) –1
(C) 1
(D) 3

QUESTION 2

When using proof by mathematical induction to show that $n(2n-1)(2n+1)$ is divisible by 3 $\forall\, n \in Z^+$, the inductive step requires proving

(A) $(k+1)(2k)(2k+2)$ is divisible by 3.
(B) $(k+1)(2k)(2k+3)$ is divisible by 3.
(C) $(k+1)(2k+1)(2k+2)$ is divisible by 3.
(D) $(k+1)(2k+1)(2k+3)$ is divisible by 3.

QUESTION 3

According to a recent census, the mean hours worked per week by all Australian workers is 35.6 hours.

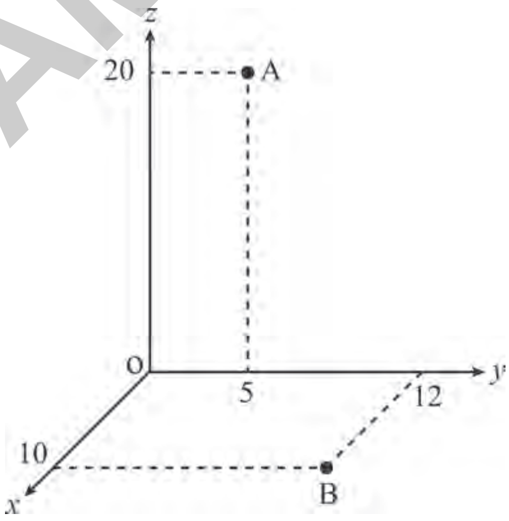
A mean of 36.1 hours worked per week is calculated from a random selection of 500 Australian workers.

Based on this data, which of the following is correct?

(A) $\bar{x} = 35.6, \mu = 36.1$
(B) $\bar{x} = 35.6, \overline{X} = 36.1$
(C) $\bar{x} = 36.1, \mu = 35.6$
(D) $\bar{x} = 36.1, \overline{X} = 35.6$

QUESTION 4

Consider points A and B as shown.




THERE ARE THREE KINDS OF PEOPLE IN THE WORLD: THOSE WHO CAN COUNT AND THOSE WHO CAN'T.
SOMETHING TO PONDER AS YOU CHECK ALL OF YOUR ANSWERS!

CHAPTER

5 ANSWERS

OMG, another cliffhanger… what on Earth could be in this chapter I wonder? You guessed it; in this chapter we provide the answers to absolutely everything! Sounds simple, I know, but to get the most out of this chapter, don't just cast an eye over the answers provided and move on.

If you really want to increase your chances of excelling on the exam, we recommend you look carefully over each of your answers in Chapters 3 and 4 and compare them with the answers in this chapter. Use the 'My mark' box under each short response question to self-assess your own answers. This will help you to get you into the habit of structuring your responses in order to receive maximum marks and to show you what the exam marker will be looking for.

Notice to students

The answers and marking advice provided in this chapter are provided for practice purposes only. Unless specifically credited, the QCAA has not written this material and does not endorse the content.

5.1 UNIT 3 TOPIC 1 – PROOF BY MATHEMATICAL INDUCTION

TECHNOLOGY FREE

MULTIPLE CHOICE ANSWERS

Question	Correct answer	Explanation
QUESTION 1	C	In the inductive step of a proof by mathematical induction, we aim to show that $P(k) \Rightarrow P(k+1)$. So, both the k^{th} and $(k+1)^{th}$ statement would appear. In the $(k+1)^{th}$ statement, all of the n terms are replaced by $(k+1)$. Given we have a sum of powers of n, we are looking for a sum of powers of $(k+1)$. Given the first power in the series is $r = 1$, the first term is $(k+1)$. Hence C. Note: If the first power was $r = 0$, then the first term would be 1, which is option D.
QUESTION 2	B	To prove the statement is true for positive even values of n, the initial value would be 2. This narrows our options to B or D. If k is even, then $k + 2$ would be the next even number. Since $2k$ is even, then $2k + 1$ would be odd. Hence B.
QUESTION 3	C	To determine which is true, substitute $n = 1$ as we are looking for positive integer values of n. A: $1^3 + (2)^2 + (2)^3 = 1 + 4 + 8 = 13 \neq 6m$ B: $1^2 = 1 \neq 1^2(2) = 2$ C: $4^3 \times 5^2 - 1 = 64 \times 25 - 1 = 1599 = 13 \times 123$ D: $\frac{1}{1+2+3} = \frac{1}{6} \neq \frac{1}{3(1-3)} = -\frac{1}{6}$ Hence C.
QUESTION 4	A	When $n = 0$, $7^0 - 4 = -3$, which is divisible by 3. When $n = 1$, $7^1 - 4 = 3$, which is divisible by 3. Hence A.
QUESTION 5	C	We are given that the series starts at $k = 1$. Hence, the value of the initial statement can be evaluated by substituting $k = 1$. $1(1 + 4) = 5$ Hence C.

- *1 mark* for each correct multiple choice answer.

SHORT RESPONSE ANSWERS

QUESTION 6 (6 marks)

Let $S(n)$ be the given statement

$\forall n \in N, \exists a \in N, 4^n + 5 = 3a$

Proof by mathematical induction:

<u>Initial statement</u>

Let $n = 1$ because it is the smallest positive integer.

$$\begin{aligned} LHS &= 4^1 + 5 \\ &= 9 \\ &= 3a \qquad \ldots\text{for } a = 3 \\ &= RHS \end{aligned}$$

Therefore, the initial statement $S(1)$ is true.

Inductive step

Assume $S(k)$ is true for some $n = k$.

$4^k + 5 = 3A, A \in N$

Let $n = k + 1$

RTP : $4^{k+1} + 5 = 3b, b \in N$

$$\begin{aligned} \text{LHS} &= 4^{k+1} + 5 \\ &= 4 \times 4^k + 5 \\ &= 3 \times 4^k + 4^k + 5 \\ &= 3 \times 4^k + 3A && \text{...using } S(k) \\ &= 3(4^k + A) \\ &= 3b && \text{...for } b = (4^k + A) \\ &= \text{RHS} \end{aligned}$$

Therefore, $S(k) \Rightarrow S(k+1)$.

Hence, by the principle of mathematical induction $S(n)$ is true $\forall n \in N$.

- *1 mark* for correctly proving the initial statement.
- *1 mark* for stating the assumption and the proof requirement for the inductive step.
- *3 marks* for proving the inductive step.
- *1 mark* for logical organisation communicating key steps.

QUESTION 7 (6 marks)

Let $S(n)$ be the given statement

$-1 + 1 + 3 + 5 + \ldots + (2n - 3) = n^2 - 2n$.

Proof by mathematical induction:

Initial statement

Let $n = 1$

$$\begin{aligned} \text{LHS} &= -1 \\ &= 1 - 2 \\ &= 1^2 - 2(1) \\ &= \text{RHS} \end{aligned}$$

Therefore, the initial statement $S(1)$ is true.

Inductive step

Assume $S(k)$ is true for $k \in N$.

$-1 + 1 + 3 + 5 + \ldots + (2k - 3) = k^2 - 2k$

Let $n = k + 1$

RTP : $-1 + 1 + 3 + 5 + \ldots + (2(k+1) - 3)$
$= (k+1)^2 - 2(k+1)$

LHS

$$\begin{aligned} &= -1 + 1 + 3 + 5 + \ldots + (2k - 3) + (2(k+1) - 3) \\ &= k^2 - 2k + (2(k+1) - 3) && \text{...using } S(k) \\ &= k^2 - 2k + 2k - 1 \\ &= k^2 + 2k - 2k - 1 \\ &= k^2 + 2k + 1 - 1 - 2k - 1 \\ &= (k+1)^2 - 2k - 2 \\ &= (k+1)^2 - 2(k+1) \\ &= \text{RHS} \end{aligned}$$

Therefore, $S(k) \Rightarrow S(k+1)$

Hence, by the principle of mathematical induction $S(n)$ is true $\forall n \in Z^+$.

- *1 mark* for correctly proving the initial statement.
- *1 mark* for stating the assumption and the proof requirement for the inductive step.
- *3 marks* for proving the inductive step.
- *1 mark* for logical organisation, communicating key steps.

QUESTION 8 (6 marks)

Let $S(n)$ be the given statement

$$\sum_{r=1}^{n}(r-1) = \frac{n^2 - n}{2}.$$

Proof by mathematical induction:

Initial statement

Let $n = 1$.

$$\begin{aligned} \text{LHS} &= (1 - 1) \\ &= 0 \\ &= \frac{1^2 - 1}{2} \\ &= \text{RHS} \end{aligned}$$

Therefore, the initial statement $S(1)$ is true.

Inductive step

Assume $S(k)$ is true for $k \in N$.

$$\sum_{r=1}^{k}(r-1) = \frac{k^2 - k}{2}$$

Let $n = k + 1$.

RTP : $\displaystyle\sum_{r=1}^{k+1}(r-1) = \frac{(k+1)^2 - (k+1)}{2}$

$$\begin{aligned} \text{LHS} &= \sum_{r=1}^{k+1}(r-1) \\ &= \sum_{r=1}^{k}(r-1) + (k + 1 - 1) \\ &= \frac{k^2 - k}{2} + (k + 1 - 1) && \text{...using } S(k) \\ &= \frac{k^2 - k}{2} + \frac{2k}{2} \\ &= \frac{k^2 + k}{2} \\ &= \frac{k^2 + (2k - k) + (1 - 1)}{2} \\ &= \frac{k^2 + 2k + 1 - k - 1}{2} \\ &= \frac{(k+1)^2 - (k+1)}{2} \\ &= \text{RHS} \end{aligned}$$

Therefore, $S(k) \Rightarrow S(k+1)$.

Hence, by the principle of mathematical induction $S(n)$ is true $\forall n \in Z^+$.


YOU'VE REACHED THE PART OF THE BOOK MOST LIKELY TO BE USELESS OR BURST... THE APPENDIX! LUCKY FOR YOU, THIS APPENDIX ISN'T USELESS AT ALL. IT CONTAINS ALL THE GOOD STUFF THAT WILL HELP YOU ACE YOUR EXAM!

APPENDIX

SPECIALIST MATHEMATICS

FORMULA SHEET

The QCAA has developed a formula sheet that will be provided for you to use to you during both examination papers. It contains a selection of useful formulae for you to refer to during the exam.

We want you to have everything you need in the one spot so that you can study effectively with this book whenever and wherever you are – on the bus, in the bath… anywhere! For that reason we've included the formula sheet here too. Shucks, that's what buddies are for!

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x)\,dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x)\,dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x)\,dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)\,g(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
volume of a solid of revolution	about the x-axis	$V = \pi\int_a^b [f(x)]^2\,dx$
	about the y-axis	$V = \pi\int_a^b [f(y)]^2\,dy$
Simpson's rule	$\int_a^b f(x)\,dx \approx \frac{w}{3}[f(x_0) + 4[f(x_1) + f(x_3) + \cdots] + 2[f(x_2) + f(x_4) + \cdots] + f(x_n)]$	
simple harmonic motion	If $\frac{d^2x}{dt^2} = -\omega^2 x$ then $x = A\sin(\omega t + \alpha)$ or $x = A\cos(\omega t + \beta)$	
	$v^2 = \omega^2(A^2 - x^2)$; $T = \frac{2\pi}{\omega}$	$f = \frac{1}{T}$
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,\mathrm{cis}(\theta)$
modulus	$\lvert z\rvert = r = \sqrt{x^2 + y^2}$
argument	$\arg(z) = \theta,\ \tan(\theta) = \frac{y}{x}, -\pi < \theta \le \pi$
product	$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\mathrm{cis}(\theta_1 - \theta_2)$
De Moivre's theorem	$z^n = r^n\mathrm{cis}(n\theta)$