SAMPLE CHAPTER

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OXFORD TEACHER REFERENCE

BUILDING ENGAGEMENT IN MIDDLE YEARS MATHEMATICS

LEARNING SEQUENCES FOR MIXED-ABILITY CLASSROOMS

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BUILDING ENGAGEMENT
IN MIDDLE YEARS MATHEMATICS

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CONTENTS

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CHAPTER 6

DECIMAL PLACE VALUE

OVERVIEW

The focus of this sequence is on developing decimal ideas and extending students' understanding of place value and fractions.

The following aspects aim to extend your usual approaches for teaching decimals. The tasks have both low floors and high ceilings, with some enabling and extending prompts suggested. For students who find these tasks straightforward, encourage them to form generalisations about the relationships between fractions, decimals and place value.

Note the critical importance of the sequence and the follow-up consolidating tasks within each aspect. The aspects are illustrative and it is assumed you will adapt these to your own classroom (while preserving the nature of the challenge). Further, all tasks allow students the opportunity to explain their thinking.

Some common mistakes or difficulties that students experience with learning decimals are from lack of conceptual understanding and include the following:

- Some students compare decimal numbers to whole numbers with the assumption that more digits means a higher number (that is, they think that 3.456 is more than 3.56).
- Some students judge decimal numbers based on their names, believing that thousandths are worth more than tenths (that is, they think 2.4 is more than 2.546).
- Some students align the right-most digit when adding and subtracting (such as when they are adding 2.4 + 2.37).
- Some students misinterpret the ways that decimal-like notations are used in practical situations such as cricket scores, time, library references.
- Some students do not connect decimals with fractions (not realising that 0.5 and $\frac{1}{2}$ are the same).
- Some students are confused by trailing zeros and place-holder zeros (that 7.50 is the same as 7.5, and that in 7.05 and 70.5 the zeros are important).

This sequence helps to address some of these misconceptions and provide opportunities to build conceptual understanding related to decimal fractions.

The parts of this learning sequence are:

- decimal place value
- addition and subtraction with decimal numbers
- word problems with decimal numbers.

THE AUSTRALIAN CURRICULUM

This sequence addresses the following descriptors from the Australian Curriculum: Mathematics.

Year	Content descriptors
5	Recognise that the place value system can be extended beyond hundredths (ACMNA104). Compare, order and represent decimals (ACMNA105).
6	Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (ACMNA128). Explore and practise efficient methods for solving problems requiring operations on decimals, to gain fluency with calculating with decimals and with recognising appropriate operations (ACMNA128).
7	Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154). Round decimals to a specified number of decimal places (ACMNA156). Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157).

The launch of any task includes posing and clarifying the language and representation of the task. There are many counting games that can be played as part of lesson introduction, such as counting sequences ascending and descending by 0.5, 0.25, etc. from whole number starting points.

You could also pose a variety of practice tasks using practical contexts like the following: I have a drink bottle that holds more than one of these and less than the other. How much might it hold?

ASSESSMENT

Students can be said to be using decimal place value when they can:

- create and describe patterns in numbers using tenths and hundredths
- read, recognise and place decimal numbers on a number line
- explain why 0.25 is between 0.2 and 0.3
- show how fractions and decimals are connected.

DECIMAL PLACE VALUE

INTRODUCTION

The idea is for students to engage with the tasks and learn about place value from discussing strategies (rather than starting by listening to explanations). The intention is that students learn about decimal place value from solving problems rather than from being told what to do. The tasks allow you to emphasise the value of the places, and the influence of the place on the number.

There is a variety of different representations (including a decimal number jigsaw that you may need to make, or the students could make their own) with the intention being to connect the corresponding representations.

A possible learning intention for this aspect:

We can compare and order decimal numbers and see patterns in decimals.

Relevant mathematical language includes:

decimals, tenths, hundredths, thousandths, ones, more, less, between, even, number line, parts of the whole, sequence, place value, rounded, digits.

The first task is as follows:

NUMBERS BETWEEN

Write down 15 numbers between 100 and 101.

Comment

The intent is to push students to go beyond tenths.

Enabling prompt

Write down five numbers between 10 and 11.

Extending prompt

Space your 15 numbers evenly on a number line.

FURTHER TASKS

NUMBERS BETWEEN

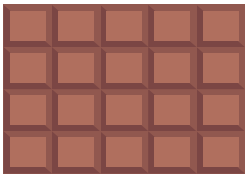
Write down 15 numbers between 104.2 and 104.5.

DECIMAL PARTS OF THE CHOCOLATE – TASK 1

Three friends are going to share all of this chocolate, but they will each receive different parts of the whole. Describe, using decimals, what parts of the chocolate each of them might get. Give as many different possible answers as you can.

DECIMAL PARTS OF THE CHOCOLATE – TASK 2

Five friends are going to share all of this chocolate, but they will each receive different parts of the whole. Describe, using decimals, what parts of the chocolate each of them might get. Give as many different possible answers as you can.



DECIMAL JIGSAW PIECE – TASK 1

This jigsaw piece fits into the larger puzzle. I know that one of the numbers on this jigsaw piece is 0.78. What might be the other numbers on this piece?

DECIMAL JIGSAW PIECE – TASK 2

The numbers 0.48 and 0.6 are on the same jigsaw piece. The piece is shaped like a letter of the alphabet. What might be the other numbers on the piece? (Give two different possible answers.)

DECIMAL BETWEEN – TASK 1

2.45 is halfway between two other numbers.

The two numbers look like __ . __

(They have units and tenths but no hundredths.) Give some different possible answers.

DECIMAL BETWEEN – TASK 2

12.65 is halfway between two other numbers.

The two numbers look like __ __ . __

(They have tens, units and tenths but no hundredths.) The units digit is not 2. Give some different possible answers.

DECIMAL SEQUENCES – TASK 1

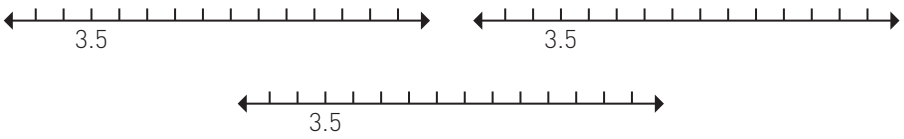
4.5 and 6 are part of a sequence. What might be the sequence? What is the 10th term in your sequence? Give some different possible answers.

DECIMAL SEQUENCES – TASK 2

12.25 and 13 are part of a sequence. What might be the sequence? What is the 10th term in your sequence? Give some different possible answers.

DECIMAL NUMBERS ON A LINE – TASK 1

The number 3.5 is marked on the number line. Where might the number 4 be on the line? Give three different possible answers. In each case, label the other marked points on the number line.



DECIMAL NUMBERS ON A LINE – TASK 2

On the number line, mark in the numbers 4 and 5.5. Label all the other marked points as well. Give two different possible answers.

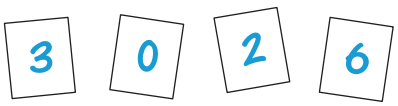
INEQUALITIES – TASK 1

Where might you place the four numbered cards in this inequation? __.__ __ < 3.__

How many different possible answers are there?

INEQUALITIES – TASK 2

Where might you place the four numbered cards in this inequation? 3.__ __ < __.__



How many different possible answers are there?

DECIMAL ROUNDING – TASK 1

A number has been rounded off to 5.8. What might be the number?

DECIMAL ROUNDING – TASK 2

A number has been rounded off to 15.25. What might be the number?

ADDITION AND SUBTRACTION WITH DECIMAL NUMBERS

INTRODUCTION

The tasks generally use different numbers of places to emphasise decisions about which digits should be used together. Often there are multiple possible answers, so let students explore the tasks prior to any instruction (which can come later). It is best for students to find rules for themselves (such as 'align tenths with tenths').

A possible learning intention for this aspect:

With addition and subtraction, it is important to add (and subtract) units with units, tenths with tenths, etc.

Relevant mathematical language includes:

decimals, tenths, hundredths, thousandths, ones, more, less, between, even, number line, parts of the whole, sequence, place value, rounded, digits.

The first task is as follows:

ADDING DECIMALS

Work out the answer to this problem in two different ways.

4.5 + 4.5 + 4.5 + 4.5 + 4.5 + 4.5 =

Comment

There are many legitimate ways of doing this so encourage all suggestions.

Enabling prompt

Work out the answer to 4.5 + 4.5 =

Extending prompt

What is the easiest way to do this? Why is that the easiest?

FURTHER TASKS

ADDING DECIMALS

Work out the answer to this problem in two different ways.

10.25 + 10.25 + 10.25 + 10.25 + 10.25 + 10.25 =

SUBTRACTING DECIMALS – TASK 1

Work out the answer to this problem in two different ways.

20 – 2.25 – 2.25 – 2.25 – 2.25 – 2.25 =

SUBTRACTING DECIMALS – TASK 2

Work out the answer to this problem in two different ways.

50 – 5.5 – 3.25 – 5.5 – 3.25 – 5.5 – 3.25 – 5.5 – 3.25 =

MISSING DECIMAL DIGITS – TASK 1

In this calculation, what might be the missing digits if they are all different?

8 = __.__ __ + __.__ __

Give as many possible answers as you can.

MISSING DECIMAL DIGITS – TASK 2

In this calculation, what might be the missing digits if they are all different?

6 = __.__ __ + __.0__

Give as many possible answers as you can.

MISSING DECIMAL DIGITS – TASK 3

In this calculation, what might be the missing digits if they are all different?

__.__ __ = 8 – __.__ __

Give as many possible answers as you can.

MISSING DECIMAL DIGITS – TASK 4

In this calculation, what might be the missing digits if they are all different?

__.__ __ = 6 – __.0__

Give as many possible answers as you can.

EQUIVALENT EQUATIONS WITH DECIMALS – TASK 1

These four equations all say the same thing.

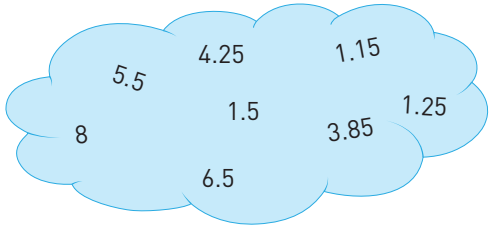
3.5 + 2.5 = 6

2.5 + 3.5 = 6

6 – 2.5 = 3.5

6 – 3.5 = 2.5

Choose three numbers from the cloud and write four equations that all say the same thing.



Note: Students can be encouraged to create their own clouds.

EQUIVALENT EQUATIONS WITH DECIMALS – TASK 2

These four equations all say the same thing.

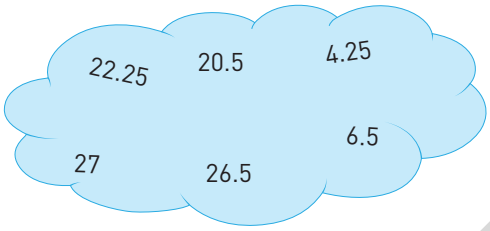
3.5 + 2.5 = 6

2.5 + 3.5 = 6

6 – 2.5 = 3.5

6 – 3.5 = 2.5

Choose three numbers from the cloud and write four equations that all say the same thing.



WORD PROBLEMS WITH DECIMAL NUMBERS

INTRODUCTION

Most of these tasks are closed, but the same pedagogies apply. That is, let the students explore the tasks prior to giving any instruction. If students complete a question, ask them to solve the question a different way. You may need to emphasise strategies that help them interpret the question (such as 'draw a diagram'). In all tasks, encourage the students to show their answer on a number line (or a bar diagram).

A possible learning intention for this aspect:

We can use knowledge of decimals to solve practical problems.

Relevant mathematical language includes:

perimeter, longer, length, width.

The first task is as follows:

PERIMETER OF A RECTANGLE

The perimeter of a rectangle is 10 cm. Neither the length nor the width are whole numbers and both are longer than 2 cm. What are some possible values for the length and the width?

Comment

Given that length plus width is 5 cm, and both dimensions are greater than 2 cm, the dimensions can be any pairs of the form 2.*(something)* + 2.*(something)* = 5. Examples include 2.1 and 2.9, 2.2 and 2.8.

Enabling prompt

The perimeter of a rectangle is 10 cm. What are some possible values for the length and the width?

Extending prompt

Write a sentence that describes all the possible answers. What is the pattern?

FURTHER TASKS

PERIMETER OF A RECTANGLE

The perimeter of a rectangle is 50 cm. Neither the length nor the width are whole numbers but both are longer than 11 cm. What are some possible values for the length and the width? (Give different options and say which you prefer.)

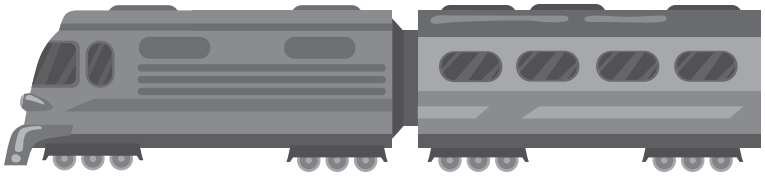
DECIMAL WORD PROBLEMS – TASK 1

In all these questions, the train (engine plus carriage) is 10 m long. Show your working on a number line.

1 If the train is 5.4 m longer than the carriage, how long is the engine?

2 If the train is 3.25 m longer than the carriage, how long is the engine?

3 If the engine is 1.7 m longer than the carriage, how long is the carriage?



DECIMAL WORD PROBLEMS – TASK 2

The perimeter of a rectangle is 12.8 cm. The length is 3.25 cm. What is the width?

DECIMAL WORD PROBLEMS – TASK 3

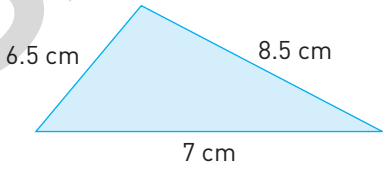
The perimeter of a rectangle is 25.4 cm. The width is 5.25 cm. What is the length?

DECIMAL WORD PROBLEMS – TASK 4

The perimeter of a rectangle is 20 cm. The length is 4.25 cm longer than the width. What are the width and the length?

EQUAL PERIMETERS – TASK 1

Use a single straight line to cut this triangle into two smaller triangles with the same perimeter.



EQUAL PERIMETERS – TASK 2

Use a single straight line to cut this triangle into two smaller triangles with the same perimeter.