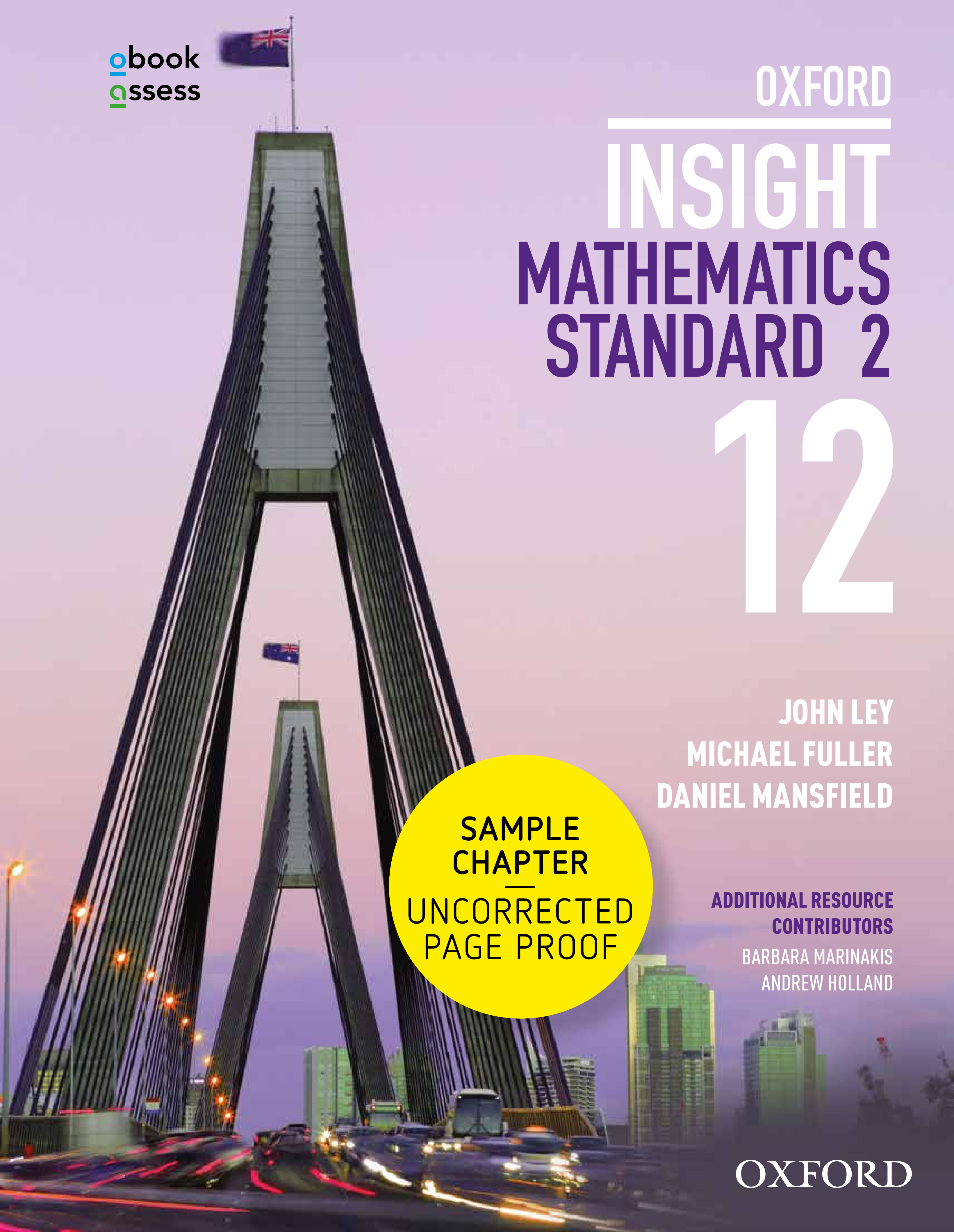

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INSIGHT MATHEMATICS STANDARD 2 12

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CONTENTS

4

Simultaneous linear equations

The main mathematical ideas in this chapter are:

- using and interpreting graphs of the form $y = mx + c$
- modelling linear relationships
- working with linear models and their graphical representation
- identifying the solution to simultaneous linear equations from tables and graphs
- solving simultaneous linear equations graphically
- solving practical problems using simultaneous linear equations
- conducting a break-even analysis.

ALGEBRA
MS-A4 Types of Relationships A4.1

ARE YOU READY?

4A **1** What is the value of y in the equation $y = 2x + 5$ when $x = -1$?

A -7 **B** -3
C 3 **D** 7

4A **2** Which table shows x and y values for $y = 3x - 4$?

A

x	2	3	4
y	2	6	9

B

x	1	3	5
y	−1	5	11

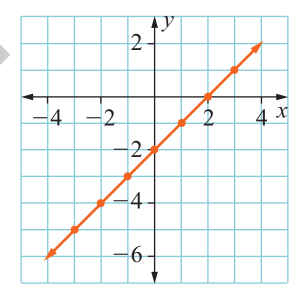
C

x	0	2	4
y	0	2	8

D

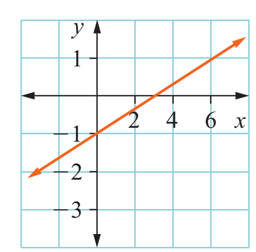
x	3	4	5
y	5	6	7

4A **3** Which equation produces the following graph?



A $y = x + 2$ **B** $y = 2x - 4$
C $y = 2 - x$ **D** $y = x - 2$

Use the following graph for questions 4, 5 and 6.



4A **4** What is the gradient of this graph?

A -1 **B** $\frac{1}{3}$
C $\frac{2}{3}$ **D** 3

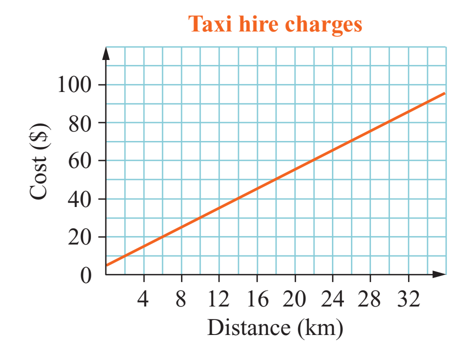
4A **5** What is the y-intercept of this graph?

A 2.5 **B** 3
C -1 **D** -3

4A **6** What is the equation of this graph?

A $y = \frac{1}{3}x - 1$ **B** $y = x + \frac{1}{3}$
C $y = \frac{1}{3}x + 1$ **D** $y = 3x - 1$

Use the following graph for questions 7, 8 and 9.



4B **7** What is the taxi hire charge for travelling 18 km?

A \$30 **B** \$40
C \$50 **D** \$60

4B **8** How far can you travel by taxi for \$70?

A 20 km **B** 22 km
C 24 km **D** 26 km

4B **9** Which equation best matches the graph of distance (d) and cost (C)?

A $C = 2.5d + 5$ **B** $C = 5d + 2.5$
C $d = 2.5C + 5$ **D** $C = 2.5d - 5$

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your obook assess.

Q1–2 **Support sheet 4A.1 Substituting for pronumerals**
Q3 **Support sheet 4A.2 Plotting points from a table of values**
Q4–6 **Support sheet 4A.3 Identifying features of a linear graph**
Q7–9 **Support sheet 4B.1 Reading values from graphs**

4A Graphs of the form $y = mx + c$

These resources are available on your obook assess:
- **Interactive 4A.1:** Explore key ideas for plotting linear graphs
- **Interactive 4A.2:** Explore gradient and intercepts of a linear graph
- **Interactive 4A.3:** Explore the drawing of graphs using gradient and y-intercept
- **Spreadsheet 4A.1:** Drawing linear graphs using a table of values
- **Spreadsheet 4A.2:** Drawing linear graphs using the gradient and y-intercept
- **assess quiz 4A:** Test your skills with an auto-correcting multiple-choice quiz

linear relationship relationship between two variables where the coordinate points describing this relationship lie in a straight line when plotted on a number plane (or Cartesian plane)

In the Year 11 Mathematics Standard course, you used straight-line graphs to model practical situations and make predictions based on the model. Graphical representations are excellent models of practical situations because they provide a visual way to view trends.

This section will revise the generation of tables of values and the graphing of a **linear relationship**. You can also explore the use of technology to produce straight-line graphs.

Example 4A–1 Completing a table of values to graph a linear relationship

a Complete this table of values for the equation $y = 5 - 2x$.

b Plot each set of ordered pairs from the table for $y = 5 - 2x$ on a number plane and draw a straight line through the points.

x	-3	-2	-1	0	1	2	3
y							

Solve

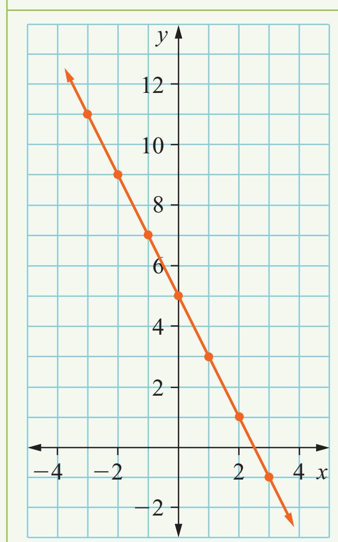
a

x	-3	-2	-1	0	1	2	3
y	11	9	7	5	3	1	-1

Think: Substitute each x-value into the equation $y = 5 - 2x$ to complete the table.

Apply: Substitute each x-value into the linear relationship to find the matching y-value.

b



Think: Plot the points on a number plane and draw a straight line through them, extending as far as possible in both directions. Include an arrowhead at each end of the line to indicate that the line continues.

Apply: Plot the points on a number plane (also called a Cartesian plane) and join them with a straight line. The line extends beyond the plotted points. The graph of a linear relationship is always a straight line.

Exercise 4A Graphs of the form $y = mx + c$

UNDERSTANDING FLUENCY AND COMMUNICATION

1 Complete the following tables using the equations provided. Plot each set of ordered pairs on separate number planes and draw a straight line through the points.

a $y = 2x + 1$

x	-2	-1	0	1	2
y					

b $y = 2x + 3$

x	-4	-2	0	2	4
y					

c $y = -3x - 4$

x	-4	-2	0	2	4
y					

d $y = 4 - 5x$

x	-2	-1	0	1	2
y					

e $y = 2x - 7$

x	-2	-1	0	1	2
y					

f $y = 5 - 2x$

x	-2	-1	0	1	2
y					

2 By using a table of values such as the one shown on the right, draw graphs of the following linear relationships on separate number planes.

x	-3	0	3
y			

a $y = x - 3$ **b** $y = x + 2$ **c** $y = 2x - 1$ **d** $y = 1 - 3x$

e $y = \frac{1}{2}x$ **f** $y = \frac{2x + 3}{2}$ **g** $y = \frac{2x}{3} - 6$ **h** $y = 4 - 2x$

3 By using a table of values, draw the graph of $y = \frac{x}{10} + 5$. Explain your choice of x-values to use in the table and the scale to use on the axes of the number plane.

! If the equation of a straight line is written in the form $y = mx + c$, the **gradient** of the line is m and the **y-intercept** is c.

Note: x is the **independent variable**, as any value may be used and y is the **dependent variable**, as it depends on the value of x.

gradient steepness of a line (defined as m) that can be measured using any two points on the line or interval using $m = \frac{\text{rise}}{\text{run}}$

y-intercept point where a line crosses the y-axis of a number plane (or Cartesian plane)

independent variable variable whose outcomes are not due to those of another variable; usually represented on the horizontal axis of a graph; e.g. for the linear relationship $y = 2x + 1$, the independent variable is x

dependent variable variable whose value depends on that of another; usually represented on the vertical axis of a graph; e.g. for the relationship $y = 2x + 1$, the dependent variable is y (as it depends on the value of x)

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Example 4A–2 Writing the gradient and y-intercept of a line from its equation

Write the gradient and y-intercept of the straight lines with the following equations.

a $y = 2x + 7$ **b** $y = -3x + 2$ **c** $y = \frac{2x}{3} - 5$

d $y = -x + 3.6$ **e** $y - \frac{x}{5} = \frac{4}{5}$ **f** $4x + 2y = 1$

	Solve	Think/Apply
a	Gradient $= 2$, y-intercept $= 7$	When the equation is in the form $y = mx + c$, the gradient is m, which is the coefficient of x (the number in front of x). The y-intercept is c (the constant term). If the equation is not in the form $y = mx + c$, it can be rearranged into this form. For part **e**, add $\frac{x}{5}$ to both sides of the equation. For part **f**, subtract $4x$ from both sides of the equation and then divide both sides by 2.
b	Gradient $= -3$, y-intercept $= 2$	
c	Gradient $= \frac{2}{3}$, y-intercept $= -5$	
d	Gradient $= -1$, y-intercept $= 3.6$	
e	$y - \frac{x}{5} = \frac{4}{5}$ becomes $y = \frac{x}{5} + \frac{4}{5}$ Gradient $= \frac{1}{5}$, y-intercept $= \frac{4}{5}$	
f	$4x + 2y = 1$ becomes $y = -2x + \frac{1}{2}$ Gradient $= -2$, y-intercept $= \frac{1}{2}$	

4 Write the gradient and y-intercept of the straight lines with the following equations.

a $y = 4x + 6$ **b** $y = -2x + 9$ **c** $y = \frac{3}{4}x - 8$

d $y = x - 2$ **e** $y = -x + 4$ **f** $y = -1.2x - 2.8$

g $y = 5.4x + 6.5$ **h** $y = -\frac{3x}{8} + \frac{9}{2}$ **i** $y = \frac{x}{2} + 7$

j $y = \frac{x}{6} + \frac{5}{3}$ **k** $y = -\frac{x}{3} - 2$ **l** $y - \frac{x}{2} = 2$

m $4x + y = 8$ **n** $3x + 2y = -4$ **o** $2x - y = 5$

Example 4A–3 Using the gradient and y-intercept to write the equation of a straight line

Write the equation of the straight line with:

a gradient 4 and y-intercept -3 **b** gradient -6 and y-intercept 1

c gradient $\frac{4}{5}$ and y-intercept -2 **d** gradient 1 and y-intercept 0.9

e gradient $\frac{1}{3}$ and y-intercept $2\frac{1}{4}$.

	Solve	Think	Apply
	The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y-intercept.	Substitute the values for m and c into $y = mx + c$.	Write the equation in the form $y = mx + c$ where m is the gradient and c is the y-intercept.
a	$y = 4x - 3$	$m = 4$, $c = -3$	
b	$y = -6x + 1$	$m = -6$, $c = 1$	
c	$y = \frac{4}{5}x - 2$	$m = \frac{4}{5}$, $c = -2$	
d	$y = x + 0.9$	$m = 1$, $c = 0.9$	
e	$y = \frac{1}{3}x + 2\frac{1}{4}$	$m = \frac{1}{3}$, $c = 2\frac{1}{4}$	

UNDERSTANDING FLUENCY AND COMMUNICATION

5 Write the equation of the straight line with:

a gradient 7 and y-intercept -1 **b** gradient -2 and y-intercept 5

c gradient $\frac{3}{7}$ and y-intercept -2 **d** gradient 1 and y-intercept 4

e gradient -1 and y-intercept 2.7 **f** gradient 0.2 and y-intercept 3

g gradient -1.4 and y-intercept -0.3 **h** gradient $\frac{5}{6}$ and y-intercept $\frac{7}{12}$

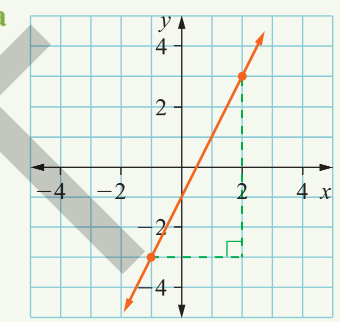
i gradient -0.56 and y-intercept -1.24 **j** gradient $1\frac{2}{5}$ and y-intercept 24.

Example 4A–4 Finding the gradient, y-intercept and equation from a linear graph

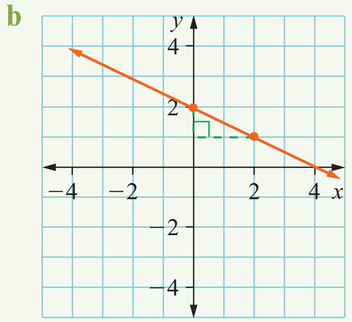
For the straight lines shown below, find the:

i gradient **ii** y-intercept **iii** equation of the line.

a



b

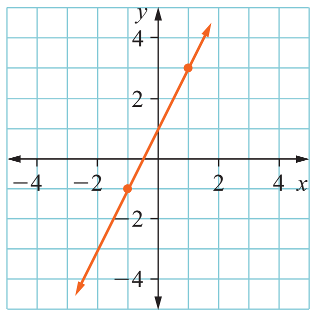


		Solve	Think	Apply
a	**i**	Gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{6}{3}$ $= 2$	Choose any two points on the line, say $(-1, -3)$ and $(2, 3)$, and draw a right-angled triangle as shown to find the gradient of the line. The vertical rise is 6 and the horizontal run is 3.	The working is easier if you can find points whose coordinates are whole numbers. Often using the points where the graph cuts an axis makes calculations easier.
	ii	y-intercept $= -1$	From the graph, the line cuts the y-axis at -1.	
	iii	The equation of the line is $y = 2x - 1$.	Use $y = mx + c$ and replace m with 2 and c with –1.	
b	**i**	Gradient $= \frac{\text{rise}}{\text{run}}$ $= -\frac{1}{2}$	Choose any two convenient points on the line, say (0, 2) and (2, 1), and draw a right-angled triangle as shown. The vertical rise is –1 and the horizontal run is 2.	The line is sloping ‘downhill’ so the gradient is negative.
	ii	y-intercept $= 2$	From the graph, the line cuts the y-axis at 2.	
	iii	The equation of the line is $y = -\frac{1}{2}x + 2$.	Use $y = mx + c$ and replace m with $-\frac{1}{2}$ and c with 2.	

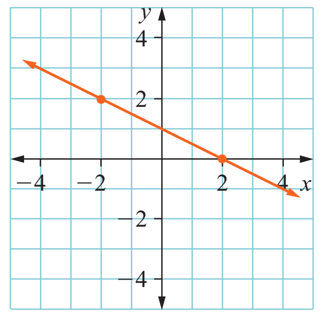
6 For the straight lines shown, find the:

i gradient **ii** y-intercept **iii** equation of the line.

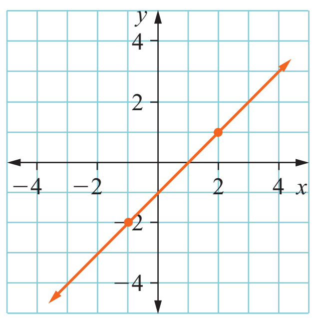
a



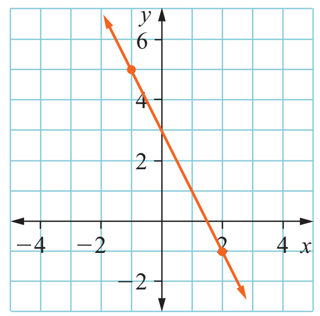
b



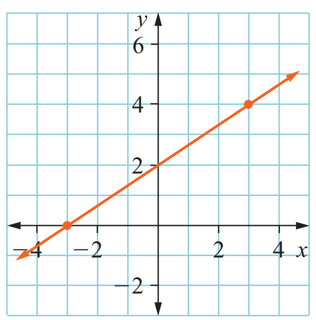
c



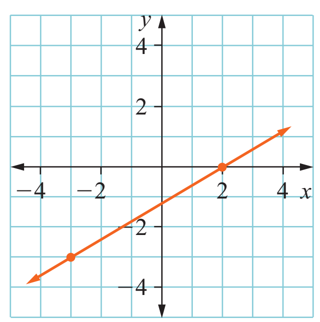
d



e



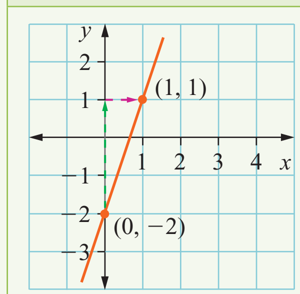
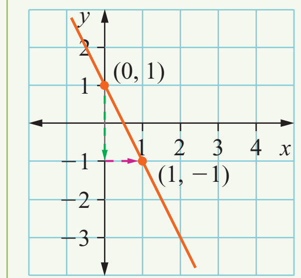
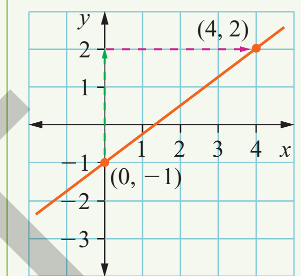
f



Example 4A–5 Drawing a linear graph using the gradient and y-intercept

Draw the following straight lines without using a table of values.

a $y = 3x - 2$ **b** $y = -2x + 1$ **c** $y = \frac{3x}{4} - 1$

	Solve	Think	Apply
a		Gradient $= 3$, y-intercept $= -2$. Start at the y-intercept $(0, -2)$. As the gradient is $\frac{3}{1}$, move up 3 units then right 1 unit from $(0, -2)$. This gives a second point $(1, 1)$ on the line. Rule a line through these two points.	Find the gradient and y-intercept of the line. Use $(0, c)$ as the first point on the line. Use Gradient $= \frac{\text{rise}}{\text{run}}$ to find a second point on the line. Plot the two points and rule a line through them.
b		Gradient $= -2$, y-intercept $= 1$. Start at the y-intercept $(0, 1)$. As the gradient is $-\frac{2}{1}$, move down 2 units then right 1 unit from $(0, 1)$. This gives a second point $(1, -1)$ on the line. Rule a line through these two points.	
c		Gradient $= \frac{3}{4}$, y-intercept $= -1$. Start at the y-intercept $(0, -1)$. As the gradient is $\frac{3}{4}$, move up 3 units then right 4 units from $(0, -1)$. This gives a second point $(4, 2)$ on the line. Rule a line through these two points.	

7 Draw each of the lines in question **4 a-f** without using a table of values.

8 **a** Use graphing technology to draw the graphs of these equations on the same set of axes.

i $y = 2x$ **ii** $y = 2x + 3$ **iii** $y = 2x - 2$ **iv** $y = 2x + 1$

b What do you notice about the graphs?

c What is the effect of the constant term?

9 **a** Use graphing technology to draw the graphs of these equations on the same set of axes.

i $y = x + 1$ **ii** $y = 2x + 1$ **iii** $y = 3x + 1$ **iv** $y = \frac{1}{2}x + 1$

b What do you notice about the graphs?

c What is the effect of changing the coefficient of x?

10 Use graphing technology to draw the graphs with the given equations in questions **1**, **2**. **3** and **4** and find specific values. Check the accuracy of your answers from the hand-drawn graphs with those obtained using technology.

11 Create a spreadsheet to produce the graph of a linear relationship from a table of values. Enter the values for x in the first column and values for y in the second column of the spreadsheet. Highlight the cells containing the x- and y-values, click **Insert** and **Charts**, and choose **Line graph**. Check some of your answers to questions **1** and **2** using your spreadsheet. You might like to use the prepared spreadsheet provided on your obook assess.

12 Create a spreadsheet to produce the graph of a linear relationship from its equation using the gradient and y-intercept. Check some of your answers to questions **1**, **2**, **3** and **4** using your spreadsheet. You might like to use the prepared spreadsheet provided on your obook assess.

production cost
cost (or expense) of producing an item for sale, which can involve both a fixed cost and a variable cost that depends on the number of items produced

revenue
amount of money (or income) received from the sale of items produced by a company, which can depend on the number of items sold

4B Linear models

These resources are available on your obook assess:
- **Video tutorial 4B:** Watch and listen to an explanation of Example 4B–1
- **Investigation 4B.1:** Model the height of water flowing into a water tank
- **Investigation 4B.2:** Model the temperature of an icy-pole placed in a freezer
- **assess quiz 4B:** Test your skills with an auto-correcting multiple-choice quiz

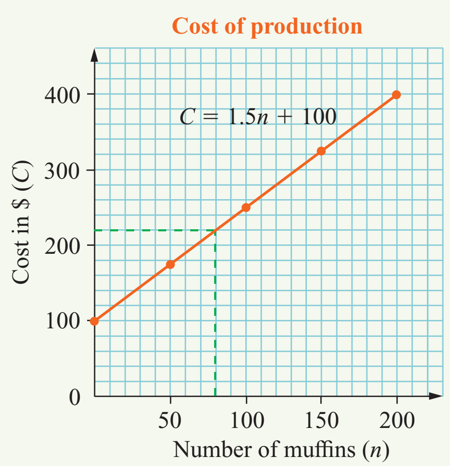
Linear relationships can be used to model practical situations. The linear equation used as a model will be of the form $y = mx + c$ where the gradient (m) and y-intercept (c) of the corresponding linear graph have practical meanings in each case. Usually the constant c is a fixed amount, and the gradient m is the rate of change. For example, a taxi fare can be calculated by adding a fixed cost to the charge per kilometre travelled.

In this section, we will also look at practical contexts involving **production cost** and **revenue**.

Example 4B–1 Drawing the graph of a cost model from its equation

The Magnificent Muffin Shop has a fixed cost of $100 per day and a variable production cost of $1.50 per muffin. The daily cost can be modelled using the equation $C = 1.5n + 100$.

a Explain each term in the equation.
b Draw the graph of $C = 1.5n + 100$ for $0 \le n \le 200$.
c Use the graph to find the number of muffins produced for $220.

<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>1.5 is the cost per muffin, 100 is the fixed cost and C is the total cost. All costs are in dollars.
n is the number of muffins made.</td><td>The gradient is the coefficient of n, which is 1.5. The constant term c is the fixed cost of 100.</td><td>The fixed cost is the intercept on the vertical axis. The variable cost relates to the gradient.</td></tr>
<tr><td>b</td><td>
</td><td>Use graphing technology or complete a table of values for n and C using C = 1.5n + 100, and plot the points to produce the graph.
<table><tr><td>n</td><td>0</td><td>50</td><td>100</td><td>150</td><td>200</td></tr><tr><td>C</td><td>100</td><td>175</td><td>250</td><td>325</td><td>400</td></tr></table></td><td>Draw the graph using technology or by completing a table of values and plotting the points. The gradient and y-intercept method of drawing a linear graph could also be used.</td></tr>
<tr><td>c</td><td>80 muffins are produced for $220.</td><td>Draw a line across from the vertical axis at 220 until it meets the line. Draw down to the horizontal axis and read the value of n.</td><td>Find values by drawing lines from one axis to the linear graph and then to the other axis.</td></tr>
</table>

UNDERSTANDING FLUENCY AND COMMUNICATION

Exercise 4B Linear models

1 The Supertight Tie Company has a fixed cost of $1200 per day and a variable production cost of $3 per tie. The daily cost can be modelled using the equation $C = 3n + 1200$.

a Explain each term in the equation $C = 3n + 1200$.
b Complete this table of values using the equation $C = 3n + 1200$.

n	0	50	100	150	200	250
C						

c Draw the graph of $C = 3n + 1200$.
d Use your graph from part **c** to find the number of ties produced when the daily cost is $1400.

2 The distance, d kilometres, travelled by a train in a time of t hours is $d = 80t$.

a Complete this table of values using the equation $d = 80t$.

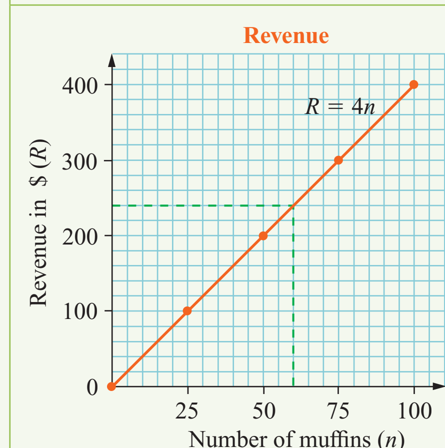
t	1	2	3	4	5
d					

b Draw the graph of $d = 80t$.
c Where does this graph cut the vertical axis? Why?
d What is the gradient of this graph? What does this value represent in this case?
e Use the graph to find how far away the train is after $3\frac{1}{2}$ hours.
f Use the graph to find when the train is 200 km away.

Example 4B–2 Drawing the graph of a revenue model from its equation

The Marvellous Muffin Shop sells its muffins for $4 each. The sales revenue can be modelled using the equation $R = 4n$.

a Explain each term in the equation $R = 4n$.
b Graph the straight line with equation $R = 4n$ for $0 \le n \le 100$.
c Use the graph to find the number of muffins that are sold if the revenue is $240.

<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>R is the revenue, n is the number of muffins sold.
The gradient is 4: this is the price per muffin in dollars.</td><td>There is no fixed amount so the graph starts at the origin. The gradient is the price of each muffin.</td><td>If there are no sales then the revenue is zero, so the origin is a point on the graph.</td></tr>
<tr><td>b</td><td>
</td><td>Use graphing technology or complete a table of values for n and R using R = 4n and plot the points to produce the graph.
<table><tr><td>n</td><td>0</td><td>25</td><td>50</td><td>75</td><td>100</td></tr><tr><td>R</td><td>0</td><td>100</td><td>200</td><td>300</td><td>400</td></tr></table></td><td>Draw the graph using technology or by completing a table of values and plotting the points.</td></tr>
</table>

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	Solve	Think	Apply
c	60 muffins must be sold for the revenue to be $240.	Draw a line across from the vertical axis at 240 to the graph and then down to the n-axis to read the value of n.	Find values by drawing a line from one axis to the graph and then to the other axis.

UNDERSTANDING FLUENCY AND COMMUNICATION

3 The Supertight Tie Company sells ties for $15 each. The sales revenue may be modelled using the equation $R = 15n$.

a Explain each term in the equation $R = 15n$.

b Complete this table of values for the equation $R = 15n$.

n	0	50	100	150	200	250
R						

c Draw the straight-line graph of the equation $R = 15n$.

d Use the graph to find the number of ties sold when the revenue is $1800.

e What is the gradient of the straight line? What does this represent?

4 Tina travels 200 km to her friend's home at an average speed of 80 km/h. The distance Tina is from her friend's home after travelling for t hours can be modelled using the linear equation $D = 200 - 80t$.

a Explain each term in the equation $D = 200 - 80t$.

b Draw the straight-line graph of the equation $D = 200 - 80t$ for t values up to 4 hours.

c Use the graph to find how far Tina is from her friend's home after 1.5 hours.

d Use the graph to find how long it takes for Tina to reach her friend's home.

e What is the value of D when t is 4 hours. Explain why this linear model can only be used for particular values of t in this context.

PROBLEM SOLVING, REASONING AND JUSTIFICATION

5 The Munchy Sandwich Shop has a monthly fixed expense of $2000 and the cost of making each sandwich is $1.20.

a Write an equation modelling this information. Use C for monthly cost and n for the number of sandwiches made.

b Complete this table of values using your equation from part **a**.

n	0	500	1000	1500	2000	2500
R						

c Draw the straight-line graph for this linear model.

d What is the value of the gradient? What does this value represent?

e Use the graph to find the number of sandwiches made in a month if the cost is $3500.

6 The Tight Squeeze Fruit Juice Company has a weekly fixed cost of $325. The cost to produce a cup of fruit juice is $1.50.

a Write a linear equation modelling this information. Use C for the weekly cost and n for the number of cups of fruit juice.

b Complete this table of values using your equation from part **a**.

n	0	200	400	600	800	1000
C						

c Draw the straight-line graph for this linear model.

PROBLEM SOLVING, REASONING AND JUSTIFICATION

d Where does the graph cut the vertical axis? What does this value represent?

e What is the gradient? What does this value represent?

f Use the graph to find the number of cups of juice made in a week if the cost is $1000.

7 The Munchy Sandwich Shop sells sandwiches for $3.50 each.

a Write an equation modelling this information. Use R for revenue and n for the number of sandwiches sold.

b Complete this table of values using your equation from part **a**.

n	0	500	1000	1500	2000	2500
C						

c Draw a straight-line graph for this linear model.

d Use the graph to find the number of sandwiches sold when the revenue is $2800.

e What is the gradient of this straight line? What does this represent?

8 The Tight Squeeze Fruit Juice Company sells juice for $4.00 per cup.

a Write an equation modelling this information. Use R for revenue and n for the number of cups of juice sold.

b Complete this table of values using your equation from part **a**.

n	0	200	400	600	800	1000
R						

c Draw the straight-line graph for this linear model.

d Use the graph to find the number of cups of juice sold when the revenue is $3000.

e What is the gradient of this straight line?

9 The cost of hiring a taxi is $6 flagfall and $3.60 per kilometre travelled.

a Write a linear equation modelling this information to relate cost and distance travelled.

b Use graphing technology or a table of values to draw the graph of this linear model for distances up to 20 km.

c Use the graph to find the cost of travelling 10 km.

d Use the graph to estimate how far you could travel for $30.

10 On a building site, columns with a fixed cross-sectional area of 0.78 m^2 are to be constructed by pouring concrete into moulds of different heights.

a Write a linear equation modelling this information to relate volume of concrete (in cubic metres) and height of a column (in metres).

b Use graphing technology or a table of values to draw the graph of this linear model for height values up to 7 m.

c Use the graph to estimate the volume of concrete needed to construct a column that is 4 m high.

d Use the graph to estimate the height of a column that can be constructed using 2.4 m^3 of concrete.

CHALLENGE

11 The conversion from temperature in degrees Celsius (C) to temperature in degrees Fahrenheit (F) is given by the equation $F = \frac{9}{5}C + 32$.

a Using graphing technology or by completing a table, draw the graph of this relationship for values of C between 0 and 100.

b A 'rule of thumb' conversion is to double the temperature in degrees Celsius and add 30. Graph this model for values of C between 0 and 100.

c Compare the two graphs. Describe the accuracy of the rule of thumb. Which would you use? When?

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4C Identifying solutions to simultaneous linear equations

These resources are available on your obook assess:
- **assess quiz 4C:** Test your skills with an auto-correcting multiple-choice quiz

Consider the following tables of values for the linear equations given.

$y = 2x + 7$

x	−3	−2	−1	0	1	2	3
y	1	3	5	7	9	11	13

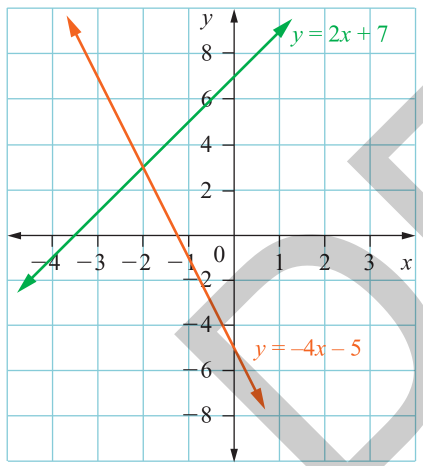
$y = -4x - 5$

x	−3	−2	−1	0	1	2	3
y	7	3	−1	−5	−9	−13	−17

The equation $y = 2x + 7$ has a solution of $x = -3$ and $y = 1$ because, when we substitute these values into the equation, the statement $1 = 2 \times -3 + 7$ is true. In fact, this equation has an infinite number of solutions as $x = -2$ and $y = 3$, $x = -1$ and $y = 5$, $x = 0$ and $y = 7$, etc. are also solutions. Similarly the equation $y = -4x - 5$ has an infinite number of solutions as $x = -3$ and $y = 7$, $x = -2$ and $y = 3$, $x = -1$ and $y = -1$, etc. are solutions.

We can see that $x = -2$ and $y = 3$ is a solution of *both* equations. We say that the linear equations $y = 2x + 7$ and $y = -4x - 5$ are solved simultaneously by the values $x = -2$ and $y = 3$. Hence, to solve a pair of linear equations simultaneously, or solve a pair of **simultaneous linear equations**, means to find the values of the variables that make both equations true.

The graphs of the two linear equations can be drawn on the same number plane as shown on the right.



What are the coordinates of the **point of intersection** of the two lines? How does this relate to the solution of the simultaneous linear equations $y = 2x + 7$ and $y = -4x - 5$?

simultaneous linear equations two or more linear equations that use the same variables

point of intersection point where two or more lines intersect or cross over one another on a graph

Exercise 4C Identifying solutions to simultaneous linear equations

1 Consider the tables given below to find the x-value and the y-value that is the solution to each pair of simultaneous linear equations.

a $y = 2x$

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

$x + y = 6$

x	−3	−2	−1	0	1	2	3
y	9	8	7	6	5	4	3

b $y = 2x - 1$

x	0	1	2	3	4
y	−1	1	3	5	7

$x + y = 8$

x	0	1	2	3	4
y	8	7	6	5	4

c $y = 3x - 5$

x	5	6	7	8	9
y	10	13	16	19	22

$x + y = 23$

x	5	6	7	8	9
y	18	17	16	15	14

Example 4C–1 Checking the solution to simultaneous linear equations

Check if each given x-value and y-value is the solution to the pair of simultaneous linear equations.

a $x = 2, y = 4$
$y = 3x - 2$ and $y = -2x + 8$

b $x = 5, y = -3$
$y = -2x + 7$ and $y = 2x - 11$

	Solve	Think	Apply
a	$y = 3x - 2$ When $x = 2$ and $y = 4$, LHS $= 4$ and RHS $= 3 \times 2 - 2 = 4$ So LHS = RHS $y = -2x + 8$ When $x = 2$ and $y = 4$, LHS $= 4$ and RHS $= -2 \times 2 + 8 = 4$ So LHS = RHS As $x = 2$, $y = 4$ satisfies both equations, it is the solution.	Substitute $x = 2$ and $y = 4$ into the left-hand side (LHS) and right-hand side (RHS) of each equation. Check if the LHS and RHS are equal for each equation. They are equal (the values make both equations true statements). Hence $x = 2$, $y = 4$ is the solution to the simultaneous linear equations.	Substitute the given x- and y-values into each equation and check if they make both equations true statements. Decide if the given x-value and y-value is the solution to the simultaneous linear equations.
b	$y = -2x + 7$ When $x = 5$ and $y = -3$, LHS $= -3$ and RHS $= -2 \times 5 + 7 = -3$ So LHS = RHS $y = 2x - 11$ When $x = 5$ and $y = -3$, LHS $= -3$ and RHS $= 2 \times 5 - 11 = -1$ So LHS ≠ RHS As $x = 5$, $y = -3$ does not satisfy both equations, this is not the solution.	Substitute $x = 5$ and $y = -3$ into the left-hand side (LHS) and right-hand side (RHS) of each equation. Check if the LHS and RHS are equal for each equation. They are equal for the first equation but not the second. Hence $x = 5$, $y = -3$ is not the solution to the simultaneous linear equations.	

2 Check if each given x-value and y-value is the solution to the pair of simultaneous linear equations.

a $x = 1, y = 5$
$y = x + 4$ and $y = -2x + 7$

b $x = -2, y = 1$
$y = x + 3$ and $y = -3x - 5$

c $x = 5, y = -2$
$y = x - 9$ and $y = -2x + 7$

d $x = 3, y = -7$
$y = -2x - 1$ and $y = -x - 4$

e $x = 4, y = 3$
$y = 2x - 5$ and $y = -x + 1$

f $x = -6, y = 0$
$y = x - 6$ and $y = -x - 6$

! To find the solution of a pair of simultaneous linear equations from a graph, find the point of intersection of the two lines. At the point of intersection, each line will simultaneously have the same x-coordinate and y-coordinate. The coordinates of the point of intersection provide the x-value and the y-value of the solution.

UNDERSTANDING FLUENCY AND COMMUNICATION

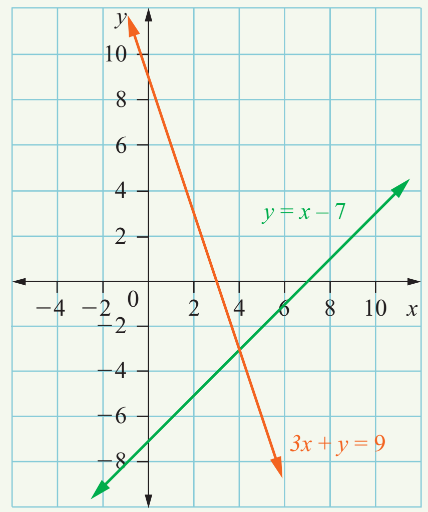
Example 4C–2 Identifying the solution to simultaneous linear equations from graphs

Consider the following pair of simultaneous linear equations and their graphs drawn on the same number plane.

$y = x - 7$ and $y = -3x + 9$

a Write the coordinates of the point of intersection of the two lines.

b Use the coordinates of this point to write the solution to the simultaneous linear equations.



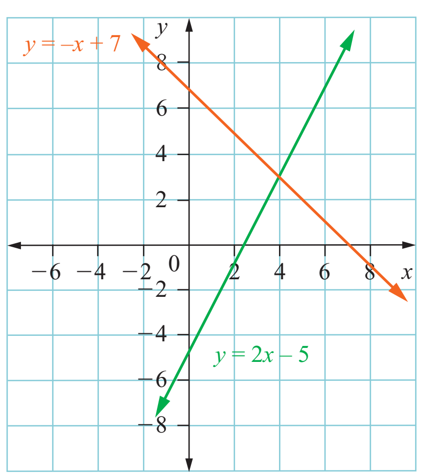
	Solve	Think	Apply
a	Point of intersection at (4, –3).	Identify the point where the two lines cross and write the x-value and the y-value of the point as coordinates.	Identify the point of intersection of the two lines and list the coordinates.
b	The solution to the simultaneous equations $y = x - 7$ and $y = -3x + 9$ is $x = 4$, $y = -3$.	Use the coordinates of the point to write the solution.	Write the solution as the x-value and the y-value that makes both equations true.

3 Consider each pair of simultaneous linear equations and their graphs.

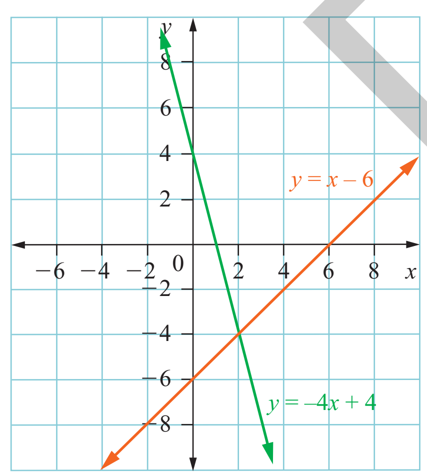
i Write the coordinates of the point of intersection of the two lines.

ii Use the coordinates of this point to write the solution to the simultaneous linear equations.

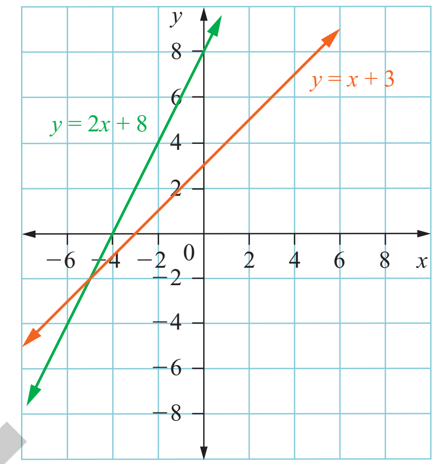
a $y = 2x - 5$ and $y = -x + 7$



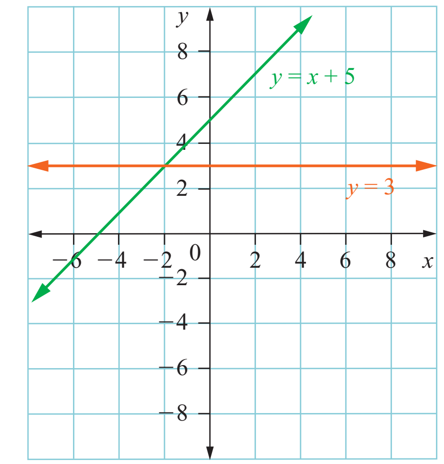
b $y = -4x + 4$ and $y = x - 6$



c $y = 2x + 8$ and $y = x + 3$



d $y = x + 5$ and $y = 3$



4 Use substitution to check your solutions to the simultaneous linear equations in question **3**.

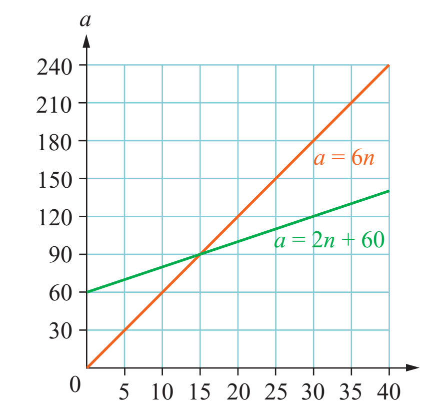
PROBLEM SOLVING, REASONING AND JUSTIFICATION

5 Suzy makes hand-painted cards. She invests \$60 in a set of paintbrushes and enough paint for 40 cards. The cost of each plain white card is \$2 and she sells the painted cards for \$6 each. The production cost and the revenue generated by the sale of the cards can be modelled with two linear equations, as shown on the graph.

a How many cards does Suzy need to sell to recover all her costs?

b How much profit does Suzy make if she sells 30 cards?

c What is the maximum profit Suzy can make?



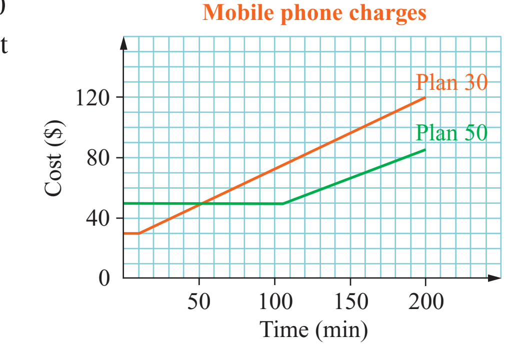
6 Daniel is deciding which mobile phone plan to use. He is comparing the Plan 30 and Plan 50 rates. Plan 30 is a minimum of \$30 per month with \$5 of free calls. Calls cost 24 cents/30 s. Plan 50 is a minimum of \$50 per month with \$40 of free calls. Calls cost 19 cents/30 s. This graph shows these plans.

a For 35 minutes of calls per month, which plan should he use? Why?

b For 120 minutes of calls per month, which plan should he use? Why?

c After how many minutes of calls per month are the costs of each plan equal? What is this cost?

d Explain why these two lines are not simultaneous linear equations.



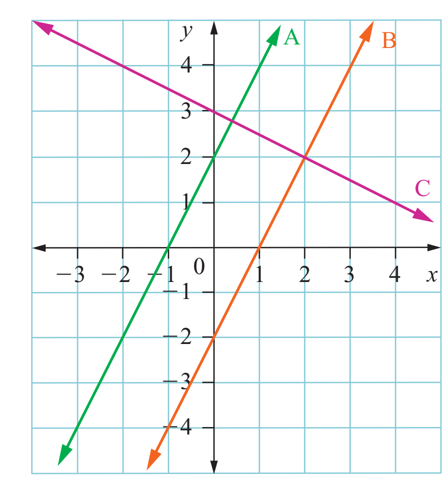
CHALLENGE

7 Consider the straight lines A, B and C shown on this graph.

a Write the coordinates of the point of intersection of lines B and C and hence give the solution to the simultaneous equations for these lines.

b Use the graph to solve the equations for lines A and C simultaneously.

c Can the equations for lines A and B be solved simultaneously? Explain why or why not.



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4D Solving simultaneous linear equations graphically

These resources are available on your obook assess:
- **Video tutorial 4D:** Watch and listen to an explanation of Example 4D–1
- **Worksheet 4D:** Practise your skills with extra problems
- **Investigation 4D:** Solve simultaneous linear equations for train travel
- **assess quiz 4D:** Test your skills with an auto-correcting multiple-choice quiz

OI

In Topic 4C, the solution to a pair of simultaneous linear equations was identified from given tables or graphs. In this section you will use your skills in drawing linear graphs to solve simultaneous linear equations graphically. You can use technology or complete tables of values to help you produce the graphs.

If using tables of values, it is easier to have only three sets of x- and y-values. Although only two points need to be plotted to produce a straight-line graph, using three (or four) points provides a check of the substitution calculations. If the plotted points do not lie in a straight line, we can easily see that an error has been made.

Example 4D–1 Drawing graphs from tables of values to solve simultaneous linear equations

a Complete the given tables below for the linear equations $y = 2x + 1$ and $y = 4 - x$.

$y = 2x + 1$

x	−2	0	2
y			

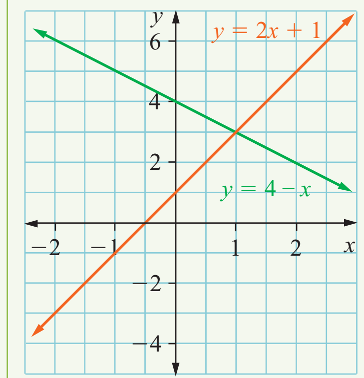
$y = 4 - x$

x	−2	0	2
y			

b Plot the values from the tables to draw the graphs of $y = 2x + 1$ and $y = 4 - x$ on the same number plane.

c Find the point of intersection of the two lines.

d Write the solution to the simultaneous linear equations $y = 2x + 1$ and $y = 4 - x$.

<table>
<tr><th></th><th>Solve</th><th>Think/Apply</th></tr>
<tr><td>a</td><td>$y = 2x + 1$
<table><tr><td>x</td><td>−2</td><td>0</td><td>2</td></tr><tr><td>y</td><td>−3</td><td>1</td><td>5</td></tr></table>$y = 4 - x$
<table><tr><td>x</td><td>−2</td><td>0</td><td>2</td></tr><tr><td>y</td><td>6</td><td>4</td><td>2</td></tr></table></td><td>Complete the table of values for each equation by substituting each x-value into the equation to find the corresponding y-value.
The coordinates of the point of intersection of the two straight lines give the solution of the simultaneous equations.</td></tr>
<tr><td>b</td><td></td><td>Use the tables of values to plot points for each equation on the same number plane and draw straight lines through them.</td></tr>
<tr><td>c</td><td>The point of intersection is (1, 3).</td><td>Identify the point where the two lines cross and write the coordinates of this point.</td></tr>
<tr><td>d</td><td>The solution is $x = 1$, $y = 3$.</td><td>As the point of intersection lies on both lines, its coordinates satisfy both equations; that is, $x = 1$ and $y = 3$ is the solution of the simultaneous equations.</td></tr>
</table>

Exercise 4D Solving simultaneous linear equations graphically

UNDERSTANDING FLUENCY AND COMMUNICATION

1 Consider the following pairs of linear equations.

- **i** Complete the given tables for the linear equations.
- **ii** Plot the values from the tables to draw the graphs of the equations on the same number plane.
- **iii** Find the point of intersection of the two lines.
- **iv** Write the solution to the simultaneous linear equations.

a $y = x + 2$

x	−2	0	2
y			

$y = 2x - 1$

x	−2	0	2
y			

Draw a number plane showing x-values from −4 to 8 and y-values from −6 to 8.

b $y = 2x + 2$

x	0	2	4
y			

$y = 3x + 1$

x	0	2	4
y			

Draw a number plane showing x-values from −4 to 8 and y-values from −2 to 14.

c $y = 2x - 2$

x	−1	0	1
y			

$y = 4 - x$

x	−1	0	1
y			

Draw a number plane showing x-values from −2 to 5 and y-values from −4 to 5.

2 Solve each of the following pairs of simultaneous linear equations graphically using the approach used in question **1**.

a $y = 6 - x$ and $y = 10 - 2x$ **b** $y = x - 3$ and $y = 1 - x$ **c** $y = x - 1$ and $y = 2x$

d $y = 4 - x$ and $y = 5 - 2x$ **e** $y = 6 - x$ and $y = 10 - 2x$ **f** $y = x + 1$ and $y = -\frac{2}{3}x + 2$

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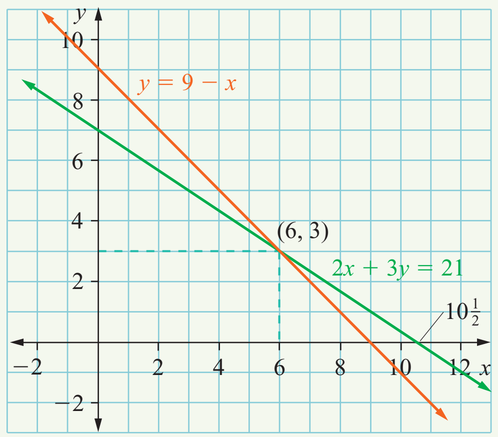
UNDERSTANDING FLUENCY AND COMMUNICATION

Example 4D–2 Solving simultaneous linear equations graphically

Use a graphical method to solve the simultaneous linear equations $2x + 3y = 21$ and $y = 9 - x$.

Solve



The point of intersection is (6, 3), so the solution to the simultaneous linear equations $2x + 3y = 21$ and $y = 9 - x$ is $x = 6$, $y = 3$.

Think

Draw the lines for the equations using graphing technology or by completing a table of values and plotting the points.

For $2x + 3y = 21$:

This equation can be rearranged to become $y = -\frac{2}{3}x + 7$. Choose x-values that make the substitution calculations easier.

x	0	3	6	9
y	7	5	3	1

For $y = 9 - x$:

x	0	2	4	8
y	9	7	5	1

Locate the coordinates of the point where the lines cross and write the solution.

Apply

Produce the linear graphs using technology or by completing a table of values and plotting the points. The gradient and y-intercept method of drawing a linear graph could also be used.

Find the point of intersection of the two lines and use the coordinates to write the solution to the simultaneous equations.

3 Use a graphical method to solve these pairs of simultaneous linear equations.

- **a** $y = x - 3$ and $y = 1 - x$
- **b** $x - y = 1$ and $y = 2x$
- **c** $4x + 3y = 10$ and $x - 2y = -3$
- **d** $3x + y = 5$ and $2x - 3y = 18$
- **e** $y = 3x - 8$ and $3x + y = -2$
- **f** $2x + y = 4$ and $2x - y = 8$
- **g** $3x + y = -5$ and $2x + 3y = -8$
- **h** $x - 3y = -9$ and $2x - 3y = -12$
- **i** $x + y = 1$ and $3x + 2y = 1$
- **j** $3x - y = 2$ and $x - y = -4$

4 Consider the linear equations $y = 2x + 3$ and $y = 8 - 3x$.

- **a** Use graphing technology to draw the graphs of the two linear equations on the same set of axes.
- **b** Find the point of intersection of these lines.
- **c** Hence write the solution to the simultaneous linear equations $y = 2x + 3$ and $y = 8 - 3x$.

5 Use graphing technology to solve the pairs of simultaneous linear equations in question **2**. Compare your answers with those obtained in question **2**.

PROBLEM SOLVING, REASONING AND JUSTIFICATION

6 The cost ($C) of hiring a car from company A is \$50 per day; that is, $C = 50d$, where d is the number of days for which the car is hired. Company B charges a flat fee of \$60 plus \$40 per day; that is, $C = 60 + 40d$.

- **a** Use graphing technology or complete tables to graph the equations on the same set of axes.
- **b** Solve the equations simultaneously using your graph. Explain the meaning of the solution.
- **c** After how many days is it cheaper to hire from company B?

7 Joanna is a salesperson who earns her income from commissions. Her supervisor offers her a choice of two methods of weekly payment.

- **i** A straight commission of 10% of her sales: $I = 0.1S$, where I is income and S is sales in dollars
- **ii** A retainer of \$300 plus 6% of sales: $I = 300 + 0.06S$

- **a** Use graphing technology or complete tables to graph each linear equation on the same set of axes.
- **b** Find the solution of the simultaneous linear equations. Explain the meaning of the solution.
- **c** What sales would Joanna have to achieve to earn more income using method **ii**?

8 A new car with a petrol engine uses 10.0 L/100 km. If the cost of petrol is \$1.50/L then the cost ($C) of fuel to drive the car is given by $C = 0.15d$, where d is the distance travelled. The same car with a diesel engine costs \$2000 more to purchase but only uses 6.0 L/100 km. The cost of diesel fuel is \$1.60/L. The cost of driving the diesel model of the car is given by $C = 2000 + 0.096d$.

- **a** Find the solution of the simultaneous linear equations using a graphical method.
- **b** How far do you have to travel before the cost of the two models is the same?

9 The perimeter of a printed rectangular photo is 50 cm.

- **a** Use l for length and w for width to write a linear equation for the perimeter of the photo.
- **b** Use l and w to write a linear equation linking the length and width of the photo given that the length is 5 cm longer than the width.
- **c** Solve the two equations simultaneously using a graphical method, and hence state the dimensions of the photo.

10 At the cinema, a large drink costs \$2 more than an ice cream. Liam buys five drinks and seven ice creams for \$70.

- **a** Write two linear equations to model this information.
- **b** Solve the equations simultaneously using a graphical method to find the cost of each item.

11 Tom rides his bicycle from Town A to Town B at an average speed of 20 km/h. Julia walks from Town B to Town A along the same road at an average speed of 5 km/h. The distance along the road between the two towns is 15 km and they each start off at the same time.

- **a** Write two linear equations to model the two journeys. In each case, relate the distance in kilometres *from Town A* to the time travelled in hours.
- **b** Solve the equations simultaneously using a graphical method to find the time at which they meet along the road between the two towns.
- **c** How far has each travelled before they meet?

12 Solve the following pairs of simultaneous linear equations graphically and explain your results.

- **a** $y = 2x + 1$ and $y = 2x - 3$
- **b** $x - 2y = 4$ and $2x - 4y = 8$

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4E Break-even analysis

break-even point
point at which the revenue (or income) and the cost of production (or expenses) are equal; it can be identified as the point of intersection of the combined graphs of cost and revenue

These resources are available on your obook assess:
- **Video tutorial 4E:** Watch and listen to an explanation of Example 4E–2
- **assess quiz 4E:** Test your skills with an auto-correcting multiple-choice quiz

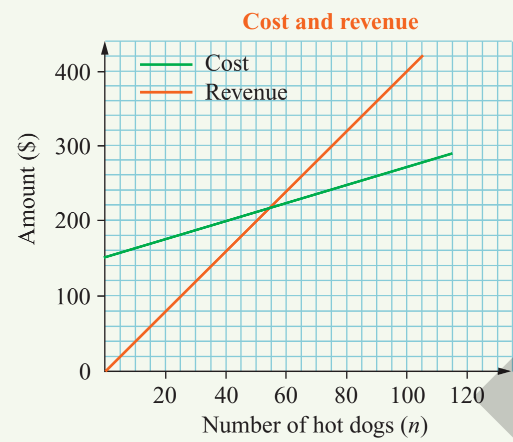
In Topic 4B, straight-line graphs were drawn to model both production cost and revenue. If we combine graphs for cost and revenue for the same business, we can find the **break-even point**. This is the point at which the graphs intersect and revenue equals cost. Sales exceeding the break-even number result in a profit for the business, and sales below the break-even number result in a loss.

This section uses simultaneous linear equations to look at break-even points, and profits and losses.

Example 4E–1 Determining the break-even point from a given graph

Holly's Hot Dogs makes and sells hot dogs. There is a fixed cost per day of \$150 and each hot dog costs \$1.20 to produce. The hot dogs are sold for \$4 each.
The graphs of the production cost and the revenue made from selling hot dogs are drawn on the same set of axes.

a How many hot dogs must be sold to break even?
b What is the revenue at the break-even point?
c Find the profit made when 90 hot dogs are sold?



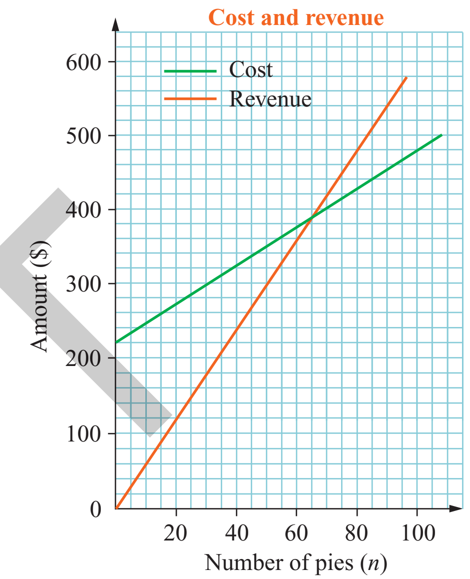
	Solve	Think	Apply
a	Approximately 54 hot dogs must be sold to break even.	To break even, the revenue has to equal the production cost. Read the value for n at the point of intersection of the two lines. *Note:* in this case we need to estimate the value.	The break-even point occurs where the lines for revenue and cost intersect.
b	At the break-even point, revenue is about \$215.	Estimate the value for the revenue from the vertical axis (Amount in dollars) when $n \approx 54$.	Read the value for the revenue at the point of intersection of the two lines.
c	When $n = 90$, revenue = \$360 cost = \$260 Profit = revenue − cost = \$360 − \$260 = \$100	Read the values from the vertical axis for the revenue line and the cost line when $n = 90$. Subtract the cost from the revenue to find the profit.	Find the values for the revenue and cost by reading across to the vertical axis from each line for the given number of hot dogs.

Exercise 4E Break-even analysis

UNDERSTANDING FLUENCY AND COMMUNICATION

1 Peter's Pie Palace makes and sells gourmet pies. There is a fixed cost per day of \$220 and each pie costs \$2.60 to produce. The pies are sold for \$6 each.

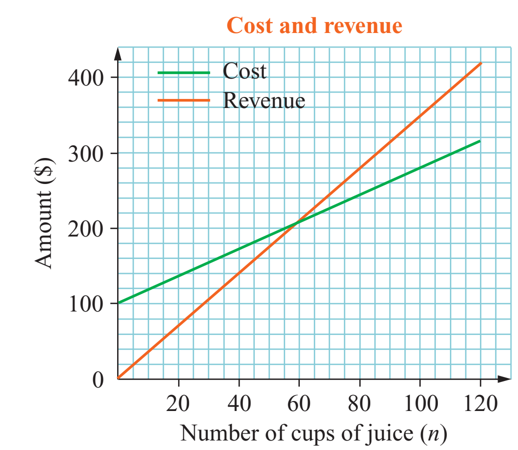
The graphs of the production cost and the revenue made from selling the pies are drawn on the same set of axes.



a How many pies must be sold to break even?
b What is the revenue at the break-even point?
c Find the profit made when 90 pies are sold?

2 Jenny's Juice Joint makes and sells fresh juices. There is a fixed cost per day of \$100 and each cup of juice costs \$1.80 to produce. The juices are sold for \$3.50 each. The graphs of production cost and revenue are drawn on the same set of axes.

a How many cups of juice must be sold to break even?
b What is the revenue at the break-even point?
c Find the profit made by Jenny's Juice Joint when 110 cups of juice are sold.
d Find the difference between the revenue and cost when 30 cups of juice are sold. Does Jenny make a profit? Explain your answer.



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UNDERSTANDING FLUENCY AND COMMUNICATION

3 A plastics company has a maximum daily production of 700 items.

a There is an initial cost of \$3000 per day plus \$8 per item produced. This can be represented by the linear equation $P = 8n + 3000$ where P is the production cost in dollars and n is the number of items. Complete this table of values for the production cost.

Number of items (n)	0	100	300	500	700
Production cost (P, \$)					

b The selling price of each item is \$15. This can be represented by the linear equation $R = 15n$ where R is the revenue in dollars on the sale of n items. Complete this table of values for the revenue.

Number of items (n)	0	100	300	500	700
Revenue (R, \$)					

c Choose a suitable scale and graph these lines on the same set of axes:

i production cost versus number of items

ii revenue versus number of items.

d Find the point of intersection of the two graphs. This is the break-even point.

e How many items must be sold each day to break even?

f What is the break-even revenue?

4 The Tin Lid factory produces sports caps. There is a fixed monthly cost of \$1800 and it costs \$3 to produce a cap. The caps are sold for \$8 each. The maximum monthly production of caps is 800.

a Use the equation $C = 3n + 1800$, where C is the production cost in dollars to produce n sports caps, to graph the cost of production.

b Use the equation $R = 8n$, where R is the revenue in dollars on the sale of n sports caps, to graph the revenue on the same set of axes as the cost equation in part **a**.

c Find the point of intersection of the two lines. This is the break-even point.

d How many caps must be sold in a month to break even?

e What is the break-even revenue?

5 The Supertight Tie Company has a fixed cost of \$1200 per day and a variable cost of \$3 per tie. This can be represented by the equation $C = 3n + 1200$ where C is the cost of producing n ties. The revenue of \$15 per tie sold can be represented by the equation $R = 15n$ where R is the revenue on the sale of n ties.

a Graph these two equations on the same set of axes for $0 \le n \le 250$.

b Find the break-even point.

c How many ties must be sold to break even?

d What is the break-even revenue?

PROBLEM SOLVING, REASONING AND JUSTIFICATION

Example 4E–2 Modelling revenue and cost for break-even analysis

The Majestic Muffin Shop has a fixed cost of \$320 per day and a variable production cost of \$2 per muffin. The selling price of each muffin is \$6. The maximum daily production is 120 muffins.

a Write an equation for the cost, \$$C$, of producing n muffins.

b Graph the linear equation for the production cost.

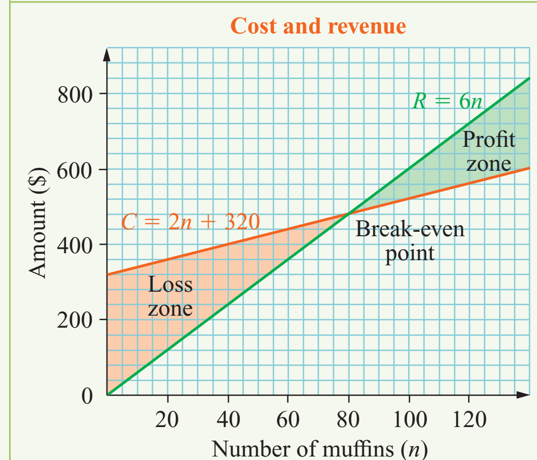
c Write an equation for the revenue, \$$R$, for the number of muffins sold, n. Draw the straight line for the revenue equation on the same set of axes as the cost equation in part **b**.

d Find the point of intersection.

e How many muffins must be sold to break even?

f What is the break-even revenue?

g On the graph drawn for parts **b** and **c**, shade the loss zone and the profit zone.

<table>
<tr><th></th><th>Solve</th><th>Think</th><th>Apply</th></tr>
<tr><td>a</td><td>The cost equation is $C = 2n + 320$.</td><td>The fixed cost is \$320.
The variable cost is $2n$ since the cost per muffin (and the gradient) is 2.</td><td>The vertical intercept is the fixed cost. The gradient relates to the variable cost.</td></tr>
<tr><td>b</td><td>
</td><td>Draw the graph using graphing technology, or the gradient (2) and y-intercept (320) method, or by completing a table of values and plotting the points.

n: 0, 40, 80, 120
C: 320, 400, 480, 560</td><td>The maximum number of muffins is 120 so this is the largest value for n.</td></tr>
<tr><td>c</td><td>The revenue equation is $R = 6n$.
The graph is drawn on the same set of axes as $C = 2n + 320$ in part b.</td><td>Draw the graph using graphing technology, or the gradient (6) and y-intercept (0) method, or by completing a table of values and plotting the points on the same set of axes.

n: 0, 40, 80, 120
R: 0, 240, 480, 720</td><td>There is no fixed cost so the equation has no constant value. Both C and R are values in dollars and so can both be represented on the vertical axis, labelled ‘Amount (\$)’.</td></tr>
<tr><td>d</td><td>The point of intersection has the coordinates (80, 480).</td><td>Find the values at the point where the lines intersect.</td><td>The break-even point is when revenue equals cost. This is the point of intersection of the two lines.</td></tr>
</table>

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PROBLEM SOLVING, REASONING AND JUSTIFICATION

	Solve	Think	Apply
e	80 muffins must be sold to break even.	Read the value for n for the point of intersection of the two lines.	
f	The break-even revenue is $480.	Read the amount (C or R) in dollars for the point of intersection of the two lines.	
g	See the graph shown in part **b**.	For the loss zone, shade the zone between the two lines to the left of the break-even point to indicate where the revenue is less than the cost for making n items. For the profit zone, shade the zone between the two lines to the right of the break-even point to indicate where the revenue is more than the cost for making n items.	The loss zone is the zone between the two lines where the revenue line is below the cost line. The profit zone is the zone between the two lines where the revenue line is above the cost line.

6 The Munchy Sandwich Shop (from Exercise 4B) had a fixed expense of $2000 per month and a cost of $1.20 per sandwich. The sandwiches were sold for $3.50 each.

a Write the linear equations that can be used to model the production cost and revenue.

b Graph straight lines modelling the cost and revenue for $0 \le n \le 1000$.

c Find the break-even point.

d How many sandwiches must be sold to break even?

e What is the break-even revenue?

f The maximum number of sandwiches that can be made in a month is 800. Explain the significance of this.

g On the graph drawn for part **b**, shade the loss zone. Explain the significance of this.

7 The Full Flavour Juice Company has a weekly fixed cost of $325 and a cost of $1.50 per cup. The cups of juice are sold for $4.

a Graph straight lines modelling the production cost and revenue for $0 \le n \le 200$.

b Find the break-even point.

c How many cups of juice must be sold to break even?

d What is the break-even revenue?

e The maximum number of cups of juice per week is 100. Is this a profitable business? Explain.

f On the graph drawn for part **a**, shade the loss zone.

8 Consider the graphs you have drawn to model the cost and revenue for each company in questions **3**, **4** and **5**.

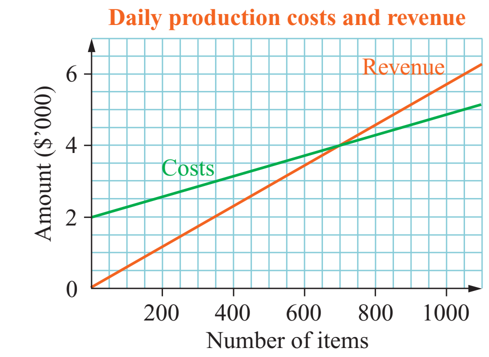
a Shade the loss zone for the graph drawn in question **3**.

b Shade the profit zone for the graph drawn in question **4**.

c Shade the loss zone for the graph drawn in question **5**.

PROBLEM SOLVING, REASONING AND JUSTIFICATION

9 The daily production costs and revenue from the sale of items is shown on the graph.



a Use the graph to determine the number of items that must be sold to break even.

b How much profit or loss is made when:

i 100 items are sold?

ii 1000 items are sold?

c How many items need to be sold for the company to make a:

i $1000 profit?

ii $1000 loss?

10 The production cost of printing books is initially $8000 plus $3000 per 1000 books printed. The books are sold for $6.50 each.

a Model the production cost and the revenue with linear equations.

b Use graphing technology or plot values from tables to draw the linear graphs for the production cost and revenue on the same set of axes.

c How many books need to be sold to break even?

d How much profit or loss is made when:

i 1000 books are sold?

ii 2500 books are sold?

iii 3200 books are sold?

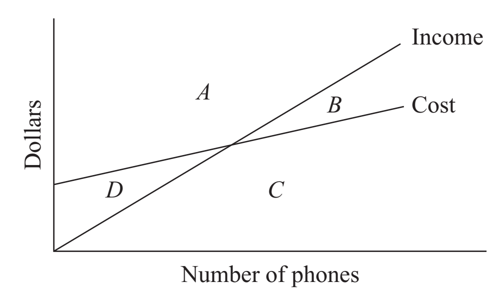
e How many books need to be sold for the company to make:

i a loss of $3000?

ii a profit of $2500?

CHALLENGE

11 A company manufactures bicycles. The company's revenue equation and cost equation are drawn on the same graph. Which region of the graph (A, B, C or D) is the profit zone?



ALGEBRA

CHAPTER 4 REVIEW SIMULTANEOUS LINEAR EQUATIONS

You should be able to:

✔ complete tables, plot points and draw graphs of the form $y = mx + c$

✔ model situations using linear equations

✔ generate tables and graph linear models

✔ use technology to graph linear models

✔ identify the solution from tables or graphs when solving linear equations simultaneously

✔ find the point of intersection between two straight-line graphs using technology

✔ solve simultaneous linear equations graphically and interpret the solution

✔ solve practical problems using simultaneous linear equations

✔ determine and interpret the break-even point of a simple business problem where cost and revenue are represented by linear equations.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

4A **1** What are the gradient and y-intercept respectively of the line $y = -2x + 3$?

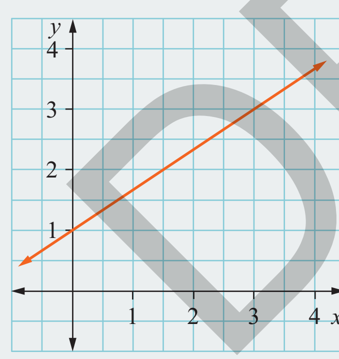
A 3 and -2 **B** -2 and 3 **C** -2 and -3 **D** -3 and -2

4A **2** What is the equation of the straight line with gradient of $\frac{1}{2}$ and y-intercept -1?

A $y = x - \frac{1}{2}$ **B** $y = -x - \frac{1}{2}$

C $y = \frac{1}{2}x - 1$ **D** $y = -\frac{1}{2}x + 1$

Use the following graph for questions **3** and **4**.



4A **3** What is the gradient of the line shown?

A $\frac{2}{3}$ **B** $\frac{3}{2}$

C 1 **D** 3

4A **4** What is the equation of the line?

A $y = \frac{2}{3}x + 1$ **B** $y = \frac{3}{2}x + 1$

C $y = x + \frac{2}{3}$ **D** $y = x + \frac{3}{2}$

4A **5** The Cactus Water Company purifies water. The weekly fixed cost is \$450 and the cost per glass is \$2. Which equation models this information where C is the total cost and n is the number of glasses?

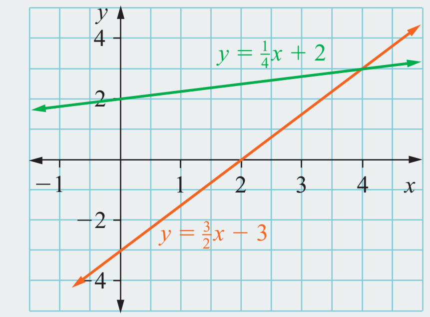
A $C = 2n + 450$ **B** $C = 450n + 2$ **C** $C = 450n - 2$ **D** $C + 450 = n$

4B **6** The equation $C = 3n + 150$ models the costs for a sandwich shop. What could the 150 represent?

A number of sandwiches sold **B** the cost per sandwich

C fixed daily cost **D** number of sandwiches made

4C **7** The straight lines for equations $y = \frac{3}{2}x - 3$ and $y = \frac{1}{4}x + 2$ are shown on the graph. What is the solution when the two equations are solved simultaneously?



A $x = -8, y = 0$

B $x = 0, y = -3$

C $x = 3, y = 4$

D $x = 4, y = 3$

4C **8** Which pair of simultaneous equations has the solution $x = -5, y = 3$?

A $y = x + 8$ and $4x - 3y = 29$ **B** $y = x + 8$ and $3x - 2y = -21$

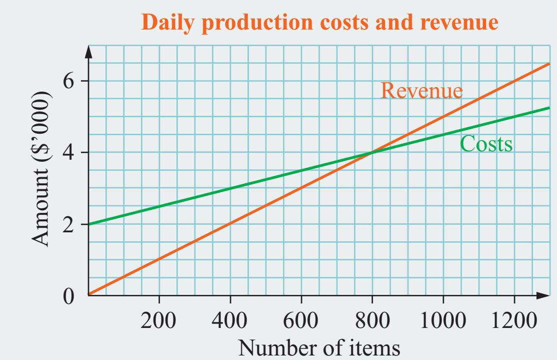
C $x + y = 2$ and $5x + y = -22$ **D** $y - x = 8$ and $4x - 3y = 26$

4D **9** What is the solution to the simultaneous equations $y = 2x - 1$ and $y = 14 - 3x$?

A $x = 5, y = 3$ **B** $x = 3, y = 5$

C $x = 2, y = -1$ **D** $x = 14, y = -3$

4E **10** From the graph, determine the number of items that need to be sold for the business to break even.



A 4

B 4000

C 700

D 650

REVIEW SET 1

1 **a** Find the gradient and y-intercept of the line $y = \frac{3}{8}x - 4$.

b Write the equation of the line with gradient of 0.08 and y-intercept 3.2.

2 **a** Complete this table of values for $y = 4x + 3$.

b Use the table of values to graph $y = 4x + 3$ on a number plane.

x	-3	-2	-1	0	1	2	3
y							

c Use technology to produce a graph of $y = 4x + 3$ and compare your answer with that for part **b**.

3 The distance, d kilometres, travelled by a train over time t hours is $d = 70t$.

a Complete this table of values for $d = 70t$.

t	1	2	3	4	5
d					

b Draw the graph of $d = 70t$.

c How far away is the train after $3\frac{1}{2}$ hours?

d When is the train 200 km away?

4 Graphically solve the simultaneous equations $y = \frac{1}{2}x + 1$ and $y = \frac{3}{4}x$.

5 Solve the equations $y = -3x - 4$ and $x + 2y = 2$ simultaneously using a graphical method.

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6 The Happy Feet Shoe Company has a weekly fixed cost of \$3725 and a cost of \$7.50 per pair of shoes. The shoes are sold for \$45 a pair.

- **a** Graph straight lines modelling the cost and revenue for $0 \le n \le 300$ on the same set of axes.
- **b** Find the break-even point.
- **c** How many pairs of shoes must be sold to break even?
- **d** What is the break-even revenue?
- **e** Shade the loss zone on the graph you produced in part a.

REVIEW SET 2

1 By plotting the points (1, 3) and (5, 8), or otherwise, find the gradient of the line through them.

2 **a** Complete this table of values for $y = 3x + 2$.

x	−3	−2	−1	0	1	2	3
y							

- **b** Use the table of values to graph $y = 3x + 2$ on a number plane.
- **c** Use technology to produce a graph of $y = 3x + 2$ and compare your answer with that for part **b**.

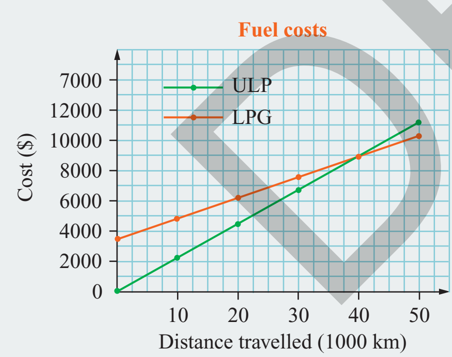
3 The Smart Tie Company has a fixed cost of \$1400 per day and a variable cost of \$4 per tie. The daily cost can be modelled using the straight-line equation $C = 4n + 1400$.

- **a** Explain each term in the equation $C = 4n + 1400$.
- **b** Complete this table of values using the equation $C = 4n + 1400$.

n	0	50	150	200	250
C					

- **c** Draw the straight-line graph of $C = 4n + 1400$ for $0 \le n \le 250$.
- **d** From the graph, determine the number of ties produced when the cost is \$1500.

4 The fuel cost of driving a large 4WD when running on unleaded petrol (ULP) is shown in the graph below. When the vehicle is converted to run on liquid petroleum gas (LPG or autogas), the cost for travelling the same distance was calculated and drawn on the same set of axes. From the graph, estimate the distance travelled before the costs are the same.



5 Solve the following pairs of simultaneous linear equations using a graphical method.

- **a** $y = 2x - 3$ and $y = 6 - x$
- **b** $2x + 3y = 0$ and $3x - y = -11$

6 The Supersheer Stocking Company has a fixed cost of \$1500 per day and a variable cost of \$2 per pair of stockings. This can be modelled by the equation $C = 2n + 1500$, where C is the cost of producing n pairs of stockings. The revenue of \$12 per pair of stockings sold can be represented by the equation $R = 12n$, where R is the revenue on the sale of n pairs of stockings.

- **a** Graph these two equations on the same set of axes for $0 \le n \le 300$.
- **b** Find the break-even point.
- **c** How many pairs of stockings must be sold to break even?
- **d** What is the break-even revenue?

REVIEW SET 3

1 **a** Complete this table of values for $y = 5x - 2$.

x	−3	−2	−1	0	1	2	3
y							

- **b** Use the table of values to graph $y = 5x - 2$ on a number plane.
- **c** Use technology to produce a graph of $y = 5x - 2$ and compare your answer with that for part **b**.

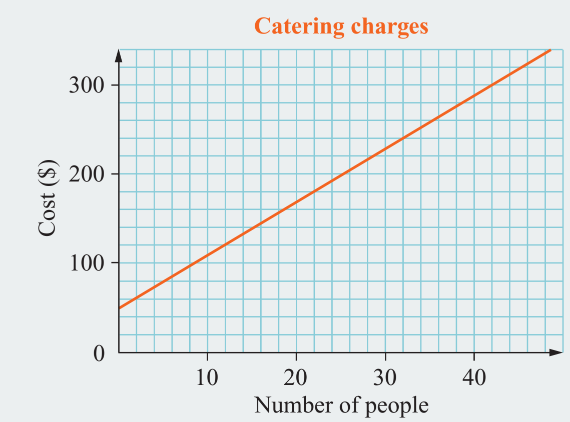
2 The Silky Scarf Company sells scarves for \$18 each. Sales revenue may be modelled using the equation $R = 18n$.

- **a** Explain each term in the equation $R = 18n$.
- **b** Complete this table of values for $R = 18n$.

n	0	50	100	200	250
R					

- **c** Draw the straight-line graph of $R = 18n$.
- **d** From the graph, find the number of scarves sold when the revenue is \$2070.

3 This graph models the cost for a company to cater for different numbers of people.



- **a** How much would it cost to cater for 35 people?
- **b** How many people could be catered for \$300?
- **c** Find the gradient. What is its meaning?
- **d** Find the intercept on the vertical axis. What is its meaning?
- **e** The model for another catering company is represented by the equation $C = 8n$, where C is the cost in dollars to cater for n people. Copy the graph above and draw the line for $C = 8n$ on the same set of axes.
- **f** For how many people is the cost of catering the same for each company? What is this cost?

4 Solve the equations $x - 3y = 11$ and $5x - 2y = 16$ simultaneously using a graphical method.

5 The cost (\$$C$) of hiring a car from company A is given by $C = 60d$, where d is the number of days the car is hired. For company B, the cost of hire is $C = 100 + 40d$.

- **a** Draw the graphs of cost versus number of days for each company on the same set of axes.
- **b** Find the solution of the simultaneous equations $C = 60d$ and $C = 100 + 40d$.
- **c** After how many days is the cost the same for both companies?

6 The Stay Dry Company produces umbrellas. There is a fixed monthly cost of \$2800 and it costs \$7 to produce an umbrella. The umbrellas are sold for \$22 each. The maximum monthly production of umbrellas is 300.

- **a** Write two linear equations that can be used to model the production cost and revenue.
- **b** Graph straight lines modelling the cost and revenue for $0 \le n \le 300$.
- **c** Find the point of intersection of the two lines. This is the break-even point.
- **d** How many umbrellas must be sold in a month to break even?
- **e** What is the break-even revenue?

REVIEW SET 4

1 **a** Complete this table of values for $y = 7 - 2x$.
b Use the table of values to graph $y = 7 - 2x$ on a number plane.

x	−3	−2	−1	0	1	2	3
y							

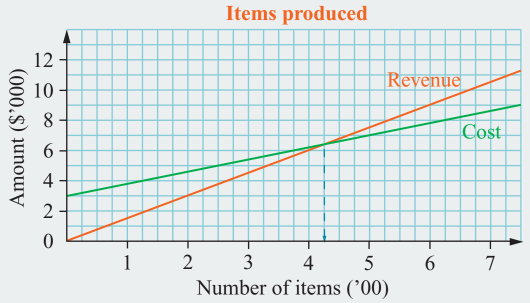
c Use technology to produce a graph of $y = 7 - 2x$ and compare your answer with that for part **b**.

2 The Tropical Delight Fruit Juice Company has a weekly fixed cost of \$400. The cost per cup of juice is \$1.75.
a Write an equation modelling this information. Use C for cost and n for the number of cups of juice.
b Complete this table of values using the cost equation from part **a**.

n	0	200	400	600	800	1000
C						

c Draw the straight-line graph of your cost equation using the values from the completed table in part **b**.
d Use the graph to find the number of cups of juice made when the cost is \$1000.

3 A plastics company has an initial cost of \$3000 per day plus \$8 per item produced. The selling price of the items is \$15 each, and the maximum daily production is 1000 items. The revenue and cost models are represented in the graph below.



a How many items need to be sold for the company to break even?
b What is the break-even revenue?

4 **a** Draw the graphs of $y = 8 - x$ and $2x + 3y = 23$ on the same number plane.
b Find the point of intersection of the two lines.
c Hence write the solution to the simultaneous equations $y = 8 - x$ and $2x + 3y = 23$.

5 The cost (\$$C$) of electricity to run a home is \$200 per month. This may be written as $C = 200m$, where m is the number of months. For the same house, the cost of installing a solar system is \$4000, but the monthly cost of electricity decreases to \$150. The total cost of electricity using a solar system is then $C = 4000 + 150m$.
a Use graphing technology or complete the following tables to graph each equation on the same number plane.

$C = 200m$

m	0	50	100
C			

$C = 4000 + 150m$

m	0	50	100
C			

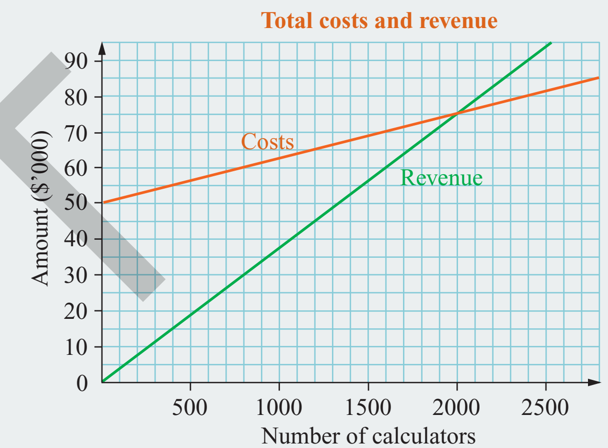
b Find the simultaneous solution of the linear equations.
c After installing the solar system, how long would it take to start saving money?

REVIEW PRACTICE EXAMINATION QUESTION

1 **a** **i** Complete this table and graph $y = 3 - 2x$. (2 marks)

x	−3	−2	−1	0	1	2	3
y							

ii What is the gradient of the line? (1 mark)
iii What is the y-intercept of the line? (1 mark)
b The graph shows the total costs and revenue for a manufacturer of calculators.



i Find the total cost to the manufacturer of producing 1000 calculators. (1 mark)
ii Will the manufacturer make a profit when 1000 calculators are produced?
Give reasons for your answers. (1 mark)
iii How many calculators must be produced for the manufacturer to break even? (1 mark)
iv How many calculators must be produced to make a profit of \$10 000? (1 mark)
v What is the initial set-up cost to the manufacturer? (1 mark)
vi What is the cost to produce each calculator, after the initial set-up costs? (1 mark)
c The Creative Cake Company makes cupcakes. The cost of production is \$200 per day and the variable production cost is \$2.50 per cupcake. Cupcakes are sold for \$7.50 each. Maximum daily production is 100 cupcakes.
i Write an equation for the cost \$$C$ of producing n cupcakes. (1 mark)
ii Write an equation for the revenue \$$R$ of selling n cupcakes. (1 mark)
iii Graph both equations for $0 \le n \le 100$ on the same set of axes. (1 mark)
iv How many cupcakes must be sold to break even? (1 mark)
v If all 100 cupcakes are sold, how much profit is made? (1 mark)

TOTAL:
15 marks

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