# Simultaneous equations

This chapter deals with solving simultaneous equations. After completing this chapter you should be able to:

- solve simultaneous equations using trial and error
- solve simultaneous equations using graphical methods
- solve simultaneous equations algebraically using substitution and elimination methods
- solve simple word problems resulting in simultaneous equations.

NSW Syllabus references: 5.2 N&A Equations Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-3WM, MA5.2-8NA NUMBER & ALGEBRA - ACMNA237, ACMNA238

# Diagnostic test

- 1 By inspection, the value of the pronumeral in x + 9 = 10 is:
  - **A** 19 **B** -1 **C** 1 **D** -19
- 2 The option that shows how the expression 2x 3 is built is:



**3** The equation 3p + 18 = 24 has been solved using backtracking techniques.

$$p \xrightarrow{\times 3} 3p \xrightarrow{+18} 3p + 18$$

$$ii \xrightarrow{\div 3} i \xrightarrow{-18} 24$$

The values of parts **i** and **ii** using this technique are:

A i	42	ii	14
Bi	3	ii	1
Ci	42	ii	12
Di	6	ii	2

- 4 The value of T in the equation 3T = -9 is:
  - **A** -3 **B** 0 **C**  $\frac{1}{3}$



- 6 The value of m in the equation 3m - 9 = 5m + 5 is: A 2 B -7 C -2 D 7
- 7 When x = 7, y = -3 and z = 0, the value of the expression z(4x 2y) is:

**D** 0

8 When x = 0, y = 3 and z = -3, the value of the expression z(4x - 2y) is:

- **A** 18 **B** 58 **C** 127 **D** 156
- **9** Ten is added to four times a certain number and the result is 18.

An equation using x as a certain number is: **A** 40 + x = 18 **B** 4x + 10 = 18 **C** 4(x + 10) = 18 **D** 40x + 10 = 18

**10** The graph that satisfies the inequality  $a - 6 \le -3$  is:

A	-	0	4	1	$1\frac{4}{5}$	2	a
B	<b>▲</b>	-2	-1	0	1	2	3 a
С	0	1	2	3	4	5	6 a
D	-1	I	0	I	1 1	$\frac{1}{4}$	2 a

The diagnostic test questions refer to outcomes ACMNA234, ACMNA235, ACMNA236 and ACMNA240.

**D** 3

# Linear equations review

Linear equations are equations of the form (or can be simplified to the form) ax + b = 0, where *a* and *b* are constants and *x* is the unknown (or variable).



## **Exercise 6A**

A

1	Solve for $x$ in the following equations.			
	<b>a</b> $x + 3 = 10$ <b>b</b> $3x = -9$	<b>c</b> $3x + 6 = 0$	<b>d</b> $3x - 4 = -6$	<b>e</b> $5x + 8 = 2$
	<b>f</b> $8x - 6 = 10$ <b>g</b> $6 + 7x = -2$	<b>h</b> $6x - 7 = -1$	<b>i</b> $6 - x = -5$	<b>j</b> $3 - 2x = 7$
	<b>k</b> $3 - 7x = -2$ <b>l</b> $11 = 3 - 2x$	<b>m</b> $8 = 3 - 2x$	<b>n</b> $-15 = 3 - 6x$	



**2** Check the given solution by substitution and state whether or not it is correct.

<b>a</b> $2x + 8 = 15$	(x = 7)	<b>b</b> $7 + 5x = 9$	(x = 2)
<b>c</b> $-15 = 6 - 7x$	(x = 3)	<b>d</b> $\frac{x}{5} - 3 = 6$	$(x = \frac{9}{5})$

#### • EXAMPLE 2

Given that y = 5x - 3, find x when y = -18.

Solve	Think	Apply
y = 5x - 3 -18 = 5x - 3 -18 + 3 = 5x - 3 + 3 -15 = 5x $\frac{-15}{5} = \frac{5x}{5}$	Substitute $y = -18$ . Add 3 to both sides. Divide both sides by 5.	Often when substituting and solving an equation, the pronumeral is on the right- hand side. Solve as normal and then write the pronumeral
-3 = x $x = -3$		on the left-hand side.

**3** a Given that 
$$y = 3x - 5$$
, find x when  $y = 5$ .

**c** Given that 
$$y = 7 - 5x$$
, find x when  $y = 0$ .

e Given that 
$$y = 5 - 7x$$
, find x when  $y = -5$ 

EXAMPLE 3

Solve the following equations. **a** 5x + 2 = 3x - 5

15 - 2x = 11 + x

**b** Given that y = 4x + 2, find x when y = 11. **d** Given that y = 4 - 3x, find x when y = -3. **f** Given that y = 3x - 5, find x when y = 8.

	Solve	Think	Apply
a	5x + 2 = 3x - 5		Eliminate the pronumeral from
	5x + 2 - 3x = 3x - 5 - 3x	Subtract $3x$ from both sides.	one side of the equation by
	2x + 2 = -5 2x + 2 - 2 = -5 - 2	Subtract 2 from both sides	adding or subtracting one of the pronumeral terms
	2x + 2 $2z = -72x = -7$	Subtract 2 from both sides.	Solve the resulting equation
	$\frac{2x}{2} = \frac{-7}{2}$	Divide both sides by 2.	in the same way as in the
	2 2		previous example.
	$x = -\frac{1}{2} = -3\frac{1}{2}$		
b	15 - 2x = 11 + x		
	15 - 2x + 2x = 11 + x + 2x	Add $2x$ to both sides.	
	15 = 11 + 3x $15 = 11 - 11 + 3x = 11$	Subtract 11 from both sides	
	4 = 3x	Subtract 11 from obth sides.	
	$\frac{4}{3} = \frac{3x}{3}$	Divide both sides by 3.	
	3 3 4		
	$\overline{3} = x$	Swap the pronumeral to the	
	$x = 1\frac{1}{3}$	icit-italia side.	

**4** Solve the following equations with integer solutions.

<b>U</b>	•	
<b>a</b> $5x + 2 = 2x + 14$	<b>b</b> $3x + 7 = 11 - x$	<b>c</b> $5 + x = 8 - 2x$
<b>d</b> $3x - 4 = 5x - 2$	<b>e</b> $3 - x = x + 7$	<b>f</b> $4 - 2x = 3 - x$
<b>g</b> $2x - 3 = x + 6$	<b>h</b> $5x - 9 = 1 + 6x$	i $3x - 5 = 7 - x$
Solve the following equations.		
<b>a</b> $8x + 7 = 4x - 2$	<b>b</b> $7x + 3 = 2x + 7$	<b>c</b> $5 + 2x = 11 - x$
<b>d</b> $x - 3 = 5x + 7$	e $3 + x = 17 + 4x$	<b>f</b> $15 - 3x = 2 - x$

d	x - 3 = 5x + 7	<b>e</b> $3 + x = 17 + 4x$	f	15 - 3x = 2 - x
g	2x + 5 = 9 - 2x	<b>h</b> $3x - 5 = 5x + 9$	i	5 - 7x = 3x + 2
j	5a + 3 = a - 1	<b>k</b> $4 - 3s = 2s + 17$	l	9x - 4 = 3 + 4x
m	11a - 7 = 5a + 12	<b>n</b> $3y - 5 = -14 - 2y$	0	7p = 15 - 3p

### EXAMPLE 4

By substituting, check the solutions to the following equations.

$$2x - 5 = 10 - 3x$$
 (*x* = 3)

5

a

**b** 5x + 2 = 2x - 7

(x = 2)

	Solve	Think	Apply
a	Does $2x - 5 = 10 - 3x$ when $x = 3$ ? LHS: $2 \times 3 - 5 = 1$ RHS: $10 - 3 \times 3 = 1$ LHS = RHS $\therefore x = 3$ is the solution.	Substitute 3 for x on both sides of the equation. Left-hand side = 1 Right-hand side = 1 x = 3 is a solution.	Substitute the value of $x$ and evaluate both sides of the equation. Both sides must give the same value for that value of $x$ to be a
b	Does $5x + 2 = 2x - 7$ when $x = 2$ ? LHS: $5 \times 2 + 2 = 12$ RHS: $2 \times 2 - 7 = -3$ $12 \neq -3$ $\therefore x = 2$ is not the solution.	Substitute 2 for x on both sides of the equation. Left-hand side = 12 Right-hand side = $-3$ This is not a solution.	solution. The actual value of the sides is not relevant.

6 By substituting, check the solutions to the following equations.

a	3x + 9 = 4 + 2x	(x = 1)	b	9a+2=7a-4	(a = -3)
c	7a - 5 = 3 - a	(a = 2)	d	15 - 2x = 6 + x	(x = 3)
e	2x - 3 = 7 - 4x	$(x=\frac{5}{3})$	f	5x - 7 = 3 + x	$(x = 3\frac{1}{2})$

### - EXAMPLE 5

Solve 5(x + 1) - 2x = 7.

Solve	Think	Apply
5(x + 1) - 2x = 7 5x + 5 - 2x = 7 3x + 5 = 7 3x + 5 - 5 = 7 - 5	Expand the brackets. Collect the like terms. Subtract 5 from both sides.	The number in front of the brackets is multiplied by each term within the brackets.
$3x = 2$ $x = \frac{2}{3}$	Divide both sides by 3.	

7 Solve for *x* in each equation.

- **a** 2(x+1) 1 = 8
- **c** 3(x+2) 7 = 11
- **e** 4(2x-1) + 7 = 0
- **g** 3 2(x + 1) = -4

#### **EXAMPLE 6**

- **b** 5(1 3x) = -4 **d** 2(x + 1) + 3(x - 1) = 6 **f** 11 - 2(x - 1) = 7**h** 7 - (2 - x) = 2x
- Given that y = 3 5(x + 4), find x when y = -32.

Solve	Think	Apply
y = 3 - 5(x + 4)		Substitute the value, simplify
-32 = 3 - 5(x + 4)	Substitute $y = -32$ .	both sides if possible, then solve
= 3 - 5x - 20	Expand.	the equation. The pronumeral is
-32 = -17 - 5x	Collect like terms.	often on the right-hand side of
-32 + 17 = -17 - 5x + 17	Add 17 to both sides.	the equation.
-15 = -5x		The most common error is to
-15 - 5x	Divide both sides by $-5$ .	multiply the second term in the
-5 -5		brackets incorrectly.
3 = x		Be vigilant with the signs.
x = 3		

- 8 a Given that y = 7 3(x + 2), find x when y = -5.
  - **b** Given that y = 5 4(x 3), find x when y = 37.
  - **c** Given that y = 4 5(2x 5), find x when y = 12
  - **d** Given that y = 14 3(2x 8), find x when y = 0.
  - e Given that y = 3x 2(5x + 1), find x when y = -16.
  - f Given that y = 4x 3(5 2x), find x when y = 8.
  - g Given that y = 3(2x 1) 4(x + 2), find x when y = -3.
  - h Given that y = 4(1 3x) 2(1 x), find x when y = 2.



# **B** Simultaneous equations

Consider the following problem. Sarah bought five pieces of fruit each of which was either an apple or a banana. The apples cost 20 cents each and the bananas cost 25 cents each. The total cost of the fruit was \$1.05. How many apples and bananas did Sarah buy?

One method of solving this problem is to introduce two unknowns and set up two equations that need to be solved simultaneously. We will solve the problem using this approach.

Suppose that there are x apples and y bananas.

Then x + y = 5 (as there are 5 pieces of fruit).

The apples cost 20 cents each, so the total cost for x apples is 20x.

Similarly, the cost for *y* bananas is 25*y*.

As the total cost is \$1.05 or 105 cents, 20x + 25y = 105.

For the equation x + y = 5, the possible values for x or y are 0, 1, 2, 3, 4 or 5, as shown in the table.

x	У	x + y	20x + 25y
0	5	5	125
1	4	5	120
2	3	5	115
3	2	5	110
4	1	5	105
5	0	5	115

This possibility satisfies both equations.

Hence, the solution is 4 apples and 1 banana.

### **Trial and error**

### EXAMPLE 1

Complete a table to find the simultaneous solution of x + y = 6 and 3x + 2y = 16, given that x and y are integers.

		Solve		Think	Apply
x 0 1 2 3	<b>y</b> 6 5 4 3	x + y 6 6 6 6	3x + 2y       12       13       14       15	List the possible values for x. As $x + y = 6$ , fill in the values of y. Next calculate the values of 3x + 2y. Keep going until the value 16 is found in the 3x + 2y column.	The <i>x</i> - and <i>y</i> -values must satisfy both equations. There is only one pair of values that will satisfy both equations.
4	2	6	16		
5	1	6	17		
6	0	6	18		
The solu	ution is <i>x</i>	x = 4 and $y$	= 2.		

## **Exercise 6B**

1 Complete this table to find the simultaneous solution of x + y = 5 and 5x + 3y = 19. Remember to look for 19 in the 5x + 3y column. Complete the *y* column first.

x	У	x + y	5x + 3y
0		5	
1		5	
2		5	
3	2	5	
4		5	
5		5	

2 Complete this table to find the simultaneous solution of x + y = 7 and 3x - 2y = 11.



#### EXAMPLE 2

By completing a table, find the integers x and y that satisfy the equations y = x + 3 and 5x - 2y = 6 simultaneously.

	Solve		Think	Apply
x 0 1 2 3 4	y = x + 3 $y = 0 + 3 = 3$ $y = 1 + 3 = 4$ $y = 2 + 3 = 5$ $y = 3 + 3 = 6$ $y = 4 + 3 = 7$	5x - 2y 5(0) - 2(3) = -6 5(1) - 2(4) = -3 5(2) - 2(5) = 0 5(3) - 2(6) = 3 5(4) - 2(7) = 6	In this example y is 3 more than x. So start with $x = 0$ and $y = 3$ . Fill in the values for $5x - 2y$ . Look for 6 in the last column.	The <i>x</i> - and <i>y</i> -values must satisfy both equations. There is only one pair of values that will satisfy both equations.
5	y = 5 + 3 = 8	5(5) - 2(8) = 9		
The so	olution is $x = 4$ and $y$	· = 7.		

3 Complete this table to find the integers x and y that satisfy the equations y = 5 - xand 3x - 2y = 10 simultaneously. The solution is  $x = \_$  and  $y = \_$ .

x	y=5-x	3x-2y
0	y = 5 - 0 = 5	3(0) - 2(5) =
1	y = 5 - 1 =	3(1) – 2() =
2		
3		
4		
5		3(5) - 2(0) = 15

4 By completing a table as in Examples 1 and 2, find integers *x* and *y* that satisfy these equations simultaneously.

**a** x + y = 4 3x + 5y = 14 **b** x + y = 11 4x + 3y = 40 **c** y = x + 2 9x - 4y = 7 **d** y = 6 + x8x - 3y = -3 **EXAMPLE 3** 

Check if the possible solution x = 5 and y = 3 is correct for the pair of simultaneous equations x - y = 2 and 3x + 4y = 17.

x - y = 2Substitute $x = 5$ and $y = 3$ Substitute the values for x $5 - 3 = 2$ This is correct.into each equation.and y into each equation. $3x + 4y = 17$ This is not correct.This is not correct.This is not correct. $3(5) + 4(3) = 27$ This is not correct.both equations.	Solve		Τ	"hink	Apply
x - 3 and $y - 5$ is not a solution.	x - y = 2 5 - 3 = 2 TI 3x + 4y = 17 3(5) + 4(3) = 27 TI ∴ x = 5 and y = 3 is not	his is correct. his is not correct. a solution.	Substitute x into each eq	= 5  and  y = 3 uation.	Substitute the values for $x$ and $y$ into each equation. The solution must satisfy both equations.

5 Complete to check if the possible solution x = 7 and y = 3 is correct for this pair of simultaneous equations. x - y = 4 and 3x - 8y = -3

x - y = 4Test: 7 - \_ = 4 3x - 8y = -3Test: 3(7) - 8(\_) = \_\_\_\_\_  $\therefore x = 7$  and y = 3 is \_\_\_\_\_.

**6** A possible solution is given for each pair of simultaneous equations. Check if the solution is correct.

 $a \quad x - y = 3$   $b \quad x + y = 9$  

 2x - y = 11 2x - y = 6 

 Solution: x = 5, y = 2 Solution: x = 5, y = 4 

  $c \quad a + b = 2$   $d \quad 2p + q = 7$  

 a - b = 8 3p + 2q = 10 

 Solution: a = 5, b = -3 Solution: p = 4, q = -1 

- **7** Find by trial and error the simultaneous solution of the following equation pairs, given that the solutions are integers.
  - **a** x + y = 4 2x - y = 5 **b** x + y = 6 2x + y = 10 **c** a - b = 1 2a + 3b = 2 **d** p - q = 35p + 2q = 29

# **C** Graphical solutions

Equations such as x + y = 9 and 2x + 3y = 21 have linear (straight-line) graphs. If the two straight-line graphs are drawn, their point of intersection is the simultaneous solution.

In this section linear graphs are used to solve simultaneous equations.

#### EXAMPLE 1

Graph y = 9 - x and 2x + 3y = 21 to find their point of intersection.



### **EXAMPLE 2**

**a** 1

Solve these equations simultaneously by drawing the graphs of $2x + 3y - 7$ and $3x - 2y11$ .					
Solve	Think	Apply			
2x + 3y = 7 2(0) + 3y = 7 3y = 7 $y = 2\frac{1}{3}$	Substitute $x = 0$ to find the <i>y</i> -intercept.	The solution to simultaneous equations can have positive or negative values.			
2x + 3y = 7 2x + 3(0) = 7 2x = 7 $x = 3\frac{1}{2}$ 5x - 2y = -11	Substitute $y = 0$ to find the <i>x</i> -intercept.	have to be integers but they are in this section to make calculations easier.			
5(0) - 2y = -11	Substitute $x = 0$ to find the				
-2y = -11	y-intercept.				
$y = 5\frac{1}{2}$	Substitute $y = 0$ to find the				
5x = 2(0) = -11 5x = -11	x-intercept.				
$x = -2\frac{1}{5}$					
Graph both lines and find the point of	Graph these points and draw the				
intersection.	lines.				
y A	The solution is $x = -1, y = 3$ .				
2x + 3y = 7 $5x - 2y = -11$	Check by substituting:				
6	2x + 3y = 2(-1) + 3(3)				
	= 7 5x - 2y = 5(-1) - 2(3)				
	= -11				
	The solution is correct.				
-6 -4 /2 2 4 6 x					
The solution is $x = -1, y = 3$ .					

# **Exercise 6C**

**1** a Use these tables of values to graph 2x - 3y = -7 and 4x + y = 7. 4x + y = 7

2x - 3y = -7

x	0	2	4
у	$2\frac{1}{3}$	$3\frac{2}{3}$	5

x	0	2	4
у	7	-1	-9

- **b** Find the point of intersection of the lines 2x 3y = -7 and 4x + y = 7.
- **c** Check that your solution from part **b** is correct.



**2** a Graph 3x - 2y = 8 and 2x + 5y = 18 using these tables of values.

3x - 2y = 8

x	0	3	6
у	-4	$\frac{1}{2}$	5

2x + 5y = 18					
x	0	4	9		
у	$3\frac{3}{5}$	2	0		

**c** 4x + 3y = 10

**f** 2x + y = 4

2x - y = 8i x - 2y = 4

2x - 4y = 8

x - 2y = -3

- **b** Find the point of intersection of these two graphs.
- **c** Check that the point from part **b** is the solution to both equations.
- **3** Use graphical methods to solve these equations simultaneously.
  - **a** y = x 3
- $\begin{array}{l} \mathbf{b} \quad x y = 1\\ y = 2x \end{array}$

**e** y = 3x - 8

**h** x - 3y = -9

3x + y = -2

2x - 3y = -12

- y = 1 x**d** 3x + y = 5
- 2x 3v = 18
- **g** 3x + y = -5
- 2x + 3y = -8

# Investigation 1 Graphics calculator

- 1 Use the graphics calculator to sketch the graphs of y = 2x + 3 and y = 8 - 3x. Find their point of intersection. These instructions are for a Casio fx-9860G AU series.
  - Step 1: Select GRAPH from the MAIN MENU.
  - Step 2: Press (F3) for graph type and choose Y = by pressing (F1).
  - Step 3: To enter the equation y = 2x + 3, press 2 (X,0,T) + 3 (the (X,0,T) key is used to enter x).

Press EXE to enter the equation. To make the calculator sketch the graph, press F6, the DRAW command. Press EXIT to return.

Step 4: To enter the equation y = 8 - 3x, press 8 - 3 X.01 (to enter x).

> Press EXE to enter the equation. To make the calculator sketch the graph, press F6, the DRAW command.

- Step 5: Both graphs now appear on the screen. The point of intersection is in the first quadrant. Use the up arrow if necessary to view.
- *Step 6:* To find the coordinates of the point of intersection use the trace function.

Press TRACE **F1** and a cursor will appear at the left of the graph entered second: y = 8 - 3x. Use the left and right arrows to move the cursor to the point of intersection. The correct solution is x = 1 and y = 5.

The coordinates are only approximate due to the accuracy of the calculator.



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**2** Use the graphics calculator to sketch the graphs in Exercise 6C.

The equations must have y as their subject. If not, then they must be rearranged. For example: 2x + 3y = 7

becomes 2x + 3y - 2x = 7 - 2x

$$3y = 7 - 2x$$
$$\frac{3y}{3} = \frac{7 - 2x}{3}$$
$$y = \frac{7 - 2x}{3}$$

Subtract 2x from both sides.

Divide both sides by 3.

D Solution by substitution

Simultaneous equations can be solved algebraically. The first method is by **substitution**. One variable is substituted into the other equation so that the expression is reduced to an equation with only one variable. The method of solution by substitution is often used when at least one of the given equations has either x or y as the subject.

As in Example 1 below, we can use substitution to solve y = 2x + 3 and 3x - 4y = 8, as the first equation has y as the subject. This method involves replacing y in the second equation with the value of y in the first equation (in this example by 2x + 3). The result is an equation containing only one unknown (x in this case).

#### EXAMPLE 1

Solve these equations simultaneously using substitution.

**a** y = 2x + 3 and 3x - 4y = 8

**b** y = 3x - 5 and y = 10 - 2x

	Solve	Think	Apply
a	Solve y = 2x + 3  (1) $3x - 4y = 8  (2)$ $3x - 4(2x + 3) = 8$ $3x - 8x - 12 = 8$ $-5x - 12 = 8$ $-5x = 20$ $x = -4$ $y = 2(-4) + 3$	Substitute $x = -4$ into equation (1)	ApplyThis method is most appropriate when one or both of the equations have 
b	y = 2(-4) + 3 = -8 + 3 = -5 The solution is x = -4 and y = -5. y = 3x - 5 1 y = 10 - 2x 2 3x - 5 = 10 - 2x 5x - 5 = 10	As both equations have <i>y</i> as the subject, put them equal to each other. Solve the resulting equation for <i>x</i> .	solution into both equations to check it is correct. This is the only method for solving simultaneous
	5x = 15 x = 3 y = 3(3) - 5 = 9 - 5 = 4 The solution is $x = 3$ and $y = 4$ .	Substitute $x = 3$ into equation (1) to obtain the <i>y</i> -value.	equations when one equation is not linear.

### **Exercise 6D**



# **E** Solution by elimination

A second method of solving simultaneous equations algebraically is by **elimination**. Solution by elimination is used for simultaneous equations such as 3x + 2y = -2 and 5x - y = 27. We make the coefficients of either x or y equal and then add (or subtract) the resulting equations.

There is always a choice as to whether we eliminate x or y. However, our choice is usually determined by the ease of eliminating one of the variables.

If we make the coefficients of one of the variables equal in size but opposite in sign, we can add these terms to eliminate them.

### **EXAMPLE 1**

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Solve 3x + 2y = -2 and 5x - y = 27 using the elimination method.

Solve	Think	Apply
3x + 2y = -2 (1) 5x - y = 27 (2) 3x + 2y = -2 10x - 2y = 54 13x = 52 x = 4 3(4) + 2y = -2 12 + 2y = -2 2y = -14 y = -7 The solution is $x = 4$ and $y = -7$ .	Label the equations 1 and 2. Multiply both sides of equation 2 by 2. Add the equations and solve for x. Substitute $x = 4$ into equation 1. <i>Check by substituting:</i> In 1: $3x + 2y = 3(4) + 2(-7)$ = 12 - 14 = -2 In 2: $5x - y = 5(4) - (-7)$ = 20 + 7 = 27	In this case only one equation had to be changed to be able to eliminate a variable. Often both equations must be multiplied by different numbers, as in the next example.

### EXAMPLE 2

Solve 5x + 3y = 12 and 7x + 2y = 19 using the elimination method.

Solvo	Think	Annhy
Solve	ТШІК	Арріу
5x + 3y = 12 (1)		In this case both
7x + 2y = 19 (2)	To eliminate the y variable:	equations must be
10x + 6y = 24	• Multiply equation (1) by 2.	multiplied by different
-21x - 6y = -57	• Multiply equation (2) by $-3$ .	numbers to be able to
$\frac{-11x = -33}{-11x = -33}$	• Add to eliminate <i>v</i> .	eliminate a variable.
x = 3	Solve for <i>x</i> .	
5(3) + 3v = 12	Substitute $x = 3$ into (1).	
15 + 3y = 12		
3y = -3		
y = -1		
Or eliminate the <i>x</i> variable.	To eliminate the <i>x</i> variable:	
35x + 21y = 84	• Multiply equation $1$ by 7.	
-35x - 10y = -95	• Multiply equation (2) by $-5$ .	
$\frac{11}{11} = -11$	• Add to eliminate <i>x</i> .	
v = -1	y = -1 and $x = 3$ by substitution.	
The solution is $x = 3$ and $y = -1$ .		

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## **Exercise 6E**

**1** Write the equation that results when: **a** 2x - 3y = 5 is multiplied by 2 **b** x + 3y = 7 is multiplied by -3c 2x + 5y = 1 is multiplied by 4 **d** 3x - 2y = 8 is multiplied by -2e 5x - y = 2 is multiplied by 5 f -2x + 5y = -1 is multiplied by -1**2** What equation results when the following pairs of equations are added? **b** 3x - y = 8**a** 3x + 2y = 6**c** x + y = 5x - 2y = 10x + y = 8x - y = 7**e** 5x - v = 6**f** -8x + 2y = 11**d** 3x - v = 4-3x + 4y = 2-5x + 3y = -88x - 3y = -7**3** Complete to solve 3x + 2y = 4 and 2x + y = 1 using the elimination method. 3x + 2y = 4(1)2x + y = 1(2)To eliminate y multiply equation (2) by \_ 3x + 2y = 4-4x - 2y = -2= 2 Add to eliminate v. x = -2Substitute  $x = \_$  into equation (1).  $3(\_) + 2y = 4$ + \_\_\_\_ + 2y = 4 + \_\_\_\_ Add to both sides. 2y =\_\_\_\_  $v = _{---}$ The solution is x = -2 and y = -2. **4** Solve the following equations simultaneously using the elimination method. **a** 3x + y = 13**b** 2x - y = 8x - y = 33x + y = 73x+4= d 5x + 2v = -19c x + 3y = 132 -x + y = 73x - 4y = -1**f** 2x + y = 1**e** 2x + 3y = 114 x x + 3y = -127x - y = 50X **g** 4x + y = 19**h** 7x + 2y = -53x + 4y = -23x - 5y = -49i 6x + 5y = -24x - 3y = 12-x + 5y = -33x - y = 13**k** 3x + 2y = 73x + 7y = 478x + 7y = 127x + 3y = 43**m** 2x + 7v = -51**n** 3x + y = -102x - y = 53x - 2y = 11**o** 2x - 7y = 145x - 7v = 34

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# F Problem solving with two unknowns

Many problems can be described in terms of two unknowns or variables, and two linear equations can be formed. These linear equations can be solved by completing tables and drawing the two lines, algebraically or by using a graphics calculator to find the point of intersection.

To solve these problems:

- 1 Call the unknowns *x* and *y*. Do not forget the units used.
- 2 Form two equations in x and y.
- 3 Solve the equations using any method.
- 4 Check your solution with the original information in the question.
- 5 Give your answer in sentence form.

### EXAMPLE 1

Two numbers have a sum of 31 and a difference of 5. Find the numbers.

Solve	Think	Apply
Let $x$ and $y$ be the unknown	Sum means add.	Define the variables and
numbers, where $x > y$ .	Difference means subtract.	form two equations.
Then $x + y = 31$ (1)		Choose the most
and $x - y = 5$ (2)		appropriate method to
2x = 36	Add to eliminate y.	solve.
x = 18		
18 + y = 31	Substitute x into equation $(1)$ to find y.	
y = 13	<i>Check:</i> $18 + 13 = 31$	
The numbers are 18 and 13	18 - 13 = 5	

### Exercise 6

1 Two numbers have a sum of 201 and a difference of 37. Form two simultaneous equations in x and y. Complete to solve the problem algebraically and find the numbers.



- **2** Find two equations in *x* and *y* and solve algebraically.
  - a The difference between two numbers is 84 and their sum is 278. What are the numbers?
  - **b** One number exceeds another by 11. The sum of the two numbers is 5. What are the numbers?
  - **c** The larger of two numbers is four times the smaller number. The sum of the two numbers is 85. What are the numbers?
  - **d** Find two integers such that three times the smaller is 33 more than twice the larger, and twice the smaller plus five times the larger equals 250.
- **3** Find two equations in *x* and *y*, draw their graphs and solve graphically.
  - **a** Find two numbers with a sum of 12 and a difference of 2.
  - **b** Find two numbers with a sum of 20 and a difference of 4.
  - **c** The sum of two numbers is 10. When one number is added to twice the other number the result is 16. Find the numbers.
  - **d** The sum of two numbers is 14. When one number is added to twice the other number the result is 25. Find the numbers.
  - e Find two numbers with a sum of 12 and with half their difference equal to 2.

### EXAMPLE 2

Five apples and 4 bananas cost a total of \$2.05, and 6 apples and 5 bananas cost of total of \$2.50. Find the cost of an apple and the cost of a banana.

Solve	Think	Apply
$5x + 4y = 205 \qquad \qquad$	Write the equations with the totals	Ensure that the units
6x + 5y = 250 (2)	in cents.	are consistent in the
25x + 20y = 1025	Multiply equation 1 by 5.	two equations.
-24x - 20y = -1000	Multiply equation $2$ by $-4$ .	
x = 25	Add to eliminate y.	
5(25) + 4y = 205	Substitute $x = 25$ into equation (1).	
125 + 4y = 205		
4y = 80		
y = 20		
An apple costs 25 cents and a banana		
costs 20 cents.		



4 Two hammers and one screwdriver cost a total of \$59, and one hammer and three screwdrivers cost a total of \$42. Complete to form simultaneous equations to find the price of each tool. Let the number of hammers be *h* and the number of screwdrivers be *d*.

Then  $2h + \underline{\phantom{a}} = 59$  (1) and  $h + \underline{\phantom{a}} = 42$  (2) Multiply equation (2) by 2 to eliminate  $\underline{\phantom{a}}$ . 2h + d = 59 (1)

$$2h + \underline{\qquad} = \underline{\qquad}$$
$$-5d = \underline{\qquad}$$
$$d = 5$$

Substitute \_\_\_\_\_ into equation (2).  $h + \___ = 42$ 

$$h = \_$$

The hammer costs \_\_\_\_\_ and the screwdriver costs \_\_\_\_\_

- **5** Form simultaneous equations and solve using any method.
  - a Theatre tickets for four adults and three children cost \$74. Theatre tickets for two adults and five children cost \$58. Calculate the price of an adult's ticket and a child's ticket.

(2)

- b Three blankets and a sheet cost \$190 altogether, and two sheets and a blanket cost a total of \$100. Calculate the cost of one blanket and one sheet.
- c Seven apples and eight oranges cost \$4.06 altogether. At the same shop three apples and one orange cost of total of \$1.06. Calculate the cost of an apple and the cost of an orange.
- **d** A purse contains \$3.75 in 5-cent and 20-cent coins. If there are 33 coins altogether, how many of each type of coin are in the purse?
- e I collect only 50-cent and \$1 coins. My collection consists of 43 coins and their total value is \$35. How many of each coin type do I have?

#### Extension

- 6 A particular brand of milk is sold in either 1 L or 2 L cartons. If a delicatessen owner orders 120 L of milk and receives 97 cartons, how many cartons of each size did they receive?
- 7 A yard holds rabbits and pheasants only. There are 35 heads and 98 feet in the yard. How many rabbits and pheasants are in the yard?





8 A baker's basket contains bread rolls and sandwich loaves. The loaves weigh 750 g and the rolls 150 g. If the total number of loaves and rolls is 16 and their total weight is 6 kg, how many rolls and how many loaves are in the basket?



- **10** An equilateral triangle has sides of length (3x y) cm, (x + 5) cm and (y + 3) cm. Find the length of each side.
- 11 *KLM* is an isosceles triangle. Find *x* and *y* and hence find the measure of the angle at *K*.
- r
- 12 Twelve years ago Jane was five times as old as Anne. In 3 years time Anne will be half Jane's age. How old is each girl at the moment?
- 13 Nine years ago a mother was three times as old as her son. In eight years from now the sum of their ages will be 78. How old are they today?
- 14 The weekly wages of Sam and Ben are in the ratio of 2 : 1. Their living costs are in the ratio of 9 : 4. If each saves \$100 per week, find their weekly incomes.

# **Investigation 2** Simultaneous equations solutions

- **a** Graph the equations y = 2x + 1 and y = 2x + 3.
  - **b** What is the solution when y = 2x + 1 and y = 2x + 3 are solved simultaneously? Why?
  - c Solve the equations simultaneously using the elimination method. What happens?
  - d Write a pair of simultaneous equations that have no solution.
- **2** a Graph the equations x + 2y = 5 and 2x + 4y = 10. What do you notice?
  - **b** How many solutions are there when x + 2y = 5 and 2x + 4y = 10 are solved simultaneously?
  - c Solve x + 2y = 5 and 2x + 4y = 10 simultaneously using the elimination method. What happens?
  - d Write a pair of simultaneous equations that have an infinite number of solutions.
- **3** Write an equation that when solved simultaneously with y = 3x + 2 gives:
  - a 0 solutions b 1 solution c an infinite number of solutions.
- **4** Write three pairs of linear equations that when solved simultaneously have:
  - a 0 solutionsb 1 solutionc an infinite number of solutions.Have your equations checked.

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### Language in mathematics

### Albert Einstein (1879–1955)

Albert Einstein was born in Ulm, Germany. His family moved to Munich where Einstein started his education at a school that instilled a very rigid discipline.

Young Albert did not take well to strict regimentation and showed very little academic enthusiasm and ability. His family moved to Milan, Italy, leaving him behind in Munich to continue his schooling. Einstein left school with poor grades in a number of subjects. It was said that the reason he could not cope was because of the stifling education system, but in fact he could not bear to be away from the rest of his family.

A few years later he recommenced studying in Switzerland and finally spent four years at the Polytechnic Academy in Zurich concentrating on Physics. His first position after graduating was as a mathematics teacher, and he later worked as an examiner in the patents office.

In later years he proved to be one of the greatest thinkers of all time. He offered explanations for many observed physical phenomena that were previously unexplained, and predicted many useful results through mathematics that ultimately led to new discoveries. Einstein published three great papers that changed theoretical physics. Einstein deduced the equation  $E = mc^2$  where *E* represents energy, *m* mass and *c* the velocity of light, in 1905 in a paper on the special theory of relativity. This equation indicated the enormous energy content of small masses, and the validity of his deduction was demonstrated in 1945 by the enormous energy in atomic explosions.

In 1933 Einstein renounced his German citizenship and moved to Princeton in the USA. He spent the last 20 years of his life working there, serving as in inspiration to other scientists around him, and campaigning for the international control of atomic power.

- **1 a** How old would Albert Einstein be today?
  - **b** Why did Einstein have difficulty at school?
  - c How can so much energy be released from something as small as an atom?
  - **d** Einstein believed that imagination is more important than knowledge. What do you think he might have meant by this?
- 2 Insert vowels to complete these terms.

<b>a</b> s_1_tn	<b>b</b> s _ m _ lt _ n s	c	$_q_{-}t_{-}n$
$\mathbf{d}  \mathbf{v} \mathbf{r} \mathbf{r} \mathbf{b} \mathbf{l}$	<b>a</b> $s_bst_t_t_n$	f	s_lv_

**3** Write a paragraph describing the methods of solving simultaneous equations.

### Terms

co: sol	nsecutive ution	elimination solve	equation substitution	linear unknown	pronumeral value	simultaneous variable
C	heck your	skills				
1	The solution to <b>A</b> $x = -\frac{1}{3}$	3x - 5 = -6 is: <b>B</b> $x =$	$\frac{1}{3}$	<b>C</b> $x = -\frac{11}{3}$	<b>D</b> $x = \frac{11}{3}$	
2	Given $y = 7 - 1$ <b>A</b> $7\frac{1}{2}$	2x  and  y = -8, x <b>B</b> $-\frac{1}{2}$	equals:	$\mathbf{C} \frac{1}{2}$	<b>D</b> $-7\frac{1}{2}$	

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- **3** The solution to 8 5x = 2x + 3 is: **A**  $x = \frac{7}{5}$  **B**  $x = \frac{5}{7}$  **C**  $x = \frac{5}{3}$  **D**  $x = -\frac{5}{3}$
- **4** x = -2 is not a solution of: **A** 3x + 5 = 3 + 2x **B** 7 - 2x = 3x + 17 **C** 4x + 1 = 7 - 5x **D** 6x + 4 = 2x - 4
- **5** The solution to the simultaneous equations y = x 3 and 5x 2y = 27 is: **A** (5, 2) **B** (6, 3) **C** (6, 5) **D** (7, 4)
- 6 The pair of simultaneous equations that has a solution of x = -5 and y = 3 is:
  - A y = x + 8<br/>2x 3y = 16B y = x + 8<br/>3x 2y = -21C x + y = 2<br/>5x + y = 28D y x = 8<br/>4x 3y = 26
- 7 The diagram showing the graphical solution of y = x 2 and 2x y = 7 is:



- 8 When solved by substituting y = x + 3 into 3x 5y = -11, the x-value is: A -3 B -2 C -1 D 0
- 9 When solving 5x 3y = 8 and 2x + y = 4 by elimination, the first line after eliminating y would be: A 7x - 2y = 12B 7x = 12C x = 4D 11x = 20
- **10** Two numbers that have a sum of 20 and a difference of 4 are:

   **A** 8 and 12
   **B** 10 and 10
   **C** 16 and 12
   **D** 16 and 4

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1-4	5,6	7	8	9	10
Section A		В	С	D	Е	F

### 6A Review set

- 1 Solve these equations for *x*.
  - **a** 2x + 5 = -3
  - **d** 3(2x+1) = 4
- **b** 4x = 2x + 11**e** 2(3 - x) = 3(x + 5)

**c** 
$$\frac{x}{3} = \frac{1}{6}$$
  
**f**  $4 - 3(2 - x) = 7$ 

**c** 4 - 3x = 2x + 1

e 3(5-2x) = 5(x-3) f 5-2(5-2x) = 3

- **2** Find y when x = -3, given that  $y = \frac{4-2x}{5}$ .
- **3** a Find y when x = 1 and 2x 3y = -7.
  - **b** What equation results when 8x 3y = -1 is multiplied by 3?
  - **c** What equation results when 3x 5y = -1 and 7x + 5y = 21 are added?
- 4 On the same set of axes, draw accurate graphs of the lines with equations y = 2x 3 and y = 2 3x.
  - **a** Use your graph to solve y = 2x 3 and y = 2 3x simultaneously.
  - **b** Check your answer to part **a** by using the substitution method.
- **5** a Solve y = 2x + 1 and y = -x 5 simultaneously.
  - **b** Are the lines with equations y = 2x + 1, y = -x 5 and y = -3x 9 concurrent? (Three or more lines are concurrent if they all meet at a common point of intersection.)
- **6** Solve the following problems.
  - a John is two years older than Paula. Three times John's age plus four times Paula's age is 55 years. How old are John and Paula?
  - **b** In my purse I have 21 coins, all of which are either 5-cent or 20-cent coins. Their total value is \$2.25. How many 20-cent coins do I have?

### **6B** Review set

- 1 Solve these equations for *x*.
  - **a** 3x 1 = 2
  - **d** 2(4x + 1) = 3
- 2 Find t when a = -2, given that  $t = \frac{5a 4}{3}$ .
- **3** a Find y when x = 2 and 3x 5y = 7.
  - **b** What equation results when 7x 2y = 8 is multiplied by 4?
  - **c** What equation results when 2x 7y = 3 and 5x + 7y = -7 are added?
- 4 On the same set of axes, draw accurate graphs of the lines with equations y = 3x 2 and y = 8 2x.

**b** 6x = 11 - 3x

- **a** Use your graph to solve y = 3x 2 and y = 8 2x simultaneously.
- **b** Check your answer to part **a** by using the substitution method.
- **5** Use the elimination method to solve 3x 5y = 21 and 5x 4y = 22 simultaneously.
- **6** Solve these problems.
  - **a** The sum of two numbers is 10. When one number is added to twice the other, the result is 16. Find the numbers.
  - **b** If I have 10-cent and 20-cent coins only with a total value of \$6.90 and I have 36 coins altogether, how many 10-cent coins do I have?
- 7 Explain why the pair of simultaneous equations x + 2y = 6 and x + 2y = 8 has no solution.

### **6C** Review set

- **1** Solve these equations.
  - **a** 4x + 5 = 12 **b** 5 - 3x = -7 **d** -3(2a + 5) = 15**e** 2(3a - 4) = a + 9
- 2 Consider  $y = \frac{2x-3}{5}$ .
  - **a** Find y when  $x = \frac{1}{2}$ .

**b** Find x when y = 3.

**c** 3x - 2 = x + 6

**c** 4x - 7 = 3x + 5**f** 4(3p - 7) = 5(1 - p)

**f** 3(2p-3) = 4(p+1)

- **3** a What equation results when 3x 7y = 5 is multiplied by -4?
  - **b** At what point does the line y = 3x 4 meet the *y*-axis?
  - c Find y when x = -5, given that 2x y = 8.
  - **d** What equation results when 3x 4y = 6 and 2x + 4y = 14 are added?
- 4 a On the same set of axes graph the lines with equations y = x + 5 and y = -4x 5.
  b Hence solve the equations y = x + 5 and y = -4x 5 simultaneously.
- **5** a Solve x = 5 y and 3x 7y = -1 simultaneously using the substitution method.
  - **b** Solve 3x 2y = 12 and 4x + 3y = -1 simultaneously using the elimination method.
- **6** a Two chocolate frogs and one lollipop cost a total of 78 cents, and three chocolate frogs and two lollipops cost \$1.32. Find the cost if I buy one chocolate frog and one lollipop.
  - **b** Four cricket balls and three tennis balls weigh a total of 1060 g. Two cricket balls and five tennis balls weigh 740 g. Find the weight of each type of ball.

### 6D Review set

- 1 Solve these equations.
- **a** 3x 2 = 5 **b** 4 - 7x = 3**c** 4(3t + 1) = t - 2
- 2 Given that  $y = \frac{3x-5}{3}$ , find:
  - **a** y when  $x = \frac{1}{3}$  **b** x when  $y = \frac{7}{3}$
- **3** a What equation results when 2x 5y = 11 is multiplied by 3?
  - **b** At what point does the line y = 3x 5 cut the y-axis?
  - c Find y when x = -3, given 3x 2y = 11.
  - **d** What equation results when 7x 3y = 8 and 5x + 3y = 7 are added?
- 4 a Graph the lines with equations y = x 3 and y = 2x 8 on the same set of axes.
  - **b** Solve y = x 3 and y = 2x 8 simultaneously.
  - c Solve simultaneously using the substitution method to check your answer.
- **5** Solve 5x 3y = 29 and 2x + y = 5 simultaneously by the elimination method.
- **6** Solve these problems.
  - **a** I wish to blend brand A coffee at \$7 per kg with brand B coffee at \$11 per kg. If the total weight is 10 kg, and the total cost is \$86, how many kilograms of each coffee do I use?
  - **b** In 5 years time Pam will be twice as old as Sam was two years ago. Pam is 8 years older than Sam. How old is Sam?