MY MATHS 7

VICTORIAN CURRICULUM

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OXFORD

Using the MyMaths series vi

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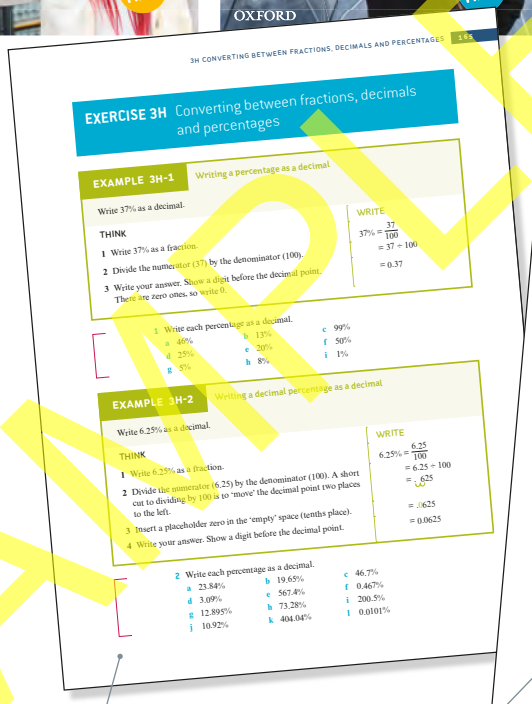
OXFORD MYMATHS VICTORIAN CURRICULUM

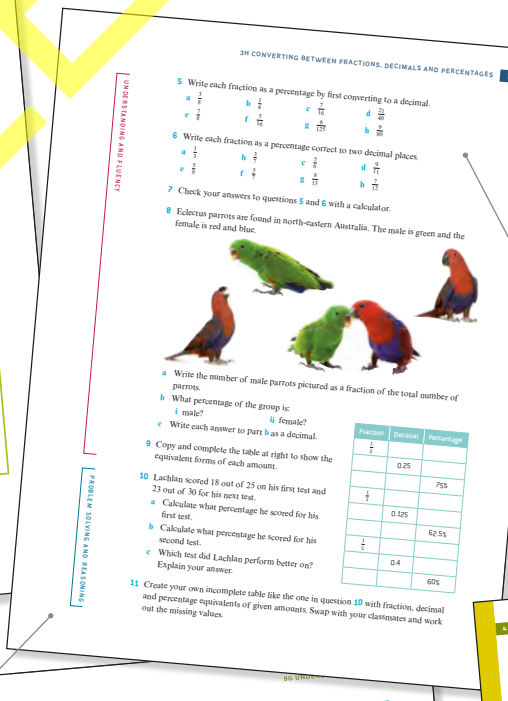
Oxford MyMaths Victorian Curriculum has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

STUDENT BOOK + OBOOK ASSESS

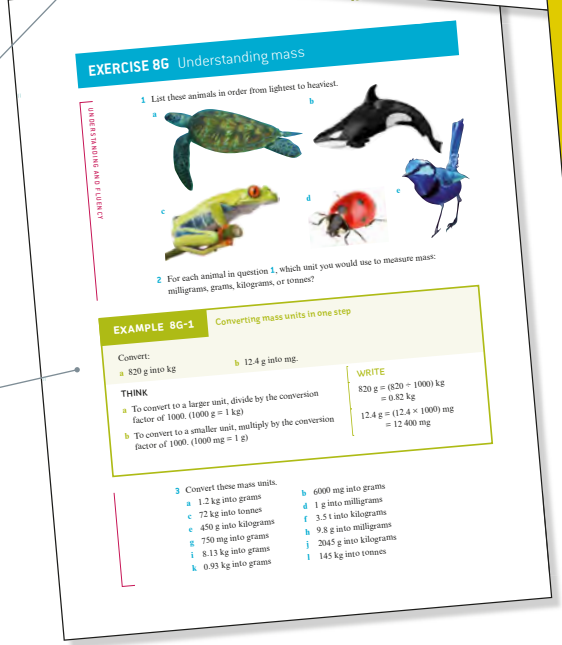
- Finely levelled exercises to ensure smooth progress
- Integrated worked examples – right where your students need them
- Learning organised around the 'big ideas' of mathematics
- Discovery, practice, thinking and problem-solving activities to promote deep understanding
- A wealth of revision material to consolidate and prove learning
- Rich tasks to apply understanding
- Highly accessible and easy to navigate
- Comprehensive digital tutorials and guided examples to support independent progress



The student book accurately and carefully delivers the course.

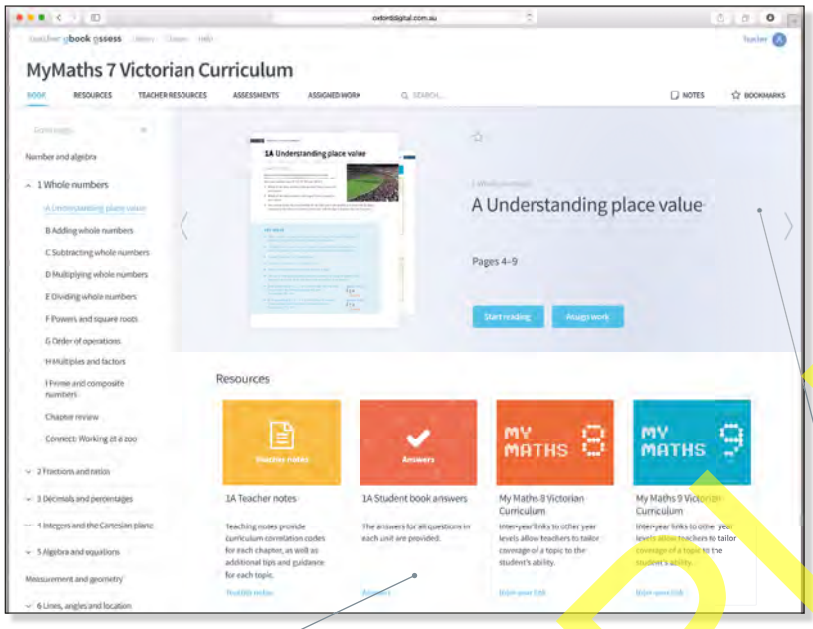


Every question is matched to the Victorian Curriculum proficiencies.



Worked examples are clearly laid out and located where students need them most.

YEAR 7–10 DIFFERENTIATION SOLUTION



TEACHER BOOK ASSESS

- Full access to all content and resources across Years 7–10: everything you need in one place!
- Select content and resources appropriate for each student in your class.
- Assign work and assessments; monitor student and class progress.

Interactive tutorials scaffold understanding of key concepts and build students' confidence.

Personalised learning: tailor the very best learning experiences for all.

Finely levelled content enables students to progress with ease.

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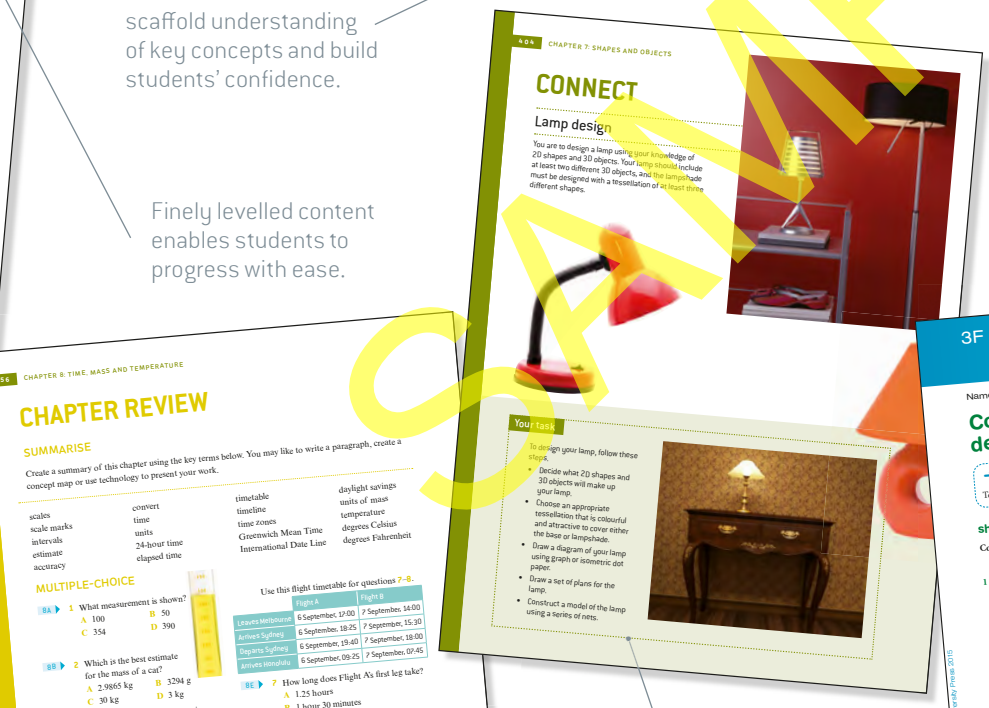
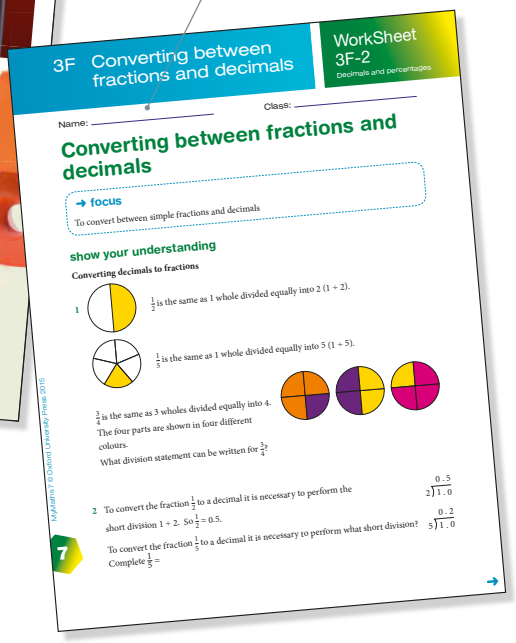


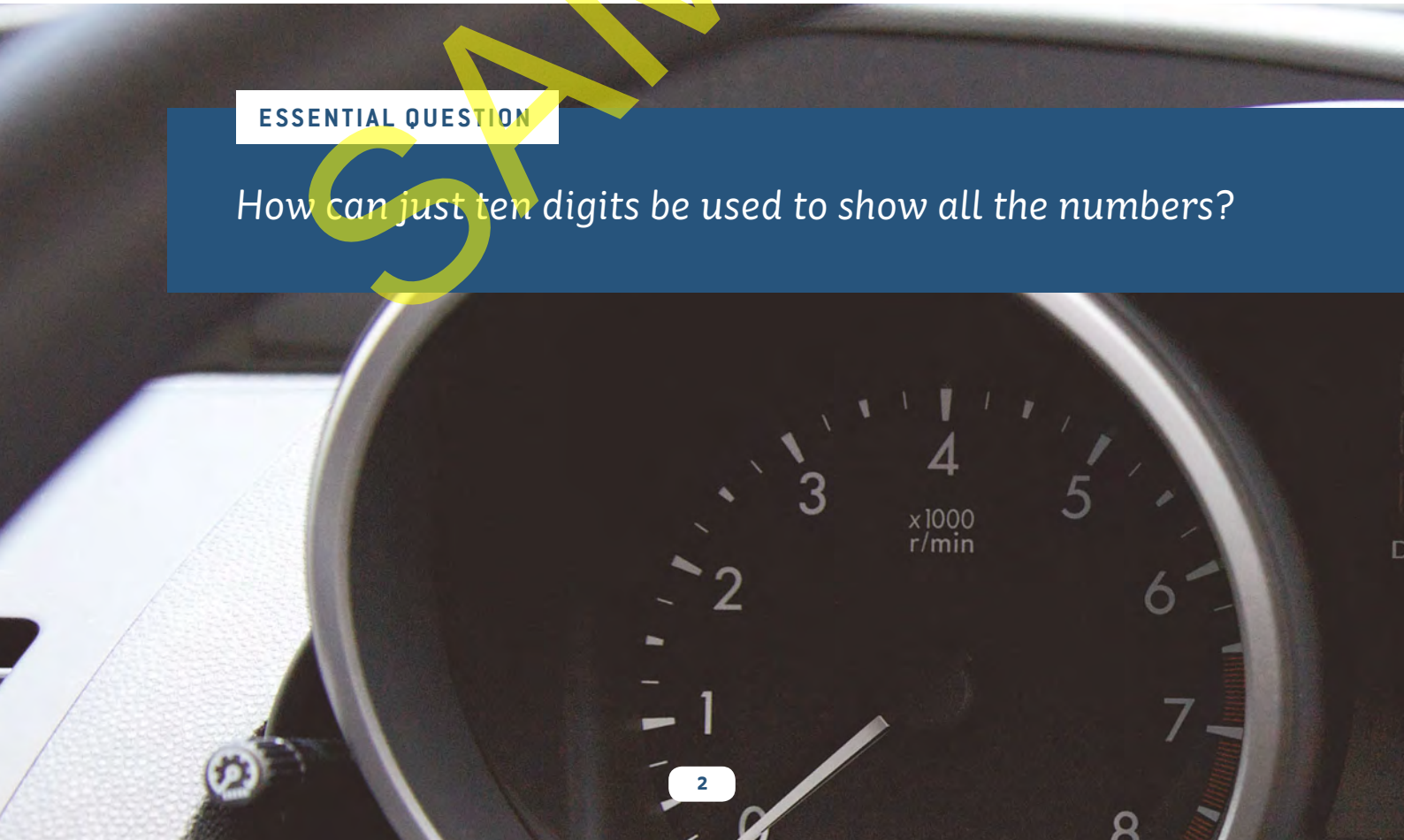


Rich tasks where students can demonstrate understanding.

Ample revision to consolidate understanding and prove that learning has happened.

1 WHOLE NUMBERS

1A Understanding place value
1B Adding whole numbers
1C Subtracting whole numbers
1D Multiplying whole numbers
1E Dividing whole numbers
1F Powers and square roots
1G Order of operations
1H Multiples and factors
1I Prime and composite numbers



ESSENTIAL QUESTION

How can just ten digits be used to show all the numbers?

Are you ready?

1A **1** Written in words, what is 82 150?

A eighty thousand, two hundred and fifteen

B eighty-two thousand, one hundred and fifteen

C twenty-eight thousand, one hundred and five

D eighty-two thousand, one hundred and fifty

1A **2** Written in digits, what is one hundred and twenty-four thousand, five hundred and eight?

A 124 580

B 124 508

C 10 024 508

D 10 020 458

1A **3** What is 236 457 written in place-value notation?

A 200 000 + 30 000 + 6000 + 400 + 50 + 7

B 2 + 3 + 6 + 4 + 5 + 7

C 200 + 30 + 6 + 400 + 50 + 7

D 200 000 + 30 000 + 6000 + 400 + 5 + 7

1A **4** Which number is the largest in each list of numbers?

a 23, 41, 18, 39

b 135, 153, 133, 150

1A **5** Which of these numbers are odd?
24, 31, 15, 50, 63, 48.

A 24, 50, 48 **B** 31, 15, 50

C 31, 15, 63 **D** 24, 15, 63

1B **6** Calculate:

a 32 + 45 **b** 18 + 57

1C **7** Calculate:

a 89 − 36 **b** 64 − 28

1D **8** Calculate:

a 32 × 4 **b** 84 × 6

1D **9** Calculate:

a 42 × 10 **b** 75 × 100

1D **10** What is 3040 × 1000?

A 34 000 **B** 304 000

C 3 040 000 **D** 30 400 000

1E **11** Calculate:

a 48 ÷ 4 **b** 5300 ÷ 10

1E **12** What is 263 ÷ 7?

A 39

B 37

C 38 remainder 3

D 37 remainder 4

1F **13** What is 2 × 2 × 2 × 2 × 2?

A 10 **B** 32

C 64 **D** 128

1F **14** What is 3 × 3 × 3 × 3?

1H **15** What are the next three numbers in the pattern 4, 8, 12, 16, …?

A 18, 20, 22

B 20, 28, 36

C 20, 24, 28

D 32, 64, 128

1H **16** Which list gives all the factors of 30?

A 1, 3, 10, 30

B 1, 2, 3, 4, 5, 6, 10, 30

C 1, 2, 5, 6, 15, 30

D 1, 2, 3, 5, 6, 10, 15, 30

1H **17** One factor pair for 12 is 3 and 4. What is another?

A 3 and 9 **B** 2 and 6

C 2 and 10 **D** 5 and 7

1A Understanding place value

Start thinking!

Huge crowds attend the AFL grand final each year at the Melbourne Cricket Ground (MCG). In three consecutive years, the crowd numbers were 97 531, 97 302 and 100 012.

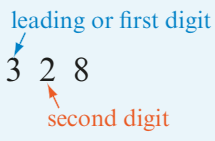
1 Which of the three numbers is the smallest? Give a reason for your answer.

2 Which of the three numbers is the largest? Give a reason for your answer.

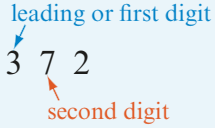
3 One student claims the crowd number for the third year is the smallest as it starts with the digit 1 compared to the other two numbers, which start with the digit 9. Explain why this is incorrect.

KEY IDEAS

- Whole numbers are made up of combinations of digits. The value of each digit depends on the place or position of the digit in the number.
- The digit 0 (zero) is important even though it shows that there is nothing in that place-value position. It keeps the other digits in the right places in the number.
- < means 'is less than' (or 'is smaller than')
- > means 'is greater than' (or 'is larger than')
- Always read mathematical statements from left to right.
- One way to write an approximate value for a number is to round the number to its leading (or first) digit. To do this, look at the second digit in the number.
- If the second digit is 0, 1, 2, 3 or 4, the first digit stays the same and each digit that follows is replaced with zero.
  For example, 328 ≈ 300.



- If the second digit is 5, 6, 7, 8 or 9, the first digit is increased by one and each digit that follows is replaced with zero.
  For example, 372 ≈ 400.



EXERCISE 1A Understanding place value

EXAMPLE 1A-1 Writing numbers in place-value notation

Write each number in place-value notation.

a 369 **b** 28 104 **c** 5070

THINK

1 You may like to show the numbers in a place-value chart. Remember to include zero in the appropriate columns.

2 Write each number in place-value notation by showing the value of each digit.

WRITE

Ten thousands 10 000	Thousands 1000	Hundreds 100	Tens 10	Ones 1
		3	6	9
2	8	1	0	4
	5	0	7	0

a 369 = 300 + 60 + 9
b 28 104 = 20 000 + 8000 + 100 + 4
c 5070 = 5000 + 70

1 Write these numbers in place-value notation.

a 56 **b** 238 **c** 4751
d 12 649 **e** 8507 **f** 63 044

2 State whether each number in question **1** is an odd or an even number.

EXAMPLE 1A-2 Writing the value of a digit in a number

Write the value of the digit 4 in each of these numbers.

a 347 **b** 48 052

THINK

a Consider the place value of 4.

b Consider the place value of 4.

WRITE

a Place value is 4 tens.
Value of 4 in 347 is 40.

b Place value is 4 ten thousands.
Value of 4 in 48 052 is 40 000.

UNDERSTANDING AND FLUENCY

3 Write the value of the digit 9 in each of these numbers.

a 298	**b** 957	**c** 39
d 59 406	**e** 7891	**f** 970 412

4 For each number shown on these signs, write the value of the digit that is listed in brackets.

a distance where animals may be near the road (9)

b height above sea level of Mt Kosciuszko (8)

c **i** distance to Alice Springs (1)

ii distance to Tennant Creek (6)

iii distance to Darwin (5)

5 Write these numbers in digit form.

a sixty-two thousand

b nine hundred and seventy-eight

c three hundred and four

d two hundred and fifty thousand, one hundred and twelve

e twelve thousand, five hundred and forty-three

f nine thousand and twenty-six

6 Write these numbers in words.

a 362	**b** 7215	**c** 45 733
d 234 601	**e** 6 420 058	**f** 55 555

EXAMPLE 1A-3 Ordering numbers

Write this list of numbers in order from smallest to largest: 23 706, 2376, 23 678.

THINK	WRITE
1 Count the number of digits in each number. The four-digit number is smallest.	23 706 has five digits, 2376 has four digits and 23 678 has five digits. So 2376 is the smallest number.
2 Compare the size of each digit in corresponding place values for the other two numbers, starting from the highest place value.	2 3 7 0 6 2 3 6 7 8 same different
3 In the hundreds place value, 7 is larger than 6 (or 700 is larger than 600), so 23 706 is larger than 23 678.	23 706 > 23 678
4 Write the numbers from smallest to largest.	2376, 23 678, 23 706

7 Write each list of numbers in order from smallest to largest.

a 4562, 439, 45 629
b 35 218, 53 176, 25 786
c 6754, 67 554, 67 454
d 9103, 9130, 9013
e 24 701, 24 007, 24 071
f 613 548, 613 583, 613 538

8 Copy and complete these number sentences by writing < or > in the space provided.

a 8530 ____ 7503
b 46 249 ____ 64 249
c 317 294 ____ 37 294
d 709 ____ 4503
e 5678 ____ 5876
f 10 462 ____ 10 248
g 360 ____ 306
h 8245 ____ 8254
i 214 007 ____ 210 007
j 989 000 ____ 998 000

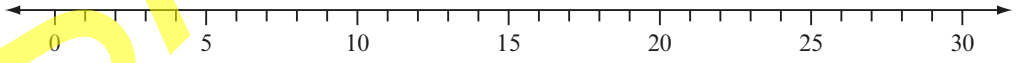
9 Write each list of numbers in ascending order (from smallest to largest).

a 58, 72, 9, 40, 88, 15, 28
b 856, 805, 890, 806, 846
c 625, 9472, 6105, 10 417, 9902
d 2374, 23 074, 23 704, 234, 2347

10 Write each list of numbers in descending order (from largest to smallest).

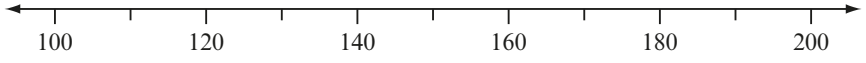
a 870, 8000, 87, 1800, 807
b 3999, 3909, 399, 309, 3099
c 72 156, 75 126, 75 561, 75 516
d 2 567 291, 256 291, 1 967 219

11 Copy this number line and mark the position of these numbers.



a 20
b 5
c 11
d 28
e 2
f 16

12 Copy this number line and mark the position of these numbers.



a 150
b 185
c 132
d 200
e 108
f 173

13 Draw a number line with a scale from 2000 to 3000 and mark the position of these numbers.

a 2500
b 2850
c 2100
d 2920
e 2250
f 2070

UNDERSTANDING AND FLUENCY

14 Sometimes when you estimate or describe amounts, you don't need to know the exact number. An approximate value can be just as valuable. For example, the crowd of 100 012 at the MCG could be described as approximately 100 000 people.

a Decide whether each number is closer to 200 or 300.
i 228 **ii** 252 **iii** 280 **iv** 219 **v** 266

b Decide whether each number is closer to 5000 or 6000.
i 5743 **ii** 5086 **iii** 5617 **iv** 5508 **v** 5499

c Explain your thinking for parts **a** and **b**.

EXAMPLE 1A-4 Rounding a number to its leading (first) digit

Write an approximation for each number by rounding to its leading digit.

a 719 **b** 4802

THINK

a Look at the second digit (1). Since it is less than 5, keep the first digit (7) and replace all other digits with zero.

b Look at the second digit (8). Since it is 5 or more, increase the first digit by one (4 + 1 = 5) and replace all other digits with zero.

WRITE

a 719 ≈ 700

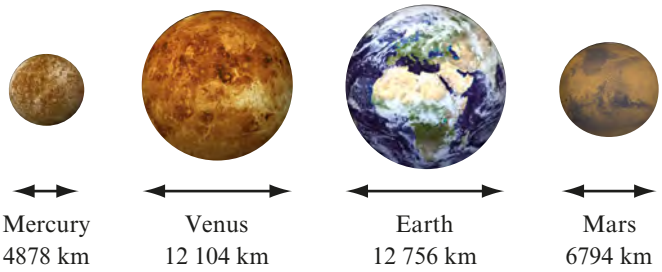
b 4802 ≈ 5000

15 Write an approximation to each number by rounding to its leading digit.

a 784 **b** 45 **c** 103 **d** 6522
e 38 405 **f** 580 **g** 8521 **h** 22 199
i 174 011 **j** 962 **k** 73 730 **l** 95 874

PROBLEM SOLVING AND REASONING

16 The four planets closest to the Sun are shown with their diameter measurements.



- **a** Write their diameters as approximate values by rounding to the leading digit.
- **b** Draw a number line and mark these approximate values on it. Label each value with the name of the planet.
- **c** Use your approximate values to write a sentence comparing the sizes of the planets.

17 **a** Write these measurements as approximate values by rounding to the leading digit.

- **i** diameter of the Moon
- **ii** distance from the Earth to the Moon, if the distance between them is 384 402 km.

b Is the distance from the Earth to the Moon approximately 10, 100 or 1000 times greater than the diameter of the Moon? Explain your thinking.

18 A novel contains 112 pages, numbered 1 to 112. How many times does the digit 1 appear in the page numbers shown on each page of the book? Show your reasoning.

19 **a** How many different two-digit numbers can you make from 3 and 5 if you cannot repeat digits? List them in ascending order.

b How many can you make if you can repeat digits? List them in descending order.

20 **a** How many different two-digit numbers can you make from 2, 4 and 7 if you cannot repeat digits? List them in descending order.

b How many can you make if you can repeat digits? List them in ascending order.

CHALLENGE

21 Consider the digits 1, 3, 6 and 9. Use these digits to write:

- **a** the largest four-digit number without repeating any digits
- **b** the smallest four-digit number if digits can be repeated
- **c** the largest even number without repeating any digits
- **d** all the four-digit numbers between 3620 and 6350 if no digits can be repeated
- **e** the third largest number if digits can be repeated
- **f** the second largest odd number if digits can be repeated.

Reflect

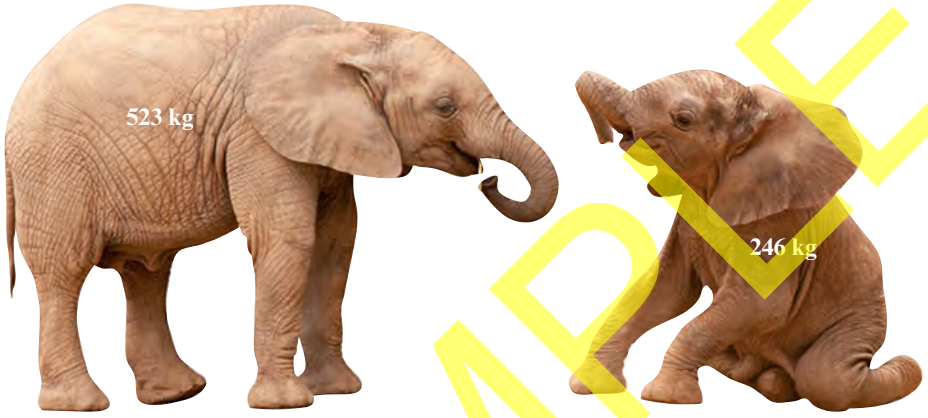
Explain why the place value of the digits is important when comparing the size of numbers.

1B Adding whole numbers

Start thinking!

Two young elephants arrive at the zoo. To work out how much food is needed, zookeepers must calculate the total mass of the elephants. Can you calculate this for them?

You need to find the **sum** of 523 and 246. This is the same as calculating 523 + 246. There are a number of ways to do this.



1 Try at least two different ways to work out 523 + 246.

2 Compare your working for each method. Which method did you find easiest to use? Explain.

3 Write your answer to the original problem. What is the total mass of the elephants?

KEY IDEAS

- Whole numbers can be added 'in your head' using a mental strategy. For example, you do not need to use pen and paper or a calculator to work out 500 + 200 = 700 or 42 + 16 = 58.
- In the **vertical addition method**, the numbers are written one under the other with the digits lined up according to their place value.

$$\begin{array}{r} 523 \\ +\ 246 \\ \hline \\ \hline \end{array} \leftarrow \text{answer}$$

- Always start by adding the digits in the ones column first, then the tens column, followed by the hundreds column and so on.
- Each digit in the answer also lines up in the appropriate column according to its place value.

EXERCISE 1B Adding whole numbers

UNDERSTANDING AND FLUENCY

1 Use a mental strategy to work out each sum.

a 50 + 30 **b** 100 + 200 **c** 3000 + 4000
d 120 + 60 **e** 360 + 20 **f** 50 + 250
g 25 + 13 **h** 16 + 61 **i** 56 + 34
j 48 + 22 **k** 65 + 35 **l** 123 + 345

2 Use a mental strategy to work out each sum.

a 20 + 40 + 10 **b** 500 + 100 + 300 **c** 4000 + 1000 + 1000
d 18 + 12 + 50 **e** 24 + 30 + 16 **f** 73 + 7 + 20
g 52 + 11 + 27 **h** 35 + 29 + 35 **i** 13 + 22 + 31
j 130 + 44 + 6 **k** 115 + 205 + 80 **l** 203 + 203 + 203

EXAMPLE 1B-1 Using the vertical addition method to add two numbers

Copy and complete this addition problem to calculate 634 + 891 using the vertical addition method.

$$\begin{array}{r} 634 \\ +\ 891 \\ \hline \\ \hline \end{array}$$

THINK

1 Ones column: 4 + 1 = 5.
2 Tens column: 3 + 9 = 12. Write 2 in the tens column of the answer and 1 in the hundreds column (shown in green).
3 Hundreds column: 1 + 6 + 8 = 15.

WRITE

$$\begin{array}{r} {\scriptstyle 1\ \ } \\ 634 \\ +\ 891 \\ \hline 1525 \end{array}$$

3 Copy and complete each addition problem using vertical addition.

a $\begin{array}{r} 243 \\ +\ 715 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 538 \\ +\ 154 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 758 \\ +\ 461 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 697 \\ +\ 805 \\ \hline \\ \hline \end{array}$

e $\begin{array}{r} 1413 \\ +\ 3706 \\ \hline \\ \hline \end{array}$ **f** $\begin{array}{r} 8520 \\ +\ 5179 \\ \hline \\ \hline \end{array}$ **g** $\begin{array}{r} 2044 \\ +\ 671 \\ \hline \\ \hline \end{array}$ **h** $\begin{array}{r} 56\,052 \\ +\ 79\,418 \\ \hline \\ \hline \end{array}$

4 Use vertical addition to calculate each of these.

a 641 + 478 **b** 157 + 296 **c** 2438 + 5160 **d** 3762 + 1489
e 2175 + 485 **f** 96 + 5743 **g** 16 407 + 782 **h** 8009 + 35 714

EXAMPLE 1B-2 Using the vertical addition method to add three numbers

Use the vertical addition method to calculate 3108 + 547 + 1619.

THINK

1 Set out the addition problem in columns according to place value.

2 Ones column: 8 + 7 + 9 = 24. Write 4 in the ones column of the answer and 2 at the top of the tens column (shown in green).

3 Tens column: 2 + 0 + 4 + 1 = 7.

4 Hundreds column: 1 + 5 + 6 = 12. Write 2 in the hundreds column of the answer and 1 at the top of the thousands column (shown in blue).

5 Thousands column: 1 + 3 + 0 + 1 = 5.

WRITE

$$\begin{array}{r} {\scriptstyle 1}\ {\scriptstyle 2} \\ 3108 \\ 547 \\ +\ 1619 \\ \hline 5274 \end{array}$$

UNDERSTANDING AND FLUENCY

5 Use vertical addition to calculate each of these.

a 385 + 461 + 723 **b** 749 + 218 + 837 **c** 516 + 705 + 192

d 927 + 56 + 614 **e** 869 + 21 + 70 **f** 24 + 503 + 988

g 2764 + 8293 + 451 **h** 7605 + 246 + 38 **i** 51 + 4032 + 876

6 Find each result without using a calculator.

a 23 748 + 61 392 + 53 709 **b** 82 407 + 9385 + 411

c 34 715 + 306 937 + 8256 **d** 947 + 600 411 + 85 103

e 78 294 + 19 + 6844 **f** 3007 + 458 + 27 090

7 Find each result without using a calculator.

a 28 + 7640 + 459 + 7 + 834 + 2406 **b** 92 762 + 547 + 85 + 2942 + 6073

c 397 + 16 + 69 005 + 8 + 6255 + 20 **d** 891 546 + 6509 + 65 + 101 + 7043

8 Check your answers to questions **6** and **7** with a calculator.

9 You decide to train for a local cycling race. On the first weekend you cycle 32 km, on the second you cycle 45 km and on the third you cycle 59 km. To find the total distance, you can write the calculation in a number of ways.

a Copy and complete these sentences to describe the same calculation in different ways.

i Add _____ and _____ and _____ together.

ii What is _____ plus _____ plus _____?

iii Find the result of _____ + _____ + _____.

iv Find the sum of _____, _____ and _____.

b Are there any other ways of describing the calculation? List them.

c Perform the calculation to find the total distance.

10 The Tour de France is an annual international cycling race that finishes in Paris, France. In 2011, Cadel Evans won the race, following the itinerary shown.

a Answer these questions without using a calculator.

i How far do the competitors ride in the first five days of the race?

ii How far do the competitors ride before their first rest day?

iii Which stage has the longest distance for a cyclist to ride in a day? Would this be the hardest day of cycling? Explain.

iv How far do the cyclists ride between leaving Gap and finishing in Paris?

v What is the total distance covered in the Tour de France?

b Check your answers using a calculator.

c Find out about this year's Tour de France. What total distance is covered?

Date in 2011	Start and finish	Distance
2 July	Passage du Gois → Mont des Alouettes	192 km
3 July	Les Essarts → Les Essarts (team time trial)	23 km
4 July	Olonne-sur-Mer → Redon	198 km
5 July	Lorient → Mûr-de-Bretagne	172 km
6 July	Carhaix → Cap Fréhel	165 km
7 July	Dinan → Lisieux	226 km
8 July	Le Mans → Châteauroux	218 km
9 July	Aigurande → Super-Besse Sancy	189 km
10 July	Issoire → Saint-Flour	208 km
11 July	Le Lioran Cantal	rest
12 July	Aurillac → Carmaux	158 km
13 July	Blaye-les-Mines → Lavaur	168 km
14 July	Cugnaux → Luz-Ardiden	211 km
15 July	Pau → Lourdes	152 km
16 July	Saint-Gaudens → Plateau de Beille	169 km
17 July	Limoux → Montpellier	192 km
18 July	Département de la Drôme	rest
19 July	Saint-Paul-Trois-Châteaux → Gap	163 km
20 July	Gap → Pinerolo	179 km
21 July	Pinerolo → Galibier Serre-Chevalier	200 km
22 July	Modane Valfréjus → Alpe-d'Huez	110 km
23 July	Grenoble → Grenoble (time trial)	42 km
24 July	Créteil → Paris Champs-Élysées	95 km

11 To enter the stadium for a soccer match, patrons used one of four gates. The number of people passing through the turnstiles of each gate is shown.

Gate	Number of people
A	8759
B	9042
C	10 365
D	11 008

a Estimate the number of people attending the match by first rounding the numbers at each gate to the leading (or first) digit.

b Calculate the exact number of people that attended the match.

c Which answer would a sports commentator be more likely to use when reporting on the match?

12 Find two whole numbers that add to 53.

13 Find two whole numbers that add to 386 and meet these conditions.

a both numbers are odd **b** both numbers are even

14 Find three whole numbers that add to 5207. Suggest another set of three numbers that add to the same total.

15 Depending on the numbers being added, a strategy can be used to make things easier. For example, to find the result of 24 + 37 + 16, you can add the first and third numbers together (24 + 16 = 40) and then add the second number to this total (40 + 37 = 77). Use this strategy to find each sum without using pen and paper or a calculator.

a 17 + 29 + 3 **b** 246 + 38 + 12

c 85 + 13 + 15 + 7 **d** 151 + 77 + 29

e 1 + 2 + 98 + 99 **f** 5 + 8 + 95 + 92

PROBLEM SOLVING AND REASONING

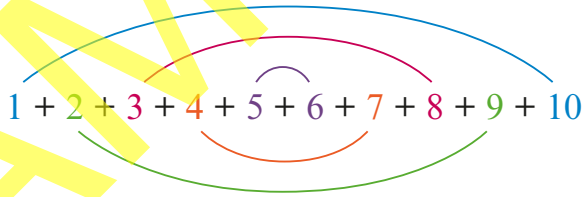
16 A ski lift is carrying two boys with their ski equipment.

- **a** What is the total mass of the boys?
- **b** What is the total mass of their boots, snowboard and skis? The mass is shown for one of each item.
- **c** What is the total mass on the ski lift?
- **d** During summer, the ski lift is used for sightseeing. Suggest what the mass of each person and their belongings, such as a picnic basket or knapsack, could be to have a total mass of 150 kg on the lift.

CHALLENGE

17 Consider adding the numbers from 1 to 10.

- **a** Add the numbers in order: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ?
- **b** Now try another way by first writing the numbers in suitable pairs. That is, write the smallest number and the largest number together, then the second smallest and second largest together and so on.



Copy and complete this calculation for adding the numbers from 1 to 10

(1 + 10) + (2 + 9) + (3 + __) + (4 + __) + (5 + __) = __ + __ + __ + __ + __ = __

18 Use the strategy of grouping numbers in suitable pairs to add the numbers from 1 to 20.

19 **a** Use the strategy of grouping numbers in suitable pairs to add these numbers.

- **i** from 1 to 9
- **ii** from 1 to 19

b Explain what is different about using this strategy with an odd number of numbers.

c Use a suitable strategy to add these numbers.

- **i** from 2 to 8
- **ii** from 3 to 17
- **iii** from 5 to 25

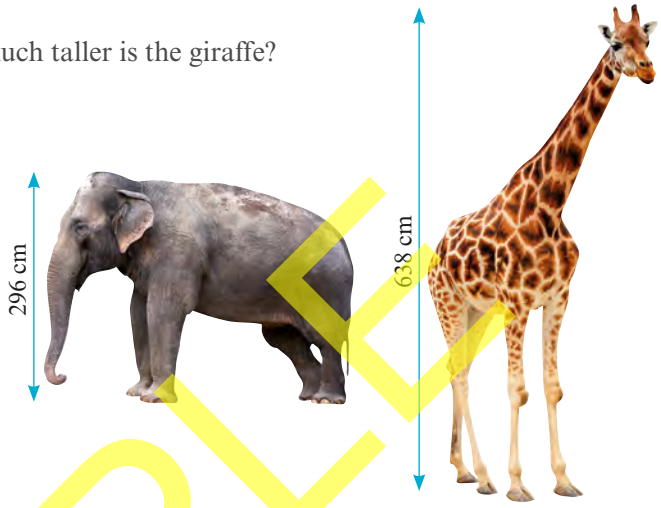
Reflect

When adding numbers together, what do you need to remember?

1C Subtracting whole numbers

Start thinking!

The height of these two animals is shown. How much taller is the giraffe?



You need to find the **difference** between 638 and 296. This is the same as calculating 638 − 296. There are a number of ways to do this.

1. Try at least two different ways to work out 638 − 296.
2. Compare your working for each method. Which method did you find easiest to use? Explain.
3. Write your answer to the original problem. How much taller is the giraffe?

KEY IDEAS

- Whole numbers can be subtracted 'in your head' using a mental strategy. For example, you do not need to use pen and paper or a calculator to work out 500 − 200 = 300 or 28 − 15 = 13.
- In the **vertical subtraction method**, the numbers are written one under the other with the digits lined up according to their place value. The larger number is written above the smaller number.
- Always start by subtracting the digits in the ones column first, then the tens column, followed by the hundreds column and so on.

$$\begin{array}{r} 638 \\ -\ 296 \\ \hline \\ \hline \end{array} \leftarrow \text{answer}$$

- If the subtraction in a particular column cannot be done, rename the top digit in the next column to the left.
- Each digit in the answer also lines up in the appropriate column according to its place value.

EXERCISE 1C Subtracting whole numbers

UNDERSTANDING AND FLUENCY

1 Use a mental strategy to work out each difference.

a 60 − 40	**b** 300 − 200	**c** 8000 − 5000
d 170 − 30	**e** 250 − 40	**f** 710 − 20
g 41 − 20	**h** 35 − 12	**i** 96 − 44
j 178 − 58	**k** 270 − 35	**l** 393 − 281

2 Use a mental strategy to work out each problem.

a 50 + 20 − 10	**b** 700 + 100 − 300	**c** 2000 + 6000 − 4000
d 14 + 16 − 20	**e** 98 + 12 − 1	**f** 570 + 25 − 80
g 28 − 8 − 5	**h** 45 − 10 − 11	**i** 120 − 40 − 30
j 36 − 12 + 8	**k** 63 − 41 + 20	**l** 400 − 50 + 85

EXAMPLE 1C-1

Using the vertical subtraction method to subtract two numbers with same number of digits

Copy and complete this subtraction problem to calculate 426 − 281 using the vertical subtraction method.

$$\begin{array}{r} 426 \\ -\ 281 \\ \hline \\ \hline \end{array}$$

THINK

1 Ones column: 6 − 1 = 5.

2 Tens column: 2 − 8. You cannot subtract 8 tens from 2 tens so rename the hundreds of the first number as 3 (shown in green) and the tens as 12 (shown in blue). So 12 − 8 = 4.

3 Hundreds column: 3 − 2 = 1.

WRITE

$$\begin{array}{r} {\scriptstyle 3\,12} \\ 4\not{2}6 \\ -\ 281 \\ \hline 145 \\ \hline \end{array}$$

3 Copy and complete each subtraction problem using vertical addition.

a 358 − 125 = ___	**b** 867 − 496 = ___	**c** 624 − 359 = ___	**d** 406 − 134 = ___
e 8527 − 2365 = ___	**f** 5031 − 4216 = ___	**g** 7349 − 1072 = ___	**h** 3415 − 2838 = ___

4 Use vertical subtraction to calculate each of these.

a 568 − 143	**b** 814 − 362	**c** 456 − 234	**d** 938 − 651
e 624 − 185	**f** 3846 − 1724	**g** 7508 − 5631	**h** 8135 − 6479

EXAMPLE 1C-2

Using the vertical subtraction method to subtract numbers that have different numbers of digits

Use the vertical subtraction method to calculate 4083 − 627.

THINK

1 Set out the problem in columns according to place value.

2 Ones column: 3 − 7. You cannot subtract 7 ones from 3 ones so rename the tens of the first number as 7 (shown in blue) and the ones as 13 (shown in green). So 13 − 7 = 6.

3 Tens column: 7 − 2 = 5.

4 Hundreds column: 0 − 6. Rename the thousands as 3 (shown in orange) and hundreds as 10 (shown in pink). So 10 − 6 = 4.

5 Thousands column: 3 − 0 = 3.

WRITE

$$\begin{array}{r} {\scriptstyle 3\ 10\ 7\ 13} \\ 4\not{0}\not{8}\not{3} \\ -\ \ 627 \\ \hline 3456 \end{array}$$

UNDERSTANDING AND FLUENCY

5 Use vertical subtraction to calculate each of these.

a 6327 − 215 **b** 5962 − 647
c 2475 − 728 **d** 7836 − 908
e 4025 − 462 **f** 5002 − 431
g 1471 − 86 **h** 9806 − 79

6 Find the result without using a calculator.

a 36 274 − 28 093 **b** 508 246 − 137 651
c 211 537 − 36 409 **d** 81 752 − 8362
e 4526 − 75 **f** 614 803 − 7024
g 920 517 − 925 **h** 100 265 − 1078

7 Check your answers to question **6** with a calculator.

8 For each calculation:

i find an estimate of the answer by first rounding each number to its leading digit
ii use pen and paper to work out the exact answer
iii use a calculator to check the result you obtained for part **ii**.

a 61 + 48 − 77 **b** 254 − 123 + 448
c 4708 − 369 + 532 **d** 29 071 + 8275 − 17 466
e 77 − 25 + 89 − 60 + 41 **f** 809 + 1252 − 754 − 36

UNDERSTANDING AND FLUENCY

9 The longest river in the world is the Nile in Africa, with a length of 6650 km. The longest river in Australia is the Darling River, with a length of 2740 km. To find the difference in length between these two rivers, you can write the calculation in a number of ways.

a Copy and complete these sentences to describe the same calculation in different ways.

i Subtract _____ from _____.

ii What is _____ minus _____?

iii Find the result of _____ − _____.

iv Find the difference between _____ and _____.

b Are there any other ways of describing the calculation? List them.

c Perform the calculation to find the difference between the two lengths.

10 Josh is comparing the distance to travel by plane from Melbourne to Rome using two different routes. One journey stops at Hong Kong for refuelling, while another stops at Singapore.

a Find the total flight distance from Melbourne to Rome if the plane stops at Hong Kong on the way.

b Find the total flight distance from Melbourne to Rome if the plane stops at Singapore on the way.

c Which flight distance is the shortest and by how much?

Flight sector	Distance by air
Melbourne–Hong Kong	7435 km
Melbourne–Singapore	6064 km
Hong Kong–Rome	9307 km
Singapore–Rome	10 048 km

PROBLEM SOLVING AND REASONING

11 The 162-storey building named Burj Khalifa, located in Dubai, United Arab Emirates, was completed in 2010.

a How does the height of this building compare with other structures? Find the difference in height between Burj Khalifa and each of the structures shown in this table.

Structures	Date completed	Height
Washington Monument (Washington DC, USA)	1884	169 m
Eiffel Tower (Paris, France)	1889	300 m
Empire State Building (New York, USA)	1931	381 m
Sydney Tower (Sydney, Australia)	1981	309 m
Petronas Towers (Kuala Lumpur, Malaysia)	1998	452 m
Taipei 101 (Taipei, Taiwan)	2003	509 m
Q1 (Gold Coast, Australia)	2005	323 m
Eureka Tower (Melbourne, Australia)	2006	297 m

b Which two structures could have their heights added to give a result closest to the height of Burj Khalifa?

c Which three structures could have their heights added to give a result closest to the height of Burj Khalifa?

12 The highest mountain in the world is Mount Everest, which has a height above sea level of 8848 m.

a How does this compare to Mount Kosciuszko, the highest mountain in Australia, at 2228 m above sea level?

b Compare the height of Mount Everest with the three mountains in our solar system listed below.

Mountain	Location	Height above surface
Mons Huygens	the Moon	4700 m
Maxwell Montes	Venus	11 000 m
Olympus Mons	Mars	21 171 m

c Find the difference in height between the mountains listed for Mars and the Moon.

d Find the difference in height between Mount Kosciuszko and Maxwell Montes.

13 There are 191 people travelling on a train. At the next station, 52 people leave the train and another 48 board the train. At the following station, 69 people get off the train and 75 get on. How many people are now on the train?

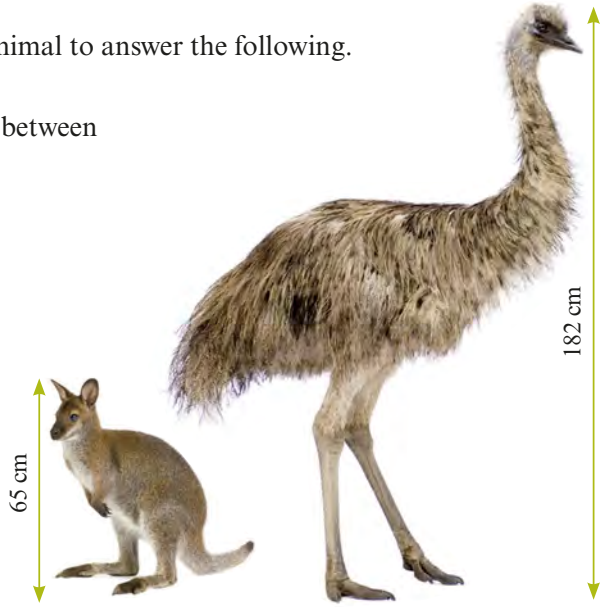
14 Work with a classmate to measure your height and your partner's height to the nearest centimetre. What is the difference in your heights?

15 Use the heights shown with each animal to answer the following. Clearly show your working.

a What is the difference in height between you and the emu?

b What is the difference in height between you and the wallaby?

c Explain why the order in which you subtract the heights in part **a** might be different from that in part **b**.



CHALLENGE

16 In May 2000, five planets in our solar system reached a point in their orbit around the Sun where they were roughly positioned in a straight line on the same side of the Sun (as viewed from the Sun). Earth was positioned on the opposite side of the Sun. The average distance from the Sun for each planet is shown.

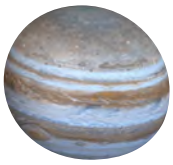
Earth
149 600 000 km

Sun

Mercury
57 909 000 km

Venus
108 200 000 km

Mars
227 940 000 km



Jupiter
778 400 000 km

Saturn
1 423 600 000 km

a Using this information, find the distance between these planets.

i Jupiter and Saturn

ii Mercury and Saturn

iii Venus and Mercury

b If the Sun has a diameter of about 1 392 000 km, what was the distance between Jupiter and Earth?

c In June 2010, Uranus, Jupiter and Mercury were roughly lined up on one side of the Sun while Venus, Mars and Saturn were lined up on the other side. At this time, what was the approximate distance between:

i Jupiter and Mercury?

ii Saturn and Venus?

iii Jupiter and Saturn?

d The film *War of the Worlds* is based on the idea that Martians travelled to Earth when the distance between Mars and Earth was the shortest. About what distance would the Martian spacecraft have travelled?

17 Find two numbers that, when subtracted from each other, produce a result of 43.

18 Find two numbers that have a difference of 168 and meet these conditions.

a both numbers are odd

b both numbers are even

19 Find two numbers that have a difference of 286 and meet these conditions.

a one number must be more than twice the other

b one number must be triple the other

Reflect

When subtracting numbers, what do you need to remember?

1D Multiplying whole numbers

Start thinking!

Pandas spend around 16 hours a day eating up to 18 kg of bamboo leaves, stems and shoots. For a group of nine pandas, how much food would be eaten in one day?

You need to find the **product** of 18 and 9. This is the same as calculating 18 × 9.

1. Work out 18 × 9. Describe the method you used to obtain your answer.
2. Write your answer to the original problem. How much food will nine pandas eat in a day?
3. Use this result to calculate how much food these pandas will eat in 28 days. Describe the method you used.

KEY IDEAS

- Use short multiplication when multiplying by a single-digit number.
- When multiplying a whole number by 10, the result will have a zero in the ones position. For example, 3 × 10 = 30.
- When multiplying a whole number by 100, the result will have a zero in both the ones and tens positions. For example, 3 × 100 = 300.
- When multiplying a whole number by 1000, the result will have a zero in the ones, tens and hundreds positions. For example, 3 × 1000 = 3000.
- Use long multiplication when multiplying by a number with more than one digit.
- In long multiplication, the two numbers are arranged vertically in columns according to place value.

$$\begin{array}{r} 34 \\ \times\ 26 \\ \hline 204 \\ 680 \\ \hline 884 \end{array}$$

← this is the same as 20 + 6
← this is the result of multiplying 34 by 6
← this is the result of multiplying 34 by 20 (Notice that you place a zero in the ones column and then multiply 34 by 2.)

The final answer is obtained by adding the two multiplication results together.

EXERCISE 1D Multiplying whole numbers

1 Use short multiplication to work out each product.

a 74 × 8	**b** 93 × 2	**c** 45 × 3	**d** 26 × 7
e 192 × 5	**f** 804 × 9	**g** 532 × 6	**h** 281 × 4
i 6158 × 3	**j** 3075 × 7	**k** 24 159 × 5	**l** 413 083 × 9

EXAMPLE 1D-1 Multiplying by 10, 100 or 1000

Write the result of each problem.

a 62 × 10 **b** 153 × 100 **c** 27 × 1000

THINK	WRITE
a Use the short cut of placing 0 in the ones position and moving 2 to the tens position and 6 to the hundreds position. Since there is one zero in 10, move the digits one place to the left.	**a** 62 × 10 = 620
b Use the short cut. Since there are two zeros in 100, move the digits two places to the left.	**b** 153 × 100 = 15 300
c Use the short cut. Since there are three zeros in 1000, move the digits three places to the left.	**c** 27 × 1000 = 27 000

UNDERSTANDING AND FLUENCY

2 Write the result of each problem.

a 5 × 10	**b** 32 × 10	**c** 639 × 10	**d** 51 623 × 10
e 7 × 100	**f** 2784 × 100	**g** 48 × 100	**h** 103 × 100
i 85 × 1000	**j** 936 × 1000	**k** 15 × 1000	**l** 7020 × 1000

3 Write the result of each problem.

a 4 × 3 × 10	**b** 6 × 7 × 10	**c** 2 × 10 × 9	**d** 15 × 4 × 100
e 28 × 6 × 100	**f** 13 × 100 × 7	**g** 8 × 5 × 1000	**h** 41 × 3 × 1000
i 356 × 1000 × 2	**j** 64 × 100 × 9	**k** 125 × 1000 × 4	**l** 7103 × 5 × 10

4 Work out each product. Use the strategy of multiplying by 10 or 100 or 1000. For example, 60 is the same as 10 × 6 or 6 × 10.

a 8 × 60	**b** 4 × 30	**c** 12 × 70	**d** 93 × 40
e 217 × 50	**f** 6345 × 20	**g** 52 × 800	**h** 428 × 300
i 7493 × 600	**j** 86 × 4000	**k** 319 × 7000	**l** 1542 × 2000

5 Rewrite each calculation so that the second number is written in place-value notation. For example, 94 × 57 = 94 × (50 + 7).

a 65 × 48 **b** 283 × 54 **c** 415 × 23 **d** 1059 × 18

EXAMPLE 1D-2 Using long multiplication to multiply by a two-digit number

Calculate 68 × 37 using long multiplication.

THINK

1. Write the numbers vertically. 37 = 30 + 7, so multiplying by 37 is the same as multiplying by 7 then 30 and adding the results.
2. Multiply 68 by 7 (68 × 7 = 476).
3. Next multiply 68 by 30. Multiply by 10 (place a zero in the ones column under the previous result) and then multiply by 3 (68 × 3 = 204).
4. Add the results of the two multiplications and write the answer underneath.

WRITE

$$\begin{array}{r} {\scriptstyle 2\ \ } \\ {\scriptstyle 5\ \ } \\ 68 \\ \times\ \ 37 \\ \hline 476 \\ 2040 \\ \hline 2516 \end{array}$$

So 68 × 37 = 2516.

UNDERSTANDING AND FLUENCY

6 Calculate each problem using long multiplication.

a 38 × 15 **b** 62 × 24 **c** 46 × 32 **d** 85 × 73
e 123 × 37 **f** 231 × 56 **g** 782 × 49 **h** 506 × 81
i 2654 × 42 **j** 9417 × 25 **k** 31 464 × 78 **l** 10 851 × 94

7 Perform the calculations listed in question **5** using long multiplication.

8 Check your answers to questions **6** and **7** using a calculator.

9 From a part time job, James has saved $37 each week. To find the amount he has saved after 28 weeks, you can write the calculation in a number of ways.

a Copy and complete these sentences to describe the same calculation in different ways.
i Multiply ____ and ____ together. **ii** What is ____ times ____?
iii Find the result of ____ × ____. **iv** Find the product of ____ and ____.
v Find ____ lots of ____.

b Are there any other ways of describing the calculation? List them.

c Perform the calculation to find the total amount that James has saved.

10 Rewrite each calculation so that the second number is written in place-value notation. For example, 716 × 245 = 716 × (200 + 40 + 5).

a 538 × 124 **b** 361 × 253 **c** 4825 × 627 **d** 9141 × 382

EXAMPLE 1D-3 Using long multiplication to multiply by a three-digit number

Calculate 543 × 286 using long multiplication.

THINK

1. Write the numbers vertically.
2. Multiply 543 by 6 (543 × 6 = 3258).
3. Next multiply 543 by 80. Multiply by 10 (place a zero in the ones column under the previous result) and then multiply by 8 (543 × 8 = 4344).
4. Next multiply 543 by 200. Multiply by 100 (place two zeros under the previous result) and then multiply by 2 (543 × 2 = 1086).
5. Add the results of the three multiplications and write the answer underneath.

WRITE

$$\begin{array}{r} {\scriptstyle 3\,2} \\ {\scriptstyle 2\,1} \\ 543 \\ \times\ 286 \\ \hline 3258 \\ 43440 \\ 108600 \\ \hline 155298 \end{array}$$

So 543 × 286 = 155 298.

UNDERSTANDING AND FLUENCY

11 Calculate each problem using long multiplication.

a 346 × 125	**b** 865 × 347	**c** 624 × 253	**d** 937 × 625
e 497 × 516	**f** 702 × 281	**g** 1302 × 374	**h** 5896 × 892
i 2475 × 403	**j** 70 219 × 210	**k** 6555 × 9385	**l** 10 461 × 7254

12 Perform the calculations listed in question **10** using long multiplication.

13 Check your answers to questions **11** and **12** using a calculator.

14 Copy this table into your workbook.

First number × second number	Product	Number of zeros in the first number	Number of zeros in the second number	Number of zeros in the product
10 × 10				
100 × 10				
1000 × 10				
10 000 × 10				

a Write your answer to each multiplication in the product column.

b Complete each row by writing the number of zeros in the first number, the second number and the product.

c Can you see a pattern? Explain how this pattern provides a quick method of doing multiplications like this.

d Use this method to calculate each product.

i 10 × 100 **ii** 100 × 100 **iii** 100 × 1000 **iv** 1000 × 1000

UNDERSTANDING AND FLUENCY

15 **a** To find 4000 × 100, the calculation can be written as 4 × 1000 × 100 or 1000 × 100 × 4.

i Calculate 1000 × 100.

ii Multiply this result by 4 to obtain your final answer.

b Use this strategy to calculate each product.

i 300 × 10 **ii** 700 × 100 **iii** 6000 × 100 **iv** 2000 × 1000

v 100 × 50 **vi** 1000 × 400 **vii** 10 000 × 8000 **viii** 100 × 9000

16 **a** One way to calculate 300 × 20 is to calculate 3 × 100 × 2 × 10 or 3 × 2 × 100 × 10.

i Calculate 3 × 2. **ii** Calculate 100 × 10.

iii Multiply the results you found in parts **i** and **ii** and write your answer to 300 × 20.

b Use this strategy to calculate each product.

i 400 × 20 **ii** 3000 × 30 **iii** 200 × 600 **iv** 9000 × 500

v 70 × 800 **vi** 600 × 4000 **vii** 30 000 × 7000 **viii** 800 × 20 000

17 Find each product.

a 40 × 10 × 200 **b** 60 × 900 × 3000 **c** 700 × 20 × 400 **d** 5000 × 300 × 80

18 Another strategy for multiplying is to look for pairs of numbers that are easy to multiply together first. For example, 25 × 18 × 4 is the same as 25 × 4 × 18. The product of 25 × 4 is 100, which makes the calculation 100 × 18 easier to work out without using long multiplication.

Calculate each of these without using a calculator or long multiplication.

a 25 × 18 × 4 **b** 5 × 679 × 2 **c** 20 × 5 × 4016 **d** 793 × 50 × 2

e 358 × 25 × 4 **f** 250 × 4 × 17 **g** 2 × 891 × 500 **h** 6055 × 5 × 200

19 **a** Estimate the result for each problem by first rounding each number to its leading digit, then multiplying. (Hint: use your strategy from question **16**.)

i 591 × 82 **ii** 2175 × 93 **iii** 7856 × 304 **iv** 63 019 × 5647

b Check how close your estimations are to the exact result.

PROBLEM SOLVING AND REASONING

20 A human heart beats around 72 times in a minute.

a How many times does it beat in one hour?

b How many times does it beat in one day?

21 To measure your pulse rate, place two fingers (not your thumb) on the inside of your wrist or at the side of your neck.

a Count the number of beats in 20 seconds, using a stopwatch or clock.

b A pulse rate is the number of beats in 1 minute. What is your pulse rate?

c Use your pulse rate to calculate the approximate number of times your heart beats in 1 hour.

d About how many times does your heart beat in 1 day?

PROBLEM SOLVING AND REASONING

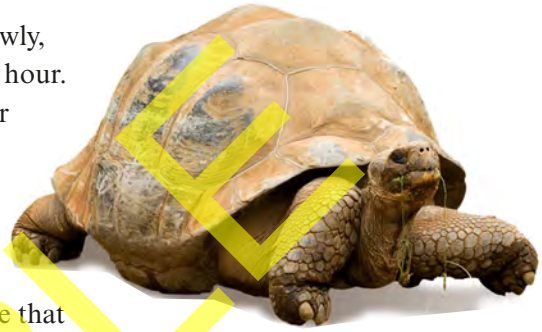
22 A school has 25 students in each of its 32 classes.

a How many students are enrolled at this school?

b On a particular day, 15 classes each have three students away from school and 11 classes have two students away. How many students are at school on this day?

23 Over the school holidays, a team of eight teenagers deliver take-away menus to homes near a pizza restaurant.
How many menus are delivered in a week if each teenager visits 46 homes each day?

24 Galápagos tortoises move extremely slowly, covering a distance of about 260 m in 1 hour. These tortoises can store food and water so well that they can go without eating or drinking for up to a year.



a What distance could a tortoise travel in 4 hours?

b Compare this result with the distance that a human could walk in 4 hours, assuming humans walk about 4500 m per hour.

c If a tortoise didn't eat or drink for 1 year, how long would this be in hours, assuming there are 365 days in a year?

d The longest lifespan on record belongs to a male tortoise kept in a British military fort for 154 years until he died from an accidental fall.
Work out how long he lived in:

i months **ii** days **iii** hours **iv** minutes **v** seconds

CHALLENGE

25 a Calculate how old you will be at your next birthday in:

i months **ii** days **iii** hours **iv** minutes **v** seconds

b Use your pulse rate from question **21** to work out the approximate number of heart beats you will have experienced in your life up to your next birthday.

> **NOTE** Assume there are 365 days in a year. As a super challenge, consider leap years in your working.

c About how many hours are there between now and your next birthday?

26 Earth travels a distance of about 2 575 200 km each day.
Assuming there are about 365 days in a year, estimate the distance Earth travels in one complete orbit around the Sun; that is, estimate how far it travels in one year.
(Hint: first round each number to its leading digit.)

Reflect

Why is the skill of multiplying by 10, 100, 1000, etc. so useful?

1E Dividing whole numbers

Start thinking!

As a treat, chimpanzees at the zoo are given some bananas to share. If there are 20 bananas to be shared among five chimpanzees, how many will each chimp get?

You need to find the result of dividing 20 by 5 or 20 ÷ 5.
This result is called the **quotient**.

1 Write the quotient for 20 ÷ 5.

2 Explain how you can check your answer using multiplication.

3 If there were six chimpanzees instead, how many bananas would each receive? Write the quotient and the remainder for 20 ÷ 6.

$$20 \div 6 = ?$$

dividend ÷ **divisor** = quotient and remainder

KEY IDEAS

- When dividing, always start with the leading or first digit. This is different from adding, subtracting and multiplying, where you start with the ones digits.
- Use short division when dividing by a number between 1 and 10.
- Use long division when dividing by a number larger than 10.
- In both methods, you set up the calculation for 871 ÷ 6 like this.

divisor → $6\overline{)871}$ ← dividend; ← quotient and remainder

- Calculating the remainder after each division stage is an important step in both methods. In the calculations shown, the remainders at each stage are in colour.
- One number is said to divide exactly into another when there is no remainder (the remainder is zero).

Short division

$$6\overline{)8^{2}7^{3}1}$$ 145 remainder 1

Long division

$$\begin{array}{r} 145 \text{ remainder } 1 \\ 6\overline{)871} \\ -6\downarrow \\ \hline 27\downarrow \\ -24 \\ \hline 31 \\ -30 \\ \hline 1 \end{array}$$

EXERCISE 1E Dividing whole numbers

UNDERSTANDING AND FLUENCY

1 For each division, identify:

i the dividend **ii** the divisor **iii** the quotient **iv** the remainder.

a 9 ÷ 2 = 4 remainder 1 **b** 17 ÷ 7 = 2 remainder 3 **c** 30 ÷ 5 = 6

2 Copy and complete this table. The first row has been done for you.

	think	quotient	quotient × divisor	remainder
7 ÷ 3	How many 3s in 7?	2	2 × 3 = 6	7 − 6 = 1
19 ÷ 5	How many 5s in 19?	3	3 × 5 = ___	19 − ___ = 4
33 ÷ 4	How many 4s in 33?		___ × 4 = ___	33 − ___ = ___
62 ÷ 15	How many 15s in 62?	4	4 × 15 = ___	62 − ___ = ___

3 Find the quotient and the remainder for each division problem.

a 13 ÷ 2 **b** 23 ÷ 5 **c** 17 ÷ 3 **d** 25 ÷ 10

e 38 ÷ 8 **f** 26 ÷ 4 **g** 66 ÷ 9 **h** 48 ÷ 6

EXAMPLE 1E-1 Using short division

Use short division to calculate each division problem. **a** 98 ÷ 4 **b** 459 ÷ 7

THINK

a **1** How many 4s in 9? (2) Write 2 above 9 in the quotient line. Work out the remainder. 4 × 2 = 8, so remainder = 9 − 8 = 1.

2 How many 4s in 18? (4) Write 4 above 8 in the quotient line. Work out the remainder. 4 × 4 = 16, so remainder = 18 − 16 = 2.

3 Write the answer.

b **1** How many 7s in 4? (none) How many 7s in 45? (6) Write 6 above 5 in the quotient line. Work out the remainder. 7 × 6 = 42, so remainder = 45 − 42 = 3.

2 How many 7s in 39? (5) Write 5 above 9 in the quotient line. Work out the remainder. 7 × 5 = 35, so remainder = 39 − 35 = 4.

3 Write the answer.

WRITE

a 2 4 remainder 2
4) 9 1 8

98 ÷ 4 = 24 remainder 2

b 6 5 remainder 4
7) 45 3 9

459 ÷ 7 = 65 remainder 4

4 Use short division to calculate each division problem.

a 538 ÷ 4 **b** 756 ÷ 6 **c** 172 ÷ 3 **d** 1229 ÷ 5

e 3048 ÷ 8 **f** 9812 ÷ 7 **g** 67 059 ÷ 2 **h** 286 347 ÷ 9

5 Another way is to write a division calculation is as a fraction. For example, 57 ÷ 3 is the same as $\frac{57}{3}$. The vinculum (horizontal line between the two numbers) replaces the division sign. Perform each division.

a $\frac{63}{9}$ **b** $\frac{1470}{6}$ **c** $\frac{658}{7}$ **d** $\frac{1251}{3}$

6 Copy this table.

First number ÷ second number	$\frac{\text{first number}}{\text{second number}}$	Quotient	Number of zeros in the first number	Number of zeros in the second number	Number of zeros in the quotient
10 ÷ 10	$\frac{10}{10}$				
100 ÷ 10	$\frac{100}{10}$				
1000 ÷ 10	$\frac{1000}{10}$				
10 000 ÷ 10	$\frac{10\,000}{10}$				
100 ÷ 100	$\frac{100}{100}$				
1000 ÷ 100	$\frac{1000}{100}$				
10 000 ÷ 100	$\frac{10\,000}{100}$				

a Write your answer to each division in the quotient column.

b Complete each row of the table by writing the number of zeros in the first number, the second number and the quotient.

c Can you see a pattern? Explain how this pattern provides a shortcut for calculations like this.

d Use this method to work out the quotient for each calculation.

i 10 000 ÷ 10 **ii** 100 000 ÷ 100

iii 1000 ÷ 1000 **iv** 10 000 ÷ 1000

7 a To work out 5000 ÷ 100, the calculation can be written as $\frac{5000}{100}$ or $\frac{5 \times 1000}{100}$ or $5 \times \frac{1000}{100}$.

i Calculate 1000 ÷ 100.

ii Multiply this result by 5 to obtain your final answer.

b Use this strategy to work out the quotient for each calculation.

i 200 ÷ 10 **ii** 600 ÷ 100

iii 9000 ÷ 100 **iv** 4000 ÷ 1000

v 800 ÷ 10 **vi** 3000 ÷ 100

vii 50 000 ÷ 1000 **viii** 6000 ÷ 10

8 **a** One way to work out 600 ÷ 20 is to write the calculation as $\frac{600}{20}$ or $\frac{6 \times 100}{2 \times 10}$ or $\frac{6}{2} \times \frac{100}{10}$.

i Calculate 6 ÷ 2. **ii** Calculate 100 ÷ 10.

iii Multiply the results obtained in parts **i** and **ii** and write your answer to 600 ÷ 20.

b Use this strategy to work out each division.

i $\frac{900}{30}$ **ii** $\frac{8000}{20}$

iii $\frac{1200}{60}$ **iv** $\frac{25\,000}{500}$

v 6000 ÷ 300 **vi** 80 000 ÷ 4000

vii 70 000 ÷ 700 **viii** 1600 ÷ 80

9 **a** Estimate the quotient to each division problem by first rounding each number to its leading digit before dividing. (Hint: use your strategy from question **8**.)

i 627 ÷ 33 **ii** 5940 ÷ 18

iii 3852 ÷ 214 **iv** 83 490 ÷ 3795

b Use a calculator to check how close your estimations are to the exact result.

EXAMPLE 1E-2 Using long division

Use long division to calculate 6492 ÷ 19.

THINK

1 How many 19s in 6? (none) How many 19s in 64? (3) Write 3 (shown in orange) above 4 in the quotient line. Work out the remainder. 19 × 3 = 57, so remainder = 64 − 57 = 7.

2 Bring down 9 and write it beside the remainder of 7. This makes the next number to divide into become 79.

3 How many 19s in 79? (4) Write 4 (shown in blue) above 9 in the quotient line. Work out the remainder. 19 × 4 = 76, so remainder = 79 − 76 = 3.

4 Bring down 2 and write it beside the remainder of 3. This makes the next number to divide into become 32.

5 How many 19s in 32? (1) Write 1 (shown in green) above 2 in the quotient line. Work out the remainder. 19 × 1 = 19, so remainder = 32 − 19 = 13.

6 Write the answer.

WRITE

```
   3
19)6492
 −57
   7
```

```
   34
19)6492
 −57↓
   79
  −76
    3
```

```
   341 remainder 13
19)6492
 −57↓
   79↓
  −76↓
    32
   −19
    13
```

6492 ÷ 19 = 341 remainder 13

10 Use long division to calculate each division problem.

a 542 ÷ 21 **b** 739 ÷ 18 **c** 884 ÷ 26 **d** 7462 ÷ 35

e 1658 ÷ 43 **f** 37 610 ÷ 50 **g** 43 803 ÷ 31 **h** 90 300 ÷ 28

11 Work out each division.

a $\frac{272}{17}$ **b** $\frac{3350}{25}$ **c** $\frac{47\,136}{32}$ **d** $\frac{36\,088}{52}$

12 Georgia is helping to arrange a birthday party for her younger brother. She has 165 sweets to share among 15 party bags. To work out the number of sweets in each party bag, you can write the calculation in a number of ways.

a Copy and complete these sentences to describe the same calculation in different ways.

i Divide _____ by _____.

ii How many times does _____ go into _____?

iii Find the result of _____ ÷ _____.

iv Find the quotient of _____ divided by _____.

b Are there any other ways of describing the calculation? List them.

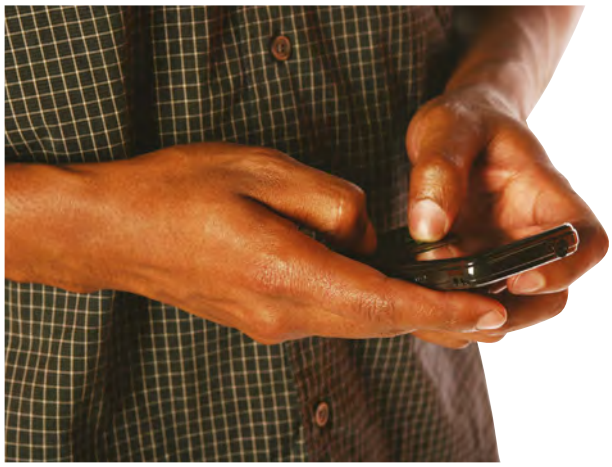
c Find the number of sweets in each party bag.

13 Tanya has 144 minutes of 'talk time' left on her mobile phone. She wants to phone six of her friends. How long should a call be if she talks to each friend for the same amount of time?

14 Chris can type messages via SMS on his mobile phone at a rate of 68 characters each minute.

a How long would it take him to type a message of 272 characters?

b Explain how you could check the answer to this division problem using multiplication.



PROBLEM SOLVING AND REASONING

15 Charlotte swam 1500 m in a 50-m swimming pool. How many laps of the pool did she complete?

16 The ancestors of the common wombat and the southern hairy-nosed wombat were the giant wombats (diprotodons) that lived in Australia from two million years ago, disappearing around 40 000 years ago. A giant wombat was about 3 m long and 180 cm high, with a mass of about 2000 kg. The mass of a common wombat is about 37 kg. The photo shows a hairy-nosed wombat and its mass.

a How many common wombats would be approximately equivalent in mass to one giant wombat?

b How many hairy-nosed wombats would be approximately equivalent in mass to one giant wombat?

17 Isobel and Conor are helping to make up food parcels for a charity organisation. There are 1350 bags of rice to be shared among 84 food parcels.

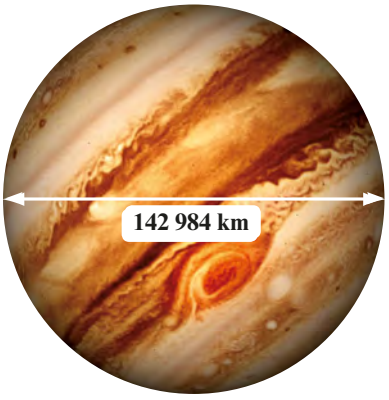
a How many bags of rice will be in each food parcel?

b How many bags of rice will be left over?

c Suggest the number of food parcels that could be made up with an equal share of the bags of rice so that no bags are left over.

CHALLENGE

18 Earth orbits the Sun in about 365 days, while the planet Jupiter completes an orbit around the Sun in about 4333 Earth days. Earth has a diameter of approximately 12 756 km.



a Estimate the number of times that Earth would complete a full orbit around the Sun for each one of Jupiter's orbits. (Hint: first round each number to its leading digit.)

b If Earth was drawn to the same scale as Jupiter is shown in the photo, about how many Earths would fit across the equator of Jupiter?

Reflect

Why is finding the remainder so useful in each step of a long division calculation?

1F Powers and square roots

Start thinking!

1 A short way of writing the repeated multiplication $2 \times 2 \times 2$ is 2^3, which is read as '2 to the power of 3'.

a What number is being repeatedly multiplied? This number is called the **base**.

b How many times has the same number (or the base) been written in the multiplication problem? This value is called the **power** or the **index**.

c Work out the product of $2 \times 2 \times 2$. This result is called the **basic numeral**.

power or index

2^3

base

2 Consider 3^5, which is a repeated multiplication written in **index form** or **index notation**.

a How is this number read?

b What is the base?

c What is the index or power?

d How many times is the base written, in the expanded form of the number?

e Carry out the repeated multiplication to find the basic numeral.

f Copy and complete this statement.

$3^5 = 3 \times \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_ = \_\_\_\_$

index form | expanded form | basic numeral

KEY IDEAS

- A repeated calculation can be written in index form or index notation.

  $2^3 = 2 \times 2 \times 2 = 8$

  index form | expanded form | basic numeral

  NOTE The plural of the word index is *indices*. For example, the powers or indices in the calculation $2^3 + 2^5$ are 3 and 5.

- The base is the number to be repeatedly multiplied and the power or index indicates the number of times the base is written. (Other names for power or index are *exponent* or *logarithm*.)

  power or index

  2^3

  base

- To square a number is to multiply it by itself or raise it to the power of 2. For example, $3^2 = 9$.

- To find the **square root** of a number is to find the number that when squared (raised to the power of 2) results in the original number. For example, $\sqrt{9} = 3$.

EXERCISE 1F Powers and square roots

1 The number 6^5 is in index form.
- **a** What does the 6 indicate?
- **b** What does the 5 indicate?

EXAMPLE 1F-1 Converting to index form

Write each repeated multiplication problem in index form.

a $8 \times 8 \times 8 \times 8 \times 8 \times 8$ **b** $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$

THINK

a Write the number that is being repeatedly multiplied as the base (8). Count the number of times the base is written and write it as the index (6).

b Notice that there are two different numbers to be repeatedly multiplied. Write the first base (3) with its power (2) multiplied to the second base (4) with its power (5).

WRITE

a $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^6$

b $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4 = 3^2 \times 4^5$

UNDERSTANDING AND FLUENCY

2 Write each repeated multiplication problem in index form.
- **a** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
- **b** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
- **c** $11 \times 11 \times 11$
- **d** $20 \times 20 \times 20 \times 20 \times 20$
- **e** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
- **f** $300 \times 300 \times 300 \times 300$

3 How would you read each answer you obtained in question **2**? Write each answer using words.

4 Write each multiplication problem in index form.
- **a** $7 \times 7 \times 7 \times 9 \times 9 \times 9 \times 9$
- **b** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2$
- **c** $3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
- **d** $8 \times 8 \times 8 \times 8 \times 13 \times 13$
- **e** $2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7$
- **f** $19 \times 19 \times 23 \times 23 \times 23 \times 23 \times 31$

EXAMPLE 1F-2 Calculating the value of a number in index form

Write 9^3 in expanded form and calculate its value.

THINK	WRITE
1 Write the repeated multiplication in expanded form. Identify the base (9) and the index (3). This means that 9 is written 3 times.	$9^3 = 9 \times 9 \times 9$
	$= 81 \times 9$
2 Calculate the basic numeral.	$= 729$

UNDERSTANDING AND FLUENCY

5 Write each of these in expanded form and calculate its value.

a 3^2 **b** 2^5 **c** 7^3 **d** 5^2

e 9^4 **f** 4^3 **g** 1^5 **h** 6^3

6 Copy and complete this table.

Index form	Base	Index or power	Expanded form	Basic numeral
2^4	2	4	$2 \times 2 \times 2 \times 2$	
6^3	6			
		2	7×7	
			$5 \times 5 \times 5$	

7 Which number is bigger: 3^5 or 5^3?

8 Arrange the numbers in each list from smallest to largest. You may like to use a calculator to help you.

a $3^2, 4^5, 2^3, 5^4$ **b** $7^6, 6^7, 1^{50}, 50^3$

c $3^{10}, 9^4, 6^5, 10^3$ **d** $8^3, 3^8, 4^6, 12^2$

EXAMPLE 1F-3 Calculating the product of two numbers in index form

Write $2^4 \times 3^2$ in expanded form and calculate its value.

THINK	WRITE
1 Write the multiplication calculation in expanded form.	$2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
2 Perform the calculation.	$= 16 \times 9$
	$= 144$

9 Write each of these in expanded form and calculate its value.

a $2^3 \times 4^2$ **b** $3^4 \times 5^2$ **c** $8^2 \times 10^3$ **d** $3^5 \times 1^4$

e $2^6 \times 7^2$ **f** $10^2 \times 3^3$ **g** $9^4 \times 11^2$ **h** $6^2 \times 3^2$

10 Calculate each of these by first working out the value of any numbers written in index form.

a $3^4 + 2^3$ **b** $10^6 + 5^2$ **c** $8^2 - 2^6$ **d** $4^3 - 1^9$

e $10^2 \div 5^2$ **f** $6^4 \div 3^3$ **g** $4^2 + 2^4 + 3^3$ **h** $9^2 + 2^2 - 1^5$

11 a Copy and complete this table.

Index form	Expanded form	Basic numeral	Number of zeros in the basic numeral
10^1	10	10	1
10^2	10×10	100	
10^3			
10^4			
10^5			

b Can you see a pattern? This pattern provides a quick method of finding the value of a number in index form with a base of 10. Describe this method.

c Use this method to write the basic numeral for each of these.

i 10^8 **ii** 10^6 **iii** 10^{15} **iv** 10^{10}

d What is the basic numeral for 10^0? Use the pattern seen in the table.

e Write each number in index form with a base of 10 and the appropriate power.

i 100 **ii** 10 000 **iii** 1 **iv** 10 000 000

12 For this task, you will need at least 25 counters. You may like to work with one or more classmates.

a Arrange four counters to make a square pattern. How many counters are there in each row and column of the square? Draw a diagram of this.

b Try a different number of counters to make another square pattern. How many counters are there in each row and column? Draw a diagram of this.

c Repeat part **b** with other groups of counters to make all the possible squares you can with the counters you have. Draw a diagram of each square pattern.

d Each number you have modelled with counters in parts **a**, **b** and **c** is called a **square number**. Why do you think this is? Write the square number next to its matching diagram.

e How many square numbers are there from 1 to 25? (Hint: is 1 a square number? Does it have the same number of counters in each row and column?)

f List the next five square numbers.

13 **a** Copy and complete this table.

Index form	Expanded form	Basic numeral
1^2	1×1	1
2^2	2×2	
3^2		9
4^2		
5^2		

b Compare the square numbers you found in question **12** with the basic numerals you have listed in the table. What do you notice?

c The number 3^2 is read as '3 to the power of 2'. It can also be read as '3 squared' or 'the square of 3'. Can you see why this is? Explain.

d Write each description in both index form and as a basic numeral.

i 9 squared **ii** 4 squared

iii the square of 10 **iv** the square of 7

14 Find the value of each of these.

a 11^2 **b** 25^2 **c** 100^2 **d** 32^2

e $3^2 + 1^2$ **f** $8^2 - 6^2$ **g** $7^2 + 9^2$ **h** $10^2 - 5^2$

15 What number, when it is squared, gives each of these results?

a 25 **b** 81 **c** 64 **d** 9

e 1 **f** 100 **g** 4 **h** 36

16 Another way of asking 'what number is squared to give a result of 25?' is 'find the square root of 25'.

a What is the square root of 25?

b A short way of writing the square root of 25 is $\sqrt{25}$. Notice that the symbol $\sqrt{}$ means 'the square root of' the given number. Write this problem and your answer using the square root symbol.

c Write each of these using the square root symbol and find its value.

i the square root of 16 **ii** the square root of 81

iii the square root of 4 **iv** the square root of 100

17 Find the value of each of these.

a $\sqrt{9}$ **b** $\sqrt{36}$ **c** $\sqrt{64}$ **d** $\sqrt{1}$

e $\sqrt{49}$ **f** $\sqrt{121}$ **g** $\sqrt{144}$ **h** $\sqrt{400}$

18 Without using a calculator, copy and complete each statement.

a $12^2 = 144$ so $\sqrt{144} =$ ___ **b** $35^2 = 1225$ so $\sqrt{1225} =$ ___

c $61^2 = 3721$ so $\sqrt{} = 61$ **d** $298^2 = 88\,804$ so $\sqrt{} = 298$

19 Explain how finding the square of a number and finding the square root of a number are related.

PROBLEM SOLVING AND REASONING

20 A colony of bacteria grows very quickly and triples its size each day.

a Copy and complete this table.

Number of days	Amount of growth	Amount of growth in index form	Number of times larger than original size
1	3	3^1	3
2	3×3	3^2	9
3	$3 \times 3 \times 3$		
4			
5			

b How many times larger is the bacteria colony after:

i 6 days? **ii** 1 week? **iii** 2 weeks?

c How long does it take for the bacteria colony to be:

i 27 times larger? **ii** 243 times larger? **iii** 6561 times larger?

21 After 1 week, the weight of a baby mouse is double its weight at birth. For the next few weeks, its weight continues to double each week.

a What number do you multiply by to double a quantity?

b After 1 week, the mouse would have a weight that is twice as big as the birth weight. After 2 weeks, the weight would be 2×2 or 4 times as big as the birth weight. Write each number in index form.

c In index form, write the number of times the weight is bigger than the birth weight after three weeks.

d How much bigger is the baby mouse after five weeks?

22 Diana sends a text message to four of her friends. Each friend forwards it to another four people, who each then send it to another four people.

a How many text messages have been sent?

b Explain how powers can be used to solve this problem.

CHALLENGE

23 Over the summer school holidays, Taylor was offered a part-time job earning her $100 per week. The job was available for 4 weeks. Taylor decided to discuss a different payment plan with her prospective boss. 'How about paying me only $5 in the first week and then in each of the other weeks paying me five times as much as the week before.' The boss thought this new plan might save her money. Which pay offer do you think the boss should go with? Show your calculations to justify your answer.

Reflect

How are powers and multiplication related?

1G Order of operations

Start thinking!

1 Consider this problem: $5 + 3 \times 4 - 6 \div 2$.

a Find the answer without using a calculator.

b Write a list of the operations (+, −, ×, etc.) you used and the order in which you used them.

c Compare your answer to part **a** with those of two classmates. Did you all get the same?

d Discuss the steps you each used to obtain your answer. Which answer do you think is correct?

To avoid confusion in calculations like this, mathematicians agree to use a particular order when performing a number of operations in the one problem. This is the **order of operations** or set of rules to follow:

First	**B**rackets (operations inside grouping symbols)
Second	**I**ndices (powers and square roots)
Third	**D**ivision and **M**ultiplication
Fourth	**A**ddition and **S**ubtraction

2 One way to remember this order is to think of **BIDMAS**. Can you see why?

NOTE Some people use **BODMAS**, which has 'O' for 'power of' in place of 'I' for 'indices'.

3 Follow the correct order of operations to calculate the answer to the problem in question **1**.

KEY IDEAS

- The order of operations is:

first:	**B**rackets (operations inside grouping symbols are always calculated first)
second:	**I**ndices (powers and square roots)
third:	**D**ivision and **M**ultiplication (work from left to right)
fourth:	**A**ddition and **S**ubtraction (work from left to right)

- Operations at the same ranking are performed in order from left to right.
- Where there is more than one set of grouping symbols in the problem, calculate the operations inside the innermost brackets first.
- One way of remembering the order is to think of **BIDMAS** (or **BODMAS**).

EXERCISE 1G Order of operations

EXAMPLE 1G-1 Order of operations with no grouping symbols

Without using a calculator, work out the result to each of these.

a $3 + 8 \div 2$ **b** $7 \times 2^3 - 9 \times 5$

THINK	WRITE
a 1 Write the problem.	**a** $3 + 8 \div 2$
2 Division is performed before addition. ($8 \div 2 = 4$)	$= 3 + 4$
3 Perform the addition.	$= 7$
b 1 Write the problem.	**b** $7 \times 2^3 - 9 \times 5$
2 Powers and square roots (indices) are worked out before $\times$, $\div$, + and −. ($2^3 = 8$)	$= 7 \times 8 - 9 \times 5$
3 Multiplication is performed before subtraction so, working from left to right, calculate 7×8 first. ($7 \times 8 = 56$)	$= 56 - 9 \times 5$
4 Perform the other multiplication next. ($9 \times 5 = 45$)	$= 56 - 45$
5 Perform the subtraction.	$= 11$

UNDERSTANDING AND FLUENCY

1 Without using a calculator, work out the result to each of these.

a $6 + 15 \div 3$ **b** $9 - 4 \times 2$

c $7 + 3 \times 6 - 4$ **d** $12 \div 4 + 7 \times 8$

e $4 \times 4 + 6 \div 2$ **f** $2 + 9 \times 4 \div 6$

g $10 \times 6 - 5 \times 9$ **h** $8^2 - 5 \times 3 + 11$

i $21 - 3^2 \times 2 + 1$ **j** $4 \times 3^3 - 6^2 \times 2 + 1$

k $\sqrt{9} + 9 \times 8 - 2^3$ **l** $22 - 2 \times 11 + 4^2 \div \sqrt{16}$

2 Perform the calculations in question **1** using a calculator. Enter each number and operation from left to right. Does your calculator perform the operations in the correct order?

3 For each calculation:

i find an estimate of the answer by first rounding each number to its leading digit

ii use a calculator to work out the exact answer.

a $57 + 216 \times 4$ **b** $41 + 572 \div 286$

c $10^2 - 33 \times 3 + 82$ **d** $3852 \div 963 + 78 \times 29$

e $51 \times 4 \div 17 + 626$ **f** $5328 \times 2 - 81 \times 38$

g $\sqrt{49} + 756 \div 42 \times 6 - 3^2$ **h** $2765 + 429 - 57 \times 5 \div 19 + 12$

EXAMPLE 1G-2 Order of operations with grouping symbols

Without using a calculator, work out the result to each of these.

a $16 \div (10 - 2) + 3 \times 7$ **b** $8 + [12 \div (9 - 5)]^2 - 1$

THINK	WRITE
a 1 Write the problem.	**a** $16 \div (10 - 2) + 3 \times 7$
2 Perform any operations inside brackets first. $(10 - 2 = 8)$	$= 16 \div 8 + 3 \times 7$
3 Perform ÷ and × before +. Working from left to right, calculate $16 \div 8$ first. $(16 \div 8 = 2)$	$= 2 + 3 \times 7$
4 Perform × before +. $(3 \times 7 = 21)$	$= 2 + 21$
5 Perform the addition.	$= 23$
b 1 Write the problem.	**b** $8 + [12 \div (9 - 5)]^2 - 1$
2 There are two sets of grouping symbols (brackets). Perform the operation in the innermost set of brackets first. $(9 - 5 = 4)$	$= 8 + [12 \div 4]^2 - 1$
3 Perform the operation in the remaining set of brackets. $(12 \div 4 = 3)$	$= 8 + 3^2 - 1$
4 Perform the operation of squaring as 'indices' comes before + and −.	$= 8 + 9 - 1$
5 Working from left to right, perform the addition then the subtraction.	$= 17 - 1$ $= 16$

UNDERSTANDING AND FLUENCY

4 Without using a calculator, work out the result to each of these.

a $28 \div (12 - 5) + 2 \times 6$ **b** $(9 + 3) \div (15 - 13)$

c $4^3 - 10 \times (3 + 1)$ **d** $6 + [18 \div (10 - 1^5)]^2 - 7$

e $\sqrt{100} \times 5 - 2 \times (2^4 + 3)$ **f** $3 \times [8 + (2 \times 9 - 4)] + 20$

5 For each calculation:

i find an estimate of the answer by first rounding each number to its leading digit

ii use a calculator to work out the exact answer.

a $69 \times (28 + 11) - 51 \times 2$ **b** $(67 + 31) \div (12 - 5)$

c $815 \times (61 - 49) + 17 \times 3$ **d** $19 \times (9 - 4) + 3 \times (52 + 27)$

e $\sqrt{100} \times 7 + 2 \times (3^2 + 42)$ **f** $[13 \times (7 - 5) + 28] \times 2$

6 The **average** of a set of scores is found by working out this calculation:

(total sum of the scores) ÷ number of scores.

For example, the average of 9, 10 and 14 is found by working out $(9 + 10 + 14) \div 3$.

a Use the correct order of operations to find the average of 9, 10 and 14.

b Find the average of each set of numbers.

i 15, 18 and 24
ii 3, 5, 6, 8, 9 and 11
iii 278 and 356
iv 7, 7, 11 and 15
v 50, 51, 52, 52 and 60
vi 50, 20, 30, 40, 60, 50 and 30

7 The set of rules called the order of operations is also related to some other mathematical laws. You will have used these laws in your calculations without realising.

a Calculate 8 + 2.
b Calculate 2 + 8.
c Compare your answers to parts **a** and **b**. Are they the same?
d Does it matter in which order you add numbers? This is called the **commutative law**. Write another example to show that the commutative law (where order does not matter) works for addition.
e Does it matter in which order you subtract numbers? For example, does 8 − 2 give the same answer as 2 − 8?
f Does it matter in which order you multiply numbers? Provide some examples to help explain your answer.
g Does it matter in which order you divide numbers? Provide some examples to help explain your answer.
h Look at your answers to parts **d**–**g**. Which operations obey the commutative law?

8 Consider what happens if you add three numbers in a different order. Let's try it with 6 + 3 + 5.

a Calculate 6 + 3 first and then add 5.
b Calculate 6 + 5 first and then add 3.
c Calculate 3 + 5 first and then add 6.
d Compare your answers to parts **a**, **b** and **c**. What does this indicate about the order in which you can add these three numbers?
e The **associative law** states that you can add numbers in any order. Apply this law to find the sum of 25 + 37 + 13. Which two numbers are easier to add together first?

9 Consider what happens if you multiply three numbers in a different order. Let's try it with 6 × 3 × 5.

a Calculate 6 × 3 first and then multiply by 5.
b Calculate 6 × 5 first and then multiply by 3.
c Calculate 3 × 5 first and then multiply by 6.
d Compare your answers to parts **a**, **b** and **c**. What does this indicate about the order in which you can multiply these three numbers?
e The associative law also states that you can multiply numbers in any order. Explain how the associative law can make the calculation 25 × 58 × 4 easier to perform.

10 Do you think that the associative law would apply to the operations of subtraction and division? Try some examples to help you decide.

UNDERSTANDING AND FLUENCY

11 **a** Calculate $3 \times (4 + 5)$ using the correct order of operations.

b Calculate $3 \times 4 + 3 \times 5$.

c Compare your answers to parts **a** and **b**. This demonstrates another law called the **distributive law**.

d Compare your answers to $7 \times (10 + 6)$ and $7 \times 10 + 7 \times 6$. Does this demonstrate the distributive law? Explain why or why not.

e Explain how the distributive law can make a calculation like 23×16 easier to perform without using long multiplication. (Hint: 23×16 is the same as $23 \times (10 + 6)$.)

f Use the distributive law to calculate each of these without using long multiplication.

i 35×14 **ii** 68×19 **iii** 41×102 **iv** 87×106

PROBLEM SOLVING AND REASONING

12 In Australian Rules football, each team scores goals and behinds. Each goal has a value of six points and each behind has a value of one point. During a match, the scoreboard usually shows the number of goals, behinds and points both teams have scored. This photo taken by a football fan does not show the number of points.

a Copy and complete these statements to show the numbers and operations needed to calculate the total points scored by each team.

Points scored by Sharks = _____ × 6 + _____ × 1

Points scored by Visitors = _____ × _____ + _____ × _____

b Calculate the number of points each team has scored.

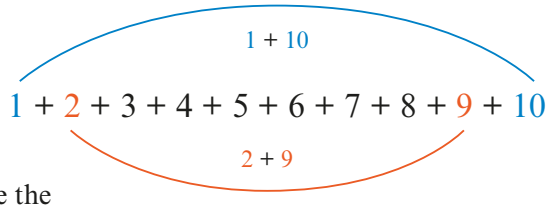
c At this stage of the match, which team is winning and by how much?

PROBLEM SOLVING AND REASONING

13 **a** Add the numbers in order from 1 to 10.

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$ _____.

b Is there a quicker way of working out the sum? Let's try a strategy where pairs of numbers are matched together. (You tried a version of this earlier in the chapter.)



Copy and complete this diagram, where the smallest number and the largest number are paired together, then the second smallest number and the second largest number are paired together and so on. Remember that you can add numbers in any order and still obtain the same total.

c What is the result when each pair of numbers is added?

d How many pairs of numbers are there?

e Use the operation of multiplication and your answers to parts **c** and **d** to calculate the sum of all the numbers from 1 to 10.

14 Use the strategy from question **13** to find the sum of all the whole numbers in these groups.

a 1 to 20 **b** 1 to 40 **c** 10 to 50 **d** 15 to 35

15 In the late 1780s, students in a class were asked to add the whole numbers from 1 to 100. A student named Johann Carl Friedrich Gauss (who later became a famous mathematician) correctly answered the question in less than one minute by using the strategy of adding pairs of numbers. Try this calculation yourself. Can you find the answer in less than one minute?

16 For your birthday, you and a group of friends go to the cinema. At the cinema kiosk, you order eight medium soft drinks, four jumbo containers of popcorn and seven ice creams.

a Write a mathematical calculation to show how to work out the total cost of the food and drink.

b Perform the operations in the calculation to find the total cost.

CHALLENGE

17 Pick a number between 1 and 10. Use this number as many times as you like with any of the operations +, −, × and ÷ to make a mathematical calculation that gives a result of 7. You can also use brackets and square root symbols or use the number as a power. For example, $(3^3 + 3) \div 3 - 3 = 7$.

Reflect

Why is it important to perform operations in the correct order?

1H Multiples and factors

Start thinking!

1 **a** Copy and complete this table for the first eight numbers of the pattern 3, 6, 9, 12, 15, …

Number	3 × ?
3	3 × 1
6	3 × 2
9	3 × ____
12	3 × ____
15	3 × ____

b What would be the 40th number in this pattern? Describe a quick way to work this out.

c These numbers are called **multiples** of 3. They are the result of multiplying 3 by another whole number. Give five more examples of numbers that are multiples of 3.

2 Multiplying the two whole numbers 4 and 9 gives a product of 36.

a List another two whole numbers that give a product of 36.

b Are there are other pairs of whole numbers that multiply to give 36? List them.

c List all the whole numbers that have been used to give the product 36. These whole numbers are called **factors**.

3 Explain how multiples and factors are different.

KEY IDEAS

- The multiples of a number result from multiplying that number by another whole number. For example, the multiples of 3 are 3, 6, 9, 12, 15, 18, …
- The factors of a number are all the whole numbers that divide exactly into that number. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- It is often easier to find factors in pairs. For example, the factor pairs that give a product of 12 are 1 and 12, 2 and 6, 3 and 4.
- A number divides exactly into another number if there is no remainder (the remainder is zero).
- The **lowest common multiple (LCM)** of two or more numbers is the smallest number that both or all the numbers divide into.
- The **highest common factor (HCF)** of two or more numbers is the highest number that will divide into both or all the numbers.

EXERCISE 1H Multiples and factors

1 Copy and complete each number pattern.

a 2, 4, 6, ___, ___, ___
b 9, 18, 27, ___, ___, ___
c 5, 10, 15, ___, ___, ___
d 4, 8, 12, ___, ___, ___
e 8, 16, 24, ___, ___, ___
f 7, 14, 21, ___, ___, ___

EXAMPLE 1H-1 Finding multiples

List the first six multiples of 7.

THINK	WRITE
1 Each multiple of 7 results from multiplying 7 by a whole number. For the first six multiples, 7 is multiplied first by 1, then 2, then 3, then 4, then 5 and then 6.	$7 \times 1 = 7$ $7 \times 2 = 14$ $7 \times 3 = 21$ $7 \times 4 = 28$ $7 \times 5 = 35$ $7 \times 6 = 42$
2 List the first six multiples of 7.	The first six multiples of 7 are 7, 14, 21, 28, 35 and 42.

UNDERSTANDING AND FLUENCY

2 List the first eight multiples of each number.

a 10 **b** 6 **c** 9 **d** 5 **e** 11 **f** 30

3 Write the multiples of 8 between 23 and 65.

4 Write the multiples of 7 between 50 and 100.

5 The first ten multiples of 3 and 4 are shown.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, …
Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …

a Copy the two lists and circle the numbers that occur in both.
b The numbers that are the same or common to each list are called **common multiples**. Write the common multiples you have found.
c Are these the only common multiples of 3 and 4? Explain.
d Write the first six common multiples of 3 and 4.

6 **a** List the first ten multiples of 4.
b List the first ten multiples of 5.
c Circle the common multiples of 4 and 5 that you can see in the lists.
d Find the first four common multiples of 4 and 5.

7 **a** List the multiples of 5 and 6 that are less than 100.
b Write the common multiples of 5 and 6 that you can see in the lists.
c What is the smallest number that is a common multiple of 5 and 6? This is called the lowest common multiple (or LCM) of 5 and 6.

EXAMPLE 1H-2 Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of 2 and 3.

THINK	WRITE
1 List the multiples of 2.	multiples of 2: 2, 4, (6), 8, 10, (12), 14, 16, (18), 20, …
2 List the multiples of 3.	multiples of 3: 3, (6), 9, (12), 15, (18), 21, …
3 Write the multiples that are the same in the two lists. You can circle them first.	common multiples: 6, 12, 18, …
4 Identify the lowest number in the list of common multiples.	LCM is 6.

UNDERSTANDING AND FLUENCY

8 Find the lowest common multiple (LCM) of each pair of numbers.
a 2 and 5 **b** 4 and 7 **c** 6 and 9 **d** 8 and 12
e 5 and 8 **f** 7 and 10 **g** 15 and 20 **h** 8 and 6

9 Find the LCM of each group of numbers.
a 2, 3 and 4 **b** 5, 6 and 20 **c** 3, 4 and 9 **d** 4, 6 and 8
e 5, 3 and 2 **f** 30, 9 and 2 **g** 6, 7 and 8 **h** 2, 3, 4 and 5

10 Copy and complete each of these.
a 15 = 1 × 15 or 3 × ___ **b** 18 = 1 × ___ or 2 × ___ or 3 × ___
c 32 = 1 × ___ or ___ × 16 or ___ × ___ **d** 28 = 1 × ___ or ___ × 14 or 4 × ___
e 49 = 1 × ___ or ___ × 7 **f** 50 = ___ × 50 or 2 × ___ or 5 × ___

11 Use your answers to question **10** to list the factors of these numbers.
a 15 **b** 18 **c** 32 **d** 28 **e** 49 **f** 50

EXAMPLE 1H-3 Finding factors

Find the factors of 48.

THINK

1 List pairs of whole numbers that multiply to give a product of 48.

2 Write the factors of 48 as a list in ascending order. These are all the whole numbers that divide exactly into 48.

WRITE

factor pairs for 48: 1×48, 2×24, 3×16, 4×12, 6×8

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

UNDERSTANDING AND FLUENCY

12 Find the factors of each number.

a 6 **b** 20 **c** 56 **d** 81 **e** 27 **f** 100

13 Which factors are common (the same) in each group of numbers? Use your answers to questions **11** and **12** to help you.

a 6 and 20 **b** 81 and 27 **c** 50 and 100

d 56 and 100 **e** 6 and 81 **f** 32 and 56

g 6, 15 and 18 **h** 20, 28 and 32 **i** 28, 49 and 56

14 The factors of 24 and 36 are shown.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

a Copy the two lists and circle the numbers that are the same in each list.

b The numbers that are the same or common to each list are called **common factors**. Write the common factors you have found.

c Are these the only factors that are common for 24 and 36? Explain.

15 **a** List the factors of 18.

b List the factors of 45.

c Circle the common factors of 18 and 45 that you can see in the lists.

d Write the common factors of 18 and 45.

16 **a** List the factors of 12.

b List the factors of 30.

c Write the common factors of 12 and 30.

d What is the largest number that is a common factor of 12 and 30? This is called the highest common factor (or HCF) of 12 and 30.

EXAMPLE 1H-4 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 24 and 36.

THINK	WRITE
1 List the factors of 24.	factors of 24: 1, (2), (3), (4), (6), 8, (12), 24
2 List the factors of 36.	factors of 36: 1, (2), (3), (4), (6), 9, (12), 18, 36
3 Write the factors that are the same in the two lists. You can circle them first.	common factors: 2, 3, 4, 6, 12
4 Identify the highest number in the list of common factors.	HCF is 12.

NOTE The HCF is also called the greatest common divisor. Can you see why?

17 Find the highest common factors (HCF) of each pair of numbers.

a 8 and 24 **b** 15 and 27 **c** 24 and 42 **d** 5 and 20
e 36 and 32 **f** 45 and 30 **g** 63 and 35 **h** 50 and 100

18 Find the HCF of each group of numbers.

a 4, 8 and 12 **b** 10, 25 and 30 **c** 16, 24 and 32 **d** 15, 6 and 27
e 42, 36 and 18 **f** 20, 30 and 50 **g** 38, 26 and 14 **h** 9, 6, 12 and 18

19 Find the LCM and the HCF of each group of numbers.

a 12 and 30 **b** 10 and 35 **c** 4, 12 and 16 **d** 2, 9 and 18

20 Every fourth paling of this fence is painted blue.

a There are 84 palings in the fence. How many of them are painted blue?
b The fence is to be extended and will now have 128 palings. How many more palings need to be painted blue?
c Have you used multiples or factors to help you find the answers?

21 According to the timetable, a train is expected to depart from the station every 12 minutes. The first train of the day departs at 5.12 am.

a List the times in the first hour that a train should depart from the station.
b When will these trains depart from the station?
i the third train of the day **ii** the sixth train of the day
c Which train will depart from the station at 8.24 am?

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

22 Max is arranging 18 pieces of sushi on a plate for his guests. He would like to arrange them so that there is the same number of pieces in each row.

- **a** Suggest one way that Max could arrange the sushi on a plate. You may like to draw a diagram to show your answer.
- **b** How many possible ways are there? Explain the other arrangements that are possible.
- **c** Have you used multiples or factors of 18 to help find the answers?

23 For a school fete, Hanna and Josh are deciding how to divide up a batch of 120 cookies so that there is an equal number of cookies in each bag. Suggest at least three ways this could be done.

24 **a** What is the lowest common multiple of 3 and 4?

- **b** Use the LCM to write a list of the first six common multiples of 3 and 4.
- **c** Is there a number that is the highest common multiple of 3 and 4? Give a reason for your answer.

25 **a** What is the highest common factor of 18 and 45?

- **b** Is there a number that is the lowest common factor of 18 and 45? If so, write the number.
- **c** Explain why the lowest common factor of two (or more) numbers is not particularly useful to us. Think about the lowest common factors for different pairs of numbers.

CHALLENGE

26 Imogen and Olivia begin jogging around an oval at exactly the same time from the same starting point. Imogen runs each lap of the oval in 6 minutes and Olivia in 8 minutes.

- **a** When will they next pass the starting point at exactly the same time?
- **b** How many laps will each of them have jogged at this time?
- **c** Assuming they keep up the same pace, how many full laps will each girl have jogged when they stop after 1 hour 30 minutes?
- **d** How many times will they have both passed the starting point at exactly the same time during their run?

27 Find the smallest whole number that has only:

- **a** one factor
- **b** two factors
- **c** three factors
- **d** four factors
- **e** five factors
- **f** six factors.

Reflect

How can you remember the difference between finding multiples and finding factors?

1I Prime and composite numbers

Start thinking!

Any counting number (1, 2, 3, 4, …), except for the number 1, can be described as either a **prime number** or a **composite number**.

1 The prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17, 19.
- **a** For each prime number listed, write its factors.
- **b** How many factors does each prime number have?

2 The composite numbers from 1 to 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.
- **a** For each composite number listed, write its factors.
- **b** How many factors does each composite number have?
- **c** Does a composite number have more, less or the same number of factors as a prime number?

3 Explain how prime numbers and composite numbers are different.

4 How many factors does the number 1 have? Is this the same number of factors that a prime number has or a composite number has? Explain why the number 1 is a special number that is neither prime nor composite.

KEY IDEAS

- A prime number is a counting number (whole number) that has exactly two factors: itself and 1.
- A composite number is a counting number that has more than two factors.
- The number 1 is neither a prime number nor a composite number.
- You can use the fact that all even numbers, except 2, are composite numbers to help you decide whether a number is prime or composite.
- A composite number can be written as the product of factors that are prime numbers. These factors are called prime factors.

For example:

$6 = 2 \times 3$

composite number; prime factors

- For larger composite numbers, it can be useful to produce a factor tree to find the prime factors.

EXERCISE 1I Prime and composite numbers

UNDERSTANDING AND FLUENCY

1 A Greek mathematician named Eratosthenes (pronounced 'E-rah-toss-thee-nees'), who lived from 276 BCE to 195 BCE, is famous for his easy method of using multiples to separate out all the composite numbers and leave only the prime numbers. This method is called 'The sieve of Eratosthenes'.

a Copy the numbers from 1 to 100 in a grid.

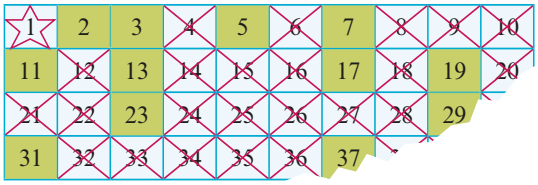
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

b Draw a star around the number 1, as it is neither a prime number nor a composite number.

c Move to the next number, which is 2, and highlight the number by drawing a circle around it or colouring the grid square containing 2. Then cross out all the multiples of 2. (That is, cross out 4, 6, 8, 10, …, 100.)

d Move to the next number, which is 3, and highlight the number. Then cross out all the multiples of 3 that have not already been crossed out.

e Move to the next number that is not crossed out and highlight it. Then cross out all the multiples of that number that have not already been crossed out.



f Repeat the step in part **e** until all the numbers from 2 to 100 have been highlighted or crossed out.

2 Look at your completed grid from question **1**. The highlighted numbers are prime numbers and those crossed out are composite numbers.

a List all the prime numbers from 20 to 100.

b List all the composite numbers from 11 to 19.

c How many one-digit prime numbers are there? List them.

d How many even prime numbers are there? List them.

e What is the largest two-digit prime number in which each digit is a prime number?

3 List the factors for each number.

a 24 **b** 49 **c** 73 **d** 125 **e** 37 **f** 82

4 Use your answers to question **3** to decide whether each number is prime or composite.

EXAMPLE 1I-1 Identifying prime and composite numbers

State whether each number is prime or composite. Give a reason to support your answer.

a 66 **b** 43 **c** 57

THINK	WRITE
a Is the number even? (yes) All even numbers except 2 are composite.	**a** 66 is a composite number since it is an even number (that is not 2) and so has more than two factors.
b Is the number even? (no) List its factors.	**b** Factors of 43 are 1, 43. 43 is a prime number since it has exactly two factors.
c Is the number even? (no) List its factors.	**c** Factors of 57 are 1, 3, 19, 57. 57 is a composite number since it has more than two factors.

UNDERSTANDING AND FLUENCY

5 State whether each number is prime or composite. Give a reason for your answer.

a 28 **b** 47 **c** 79 **d** 145
e 131 **f** 200 **g** 203 **h** 303

6 Find two prime numbers that add to give each sum.

a 24 **b** 36 **c** 82 **d** 144

7 Find two prime numbers that multiply to give each product.

a 21 **b** 55 **c** 26 **d** 115

8 Repeat the method used in question **1** to find all the prime numbers from 101 to 200.

9 Answer true or false to each statement. Give a reason for your answer.

a All even numbers are prime numbers.
b All odd numbers are prime numbers.
c The first two prime numbers are 2 and 3.
d All composite numbers are even numbers.
e The sum of two prime numbers is always even.
f The product of two prime numbers is always a composite number.
g The number 1 is neither a prime number nor a composite number.
h Between 1 and 30, there are more composite numbers than prime numbers.

EXAMPLE 1I-2 Writing a composite number as the product of prime factors

For each composite number:

i find its prime factors

ii write it as a product of prime factors.

a 8 b 12 c 30

THINK

a **1** Find the factors of 8.

2 Identify the prime factors of 8.

3 Use the prime factor to write 8 as a product.

b **1** Find the factors of 12.

2 Identify the prime factors of 12.

3 Use the prime factors to write 12 as a product.

c **1** Find the factors of 30.

2 Identify the prime factors of 30.

3 Use the prime factors to write 30 as a product.

WRITE

a factors of 8: 1, 2, 4, 8

i prime factor of 8: 2

ii $8 = 2 \times 2 \times 2$

b factors of 12: 1, 2, 3, 4, 6, 12

i prime factors of 12: 2 and 3

ii $12 = 2 \times 2 \times 3$

c factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

i prime factors of 30: 2, 3 and 5

ii $30 = 2 \times 3 \times 5$

UNDERSTANDING AND FLUENCY

10 For each composite number:

i find its prime factors

ii write it as a product of prime factors.

a 4 **b** 10 **c** 18 **d** 35

e 16 **f** 20 **g** 27 **h** 40

11 From earlier work in this chapter, you will have seen that repeated factors can be expressed in a form using powers. Factors that are not repeated remain the same.

For example, the composite numbers in Example 1I-2 can be written as:

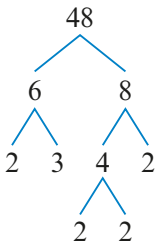
$8 = 2 \times 2 \times 2 = 2^3$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$30 = 2 \times 3 \times 5$

For each composite number in question **10**, write the product of prime factors using powers.

12 For larger composite numbers, it is useful to produce a factor tree to find the prime factors. The factor tree for 48 starts with two branches showing a product of two factors (say, 6×8) and continues with pairs of branches until all factors are prime numbers.



Use the factor tree to answer the following.

a What is the pair of factors branching from 6? Are they prime numbers?

b What is the pair of factors branching from 8? Are they prime numbers?

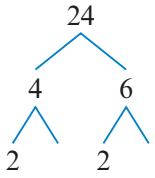
c Explain why you need to have a further pair of factors branching from 4 but not from the other numbers.

d List the prime factors of 48.

e Write 48 as a product of its prime factors.

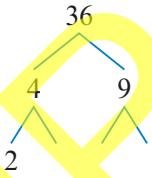
13 Copy and complete each factor tree and show each number as a product of prime factors.

a



Prime factors of 24 are:
2 and ___
$24 = 2 \times 2 \times$ ___ $\times$ ___
$= 2^3 \times$ ___

b



Prime factors of 36 are:
___ and ___
$36 = 2 \times$ ___ $\times$ ___ $\times$ ___
$= 2^2 \times$ ___

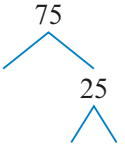
c

Prime factors of 45 are:
___ and 5
$45 =$ ___ $\times 3 \times$ ___
$=$ ___ $\times$ ___

d

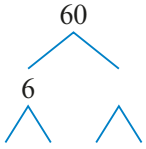
Prime factors of 99 are:
3 and ___
$99 = 3 \times$ ___ $\times$ ___
$=$ ___ $\times$ ___

e



Prime factors of 75 are:
___ and ___
$75 =$ ___ $\times$ ___ $\times$ ___
$= 3 \times$ ___

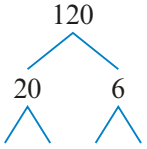
f



Prime factors of 60 are:
___, ___ and ___
$60 =$ ___ $\times$ ___ $\times$
___ $\times$ ___
$=$ ___ $\times$ ___ $\times$ ___

14 Complete each factor tree to show that the same product of prime factors will be obtained for 120 regardless of the factors you begin with.

a



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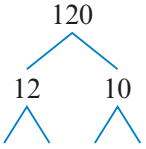
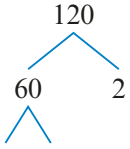


c



15 For each composite number:

i find its prime factors by first drawing a factor tree

ii write it as a product of prime factors.

a 28	**b** 56	**c** 44	**d** 80
e 132	**f** 52	**g** 250	**h** 90
i 72	**j** 210	**k** 100	**l** 400

PROBLEM SOLVING AND REASONING

16 Look at the pizza shown. It is to be divided into 12 equal pieces. Writing 12 as the product of prime factors shows you an easy way of cutting the pizza. As $12 = 2 \times 2 \times 3$, you can use the numbers 2, 2 and 3 to guide you in dividing the pizza. Here are the steps to follow.

a Use this method to work out how to divide the pizza into six equal pieces. Draw a diagram to show each step.

b Describe how this method could be used to divide the pizza into 18 equal pieces.

c Does it matter in which order you use the prime factors to divide the pizza? Explain your answer.

Step 1:
cut the pizza into 2 equal pieces.

Step 2:
cut each piece into 2 equal pieces.

Step 3:
cut each piece into 3 equal pieces.

17 A chocolate cake mixture has been baked in a large rectangular container. The chef wishes to cut the cake into 36 individual serves.

a Describe how prime factors can be used to divide the cake into 36 equal pieces.

b Draw a diagram to show a way of cutting the cake.

c Are there different ways to cut the cake into 36 equal pieces? If so, draw diagrams to show another two possible ways.

CHALLENGE

18 A pair of prime numbers that differ by 2 are called twin primes. For example, 3 and 5 are twin primes, as $5 - 3 = 2$.

a List the three other pairs of twin primes that occur for numbers from 1 to 20.

b How many pairs of twin primes occur for numbers from 21 to 100? List them.

c How many pairs of twin primes occur for numbers from 101 to 200? List them.

Reflect

How can you identify which numbers are prime and which are composite?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

place value	sum	long division	order of operations
odd	vertical addition	power	multiples
even	difference	index	factors
ascending order	vertical subtraction	base	lowest common multiple
descending order	product	index form	highest common factor
number line	long multiplication	expanded form	prime number
approximate	quotient	basic numeral	composite number
leading digit	remainder	square numbers	prime factors
rounding	short division	square root	factor tree

MULTIPLE-CHOICE

1A **1** Which list of numbers is written in order from smallest to largest?

A 1053, 531, 510, 501
B 345, 354, 3045, 3405
C 2461, 2614, 2641, 2611
D 85, 83, 81, 82, 87, 89

1B **2** What is 14 985 + 486 + 1987 + 9135?

A 14 373 **B** 25 483
C 26 593 **D** 174 805

1C **3** Which calculation gives an answer of 42?

A 96 − 42 **B** 1569 − 584
C 7859 − 7813 **D** 9026 − 8984

1D **4** Which multiplication problem produces the largest answer?

A 48 × 49 **B** 13 × 78
C 26 × 54 **D** 89 × 21

1E **5** Which division problem does *not* have a remainder of 4?

A 79 ÷ 5 **B** 36 ÷ 8
C 43 ÷ 10 **D** 22 ÷ 6

1F **6** What is 5 × 5 × 5 × 8 × 8 in index form?

A $3^5 \times 2^8$ **B** $5^3 + 8^2$
C 40^5 **D** $5^3 \times 8^2$

1G **7** What is 5 + 4 × 9 − 3?

A 15 **B** 38 **C** 54 **D** 78

1H **8** What is the highest common factor of 10 and 25?

A 5 **B** 10 **C** 25 **D** 50

1I **9** Which number is *not* a prime number?

A 11 **B** 25 **C** 37 **D** 53

1I **10** How is 60 written as the product of prime factors?

A 3 × 4 × 5 **B** 2 × 2 × 15
C 2 × 2 × 3 × 5 **D** 2 × 3 × 10

SHORT ANSWER

1A **1** Decide whether each statement is true or false.
- **a** $467 > 647$ **b** $0 < 78$
- **c** $930 < 9030$ **d** $15\,452 > 15\,542$

1A **2** Write each list of numbers in ascending order.
- **a** 5347, 547, 53 047, 57
- **b** 87 605, 87 506, 87 056

1A **3** Write an approximation to each number by rounding to its leading digit.
- **a** 392 **b** 9488
- **c** 554 **d** 12 345

1B **4** Calculate each of these.
- **a** 467 + 56 + 7801 + 943
- **b** 383 604 + 2557 + 16 092

1B **5** Find two whole numbers that add to give 478 if:
- **a** both numbers are odd
- **b** both numbers are even.

1C **6** Calculate each of these.
- **a** 9564 − 5381 **b** 371 625 − 38 047
- **c** 17 659 − 9816 **d** 101 011 − 59 678

1C **7** Calculate each of these.
- **a** 4895 − 1625 − 325
- **b** 978 − 486 − 239

1D **8** Calculate each of these.
- **a** 54×37 **b** 6135×429
- **c** 9856×11 **d** $12\,652 \times 43$

1D **9** Use a strategy to calculate each of these.
- **a** 389×100 **b** 412×6000
- **c** 3400×200 **d** $500 \times 300 \times 40$
- **e** $25 \times 48 \times 4$ **f** $931 \times 50 \times 2$

1E **10** Calculate each of these.
- **a** $567\,204 \div 7$ **b** $850 \div 16$
- **c** $\frac{7521}{23}$ **d** $\frac{68\,445}{45}$

1E **11** Use a strategy to calculate each of these.
- **a** $7000 \div 10$ **b** $80\,000 \div 2000$

1F **12** Write each in expanded form and calculate its value.
- **a** 2^6 **b** $5^3 \times 3^2$

1F **13** Write 10 000 in index form with a base of 10.

1F **14** Write each of these in both index form and as a basic numeral.
- **a** seven squared **b** the square of six

1F **15** Find the value of each of these.
- **a** $\sqrt{25}$ **b** $\sqrt{81} + 4^2$

1G **16** Calculate each of these.
- **a** $7^2 - 6 \times 4 + 3$
- **b** $8 \times (19 - 13) + 2 \times \sqrt{36}$
- **c** $4 + 3 \times (5 - 2)^2$
- **d** $16 - 2 \times (3 + 4) + 9$

1H **17** Write the multiples of 4 between 22 and 45.

1H **18** Write all the factors of 60.

1H **19** Find the lowest common multiple of each group of numbers.
- **a** 12 and 18 **b** 6, 9 and 15

1H **20** Find the highest common factor of each group of numbers.
- **a** 12 and 18 **b** 6, 9 and 15

1H **21** Find three numbers that have:
- **a** a factor of 7
- **b** four different factors
- **c** an odd number of factors.

1I **22**
- **a** Is 40 a prime or composite number? Give a reason to support your answer.
- **b** Draw a factor tree and list the prime factors of 40.
- **c** Write 40 as a product of its prime factors in index notation.

NAPLAN-STYLE PRACTICE

1 Which number is six thousand and forty-eight?

- ◯ 6480
- ◯ 6048
- ◯ 6408
- ◯ 6084

2 Which group of numbers is listed in ascending order?

- ◯ 236, 245, 254, 263, 236
- ◯ 7503, 7053, 7350, 7530
- ◯ 4965, 4695, 4659, 4569
- ◯ 603, 623, 630, 632, 662

3 Jane's total electricity bill this year is $2386. Last year it was $1857. What is the difference in the two yearly amounts?

☐

Questions **4** and **5** refer to this road sign.

4 What is the distance between Miles and Surat?

☐ km

5 If a person drives from this intersection to Miles, then to Injune and then to Mitchell, how far have they travelled in total?

☐ km

6 Hayden kept a record of how many laps of the pool he swam each day.

What is the total number of laps he swam in a week?

☐

Day	Number of laps
Monday	18
Tuesday	25
Wednesday	19
Thursday	32
Friday	28
Saturday	15
Sunday	24

7 A set of DVDs contains 14 episodes of a television show. Each episode runs for 86 minutes. What is the total running time?

☐

8 Alexis bought five books, three photo frames, seven magazines and two board games. If the books cost $11 each, photo frames $14 each, magazines $6 each and board games $21 each, how much did she spend?

$ ☐

9 To estimate the value of 751 × 329, each number is first rounded to its leading digit.

Which calculation must be performed?

- ◯ 700 × 300
- ◯ 700 × 400
- ◯ 800 × 300
- ◯ 800 × 400

10 This carton contains eggs of about the same size. The total mass inside the carton is 660 g.

What is the mass of one egg?

55 g	66 g	648 g	672 g
◯	◯	◯	◯

11 Two families paid $10 each to buy one lottery ticket. Their ticket was one of five winners in a $1 000 000 prize. If each family has four members, how much did each person receive?

- ◯ $250 000
- ◯ $125 000
- ◯ $50 000
- ◯ $25 000

12 Zahra goes for a bike ride and after 17 minutes she has ridden 4250 metres. How many kilometres will she cover in an hour?

15	17	72	255
◯	◯	◯	◯

13 What is another way of writing 4^3?

- ◯ 3 × 3 × 3 × 3
- ◯ 4 × 3
- ◯ 4 × 4 × 4
- ◯ 4 + 4 + 4

14 What is the value of $\sqrt{100}$?

15 If $17^2 = 289$, what is the value of $\sqrt{289}$?

2 ○ 17 ○ 289 ○ 578 ○

16 Which calculation produces the largest result?

○ $2^4 \times 5^2$ ○ $5^2 \times 3^2$
○ $4^2 \times 6^2$ ○ $3^2 \times 2^5$

17 Which calculation has the same value as 12×4?

○ $3 + 9 \times 5$ ○ $6 \times 9 - 5$
○ $6 \times 3 + 5$ ○ $(3 + 6) \times 5$

18 What is the value of $8 \times (3 + 6^2 \div 9) - 2$?

19 Which number comes next in this sequence?
48, 72, 96, 120, 144, ___

148 ○ 160 ○ 168 ○ 192 ○

20 Christos walks around an athletic track in 6 minutes while Lisa jogs around the same track in 4 minutes. They both begin their laps of the oval at the same time and from the same starting position. When will they next pass the starting position at the same time?

21 Which of these is *not* a prime number?

2 ○ 43 ○ 51 ○ 67 ○

22 How is 72 written as a product of its prime factors?

○ 8×9 ○ $2 \times 2 \times 2 \times 3 \times 3$
○ $3 \times 4 \times 6$ ○ $2 \times 3 \times 3 \times 4$

23 How many prime factors does 30 have?

1 ○ 2 ○ 3 ○ 8 ○

ANALYSIS

Jordan wrote a list of the type and quantity of food he needed for his birthday party.
He carefully noted the price per item during his shopping trip.

Item	Quantity	Price per item
Party Pie 24 pack	4	$4
Allen's mixed lollies	3	$3
Chips 100 g	4	$2
M&Ms 250 g	2	$4
Freddo frogs 20 pack	2	$5
Cocktail sausages 2 kg pack	1	$6
Frozen pizza	1	$10
Mudcake	3	$4

a Work out how much Jordan spent on each item.

b How much did he spend in total?

c If he paid with two $50 notes, how much change did he get?

d What is the cheapest item on the list?

e Which item did he spend the least money on? Is this different from the answer to part **d**? Explain why or why not.

f Each bag of Allen's mixed lollies has 40 lollies. How many lollies in total does Jordan have?

g Jordan wants lolly bags for himself and his friends with at least 10 lollies in each bag. What is the maximum number of friends he can invite?

h Jordan adds Freddo frogs to the lolly bags. How many Freddos does each friend get? How many Freddos are left over?

i Jordan and his friends need to be divided into equal groups for games. Using your answer from part **g**, what size could groups be if there must be at least three people in each?

j Explain how Jordan can easily cut the pizza into 12 equal slices.

CONNECT

Working at a zoo

Many zoos offer the opportunity to work at a zoo for the day. A zookeeper's role includes many responsibilities. Here is some information about a few of the animals that you might look after at a zoo.

MEERKATS

Meerkats:

- eat a wide variety of food, including termites, crickets, spiders, eggs and mice
- have a mass at birth of about 35 g
- have a mass as an adult of about 950 g
- have an adult body length of about 30 cm
- have a lifespan in the wild of about 10 years, up to 13 years in a zoo
- are given special treats such as toys, live insects scattered in the sand and food hidden in logs or wrapped in special parcels to keep them occupied and active in zoos.

HIPPOPOTAMUSES

Hippos:

- eat about 40 kg of grass and fruit each night
- in zoos are often fed herbivore pellets (pressed hay), alfalfa, lettuce and, as a special treat, melons
- have a mass at birth of 25–45 kg
- have an adult mass of 1600–4500 kg for males, and about 1400 kg for females
- have a lifespan of about 45 years
- can run up to 30 km/h on land and hold their breath underwater for up to 30 minutes
- like to mark their territory by spreading their faeces and urine around with their tails, so it can be quite a messy job for zoo keepers to clean up!

Your task

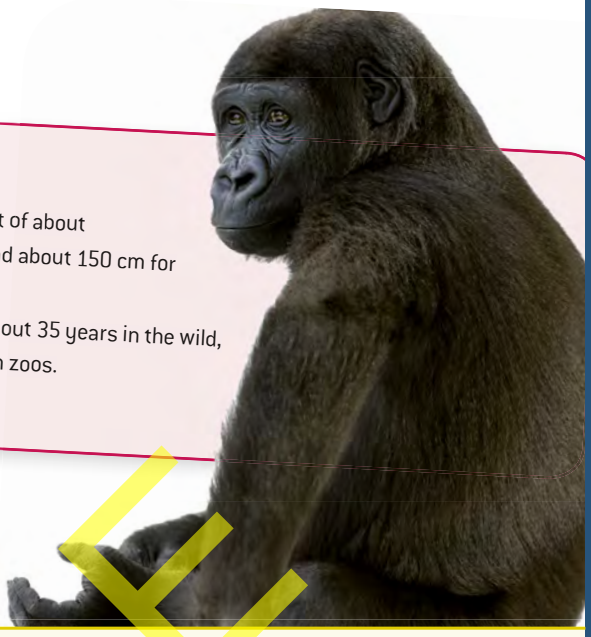
You are to investigate some of the ways animals are looked after at zoos. This includes consideration of the food they eat, the environment they live in and other factors that make their lives comfortable. Use the information provided for the animals described here, and then research information on a further three animals.

As a starting point, consider:

- the type of food each animal needs
- the amount of food needed to feed all the animals in the one enclosure
- the height requirements for an animal enclosure
- the amount of living space needed for different animals
- any special environmental requirements
- any special treats that zookeepers give animals to keep them happy.

You will need to show your calculations as evidence to back up your findings.

GROUPS



GORILLAS

Gorillas:

- eat up to 18 kg of fruits and vegetables each day
- have a mass at birth of about 2 kg
- have an adult mass of 136–227 kg for males and 60–91 kg for females
- have an adult height of about 175 cm for males and about 150 cm for females
- have a lifespan of about 35 years in the wild, and up to 50 years in zoos.

CAMELS

Camels:

- eat vegetation, including hay, and alfalfa pellets and carrots
- have a mass at birth of 37 kg
- have an adult mass of 300–690 kg
- can easily carry 90 kg and walk 32 km in a day
- can drink up to 145 L of water at one drinking session
- can survive a week or more in the wild without water, and several months without food.

GIRAFFES

Giraffes:

- eat up to 80 kg of acacia leaves each day
- have a height at birth of 200 cm
- have an adult height of about 500 cm for males
- can run at speeds up to 60 km/h.

Groups of animals

Collective nouns are used to describe groups of animals. In some cases, there can be more than one noun used to describe a particular animal. Here are just a few.

- a mob of emus
- a band of gorillas
- a herd of elephants
- an ambush of tigers
- a dazzle of zebras
- a crash of rhinoceroses
- a mob of meerkats
- a colony of penguins
- a coalition of cheetahs
- a bask of crocodiles

Complete the 1 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a PowerPoint presentation
- a newspaper article
- a video diary
- other (check with your teacher).