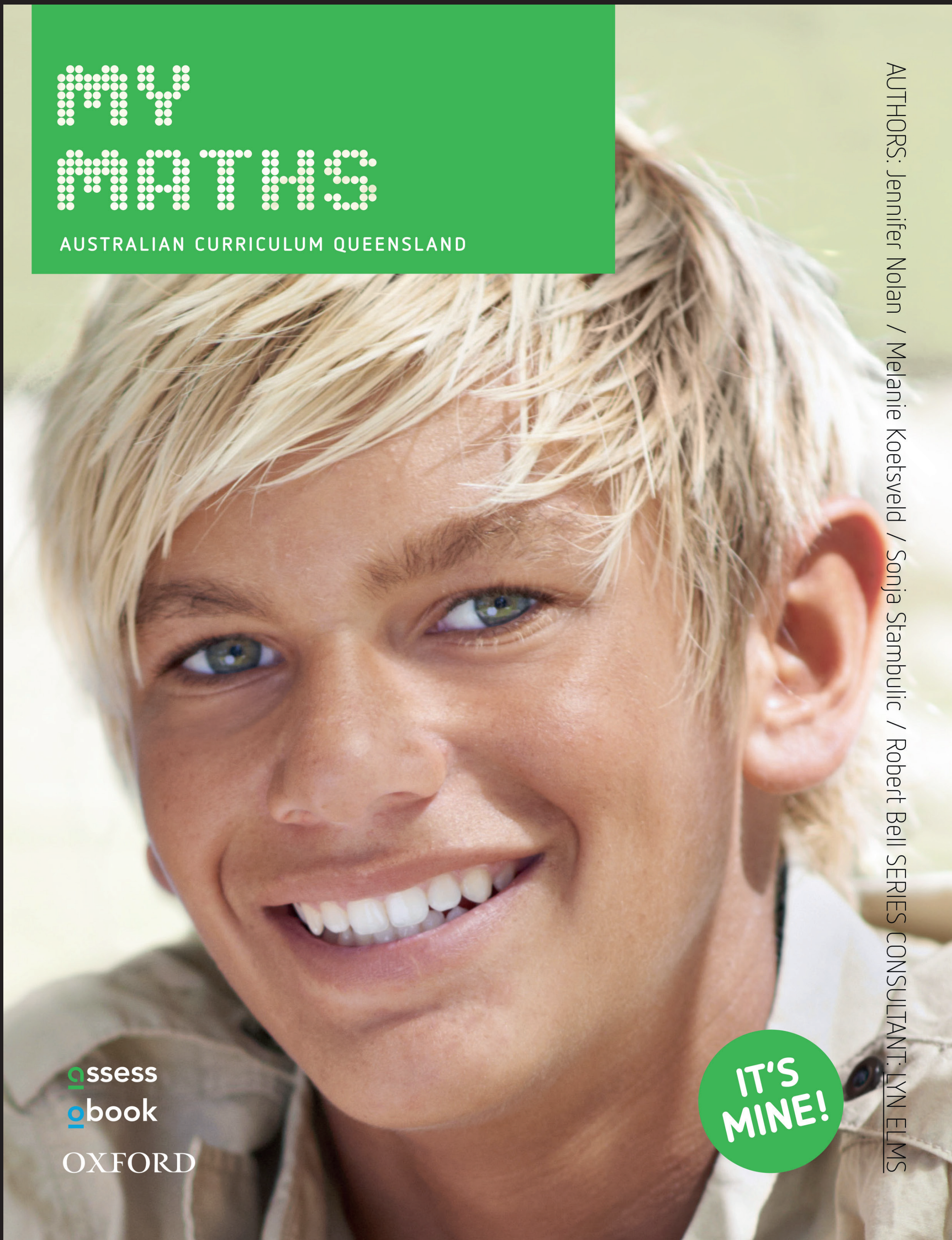

Helping you implement your product

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Contents*

Teacher resources

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*This is the contents from your Oxford Digital Dashboard.

Australian Curriculum: Mathematics Year 7

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the units.

Number and Algebra

Number and place value	**Elaborations**	**MyMaths 7**
Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)	• defining and comparing prime and composite numbers and explaining the difference between them • applying knowledge of factors to strategies for expressing whole numbers as products of powers of prime factors, such as repeated division by prime factors or creating factor trees • solving problems involving lowest common multiples and greatest common divisors (highest common factors) for pairs of whole numbers by comparing their prime factorisation	1H Multiples and factors 1I Prime and composite numbers
Investigate and use square roots of perfect square numbers (ACMNA150)	• investigating square numbers such as 25 and 36 and developing square-root notation • investigating between which two whole numbers a square root lies	1F Powers and square roots
Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)	• understanding that arithmetic laws are powerful ways of describing and simplifying calculations	1G Order of operations
Compare, order, add and subtract integers (ACMNA280)		4A Understanding negative numbers 4B Adding integers 4C Subtracting integers 4D Simplifying addition and subtraction of integers
Real numbers	**Elaborations**	**MyMaths 7**
Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)	• exploring equivalence among families of fractions by using a fraction wall or a number line (for example, by using a fraction wall to show that $\frac{2}{3}$ is the same as $\frac{4}{6}$ and $\frac{6}{9}$)	2A Understanding fractions 2B Equivalent fractions 4F Negative numbers and the Cartesian plane
Solve problems involving	• exploring and developing efficient strategies to solve additive problems involving fractions	2C Adding and subtracting

addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)	(for example, by using fraction walls or rectangular arrays with dimensions equal to the denominators)	fractions
Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)	• investigating multiplication of fractions and decimals, using strategies including patterning and multiplication as repeated addition, with both concrete materials and digital technologies, and identifying the processes for division as the inverse of multiplication	2D Multiplying fractions 2E Dividing fractions 2F Powers and square roots of fractions 3C Multiplying decimals 3D Dividing decimals by a whole number 3E Dividing decimals by a decimal
Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)	• using authentic examples for the quantities to be expressed and understanding the reasons for the calculations	2A Understanding fractions
Round decimals to a specified number of decimal places (ACMNA156)	• using rounding to estimate the results of calculations with whole numbers and decimals, and understanding the conventions for rounding	1A Understanding place value 3A Understanding decimals
Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)	• justifying choices of written, mental or calculator strategies for solving specific problems including those involving large numbers • understanding that quantities can be represented by different number types and calculated using various operations, and that choices need to be made about each • calculating the percentage of the total local municipal area set aside for parkland, manufacturing, retail and residential dwellings to compare land use	3F Converting between fractions and decimals 3H Converting between fractions, decimals and percentages
Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)	• using authentic problems to express quantities as percentages of other amounts	3G Understanding percentages 3I Calculating percentages
Recognise and solve problems involving simple ratios (ACMNA173)	• understanding that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem	2G Understanding ratios 2H Working with ratios

Money and financial mathematics	**Elaborations**	**MyMaths 7**
Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)	• applying the unitary method to identify ‘best buys’ situations, such as comparing the cost per 100 g	3D Dividing decimals by a whole number 3E Dividing decimals by a decimal
Patterns and algebra	**Elaborations**	**MyMaths 7**
Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)	• understanding that arithmetic laws are powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra	5A Understanding rules 5B Using pronumerals 5C Terms, expressions and equations
Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)	• using authentic formulas to perform substitutions	5D Evaluating expressions
Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)	• identifying order of operations in contextualised problems, preserving the order by inserting brackets in numerical expressions, then recognising how order is preserved by convention • moving fluently between algebraic and word representations as descriptions of the same situation	5B Using pronumerals 5C Terms, expressions and equations 5F Using flowcharts 5G Building expressions using flowcharts
Linear and non-linear relationships	**Elaborations**	**MyMaths 7**
Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)	• plotting points from a table of integer values and recognising simple patterns, such as points that lie on a straight line	4E Introducing the Cartesian plane 4F Negative numbers and the Cartesian plane 5B Using pronumerals
Solve simple linear equations (ACMNA179)	• solving equations using concrete materials, such as the balance model, and explain the need to do the same thing to each side of the equation using substitution to check solutions • investigating a range of strategies to solve equations	5E Strategies for solving equations 5H Solving equations using backtracking 5I Solving equations using a balance model
Investigate, interpret and analyse	• using travel graphs to investigate and compare the distance travelled to and from school	4G Interpreting graphs

graphs from authentic data (ACMNA180)	• interpreting features of travel graphs such as the slope of lines and the meaning of horizontal lines • using graphs of evaporation rates to explore water storage	

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 7**
Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)	• building on the understanding of the area of rectangles to develop formulas for the area of triangles • establishing that the area of a triangle is half the area of an appropriate rectangle • using area formulas for rectangles and triangles to solve problems involving areas of surfaces	9C Understanding area 9D Area of a rectangle 9E Area of a parallelogram 9F Area of a triangle 9G Surface area
Calculate volumes of rectangular prisms (ACMMG160)	• investigating volumes of cubes and rectangular prisms and establishing and using the formula $V = l \times b \times h$ • understanding and using cubic units when interpreting and finding volumes of cubes and rectangular prisms	9H Volume and capacity
Shape	**Elaborations**	**MyMaths 7**
Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)	• using aerial views of buildings and other 3-D structures to visualise the structure of the building or prism	7E Drawing 2D shapes and 3D objects 7F Planning and constructing 3D objects
Location and transformations	**Elaborations**	**MyMaths 7**
Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)	• describing patterns and investigating different ways to produce the same transformation such as using two successive reflections to provide the same result as a translation • experimenting with, creating and re-creating patterns using combinations of reflections and rotations using digital technologies	7G Symmetry of 2D shapes and 3D objects 7H Describing transformations 7I Performing transformations
Geometric reasoning	**Elaborations**	**MyMaths 7**
Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)	• defining and classifying pairs of angles as complementary, supplementary, adjacent and vertically opposite	6B Types of angles 6D Angles around a point 6E Angles and parallel lines
Investigate conditions for two	• constructing parallel and perpendicular lines using their properties, a pair of compasses and	6A Lines, rays and segments

lines to be parallel and solve simple numerical problems using reasoning (ACMMG164)	a ruler, and dynamic geometry software • defining and identifying the relationships between alternate, corresponding and co-interior angles for a pair of parallel lines cut by a transversal	6E Angles and parallel lines
Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)	• using concrete materials and digital technologies to investigate the angle sum of a triangle and quadrilateral	7A Classifying triangles 7B Classifying quadrilaterals
Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)	• identifying side and angle properties of scalene, isosceles, right-angled and obtuse-angled triangles • describing squares, rectangles, rhombuses, parallelograms, kites and trapeziums	7A Classifying triangles 7B Classifying quadrilaterals

Statistics and Probability

Chance	**Elaborations**	**MyMaths 7**
Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)	• discussing the meaning of probability terminology (for example, probability, sample space, favourable outcomes, trial, events and experiments) • distinguishing between equally likely outcomes and outcomes that are not equally likely	10H Describing probability 10J Experimental probability
Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)	• expressing probabilities as decimals, fractions and percentages	10H Describing probability 10I Theoretical probability 10J Experimental probability
Data representation and interpretation	**Elaborations**	**MyMaths 7**
Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)	• obtaining secondary data from newspapers, the Internet and the Australian Bureau of Statistics • investigating secondary data relating to the distribution and use of non-renewable resources around the world	10A Collecting data 10B Interpreting data
Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)	• understanding that some data representations are more appropriate than others for particular data sets, and answering questions about those data sets • using ordered stem-and-leaf plots to record and display numerical data collected in a class	10B Interpreting data 10C Dot plots, column and bar graphs 10D Pie graphs

	investigation, such as constructing a class plot of height in centimetres on a shared stem-and-leaf plot for which the stems 12, 13, 14, 15, 16 and 17 have been produced	10E Line graphs and scatterplots 10F Stem-and-leaf plots
Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)	• understanding that summarising data by calculating measures of centre and spread can help make sense of the data	10G Summary statistics
Describe and interpret data displays using median, mean and range (ACMSP172)	• using mean and median to compare data sets and explaining how outliers may affect the comparison • locating mean, median and range on graphs and connecting them to real life	10G Summary statistics

Year 7 achievement standard

By the end of Year 7, students solve problems involving the comparison, addition and subtraction of integers. They make the connections between whole numbers and index notation and the relationship between perfect squares and square roots. They solve problems involving percentages and all four operations with fractions and decimals. They compare the cost of items to make financial decisions. Students represent numbers using variables. They connect the laws and properties for numbers to algebra. They interpret simple linear representations and model authentic information. Students describe different views of three-dimensional objects. They represent transformations in the Cartesian plane. They solve simple numerical problems involving angles formed by a transversal crossing two lines. Students identify issues involving the collection of continuous data. They describe the relationship between the median and mean in data displays.

Students use fractions, decimals and percentages, and their equivalences. They express one quantity as a fraction or percentage of another. Students solve simple linear equations and evaluate algebraic expressions after numerical substitution. They assign ordered pairs to given points on the Cartesian plane. Students use formulas for the area and perimeter of rectangles and calculate volumes of rectangular prisms. Students classify triangles and quadrilaterals. They name the types of angles formed by a transversal crossing parallel line. Students determine the sample space for simple experiments with equally likely outcomes and assign probabilities to those outcomes. They calculate mean, mode, median and range for data sets. They construct stem-and-leaf plots and dot plots.

Number and Algebra

4 Integers and the Cartesian plane

4 Integers and the Cartesian plane

Teaching support for pages 182–3

Syllabus links

Content descriptions and elaborations

Number and place value

ACMNA280: Compare, order, add and subtract integers

Real numbers

ACMNA152: Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line

- exploring equivalence among families of fractions by using a fraction wall or a number line (for example, by using a fraction wall to show that

  $\frac{2}{3}$ is the same as $\frac{4}{6}$ and $\frac{6}{9}$)

Linear and non-linear relationships

ACMNA178: Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point

- plotting points from a table of integer values and recognising simple patterns, such as points that lie on a straight line

ACMNA180: Investigate, interpret and analyse graphs from authentic data

- using travel graphs to investigate and compare the distance travelled to and from school
- interpreting features of travel graphs such as the slope of lines and the meaning of horizontal lines
- using graphs of evaporation rates to explore water storage

The proficiency strands **Understanding, Fluency, Problem solving** and **Reasoning** are fully

Australian Curriculum: Mathematics Year 8

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the units.

Number and Algebra

Number and place value	**Elaborations**	**MyMaths 8**
Use index notation with numbers to establish the index laws with positive integral indices and the zero index (ACMNA182)	• evaluating numbers expressed as powers of positive integers	1H Powers and roots 1I Index laws 3G Powers of directed numbers
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)	• using patterns to assist in finding rules for the multiplication and division of integers • using the number line to develop strategies for adding and subtracting rational numbers	1B Order of operations 1D Operations with fractions 1F Operations with decimals 3B Adding integers 3C Subtracting integers 3D Simplifying addition and subtraction of integers 3E Multiplying and dividing integers 3F Operations with directed numbers
Real numbers	**Elaborations**	**MyMaths 8**
Investigate terminating and recurring decimals (ACMNA184)	• recognising terminating, recurring and non-terminating decimals and choosing their appropriate representations	1G Terminating, non-terminating and recurring decimals
Investigate the concept of irrational numbers, including π (ACMNA186)	• understanding that the real number system includes irrational numbers	1G Terminating, non-terminating and recurring decimals
Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187)	• using percentages to solve problems, including those involving mark-ups, discounts and GST • using percentages to calculate population increases and decreases	2C Percentage calculations
Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188)	• understanding that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem • calculating population growth rates in Australia and Asia and explaining their difference	2E Understanding ratios 2F Working with ratios 2G Dividing a quantity in a given ratio 2H Understanding rates

Money and financial mathematics	**Elaborations**	**MyMaths 8**
Solve problems involving profit and loss, with and without digital technologies (ACMNA189)	• expressing profit and loss as a percentage of cost or selling price, comparing the difference • investigating the methods used in retail stores to express discounts	2D Financial calculations
Patterns and algebra	**Elaborations**	**MyMaths 8**
Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)	• applying the distributive law to the expansion of algebraic expressions using strategies such as the area model	4F Working with brackets
Factorise algebraic expressions by identifying numerical factors (ACMNA191)	• recognising the relationship between factorising and expanding • identifying the greatest common divisor (highest common factor) of numeric and algebraic expressions and using a range of strategies to factorise algebraic expressions	4G Factorising expressions
Simplify algebraic expressions involving the four operations (ACMNA192)	• understanding that the laws used with numbers can also be used with algebra	4C Simplifying expressions containing like terms 4D Multiplying algebraic terms 4E Dividing algebraic terms
Linear and non-linear relationships	**Elaborations**	**MyMaths 8**
Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)	• completing a table of values, plotting the resulting points and determining whether the relationship is linear • finding the rule for a linear relationship	3H The Cartesian plane 5A Understanding equations 5H Plotting graphs of linear relationships
Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution. (ACMNA194)	• solving real life problems by using variables to represent unknowns	5B Solving equations using tables 5D Solving equations using backtracking 5E The balance model and equivalent equations 5F Solving equations by performing the same operation on both sides 5G Solving equations with the unknown on both sides 5I Solving linear equations using graphs

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 8**
Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195)	• choosing units for area including mm^2, cm^2, m^2, hectares, km^2, and units for volume including mm^3, cm^3, m^3 • recognising that the conversion factors for area units are the squares of those for the corresponding linear units • recognising that the conversion factors for volume units are the cubes of those for the corresponding linear units	8C Area of rectangles and triangles 8D Area of other quadrilaterals 8E Area of circles 8F Surface area 8G Volume of prisms 8H Converting units of area and volume
Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196)	• establishing and using formulas for area such as trapeziums, rhombuses and kites	8A Length and perimeter 8D Area of other quadrilaterals 8F Surface area
Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)	• investigating the circumference and area of circles with materials or by measuring, to establish an understanding of formulas • investigating the area of circles using a square grid or by rearranging a circle divided into sectors	8B Circumference of circles 8E Area of circles 8F Surface area
Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198)	• investigating the relationship between volumes of rectangular and triangular prisms	8G Volume of prisms
Solve problems involving duration, including using 12- and 24- hour time within a single time zone (ACMMG199)	• identifying regions in Australia and countries in Asia that are in the same time zone	6F Angles and time zones 6G Working with time zones
Geometric reasoning	**Elaborations**	**MyMaths 8**
Define congruence of plane shapes using transformations (ACMMG200)	• understanding the properties that determine congruence of triangles and recognising which transformations create congruent figures • establishing that two figures are congruent if one shape lies exactly on top of the other after one or more transformations (translation, reflection, rotation), and recognising that the matching sides and the matching angles are equal	7F Translations, rotations and reflections 7G Understanding congruence 7H Using congruence

Develop the conditions for congruence of triangles (ACMMG201)	• investigating the minimal conditions needed for the unique construction of triangles, leading to the establishment of the conditions for congruence (SSS, SAS, ASA and RHS) • solving problems using the properties of congruent figures • constructing triangles using the conditions for congruence	7G Understanding congruence 7H Using congruence
Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202)	• establishing the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites • identifying properties related to side lengths, parallel sides, angles, diagonals and symmetry	7A Triangle properties 7B Quadrilateral properties 7I Dilations

Statistics and Probability

Chance	**Elaborations**	**MyMaths 8**
Identify complementary events and use the sum of probabilities to solve problems (ACMSP204)	• identifying the complement of familiar events • understanding that probabilities range between 0 to 1 and that calculating the probability of an event allows the probability of its complement to be found	10B Theoretical probability 10C Tree diagrams
Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’ (ACMSP205)	• posing ‘and’, ‘or’ and ‘not’ probability questions about objects or people	10B Theoretical probability 10F Experimental probability 10G Simulations and long term trends
Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292)	• using Venn diagrams and two-way tables to calculate probabilities for events, satisfying ‘and’, ‘or’ and ‘not’ conditions • understanding that representing data in Venn diagrams or two-way tables facilitates the calculation of probabilities • collecting data to answer the questions using Venn diagrams or two-way tables	10D Two-way tables 10E Venn diagrams 10F Experimental probability 10G Simulations and long term trends
Data representation and interpretation	**Elaborations**	**MyMaths 8**
Investigate techniques for collecting data, including census, sampling and observation (ACMSP284)	• identifying situations where data can be collected by census and those where a sample is appropriate	9A Sampling data
Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206)	• investigating the uses of random sampling to collect data	9A Sampling data 9B Collecting data 9C Presenting data in graphs

		9D Stem-and-leaf plots and dot plots 9E Presenting grouped data
Explore the variation of means and proportions in random samples drawn from the same population (ACMSP293)	• using sample properties to predict characteristics of the population	9F Summary statistics 9G Analysing data
Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207)	• using displays of data to explore and investigate effects	9F Summary statistics 9G Analysing data

Year 8 achievement standard

By the end of Year 8, students solve everyday problems involving rates, ratios and percentages. They describe index laws and apply them to whole numbers. They describe rational and irrational numbers. Students solve problems involving profit and loss. They make connections between expanding and factorising algebraic expressions. Students solve problems relating to the volume of prisms. They make sense of time duration in real applications. They identify conditions for the congruence of triangles and deduce the properties of quadrilaterals. Students model authentic situations with two-way tables and Venn diagrams. They choose appropriate language to describe events and experiments. They explain issues related to the collection of data and the effect of outliers on means and medians in that data.

Students use efficient mental and written strategies to carry out the four operations with integers. They simplify a variety of algebraic expressions. They solve linear equations and graph linear relationships on the Cartesian plane. Students convert between units of measurement for area and volume. They perform calculations to determine perimeter and area of parallelograms, rhombuses and kites. They name the features of circles and calculate the areas and circumferences of circles. Students determine the probabilities of complementary events and calculate the sum of probabilities.

Number and Algebra

4 Algebra

4 Algebra

Teaching support for pages 176–7

Syllabus links

Content descriptions and elaborations

Patterns and algebra

ACMNA190: Extend and apply the distributive law to the expansion of algebraic expressions.

- Applying the distributive law to the expansion of algebraic expressions using strategies such as the area model

ACMNA191: Factorise algebraic expressions by identifying numerical factors.

- Recognising the relationship between factorising and expanding
- Identifying the greatest common divisor (highest common factor) of numeric and algebraic expressions

ACMNA192: Simplify algebraic expressions involving the four operations

- Understanding that the laws used with numbers can also be used with algebra

The proficiency strands Understanding, Fluency, Problem solving and Reasoning are fully integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Direct students to examine the opening photo for this chapter on pages 176 and 177 of their Student Book.
- Ask the students to think about the different relationships that they can link to the photograph shown. (Possible answers: distance fallen over time, height from the ground over time, speed of fall over time.)

Australian Curriculum: Mathematics Year 9

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Number and Algebra

Real numbers	**Elaborations**	**MyMaths 9**
Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)	• identifying direct proportion in real-life contexts	1A Working with whole numbers 1B Working with decimals 1C Working with ratios 4G Relationships and direct proportion
Apply index laws to numerical expressions with integer indices (ACMNA209)	• simplifying and evaluating numerical expressions, using involving both positive and negative integer indices	2B Index laws
Express numbers in scientific notation (ACMNA210)	• representing extremely large and small numbers in scientific notation, and numbers expressed in scientific notation as whole numbers or decimals	2D Scientific notation
Money and financial mathematics	**Elaborations**	**MyMaths 9**
Solve problems involving simple interest (ACMNA211)	• understanding that financial decisions can be assisted by mathematical calculations	1D Percentage of an amount 1E Writing one quantity as a percentage of another 1F Understanding simple interest 1G Working with simple interest

Patterns and algebra	**Elaborations**	**MyMaths 9**
Extend and apply the index laws to variables, using positive integer indices and the zero index (ACMNA212)	• understanding that index laws apply to variables as well as numbers	2B Index laws 2C Negative indices
Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)	• understanding that the distributive law can be applied to algebraic expressions as well as numbers • understanding the relationship between expansion and factorisation and identifying algebraic factors in algebraic expressions	2A Working with algebraic terms 2E Expanding algebraic expressions 2F Factorising using common factors 2G Factorising quadratic expressions
Linear and non-linear relationships	**Elaborations**	**MyMaths 9**
Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214)	• investigating graphical and algebraic techniques for finding distance between two points • using Pythagoras' theorem to calculate distance between two points	3G Midpoint and length of line segments
Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294)	• investigating graphical and algebraic techniques for finding midpoint and gradient • recognising that the gradient of a line is the same as the gradient of any line segment on that line	3D Gradient and intercepts 3G Midpoint and length of line segments

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)	• determining linear rules from suitable diagrams, tables of values and graphs and describing them using both words and algebra	3A Solving linear equations 3B Solving linear equations with the unknown on both sides 3C Plotting linear graphs 3E Sketching linear graphs using gradient and *y*-intercept 3F Sketching linear graphs using *x*- and *y*-intercepts
Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (ACMNA296)	• graphing parabolas, and circles connecting *x*-intercepts of a graph to a related equation	4A Solving quadratic equations 4B Plotting quadratic relationships 4C Parabolas and transformations 4D Sketching parabolas using transformations 4E Sketching parabolas using intercepts 4F Circles and other non-linear relationships 4G Relationships and direct proportion

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 9**
Calculate the areas of composite shapes (ACMMG216)	• understanding that partitioning composite shapes into rectangles and triangles is a strategy for solving problems involving area	7C Area of simple shapes 7D Area of composite shapes
Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)	• analysing nets of cylinders to establish formulas for surface area • connecting the volume and capacity of a cylinder to solve authentic problems	7F Surface area of cylinders 7G Volume
Solve problems involving the surface area and volume of right prisms (ACMMG218)	• solving practical problems involving surface area and volume of right prisms	7E Surface area 7G Volume

Investigate very small and very large time scales and intervals (ACMMG219)	• investigating the usefulness of scientific notation in representing very large and very small numbers	7A Understanding and representing measurement
Geometric reasoning	**Elaborations**	**MyMaths 9**
Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220)	• establishing the conditions for similarity of two triangles and comparing this to the conditions for congruence • using the properties of similarity and ratio, and correct mathematical notation and language, to solve problems involving enlargement (for example, scale diagrams) • using the enlargement transformation to establish similarity understanding that similarity and congruence help describe relationships between geometrical shapes and are important elements of reasoning and proof	5C Transformations 5D Congruent figures 5E Dilation and scale factor 5F Similar figures 5G Similar triangles
Solve problems using ratio and scale factors in similar figures (ACMMG221)	• establishing the relationship between areas of similar figures and the ratio of corresponding sides (scale factor)	5E Dilation and scale factor 5F Similar figures 5G Similar triangles 5H Scale factor and area
Pythagoras and trigonometry	**Elaborations**	**MyMaths 9**
Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles (ACMMG222)	• understanding that Pythagoras' Theorem is a useful tool in determining unknown lengths in right-angled triangles and has widespread applications • recognising that right-angled triangle calculations may generate results that can be integers, fractions or irrational numbers	6A Understanding Pythagoras' theorem 6B Using Pythagoras' theorem to find the length of the hypotenuse 6C Using Pythagoras' theorem to find the length of a shorter side
Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223)	• developing understanding of the relationship between the corresponding sides of similar right-angled triangles	6D Understanding trigonometry
Apply trigonometry to solve right-angled triangle problems (ACMMG224)	• understanding the terms 'adjacent' and 'opposite' sides in a right-angled triangle • selecting and accurately using the correct trigonometric ratio to find unknown sides (adjacent, opposite and hypotenuse) and angles in right-angled triangles	6E Using trigonometry to find lengths 6F Using trigonometry to find angles 6G Applications involving right-angled triangles

Statistics and Probability

Chance	**Elaborations**	**MyMaths 9**
List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225)	• conducting two-step chance experiments • using systematic methods to list outcomes of experiments and to list outcomes favourable to an event • comparing experiments which differ only by being undertaken with replacement or without replacement	9A Theoretical probability 9C Tree diagrams 9D Two-way tables 9F Experiments with replacement 9G Experiments without replacement
Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (ACMSP226)	• using Venn diagrams or two-way tables to calculate relative frequencies of events involving ‘and’, ‘or’ questions • using relative frequencies to find an estimate of probabilities of ‘and’, ‘or’ events	9B Experimental probability and relative frequency 9D Two-way tables 9E Venn diagrams
Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (ACMSP227)	• investigating a range of data and its sources, for example the age of residents in Australia, Cambodia and Tonga; the number of subjects studied at school in a year by 14-year-old students in Australia, Japan and Timor-Leste	8G Comparing data

Data representation and interpretation	**Elaborations**	**MyMaths 9**
Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (ACMSP228)	• comparing the annual rainfall in various parts of Australia, Pakistan, New Guinea and Malaysia	8E Collecting data
Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282)	• using stem-and-leaf plots to compare two like sets of data such as the heights of girls and the heights of boys in a class • describing the shape of the distribution of data using terms such as ‘positive skew’, ‘negative skew’ and 'symmetric' and 'bimodal'	8A Understanding and representing data 8B Grouped data 8F Describing data
Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283)	• comparing means, medians and ranges of two sets of numerical data which have been displayed using histograms, dot plots, or stem and leaf plots	8C Summary statistics 8D Summary statistics from displays 8G Comparing data

Year 9 achievement standard

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data from primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras’ Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Number and Algebra

4 Non-linear relationships

4 Non-linear relationships

Teaching support for pages 152–153

Syllabus links

Content descriptions and elaborations

Linear and non-linear relationships

ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations

- graphing parabolas, and circles connecting *x*-intercepts of a graph to a related equation

Real numbers

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems

- identifying direct proportion in real-life contexts

The proficiency strands **Understanding, Fluency, Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Direct students to examine the opening photo for this chapter.
- Ask students to consider a basketball game and a player taking a free throw. The path of the ball is curved but is it possible to write an equation to track the path of the ball?
- In Chapter 3, we looked at linear relationships but the path of the basketball will need a different type of relationship.
- If a relationship is not linear, then it is called non-linear; there are several different types of non-linear relationships that are considered in this chapter.

Essential question

Australian Curriculum: Mathematics Year 10

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Year 10

Number and Algebra

Money and financial mathematics	**Elaborations**	**MyMaths 10+10A**
Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)	• working with authentic information, data and interest rates to calculate compound interest and solve related problems	1C Understanding simple interest 1D Working with simple interest 1E Understanding compound interest 1F The compound interest formula 1G Working with compound interest
Patterns and algebra	**Elaborations**	**MyMaths 10+10A**
Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)	• using the distributive law and the index laws to factorise algebraic expressions • understanding the relationship between factorisation and expansion	2D Factorising algebraic expressions
Simplify algebraic products and quotients using index laws (ACMNA231)	• applying knowledge of index laws to algebraic terms, and simplifying algebraic expressions using both positive and negative integral indices	2B Review of index laws

Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)	• expressing the sum and difference of algebraic fractions with a common denominator • using the index laws to simplify products and quotients of algebraic fractions	2F Working with algebraic fractions
Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)	• exploring the method of completing the square to factorise quadratic expressions and solve quadratic equations • identifying and using common factors, including binomial expressions, to factorise algebraic expressions using the technique of grouping in pairs • using the identities for perfect squares and the difference of squares to factorise quadratic expressions	2C Expanding algebraic expressions 2E Factorising quadratic trinomials of the form $x^2 + bx + c$
Substitute values into formulas to determine an unknown (ACMNA234)	• solving simple equations arising from formulas	4A Solving linear equations
Linear and non-linear relationships	**Elaborations**	**MyMaths 10+10A**
Solve problems involving linear equations, including those derived from formulas (ACMNA235)	• representing word problems with simple linear equations and solving them to answer questions	4A Solving linear equations
Solve linear inequalities and graph their solutions on a number line (ACMNA236)	• representing word problems with simple linear inequalities and solving them to answer questions	4B Solving linear inequalities

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology (ACMNA237)	• associating the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs	4F Solving linear simultaneous equations graphically 4G Solving linear simultaneous equations algebraically
Solve problems involving parallel and perpendicular lines (ACMNA238)	• solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel • solving problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular	4E Parallel and perpendicular lines
Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)	• sketching graphs of parabolas, and circles • applying translations, reflections and stretches to parabolas and circles • sketching the graphs of exponential functions using transformations	5C Sketching parabolas using intercepts 5D Sketching parabolas using transformations 5E Graphs of circles 5F Graphs of exponential relationships
Solve linear equations involving simple algebraic fractions (ACMNA240)	• solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution • representing word problems, including those involving fractions, as equations and solving them to answer the question	4A Solving linear equations
Solve simple quadratic equations using a range of strategies (ACMNA241)	• using a variety of techniques to solve quadratic equations, including grouping, completing the square, the quadratic formula and choosing two integers with the required product and sum	5A Solving quadratic equations 5B Solving quadratic equations using the quadratic formula

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 10+10A**
Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)	• investigating and determining the volumes and surface areas of composite solids by considering the individual solids from which they are constructed	9C Surface area of prisms and cylinders 9D Volume of prisms and cylinders
Geometric reasoning	**Elaborations**	**MyMaths 10+10A**
Formulate proofs involving congruent triangles and angle properties (ACMMG243)	• applying an understanding of relationships to deduce properties of geometric figures (for example the base angles of an isosceles triangle are equal)	7A Geometry review 7B Congruence 7D Understanding proofs 7E Proofs and triangles
Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)	• distinguishing between a practical demonstration and a proof (for example demonstrating triangles are congruent by placing them on top of each other, as compared to using congruence tests to establish that triangles are congruent) • performing a sequence of steps to determine an unknown angle giving a justification in moving from one step to the next • communicating a proof using a sequence of logically connected statements	7C Similarity 7E Proofs and triangles 7F Proofs and quadrilaterals
Pythagoras and trigonometry	**Elaborations**	**MyMaths 10+10A**
Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245)	• applying Pythagoras' Theorem and trigonometry to problems in surveying and design	8A Finding lengths using Pythagoras' theorem 8B Finding lengths using trigonometry 8C Finding angles using trigonometry 8D Applications of trigonometry

Statistics and Probability

Chance	Elaborations	MyMaths 10+10A
Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP246)	• recognising that an event can be dependent on another event and that this will affect the way its probability is calculated	11A Review of theoretical probability 11B Tree diagrams 11C Experiments with and without replacement 11D Independent and dependent events
Use the language of 'if ....then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language (ACMSP247)	• using two-way tables and Venn diagrams to understand conditional statements • using arrays and tree diagrams to determine probabilities	11B Tree diagrams 11E Conditional probability with two-way tables and tree diagrams 11F Conditional probability and Venn diagrams
Data representation and interpretation	**Elaborations**	**MyMaths 10+10A**
Determine quartiles and interquartile range (ACMSP248)	• finding the five-number summary (minimum and maximum values, median and upper and lower quartiles) and using its graphical representation, the box plot, as tools for both numerically and visually comparing the centre and spread of data sets	10A Measures of centre 10B Measures of spread 10D Box plots

Construct and interpret box plots and use them to compare data sets (ACMSP249)	• understanding that box plots are an efficient and common way of representing and summarising data and can facilitate comparisons between data sets • using parallel box plots to compare data about the age distribution of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole	10D Box plots
Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250)	• investigating data in different ways to make comparisons and draw conclusions	10D Box plots
Use scatterplots to investigate and comment on relationships between two numerical variables (ACMSP251)	• using authentic data to construct scatterplots, make comparisons and draw conclusions	10E Scatterplots and bivariate data
Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252)	• investigating biodiversity changes in Australia since European occupation • constructing and interpreting data displays representing bivariate data over time	10G Time series 10 Connect task
Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253)	• investigating the use of statistics in reports regarding the growth of Australia's trade with other countries of the Asia region • evaluating statistical reports comparing the life expectancy of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole	10H Analysing reported statistics 10 Connect task

Year 10 Achievement Standard

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Year 10A

Number and Algebra

Real numbers	Elaborations	MyMaths 10+10A
Define rational and irrational numbers and perform operations with surds and fractional indices (ACMNA264)	• understanding that the real number system includes irrational numbers • extending the index laws to rational number indices • performing the four operations with surds	3A Understanding rational and irrational numbers 3B Multiplying and dividing surds 3C Simplifying surds 3D Adding and subtracting surds 3E Writing surd fractions with a rational denominator 3F Fractional indices
Use the definition of a logarithm to establish and apply the laws of logarithms (ACMNA265)	• investigating the relationship between exponential and logarithmic expressions • simplifying expressions using the logarithm laws	3G Understanding logarithms 3H Working with logarithms

Patterns and algebra	Elaborations	MyMaths 10+10A
Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems (ACMNA266)	• investigating the relationship between algebraic long division and the factor and remainder theorems	6A Understanding polynomials 6B Division of polynomials 6C Remainder and factor theorems 6D Solving polynomial equations

Linear and non-linear relationships	**Elaborations**	**MyMaths 10+10A**
Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267)	• applying transformations, including translations, reflections in the axes and stretches to help graph parabolas, rectangular hyperbolas, circles and exponential functions	5D Sketching parabolas using transformations 5H Graphs of hyperbolas 5I Sketching non-linear relationships using transformations
Solve simple exponential equations (ACMNA270)	• investigating exponential equations derived from authentic mathematical models based on population growth	5G Solving exponential equations
Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation (ACMNA268)	• investigating the features of graphs of polynomials including axes intercepts and the effect of repeated factors	6E Graphs of polynomial relationships 6F Polynomials and transformations
Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)	• writing quadratic equations that represent practical problems	2G Factorising quadratic trinomials of the form $ax^2 + bx + c$ 5A Solving quadratic equations 5B Solving quadratic equations using the quadratic formula

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 10+10A**
Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids (ACMMG271)	• using formulas to solve problems • using authentic situations to apply knowledge and understanding of surface area and volume	9E Surface area of pyramids and cones 9F Volume of pyramids and cones 9G Surface area and volume of spheres 9H Surface area and volume of composite shapes
Geometric reasoning	**Elaborations**	**MyMaths 10+10A**
Prove and apply angle and chord properties of circles (ACMMG272)	• performing a sequence of steps to determine an unknown angle or length in a diagram involving a circle, or circles, giving a justification in moving from one step to the next • communicating a proof using a logical sequence of statements • proving results involving chords of circles	7G Circle geometry: circles and angles 7H Circle geometry: chords 7I Circle geometry: tangents and secants
Pythagoras and trigonometry	**Elaborations**	**MyMaths 10+10A**
Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)	• applying knowledge of sine, cosine and area rules to authentic problems such as those involving surveying and design	8F Sine and area rules 8G Cosine rule
Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)	• establishing the symmetrical properties of trigonometric functions • investigating angles of any magnitude • understanding that trigonometric functions are periodic and that this can be used to describe motion	8H Unit circle and trigonometric graphs
Solve simple trigonometric equations (ACMMG275)	• using periodicity and symmetry to solve equations	8I Solving trigonometric equations

Apply Pythagoras' Theorem and trigonometry to solving three-dimensional problems in right-angled triangles (ACMMG276)	• investigating the applications of Pythagoras's Theorem in authentic problems	8E Three-dimensional problems

Statistics and Probability

Chance	**Elaborations**	**MyMaths 10+10A**
Investigate reports of studies in digital media and elsewhere for information on their planning and implementation (ACMSP277)	• evaluating the appropriateness of sampling methods in reports where statements about a population are based on a sample • evaluating whether graphs in a report could mislead, and whether graphs and numerical information support the claims	11G Sampling and reporting
Data representation and interpretation	**Elaborations**	**MyMaths 10+10A**
Calculate and interpret the mean and standard deviation of data and use these to compare data sets (ACMSP278)	• using the standard deviation to describe the spread of a set of data • using the mean and standard deviation to compare numerical data sets	10C Standard deviation

Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation (ACMSP279)	• investigating different techniques for finding a 'line of best fit'	10F Bivariate relationships

Number and Algebra

4 Linear relationships

Teaching support for pages 158–159

Syllabus links

Content descriptions and elaborations

Patterns and algebra

ACMNA234: Substitute values into formulas to determine an unknown

- solving simple equations arising from formulas

Linear and non-linear relationships

ACMNA235: Solve problems involving linear equations, including those derived from formulas

- representing word problems with simple linear equations and solving them to answer questions

ACMNA236: Solve linear inequalities and graph their solutions on a number line

- representing word problems with simple linear equations and solving them to answer questions

ACMNA237: Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology

- associating the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs

ACMNA238: Solve problems involving parallel and perpendicular lines

- solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel
- solving problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular

ACMNA240: Solve linear equations involving simple algebraic fractions

- solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution
- representing word problems, including those involving fractions, as equations and solving them to answer the question

The proficiency strands **Understanding, Fluency, Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Direct students to examine the opening photo for this chapter.
- Ask students to consider the acceleration of the two cars in the photo. Would the speed of the cars as they accelerate be a linear or non-linear model?

Essential question

The graph of speed versus time will tell you the speed of the car at any time but also the gradient of this graph will tell you the acceleration of the car at any point.

Are you ready?

Prerequisite knowledge and skills can be tested by completing **Are you ready?**. This will give you an indication of the differentiated pathway each student can follow.

Students need to be able to:

- solve simple one-step equations
- substitute into algebraic expressions
- expand a single pair of brackets using the distributive law
- simplify an expression by collecting like terms
- perform the four operations with fractions
- use inequality signs to identify the greater and lesser value
- graph an inequality on a number line
- recognise the *x*-intercept, *y*-intercept and gradient on a linear graph
- identify the gradient and *y*-intercept from a linear rule in the form $y = mx + c$
- find the *x*- and *y*-intercepts from a linear rule

- find equivalent equations by multiplying a given equation by an integer.

At the beginning of each topic, there is a suggested differentiated pathway that allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to select the best pathway.

Support Strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

CHAPTER 4 LINEAR RELATIONSHIPS

4 Are you ready?

1 **a** $x = 13$ **b** $x = -8$ **c** $x = 5$ **d** $x = -45$

2 **a** 11 **b** −9 **c** −12 **d** 5

3 **a** $4x + 28$ **b** $-15x + 6$

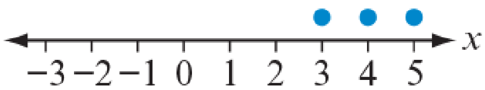
4 **a** $11x + 8$ **b** $2 - 3x$ **c** $5x - 2$ **d** $3x - 9$

5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{13}{10}$ or $1\frac{3}{10}$ **d** $\frac{1}{6}$

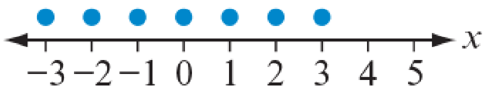
6 **a** true **b** true **c** true **d** false

7 **a**

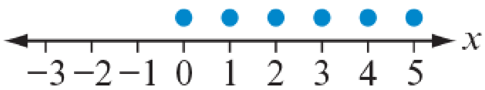
b



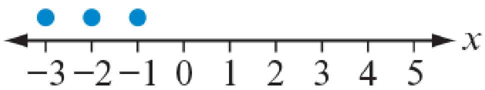
c



d



e



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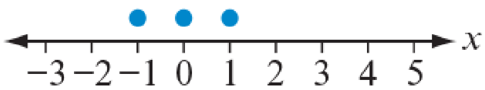


8 **a** 2 **b** −4 **c** 2

9 **a** **i** $c = -4, m = 3$ **ii** $c = 2, m = -1$

iii $c = 0, m = 7$ **iv** $c = -1, m = -\frac{2}{3}$

b **i** rise = 3, run = 1 **ii** rise = −1, run = 1

iii rise = 7, run = 1 **iv** rise = −2, run = 3

10 **a** x-intercept = 1, y-intercept = 4

b x-intercept = 9, y-intercept = 3

c x-intercept = 10, y-intercept = −5

d x-intercept = −3, y-intercept = −6

11 **a** $6x + 3y = 12$ **b** $2x - 6y = 2$

c $-x + 8y = -6$ **d** $-16x + 20y = 0$

Resources

assess: assessments

Each topic of the *MyMaths 10+10A* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students' confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

4A Solving linear equations

Teaching support for pages 160–165

Teaching strategies

Learning focus

To solve a variety of linear equations including those that have the pronumeral on both sides, equations with brackets and equations involving algebraic fractions

Start thinking!

The task guides students to:

- recognise a linear equation
- see how the balance method is used to solve a one-step equation
- consider the order in which the operations need to be performed in solving a two-step equation using the balance method
- follow the step involved in using the balance method to solve a two-step equation involving fractions and negative coefficients.

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 4a–f, 5a–f, 6–8, 10a–f	1–4, 5a–j, 6–8, 9a–f, 10a–l, 11, 12, 15	2, 3, 4e–i, 5e–l, 7, 8d–f, 9d–i, 10g–o, 12–14, 16–19
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q1–5

Focus: To solve simple equations, expand brackets, collect like terms and perform operations with fractions

- Direct students to complete **SS 4A-1 Solving simple linear equations** (see Resources) if they had difficulty with Q1 or require more practice at this skill.
- Direct students to complete **SS 4A-2 Evaluating simple algebraic expressions** (see Resources) if they had difficulty with Q2 or require more practice at this skill.
- Direct students to complete **SS 4A-3 Expanding and simplifying algebraic expressions** (see Resources) if they had difficulty with Q3 and Q4 or require more practice at this skill.

- Direct students to complete **SS 4A-4 Working with fractions** (see Resources) if they had difficulty with Q5 or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to:
  - solve a one-step equation
  - substitute into an expression and evaluate
  - use the distributive law to expand brackets
  - simplify an expression by collecting like terms
  - perform four operations with fractions.

At Level

At Level
1–4, 5a–j, 6–8, 9a–f, 10a–l, 11, 12, 15

- Revise the fact that an equation is an incomplete number sentence and the task is to find the value of the unknown pronumeral.
- An equation is a balance, and in solving the equation whatever is done to one side must also be done to the other in order to maintain the equality. Hence, the balance method is used to solve equations.
- Discuss how in using the balance method the operations that have been used to form the equation must be undone by using the opposite operations in the correct order.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4A-1**. It shows how to solve linear equations and will help students to complete Q1 and Q2.
- **Example 4A-2** shows how to solve an equation when the variable is in the denominator. This will help students to complete Q5.
- **Example 4A-3** shows how to solve an equation with an unknown on both sides. This will help students to complete Q6.
- For Q8, explain that multiplying both sides of the equation by the common denominator produces an equation without algebraic fractions.

- **Example 4A-4** shows how to solve an equation with algebraic fractions on both sides. This will help students to complete Q9.
- For Q10, students need to consider all pronumerals other than x as numbers and move them to the other side of the equation using equation-solving techniques.
- For Q11 onwards, where students write an equation from worded information, remind them to define the pronumeral/s used.
- You may like students to use the ‘solve’ function of a calculator or other digital technology to check whether they obtain the same answers.
- For additional practice, students can complete Q1–7 of **WS 4-1 Solving linear equations and inequalities** (see Resources). Additional questions similar to Exercise 4A Q1–7 are provided. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Calendar capers** (see Resources).

Below Level

Below Level
1, 2, 4a–f, 5a–f, 6–8, 10a–f

- To complete this topic, students may need their calculators.
- Students may need to complete **SS 4A-1 Solving simple linear equations** (see Resources).
- Students may need to complete **SS 4A-2 Evaluating simple algebraic expressions** (see Resources).
- Students may need to complete **SS 4A-3 Expanding and simplifying algebraic expressions** (see Resources).
- Students may need to complete **SS 4A-4 Working with fractions** (see Resources).
- Students may need help in some questions to work out the correct order of operations. Have them make a simple flowchart of the expression on the left side of the equation.
- In Q4, remind students to expand expressions to remove brackets and collect like terms before solving each equation.
- In Q6, guide students to remove the pronumeral from the right side of the equation. If this leaves a negative coefficient of x on the left side, explain when and how to change the sign on both sides.

- For Q8, explain that multiplying both sides of the equation by the common denominator produces an equation without fractions.
- For students who do not progress past Q7, direct them to Q1–7 of **WS 4-1 Solving linear equations and inequalities** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.
- Students sometimes struggle with what exactly an equation is and what they are doing when they solve an equation. This can be overcome to some extent by writing the equation, replacing the pronumeral with a box. The task is to write the number in the box to make a true statement.
- Once established that the pronumeral takes the place of the box, have students check each of their answers by substitution. Explain that if they do this, they will know immediately if they are correct or incorrect.

Above Level

Above Level
2, 3, 4e–i, 5e–l, 7, 8d–f, 9d–i, 10g–o, 12–14, 16–19

- For Q8, explain that multiplying both sides of the equation by the common denominator produces an equation without algebraic fractions. In Q9, students first need to obtain equivalent fractions with the same denominator before proceeding with the method used in Q8.
- For Q10, students need to consider all pronumerals other than x as numbers and move them to the other side of the equation using equation-solving techniques.
- For Q11 onwards, where students write an equation from worded information, remind them to define the pronumeral/s used.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Calendar capers** (see Resources).

Extra activities

1 **Quick Questions**

a Expand $3(2x - 7)$. $(6x - 21)$

b Simplify $2x - 4 + 9 - 11x$. $(5 - 9x)$

c Calculate $\frac{1}{2}+\frac{3}{8}$. $\left(\frac{7}{8}\right)$

d Calculate $\frac{2}{3} \times \frac{4}{5}$. $\left(\frac{8}{15}\right)$

2 A formula is used to calculate a quantity by some given rule. Most formulas are designed to calculate the subject. For example, in the formula $A = l \times w$, A is the subject and is used to find the area of a rectangle. If we are to use the formula to calculate, a different variable and equation will be formed. For example, if the area of a rectangle is 56 and the length is 7, the equation $56 = 7w$ can be used to find the width.

In each of the following, substitute into the given formula and solve the resulting equation.

a $A = \frac{1}{2}bh$ where $A = 72$, $b = 12$ $(h = 12)$

b $E = mc^2$ where $E = 40$, $c = 2$ $(m = 10)$

c $C = \frac{5}{9}(F - 32)$ where $F = 100$ $(C = 37\frac{7}{9})$

Answers

4A Solving linear equations

4A Start thinking!

1 a, c, f and g are linear equations

2 a subtract 9 b $x = 7$

3 a i add 4 ii divide by 3

b You would get an incorrect solution if you did not perform the 'undo' operations in the reverse order to that used to form the equation.

c
$$\begin{aligned} 3x - 4 &= 2 \\ 3x - 4 + 4 &= 2 + 4 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

d
$$\begin{aligned} 3x - 4 &= 2 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

e Substitute the solution into the original equation to check left side equals right side.
LS = $3 \times 2 - 4 = 2$ = RS

4 a multiply by 5 then add 2; $x = -18$

b subtract 7 then divide by -4; $x = -2$

Exercise 4A Solving linear equations

ANSWERS

1 a $x = 5$ b $x = 1$ c $x = 15$ d $x = -5$
e $x = 8$ f $x = 10$ g $x = 6$ h $x = -4$
i $x = 9$ j $x = -7$ k $x = 3$ l $x = 2$
m $x = 8$ n $x = -1$ o $x = 0$

2 a $x = 10$ b $x = -4$ c $x = 5$ d $x = -8$
e $x = 7$ f $x = 6$ g $x = 8$ h $x = -2$
i $x = 3$

3 a $x = -8\frac{1}{2}$ b $x = 1.7$ c $x = 4$
d $x = -2.464$ e $x = 2.4$
f $x = -9\frac{1}{2}$ or -9.5

4 a $x = -5$ b $x = 3$ c $x = -2$ d $x = 6$
e $x = -\frac{1}{3}$ f $x = 7$ g $x = 3$ h $x = \frac{1}{9}$
i $x = 6$

5 a $x = 2$ b $x = -5$ c $x = -6$ d $x = -4$
e $x = 4$ f $x = 7.5$ g $x = 3\frac{1}{3}$ h $x = \frac{2}{35}$
i $x = 2$ j $x = -1$ k $x = \frac{1}{4}$ l $x = -2\frac{2}{5}$

6 a $x = 2$ b $x = 5$ c $x = -3$ d $x = 1$
e $x = -7$ f $x = -2$ g $x = 3$ h $x = 4$
i $x = \frac{3}{7}$ j $x = -4$ k $x = 2$ l $x = -5$

7 a $x = 5$ b $x = -2$ c $x = 3$ d $x = -1$
e $x = 6$ f $x = 2$ g $x = -4$ h $x = 8$
i $x = 4$ j $x = -1$ k $x = 5$ l $x = 9$

8 a $x = -8$ b $x = 4$ c $x = -2$ d $x = 2$
e $x = -1$ f $x = 4$

9 a $x = -8$ b $x = 31$ c $x = 8$ d $x = 1$
e $x = 17$ f $x = 13$ g $x = -4\frac{2}{3}$ h $x = 6\frac{1}{7}$
i $x = -2\frac{1}{8}$

10 a $x = b - a$ b $x = k + p$ c $x = \frac{d}{c}$
d $x = \frac{h - g}{3}$ e $x = \frac{y - 5}{4}$ f $x = mn - a$
g $x = \frac{5 + 2w}{7}$ h $x = a(b - c)$ i $x = \frac{h}{k}$
j $x = \frac{e + f}{2}$ k $x = \frac{v - 2y}{2}$ l $x = p(w - q)$
m $x = \frac{a}{b + c}$ n $x = \frac{2ny + m}{k}$ o $x = \frac{5c - 3d}{8}$

11 a let n = number of weeks of saving
b $24n + 105 = 219$ c $n = 4.75$
d It will take Tom 5 weeks to save for his new tennis racquet.

12 a \$89.50 b length: 28 m, width: 16 m
c Lisa: 14 goals, Nicole: 9 goals, Tania: 6 goals
d 51 people

13 a $6d + 3.8 = 4d + 9.2$, where d is cost of a dumpling
b \$2.70

14 a $A = 50\text{ m}^2$ b $h = 7\text{ mm}$ c $a = 7\text{ m}$
d $a = \frac{2A}{h} - b$
e i $a = 52\text{ cm}$ ii $a = 7.6\text{ m}$

15 a $v = 34\text{ m/s}$ b $t = 5.5\text{ s}$

16 15, 17, 19, 21, 23

17 $n - 25 - \frac{n - 25}{3} = 40$, where n is number of cupcakes made; 25 chocolate, 20 lemon, 40 without icing and 85 in total

18 a $x = -6$ b $x = 8$ c $x = -1$ d $x = 13$
e $x = \frac{1}{8}$ f $x = -2\frac{7}{10}$

Reflect

Possible answer: When solving equations, operations are performed in the reverse order to how the expression on the left side was formed.

Resources

SupportSheet

SS 4A-1 Solving simple linear equations

Focus: To use the balance method to solve one-step linear equations

Students review equations and develop an understanding of what solving an equation means. They are guided to solve simple one-step linear equations using the balance method.

SS 4A-2 Evaluating simple algebraic expressions

Focus: To substitute values into a variety of algebraic expressions

Students review substitution into an algebraic expression. They determine the value of expressions by substituting given values into the expressions and evaluating.

SS 4A-3 Expanding and simplifying algebraic expressions

Focus: To expand algebraic expressions by using the distributive law to remove brackets and to simplify expressions by collecting like terms

Students expand algebraic expressions using the distributive law to remove a pair of brackets and simplify expressions by adding and subtracting like terms.

SS 4A-4 Working with fractions

Focus: To perform the four basic operations (addition, subtraction, multiplication and division) on fractions

Students review the requirements for performing the four operations on fractions (addition, subtraction, multiplication and division) in preparation for solving equations involving algebraic fractions.

WorkSheet

WS 4-1 Solving linear equations and inequalities

Focus: To solve a variety of linear equations and inequalities

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4A and 4B. Q1–7 relate to Exercise 4A.

Students solve a range of linear equations involving two-step equations, three step equations, equations with the unknown in the denominator and equations where the unknown appears on both sides. They represent inequalities on a number line and solve linear inequalities.

Investigation

INV 4-1 Calendar capers

Focus: To discover some interesting relationships between dates in any month of a calendar

Resources: calculator

Students look at the calendar and consider the relationships in the sum of the dates in 2 by 2 squares and 3 by 3 squares. Students then form equations to solve that will determine the exact dates in any such squares.

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4B Solving linear inequalities

Teaching support for pages 166–171

Teaching strategies

Learning focus

To solve linear inequalities and graph the solutions on a number line

Start thinking!

The task guides students to:

- use the symbols >, <, ≥ and ≤ to compare numbers
- understand the meaning of a statement about an unknown using one of the above symbols
- consider how an inequality might be represented on a number line
- consider different values of x that will make an inequality statement true
- consider the meaning of an inequality of the form $a < x < b$.

Differentiated pathways

Below Level	At Level	Above Level
1–9, 13, 14	1d–h, 2–10, 11a–c, 12–16	1g, h, 2–4, 6–12, 15–19
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q6 and Q7

Focus: To compare numbers using inequality signs and represent inequalities on a number line

- Direct students to complete **SS 4B-1 Understanding inequality statements** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to use:

- the signs $<$ and $>$ to show a comparison in the size of two numbers
- graphing conventions to show numbers on a number line.

At Level

At Level
1d–h, 2–10, 11a–c, 12–16

- Explain to students that a linear equation has a single solution while an inequality (or inequation) has a range of values that will satisfy it.
- Demonstrate that the balance method is still used in exactly the same way to find the solution to an inequality.
- Explain that the major difference between solving equations and inequalities is that when multiplying or dividing by a negative number, the inequality signs need to be reversed. This can be demonstrated as follows.

  Start with: $4 > 1$

  Adding 10 to both sides: $14 > 11$

  Subtracting 5 from both sides: $9 > 6$

  Multiplying both sides by 2: $18 > 12$

  Dividing both sides by –3: $-6 < -4$

  The inequality remained in the same direction until both sides were divided by a negative number. Similarly show what happens when an inequality like $4 > 1$ is multiplied by a negative number.
- Show students how to graph a solution on a number line. Explain that a circle is put on the number line where the boundary of the solution lies and an arrow is drawn in the direction of the inequality. For $<$ or $>$ an open circle (○) should be used while for $\leq$ or $\geq$ a closed circle (●) should be used.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4B-1**. It shows how to write inequalities from a number line and will help students to complete Q2 and Q3.
- In Q2, students need to be particularly aware of the closed circle or open circle and may need to have the 'between' solution explained further.

- Have students work through the demonstrations in Q4 carefully as they show the reason why the inequality sign needs to be reversed (to keep statements true) when multiplying or dividing by a negative number.
- **Example 4B-2** shows how to solve inequalities when multiplying or dividing by a positive number. This will help students to complete Q6.
- **Example 4B-3** shows how to solve inequalities when multiplying or dividing by a negative number. This will help students to complete Q7 and Q8.
- In Q8, point out that students must consider the order of operations in the different questions and that in some problems the inequality will need to be reversed and in others it will not.
- **Example 4B-4** shows how to solve inequalities with the unknown on both sides. This will help students to complete Q9 and Q10.
- In Q10, students will need to expand expressions to remove brackets. You can demonstrate that they can avoid using negatives by reversing the direction of an inequality.

  e.g. $6x – 15 < 7x + 4$ can be written as

  $7x + 4 > 6x – 15$ then they can subtract $6x$ from both sides

  $x + 4 > –15$

- Q13 requires students to write an appropriate inequality statement. Some students may need an explanation of what is meant by reserve price. Have them define the pronumeral used for the unknown quantity in each case.
- You may like students to use the ‘solve’ function of a calculator or other digital technology to check whether they obtain the same answers.
- For additional practice, students can complete Q8–11 of **WS 4-1 Solving linear equations and inequalities** (see Resources). Additional questions similar to Exercise 4B Q2, Q3, Q6 and Q7 are provided. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.

Below Level

Below Level
1–9, 13, 14

- To complete this topic, students may need their calculators.

- Students may need to complete **SS 4B-1 Understanding inequality statements** (see Resources).
- For Q1, ensure that students understand the meaning of each of the four inequality symbols. Specifically, they may need to have explained:
  - the direction in which the arrow of the inequality points. Explain it points to the smaller number
  - the difference between ‘less than’ and ‘less than or equal to’
  - the meaning of a ‘between’ inequality.
- In Q2, emphasise the meaning of the closed circle or open circle. You may need to explain the ‘between’ solution further.
- Have students work through the demonstrations in Q4 carefully as they show the reason why the inequality sign needs to be reversed (to keep statements true) when multiplying or dividing by a negative number.
- In Q8, point out that students must consider the order of operations in the different questions and that in some problems the inequality will need to be reversed and in others it will not.
- For students who do not progress past Q7, direct them to Q8–11 of **WS 4-1 Solving linear equations and inequalities** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.
- Below Level students will need some support in remembering the direction of the inequality signs.
- Support them by having them use exactly the same solving methods that they would use for an equation with the different inequality sign used in place of the equals sign.
- They will have the greatest difficulty understanding that the direction of the inequality needs to be turned around when multiply or dividing by a negative number. Demonstrate this several times with examples using numbers to reinforce the concept.

Above Level

Above Level
1g, h, 2–4, 6–12, 15–19

- In Q2, emphasise the meaning of the closed circle or open circle. You may need to explain the ‘between’ solution further.

- Have students work through the demonstrations in Q4 carefully as they show the reason why the inequality sign needs to be reversed (to keep statements true) when multiplying or dividing by a negative number.
- Q8 contains a mixture of inequalities to solve. Remind students that, in some problems, the inequality sign will need to be reversed and in others it will not.
- In Q17, students need to take care with their algebraic manipulation. Common denominators may need to be applied to whole numbers as well as fractional terms.
- In Q18, the inequalities have three parts. When using the balance method students will need to balance all three parts of the inequality. In part b, when dividing by a negative number, the inequality will also need to be written backwards to maintain the correct 'between' concept.

Extra activities

1 **Quick Questions**

Solve each equation.

a $2x + 6 = 18$ $(x = 6)$

b $3x - 5 = 2x + 11$ $(x = 16)$

c $5(2x - 12) = 80$ $(x = 14)$

d $\frac{x+5}{9} = 11$ $(x = 94)$

2 A surfboard company has found that it is only able to sell surfboards for a minimum price of $250. They have also found that when surfboards are priced at over $4325 they will not be able to be sold. Graph the range of suitable prices.

Julie goes running each day but will only do so in a cool temperature. Julie will go before 8.00 am or after 5.00 pm. Graph the suitable times for Julie to go running on a number line. (Consider midnight to be 0 and the number line up to 24.)

Answers

4B Solving linear inequalities

4B Start thinking!

1 < (less than), ≤ (less than or equal to), > (greater than), ≥ (greater than or equal to)

2 a > b < c < d >

3 a x is greater than 2
  b x is less than or equal to 3
  c x is greater than or equal to 5
  d x is less than −4

4 Each inequality represents a region on the number line and not a single point.

5 Some possible answers are given.
  a 7, 30.5, 122 b 4, 0, −5
  c −3, −1, 10.6 d −5, −10, −36.4

6 x is greater than 2, and less than 9. Some possible values are: 4, 5.7, 8.2.

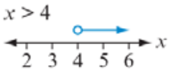
7 −4, −3, −2, −1, 0, 1, 2

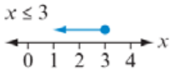
Exercise 4B Solving linear inequalities

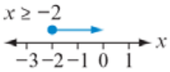
ANSWERS

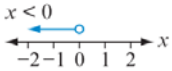
1 **a** 2.4, 3, 7, 8.3, $6\frac{4}{5}$
b $-5, -1.2, 0, -\frac{3}{4}, -10, -4.9$
c 7, 8.3, $6\frac{4}{5}$ **d** $-5, -10, -4.9$
e 2.4, 3, 7, $6\frac{4}{5}$ **f** 2.4, $\frac{1}{2}$, 3, $6\frac{4}{5}$
g $-5, 2.4, -1.2, \frac{1}{2}, 0, -\frac{3}{4}, -4.9$
h $-5, -1.2, -\frac{3}{4}, -4.9$

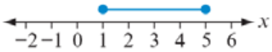
2 **a** $x \le 5$ **b** $x > 1$ **c** $x \ge -4$
d $x < -2$ **e** $3 \le x \le 7$ **f** $-5 < x \le 0$

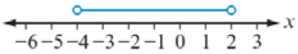
3 **a** $x > 4$


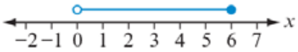
b $x \le 3$


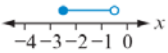
c $x \ge -2$


d $x < 0$


e $1 \le x \le 5$


f $-4 < x < 2$


g $0 < x \le 6$


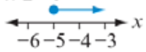
h $-2.5 \le x < -0.5$


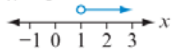
4 **a** **i** $3 < 7$; true **ii** $5 < 9$; true
iii $9 < 13$; true **iv** $-2 < 2$; true
b **i** $1 < 5$; true **ii** $-1 < 3$; true
iii $-5 < -1$; true **iv** $6 < 10$; true
c **i** $4 < 12$; true **ii** $-4 < -12$; not true
iii $6 < 18$; true **iv** $-2 < -6$; not true
d **i** $1 < 3$; true **ii** $-1 < -3$; not true
iii $\frac{2}{3} < 2$; true **iv** $-2 < -6$; not true
e Inequality sign stays the same when adding or subtracting any number. Also stays the same when multiplying or dividing by a positive number. However, direction of inequality sign is reversed when multiplying or dividing by a negative number.
f Some possible answers are given.
i $4 > -8$, $4 + 2 > -8 + 2$ $(6 > -6)$,
$4 - 1 > -8 - 1$ $(3 > -9)$
ii $4 > -8$, $4 \times 2 > -8 \times 2$ $(8 > -16)$,
$4 \div 2 > -8 \div 2$ $(2 > -4)$
iii $4 > -8$, $4 \times -2 < -8 \times -2$ $(-8 < 16)$,
$4 \div -2 < -8 \div -2$ $(-2 < 16)$

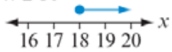
5 **a** $x + 3 > 8$ **b** $-2x \ge 8$ **c** $x \le 10$
d $x < 4$ **e** $-x \ge -9$ **f** $x > -16$
g $x \ge 7$ **h** $-x > 3$ **i** $-6x > -5$

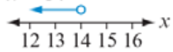
6 **a** $x < 5$ **b** $x \ge 7$ **c** $x \le -6$
d $x > 6$ **e** $x \le -5$ **f** $x < 50$
g $x \ge -20$ **h** $x \le 2$ **i** $x > -5$

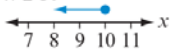
7 **a** $x \ge -6$ **b** $x < -2$ **c** $x \le 3$
d $x > -24$ **e** $x < -7$ **f** $x \ge 1$
g $x > -24$ **h** $x < 7$ **i** $x \le 10$

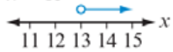
8 **a** $x \ge -5$


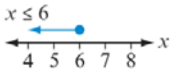
b $x > 1$


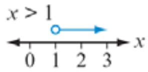
c $x \ge 18$


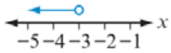
d $x < 14$


e $x \le 10$


f $x > 13$


g $x \le 6$


h $x > 1$


i $x < -3$


9 **a** $x > 4$ **b** $x \le -2$ **c** $x \ge 3$
d $x > 4$ **e** $x < 1$ **f** $x \ge -1$
g $x \le 5$ **h** $x < 2$ **i** $x > \frac{1}{2}$

10 **a** $x > 11$ **b** $x \le 13$ **c** $x \ge -2$
d $x > -19$ **e** $x > -15$ **f** $x \le 1$
g $x \le 3$ **h** $x > -\frac{1}{4}$ **i** $x \ge -6.5$

11 **a** $x \le -7$ **b** $x > 1$ **c** $x \le -3$
d $x < 1$ **e** $x \ge 4$ **f** $x > -5$

12 **a** one
b Linear inequalities can have many solutions within a given range.

13 **a** $p \ge 650\,000$, where p is selling price in \$
b $h > 97$, where h is height of a person in cm
c $s \le 60$, where s is speed in km/h
d $125 \le h \le 196$, where h is height of a person in cm

14 **a** $3p \le 20$, where p is cost of pack of gum in \$; $p \le 6\frac{2}{3}$
b Todd could buy 1, 2, 3, 4, 5 or 6 packs of gum.

15 **a** $m = \frac{1}{2}(20 - x)$, where m is number of watermelons they will each take home.
b $\frac{1}{2}(20 - x) \le 3$; $x \ge 14$
c 14 or more watermelons

16 **a** 25 min **b** 16 min **c** 19 min to 29 min

17 **a** $x \ge -11$ **b** $x < 4$ **c** $x \ge 2$ **d** $x < -8$
e $x \le 2$ **f** $x \le -3$

18 **a** $1 \le x \le 6$ **b** $-4 > x > -7$

Reflect

Possible answer: The method of solving inequalities is much the same as for linear equations; however, care must be taken with the direction of the inequality sign when multiplying or dividing by a negative number.

Resources

SupportSheet

SS 4B-1 Understanding inequality statements

Focus: To compare the size of numbers and to represent inequality statements on a number line

Resources: ruler

Students review the inequality symbols ($<$, $\leq$, $>$ and $\geq$) and use these symbols when comparing numbers. They represent statements involving x and these symbols on a number line.

WorkSheet

WS 4-1 Solving linear equations and inequalities

Focus: To solve a variety of linear equations and inequalities

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4A and 4B. Q8–11 relate to Exercise 4B.

Students solve a range of linear equations involving two-step equations, three-step equations, equations with the unknown in the denominator and equations where the unknown appears on both sides. They represent inequalities on a number line and solve linear inequalities.

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4C Sketching linear graphs

Teaching support for pages 172–178

Teaching strategies

Learning focus

To sketch linear graphs using the gradient-intercept method and the *x*- and *y*-intercept method

Start thinking!

The task guides students to:

- describe the graph of a linear relationship
- recognise that two points are needed to graph a linear relationship
- recognise that a linear relationship has a gradient and a *y*-intercept.
- use the gradient and *y*-intercept to sketch the linear relationship.

Differentiated pathways

Below Level	At Level	Above Level
1, 2a–i, 3a–i, 4, 5a, b, 6, 7a, b, 8, 9a–f, 10, 11, 13, 14a, b, 15a–d	1d–i, 2–8, 9d–j, 10, 12–14, 15a–d, 17–22	1f–i, 2f–l, 3f–l, 4, 5b, d, 6, 7c, d, 8, 9g–l, 10, 12–14, 15e–h, 16–24
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q8–10

Focus: To understand the concepts of gradient, *x*-intercept and *y*-intercept as they apply to linear relationships

- Direct students to complete **SS 4C-1 Identifying features of a linear graph** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to:
  - identify the gradient, *x*- and *y*-intercepts from a graph
  - identify the gradient and *y*-intercept from a rule in the form $y = mx + c$

- find the *x*- and *y*-intercepts from a given rule.

At Level

At Level
1d–i, 2–8, 9d–j, 10, 12–14, 15a–d, 17–22

- To complete this topic, students will need a ruler and pencil, and may like to use grid or graph paper. You may want to provide students with copies of the BLM **1-cm grid paper** (see Resources).
- This topic may need to be split up over 2 or 3 lessons.
- Ensure that students understand the idea of gradient.
- Students may use technology to complete some questions if you need to complete the exercise quickly. Or they may use technology to check their answers.
- Students may be assisted by using the BLM **Cartesian plane grids** (see Resources).
- You may want students to draw several graphs of lines per Cartesian plane as they go through this topic.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4C-1**. It shows how to use the gradient–intercept method to sketch a linear graph and this will help students to complete Q1–4.
- In Q2, students may need to be reminded of their equation-solving methods to rearrange each of the formulas. Encourage them to expand any expressions containing brackets first and then move everything else except *y* from the left side of the equation.
- Explain that in Q4, any linear rule without a constant term has the origin as both its *x*- and *y*-intercept.
- For Q6–8, discuss the form of:
  - a horizontal line that shows all points with the same *y* value
  - a vertical line that shows all points with the same *x* value.
- **Example 4C-2** shows how to use the *x*- and *y*-intercept method to sketch a linear graph. This will help students to complete Q9–12.
- For Q9–12, emphasise to students that the:
  - *x*-intercept is the value of *x* when $y = 0$
  - *y*-intercept is the value of *y* when $x = 0$.

- You may like to explain the difference between writing, for example, ‘the *x*-intercept is 3’ and ‘the coordinates of the *x*-intercept are (3, 0)’.
- **Example 4C-3** shows how to sketch horizontal and vertical linear graphs and this will help students to complete Q15.
- For Q18, discuss with students when it is most appropriate to use the gradient-intercept method or the *x*- and *y*-intercept method.
- Students may need some explanation of Q19. They will need to be shown that they would normally draw a linear relationship with arrows on each end indicating that the relationship continues infinitely. In Q19, the line is only drawn for the *x* or *y* values indicated.
- Explain that Q21 and Q22 are examples of how linear relationships are used to model a real life situation and that these models are then used to make predictions.
- For additional practice, students can complete Q1–6 of **WS 4-2 Working with linear relationships** (see Resources). Additional questions similar to Exercise 4C Q1–5, Q9 and Q10 are provided. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.

Below Level

Below Level
1, 2a–i, 3a–i, 4, 5a, b, 6, 7a, b, 8, 9a–f, 10, 11, 13, 14a, b, 15a–d

- Students may need to complete **SS 4C-1 Identifying features of a linear graph** (see Resources).
- Students who are struggling with the gradient-intercept method for Q1 should use the method of finding three points (substitute $x = 0$, $x = 1$, $x = 2$) and then observe the *y*-intercept and gradient.
- In Q2, students may need to be reminded of their equation-solving methods to rearrange each of the formulas. Encourage them to expand any expressions containing brackets first and then move everything else except *y* from the left side of the equation.
- Explain that in Q4, any linear rule without a constant term has the origin as both its *x*- and *y*-intercept.
- For Q6–8, discuss the form of:
  - a horizontal line that shows all points with the same *y* value
  - a vertical line that shows all points with the same *x* value.

- For Q9, emphasise to students that:
  - the *x*-intercept is the value of *x* when $y = 0$. They will need to substitute $y = 0$ into the linear rule and solve for *x*
  - the *y*-intercept is the value of *y* when $x = 0$. They will need to substitute $x = 0$ into the linear rule and solve for *y*.

POTENTIAL DIFFICULTY
When the *x*- and *y*-intercepts are the origin, students may struggle to see a second point to use to draw their line. They may need help with the reading of the steps in Q13 to work through the solution to this problem.

- For students who do not progress past Q10, direct them to Q1–6 of **WS 4-2 Working with linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.
- Make use of the BLM **Cartesian plane grids** (see Resources) to save students time with ruling up their own Cartesian planes.
- Where necessary, provide students with the technology to help them with their graphing or to check their answers.

Above Level

Above Level
1f–i, 2f–l, 3f–l, 4, 5b, d, 6, 7c, d, 8, 9g–l, 10, 12–14, 15e–h, 16–24

- Explain that in Q4 any linear rule without a constant term has the origin as both its *x*- and *y*-intercept.
- You may like to explain the difference between writing, for example, ‘the *x*-intercept is 3’ and ‘the coordinates of the *x*-intercept are (3, 0)’.
- In Q16, students must consider that:
  - all vertical and horizontal lines have one axis intercept
  - all other lines will have two axis intercepts although the two axis intercepts can be at the same point for graphs passing through the origin.
- For Q18, discuss with students when it is most appropriate to use the gradient-intercept method or the *x*- and *y*-intercept method.
- Students may need some explanation of Q19. They will need to be shown that they would normally draw a linear relationship with arrows on each end indicating that the

relationship continues infinitely. In Q19, the line is only drawn for the x or y values indicated.

- Explain that Q21 and Q22 are examples of how linear relationships are used to model a real life situation and that these models are then used to make predictions.

Extra activities

1 Quick Questions

Find the value of x when $y = 0$ in the following equations.

a $y = 2x - 6$ (3)

b $2x - 3y = 6$ (3)

Find the value of y when $x = 0$ in the following equations.

c $y = 4 - 2x$ (4)

d $5x - 2y + 10 = 0$ (5)

2 Conversion graphs

When measuring temperature, the convention is that we use degrees Celsius (°C). Until 1972, temperature was measured in degrees Fahrenheit (°F) and this scale is still used in many countries today including the USA.

To convert a temperature from °F to °C, the linear relationship $C = \frac{5}{9}(F - 32)$ can be used.

a By placing F on the horizontal axis and C on the vertical axis draw a graph of this relationship.

b Use your graph to find the value of:

i C when $F = 32$ ($C = 0$)

ii C when $F = 100$ ($C \approx 38$)

iii F when $C = 100$ ($F = 212$)

iv F when $C = 25$. ($F = 77$)

Answers

4C Sketching linear graphs

4C Start thinking!

1 The graph of a linear relationship is a straight line.

2 A straight line connects two points, and this will define the linear relationship.

3 a i $m = 4, c = 1$ ii $m = -4, c = 1$
iii $m = \frac{1}{4}, c = 1$

b i (0, 1) ii (0, 1) iii (0, 1)

c i 4 ii -4 iii $\frac{1}{4}$

d i rise = 4, run = 1 ii rise = -4, run = 1
iii rise = 1, run = 4

e i $y = 4x + 1$ matches graph C
ii $y = -4x + 1$ matches graph A
iii $y = \frac{1}{4}x + 1$ matches graph B

4 Starting at y-intercept, use rise and run to locate a second point. Ruling a straight line passing through these two points will represent the graph of the linear relationship.

Exercise 4C Sketching linear graphs

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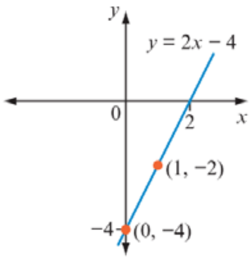

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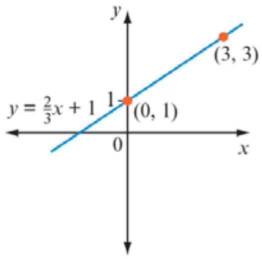

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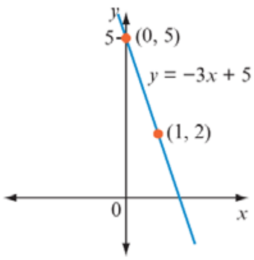

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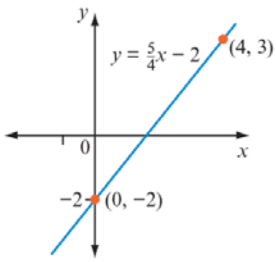

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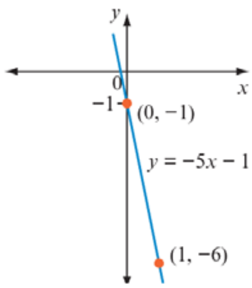

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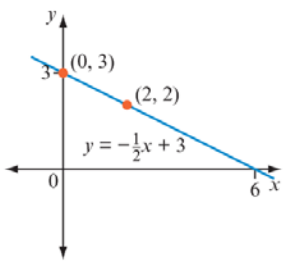

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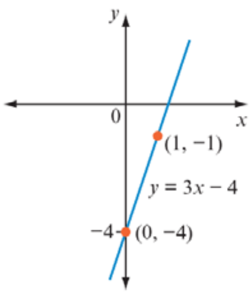

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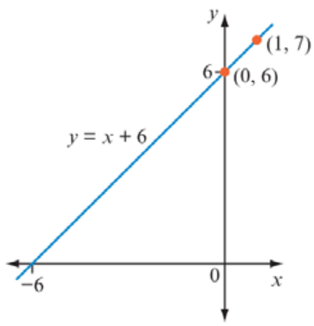

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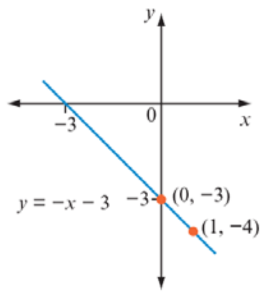


2 a $y = 2x + 1$, $m = 2$, $c = 1$

b $y = -3x + 4$, $m = -3$, $c = 4$

c $y = -\frac{3}{4}x - 3$, $m = -\frac{3}{4}$, $c = -3$

d $y = 3x - 6$, $m = 3$, $c = -6$

e $y = -2x + 6$, $m = -2$, $c = 6$

f $y = -5x - 2$, $m = -5$, $c = -2$

g $y = -3x + 2$, $m = -3$, $c = 2$

h $y = 4x - 5$, $m = 4$, $c = -5$

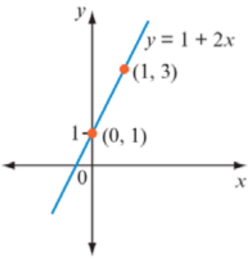
i $y = -2x + 3$, $m = -2$, $c = 3$

j $y = \frac{4}{5}x + 3$, $m = \frac{4}{5}$, $c = 3$

k $y = \frac{3}{2}x + 2$, $m = \frac{3}{2}$, $c = 2$

l $y = -\frac{1}{3}x - 2$, $m = -\frac{1}{3}$, $c = -2$

3 a



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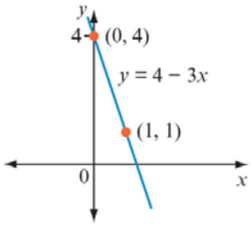

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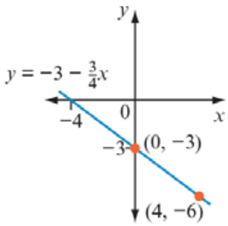

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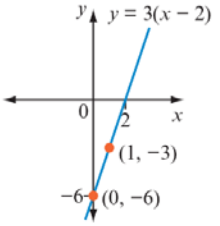

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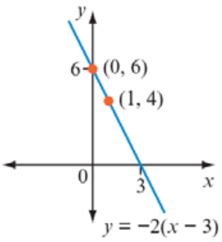

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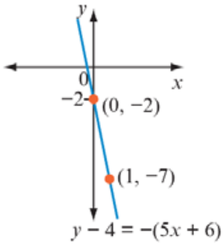

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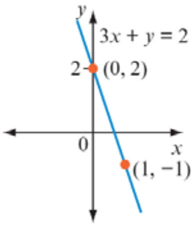

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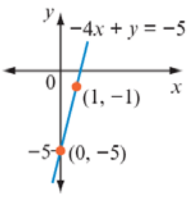

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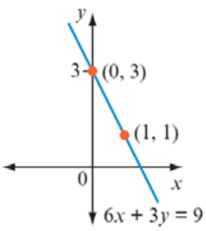

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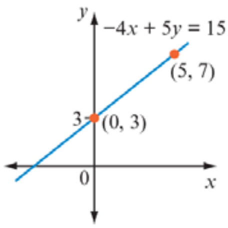

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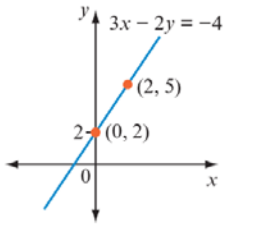

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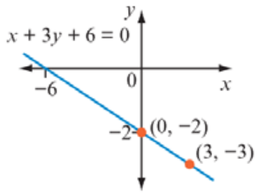
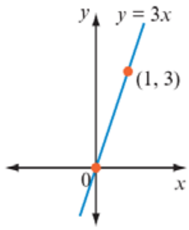


4 a $y = 3x + 0$

b i $m = 3$

ii $c = 0$

c



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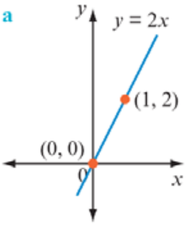

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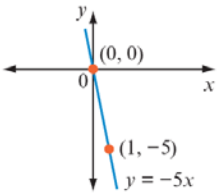

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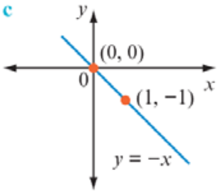

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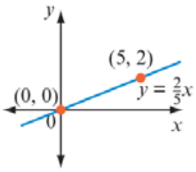

6 a $y = 0x + 4$

b i $m = 0$

ii $c = 4$

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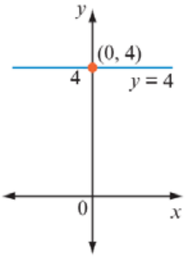

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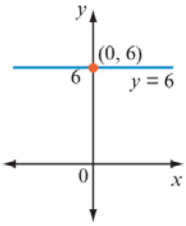

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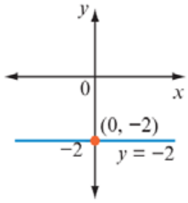

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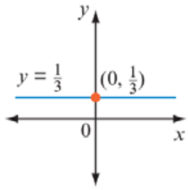

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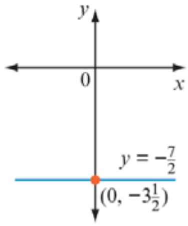
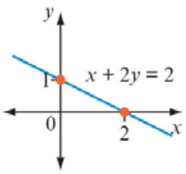


8 The rule $x = 4$ cannot be written in the form $y = mx + c$ since there is no y value specified.

9 a



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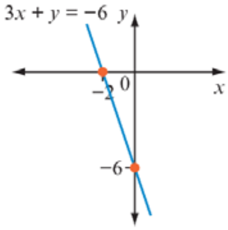

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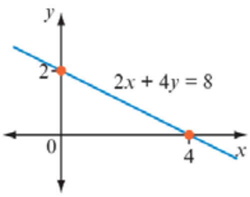

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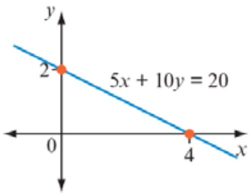

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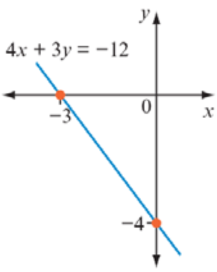

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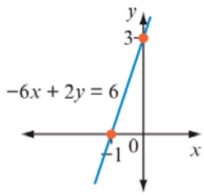

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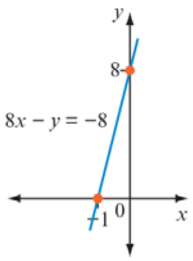

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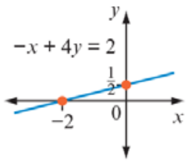

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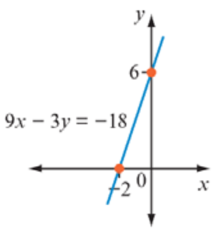

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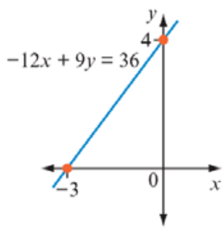

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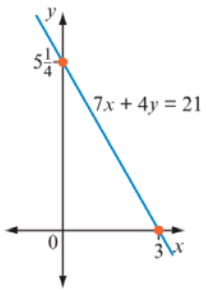

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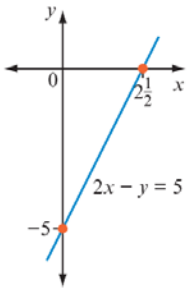

10 a x-intercept: $(-5, 0)$, y-intercept: $(0, 5)$
b x-intercept: $(4, 0)$, y-intercept: $(0, -4)$
c x-intercept: $(-2, 0)$, y-intercept: $(0, 6)$
d x-intercept: $(4, 0)$, y-intercept: $(0, -8)$
e x-intercept: $(2, 0)$, y-intercept: $(0, 2)$
f x-intercept: $(-\frac{1}{2}, 0)$, y-intercept: $(0, -3)$

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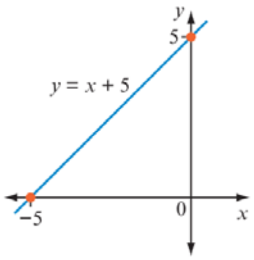

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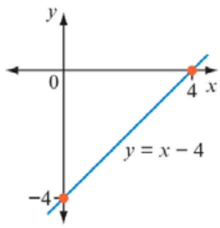

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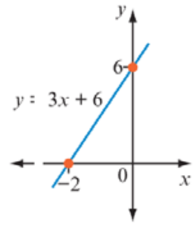

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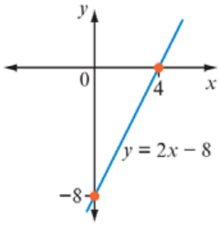

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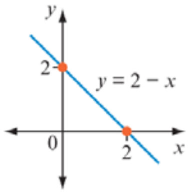

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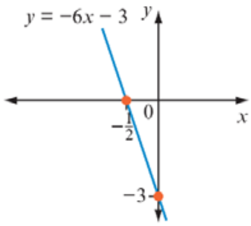
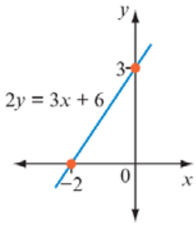


12 a



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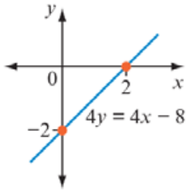

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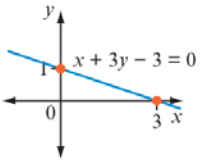

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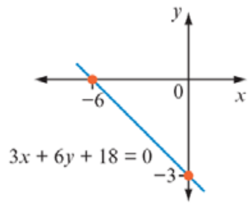

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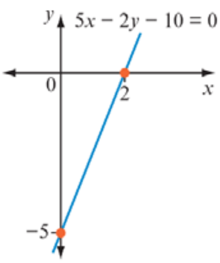

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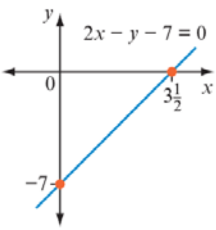
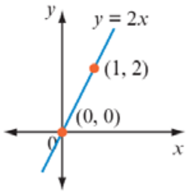
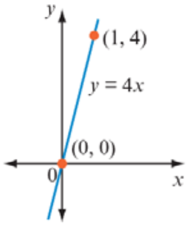


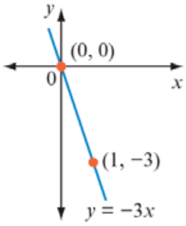
13 a **i** x-intercept: (0, 0)
ii y-intercept: (0, 0)
b no; only have one point
c coordinates of one other point on the line
d $y = 2$
e (0, 0) and (1, 2)
f



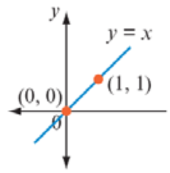
14 **a**



b



c



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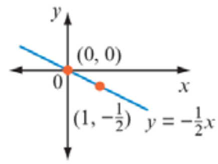
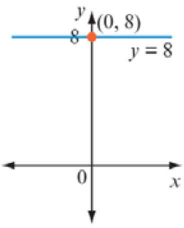


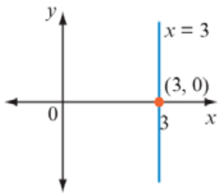
15 **a** **i** y-intercept: (0, 8), no x-intercept

ii



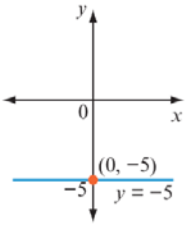
b **i** x-intercept: (3, 0), no y-intercept

ii



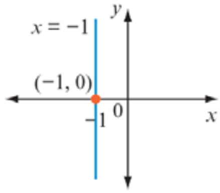
c **i** y-intercept: (0, −5), no x-intercept

ii



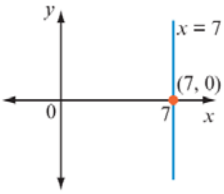
d **i** x-intercept: (−1, 0), no y-intercept

ii



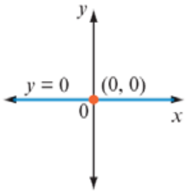
e **i** x-intercept: (7, 0), no y-intercept

ii



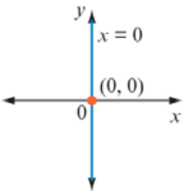
f **i** y-intercept: (0, 0), no x-intercept

ii



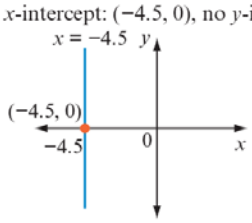
g **i** x-intercept: (0, 0), no y-intercept

ii



h **i** x-intercept: (−4.5, 0), no y-intercept

ii

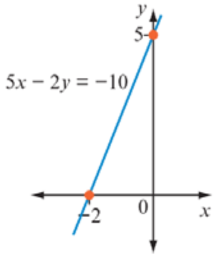


16 A linear graph can have one or two axis intercepts (or perhaps lie along the x-axis or y-axis).

- a x-intercept: (2, 0), y-intercept: (0, −4), two intercepts
- b no x-intercept, y-intercept: (0, −7), one intercept
- c x-intercept: (3, 0), y-intercept: (0, 2), two intercepts
- d x-intercept: (0, 0), y-intercept: (0, 0), one intercept
- e x-intercept: (10, 0), y-intercept: (0, −50), two intercepts
- f x-intercept: (−5, 0), y-intercept: (0, 3), two intercepts
- g x-intercept: (28, 0), no y-intercept, one intercept
- h x-intercept: (6, 0), y-intercept: (0, −2), two intercepts
- i no x-intercept, y-intercept: (0, 5.6), one intercept

17
- a gradient–intercept method
- b x- and y-intercept method
- c Find the coordinates of another point, and draw the straight line through that point and the origin.
- d Locate the y-intercept, and draw a line through that point parallel to the x-axis.
- e Locate the x-intercept, and draw a line through that point parallel to the y-axis.

18 **a**



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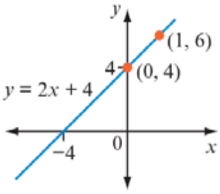

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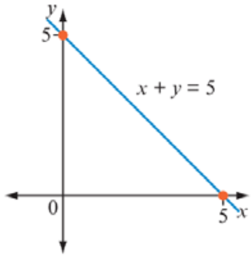

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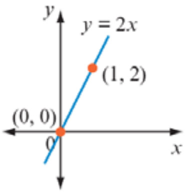

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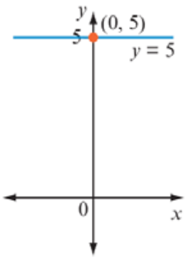

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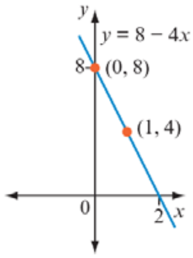

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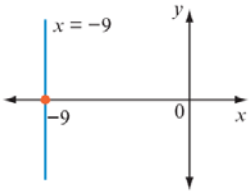

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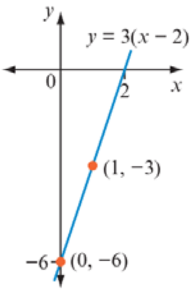

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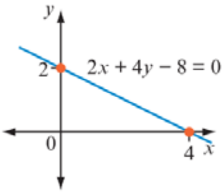

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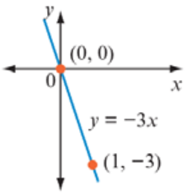

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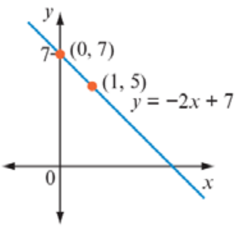

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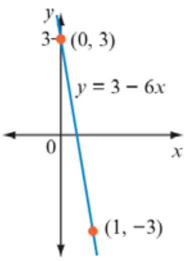
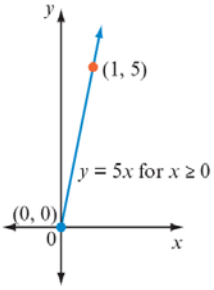


19 a



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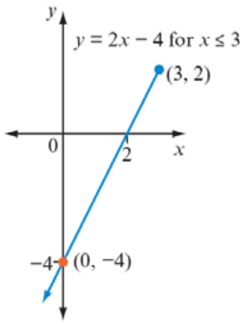

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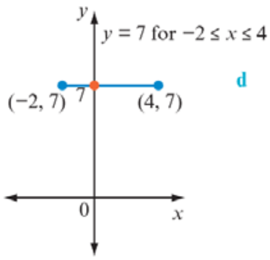

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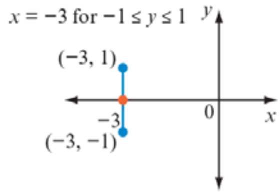

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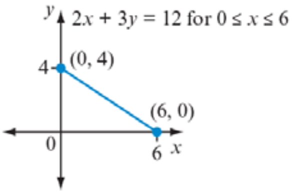

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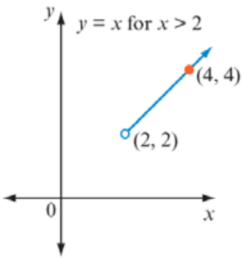

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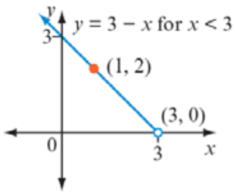

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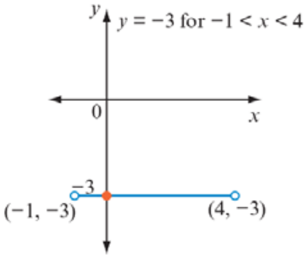

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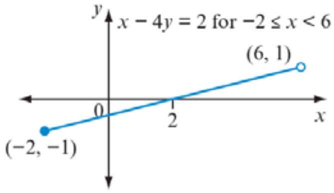

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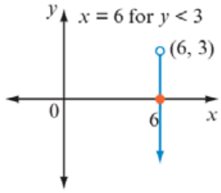
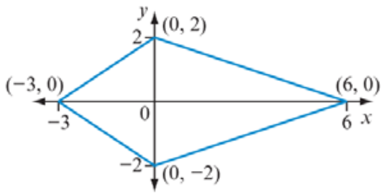


20 a



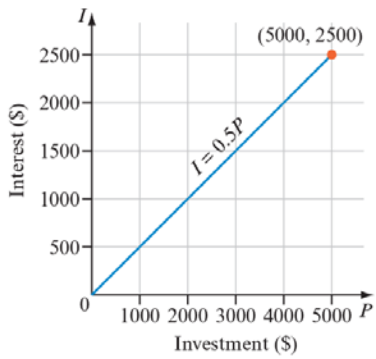
b kite

c **i** 19.9 units **ii** 18 square units

d coordinates of midpoints: (3, 1), (3, −1), (−1.5, −1) and (−1.5, 1)

e perimeter =13 units, area = 9 square units

21 a

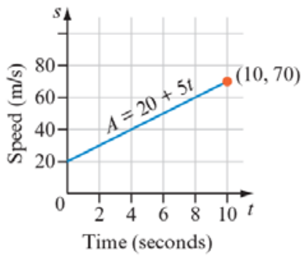


b **i** \$500 **ii** \$1250 **iii** \$2125

c **i** \$1500 **ii** \$2000 **iii** \$4500

d Gradient represents value of RT. This is the rate at which the investment grows over a 10-year period. $RT = 0.05 \times 10 = 0.5$.

22 a



b initial constant speed

c rate of increase in speed

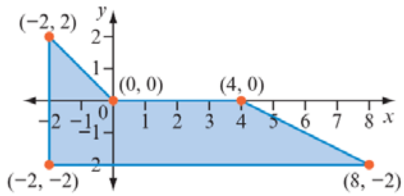
d **i** 35 m/s **ii** 45 m/s **iii** 57.5 m/s

e **i** 126 km/h **ii** 162 km/h **iii** 207 km/h

f **i** 2 s **ii** 7 s **iii** 9 s

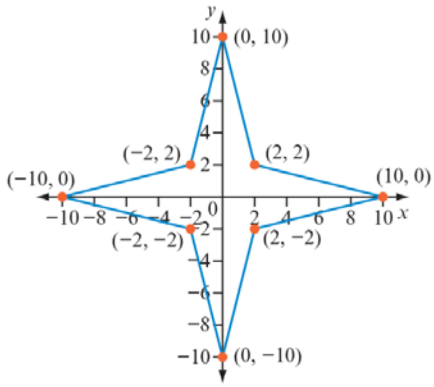
g 8 s **h** 252 km/h reached after 10 s

23 a



b 25.3 units **c** 18 square units

24 a



b 66 units **c** 80 square units

d $y = x$ **e** $y = -x$

Reflect

Possible answer: The common goal of the two methods is to locate two points on a Cartesian plane to enable a line to be ruled through them to represent the linear relationship.

Resources

SupportSheet

SS 4C-1 Identifying features of a linear graph

Focus: To review common features of a linear graph including gradient and the *x*-and *y*-intercepts

Resources: ruler (optional), **1-cm grid paper** (BLM) or graph paper (optional)

Students review the definitions of linear graph, gradient, rise, run, *x*-intercept and *y*-intercept. They work out the vertical rise and horizontal run to calculate the gradient and identify the *x*- and *y*-intercepts from a given linear graph. Students use the general rule $y = mx + c$ and their information for m and c to write the rule for these linear graphs. They also find the gradient and *x*- and *y*-intercepts for linear graphs from given rules.

WorkSheet

WS 4-2 Working with linear relationships

Focus: To sketch linear graphs and to determine the rule for linear relationships given relevant information.

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4C–E. Q1–6 relate to Exercise 4C.

Students sketch linear graphs using the gradient-intercept method and also by using the *x*- and *y*-intercept method. They perform calculations to determine the gradient of a line and to find the rule for a linear graph using $y = mx + c$ and $y - y_1 = m(x - x_1)$. Students also work with parallel and perpendicular lines.

BLMs

1-cm grid paper

Cartesian plane grids

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4D Finding the rule for a linear relationship

Teaching support for pages 178–183

Teaching strategies

Learning focus

To determine the rule for a linear relationship

Start thinking!

The task guides students to:

- determine the y-intercept and the gradient from a linear graph
- substitute this information into the $y = mx + c$ form of a linear relationship to find the rule
- apply this method to find the rule for more linear graphs given the y-intercept and gradient.

Differentiated pathways

Below Level	At Level	Above Level
1, 3–6, 8–11, 15	1–5, 6a–c, 7, 8a–c, 9–18	1–4, 6d–f, 7, 8d–f, 9, 10, 12–21
Students complete the **assessment** for this topic		

At Level

At Level
1–5, 6a–c, 7, 8a–c, 9–18

- Students will need to revise the rule $y = mx + c$ for linear relationships.
- Explain that, to find the rule, students will need to know the gradient and the y-intercept.
- Students will be familiar with the formula $m = \frac{\text{rise}}{\text{run}}$ but need to be introduced to the more formal rule $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Direct students to the **Key ideas**. You may like them to copy this summary.

- Direct students to **Example 4D-1**. It shows how to find the gradient of a line through two given points and will help students to complete Q1.
- In Q1, ensure that students set out their working properly, writing the formula at the top of each problem and labelling the values for x_1, y_1, x_2 and y_2.
- In Q3, students are to find the gradient but do not need to use a formula. As the graphs are provided, they should be able to work out the gradient from each line.
- **Example 4D-2** shows how to find the rule given the y-intercept and a point. This will help students to complete Q4 and Q5.
- In Q4, some students may need help in identifying which point is the y-intercept before substituting for c in the rule $y = mx + c$.
- **Example 4D-3** shows how to find the rule given the gradient and a point. This will help students to complete Q6.
- For Q6, explain that, although students have not being given the y-intercept, they can substitute the coordinates of the given point and the gradient into the rule $y = mx + c$ and solve to find c.
- You may like to work through the steps of Q7 as a whole class to produce the point-gradient rule $y - y_1 = m(x - x_1)$.
- **Example 4D-4** shows how to find the rule given two points using $y - y_1 = m(x - x_1)$. This will help students to complete Q9–11.
- In Q10, after first finding the gradient, have some students find the rule using the first point in the pair and others find the rule using the second point. Students will see that the same answer is obtained and so it does not matter which point is used in the formula. This will confirm their findings in Q9.
- In Q13, guide students to write the x- and y-intercepts as pairs of coordinates so they can use the point-gradient formula to find the rule.
- In Q15 and Q16, some students may need to draw the linear relationship to find the rule.
- For additional practice, students can complete Q7–9 of **WS 4-2 Working with linear relationships** (see Resources). Additional questions similar to Exercise 4D Q1, Q3 and Q10 are provided. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.
- For more problem-solving tasks and investigations, direct students to **INV 4-2 The human body** (see Resources).

Below Level

Below Level
1, 3–6, 8–11, 15

- In Q1, encourage students to identify and then write the values for x_1, y_1, x_2 and y_2 to ensure that students substitute the values correctly in the formula for gradient. They should clearly show all working out.
- In Q3, students may need to be guided to see that they can work out the gradient directly from each linear graph.
- In Q4, some students may need help in identifying which point is the *y*-intercept before substituting for *c* in the rule $y = mx + c$.
- For Q6, explain that, although students have not being given the *y*-intercept, they can substitute the coordinates of the given point and the gradient into the rule $y = mx + c$ and solve to find *c*.
- You may like to work through the steps of Q7 as a whole class to produce the point-gradient rule $y - y_1 = m(x - x_1)$. Below Level students may find it easier to use the provided formula. However, all the pronumerals will need to be carefully explained.
- When students get to Q9 and Q10, explain that the formula $y - y_1 = m(x - x_1)$ is called the point-gradient formula. In each problem, have students calculate the gradient and label one of the points x_1 and y_1 before substituting.
- In Q10, after first finding the gradient, have some pairs of students find the rule using the first point and other pairs of students find the rule using the second point. Students will see that the same answer is obtained and so it does not matter which point is used in the formula. This will confirm their findings in Q9.
- In Q15, students may benefit from drawing each linear relationship first to assist them in finding the rule.
- For students who do not progress past Q10, direct them to Q7–9 of **WS 4-2 Working with linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.
- Students need to be confident in their understanding that the graph represents all points that satisfy a given rule for a linear relationship.

- They will need to have the two key formulas $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and $y - y_1 = m(x - x_1)$ at their fingertips.
- Where appropriate, allow students to use technology to check their answers.

Above Level

Above Level
1–4, 6d–f, 7, 8d–f, 9, 10, 12–21

- In Q1, an easy mistake to make is to confuse the order of the x and y values in the gradient formula. Encourage students to label the values for x_1, y_1, x_2 and y_2 before substituting into the formula.
- For Q7, explain that the formula $y - y_1 = m(x - x_1)$ is called the point-gradient formula.
- In Q10, after first finding the gradient, have some students find the rule using the first point in the pair and others find the rule using the second point. Students will see that the same answer is obtained and so it does not matter which point is used in the formula. This will confirm their findings in Q9.
- In Q13, guide students to write the x- and y-intercepts as pairs of coordinates so they can use the point-gradient formula to find the rule.
- In Q19, students will obtain a fractional answer for the gradient. Explain that the best way to express the rule without fractions is to multiply both sides of the equation by the denominator before expanding as shown below for part a.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = \tfrac{1}{2}(x - 20)$$

$$2y - 16 = 1(x - 20)$$

$$2y - 16 = x - 20$$

$$2y = x - 4$$

- For more problem-solving tasks and investigations, direct students to **INV 4-2 The human body** (see Resources).

Extra activities

1 Quick Questions

Solve each equation.

a $7 = 2 \times 3 + c$ ($c = 1$)

b $-4 = 3 \times 4 + c$ ($c = -16$)

c $9 = 5 \times (-4) + c$ ($c = 29$)

d $10 = c - 4 \times 3$ ($c = 22$)

2 Jaimee is organising a school disco. It will cost $480 to stage the disco and she is planning to charge $12 per ticket.

a Write a rule that will represent the profit (or loss) that Jaimee will make.
($P = 12n - 480$ where P is the profit and n is the number of tickets sold)

b The number of people that will attend can be found using the rule $N = 950 - 12c$ where c is the cost of attending in dollars. Find the number of people who will attend the disco and Jaimee’s profit or loss.
($N = 950 - 12 \times 12 = 806$; $P = 12 \times 806 - 480 = \$9192$)

Answers

4D Finding the rule for a linear relationship

4D Start thinking!

1 a Graph shows line drawn through two points on the Cartesian plane.

b Rule can be calculated from gradient and y-intercept.

c 3 d (0, 2) e $m = 3, c = 2$

f $y = 3x + 2$

2 c -2 d (0, 3) e $m = -2, c = 3$

f $y = -2x + 3$

3 General rule for straight line is $y = mx + c$. Gradient (m) is coefficient of x, and y-coordinate of y-intercept represents value of c.

4 a $y = 3x - 2$

b $y = -\frac{5}{3}x + 1$ or $5x + 3y = 3$

Exercise 4D Finding the rule for a linear relationship

1 **a** 4 **b** −2 **c** $\frac{1}{2}$ **d** −1 **e** 5 **f** −3

2 No. Consider the two points (2, 6) and (1, 4).
Taking (2, 6) as (x_1, y_1) and (1, 4) as (x_2, y_2),
$m = \frac{4-6}{1-2} = \frac{-2}{-1} = 2$.
Taking (1, 4) as (x_1, y_1) and (2, 6) as (x_2, y_2),
$m = \frac{6-4}{2-1} = \frac{2}{1} = 2$.

3 **a** $y = 2x + 1$ **b** $y = 4x - 3$ **c** $y = -3x + 5$
d $y = -x - 1$ **e** $y = \frac{3}{5}x$ **f** $y = -\frac{1}{2}x + 2$

4 **a** $y = 4x + 2$ **b** $y = x - 7$ **c** $y = \frac{1}{2}x + 5$
d $y = 10x + 6$ **e** $y = -3x - 5$ **f** $y = -\frac{1}{4}x + 4$

5 **a** $y = 5x - 3$ **b** $y = -2x + 4$ **c** $y = \frac{4}{5}x + 2$

6 **a** $y = 3x + 1$ **b** $y = 2x + 3$ **c** $y = -4x - 2$
d $y = -x + 9$ **e** $y = 5x + 9$ **f** $y = -3x - 1$

7 **a** $c = y_1 - mx_1$ **b** $y = mx + y_1 - mx_1$
c $y - y_1 = mx - mx_1, y - y_1 = m(x - x_1)$

8 **a** $y = 3x + 1$ **b** $y = 2x + 3$ **c** $y = -4x - 2$
d $y = -x + 9$ **e** $y = 5x + 9$ **f** $y = -3x - 1$

9 The answer is the same. Either of the two points can be used.

10 **a** $y = 2x + 1$ **b** $y = 6x - 28$ **c** $y = -3x + 10$
d $y = -2x + 4$ **e** $y = x - 1$ **f** $y = -x - 6$

11 **a** $y = 3x - 11$ **b** $y = -2x + 6$ **c** $y = x + 2$

12 $m = \frac{7-3}{3-1} = 2$ so $y = 2x + c$
Substituting the point (1, 3) gives $3 = 2 + c$ or $c = 1$.
So the rule is $y = 2x + 1$.

13 **a** $y = -2x + 4$ **b** $y = x + 3$ **c** $y = 2x - 6$
d $y = -5x + 5$

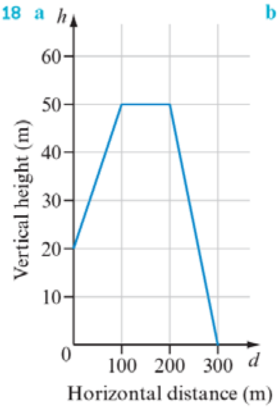
14 **a** $y = -3x + 5$ **b** $y = 2x + 4$ **c** $y = 8$
d $y = \frac{1}{3}x$ **e** $x = -\frac{5}{2}$ **f** $y = 2x - 1$

15 **a** $y = \frac{1}{200}x + 50$ **b** $y = -\frac{1}{15}x + 6$ **c** $y = \frac{1}{5}x$

16 **a** $a = 20n + 80$ **b** 21 weeks

17 **a** $w = -195n + 1560$ **b** $975
c 8 months

18 **a**

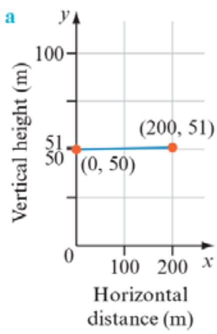


b In order of the flight:
$h = 0.3d + 20$, $h = 50$,
$h = -0.5d + 150$

c **i** 29 m **ii** 44 m **iii** 50 m **iv** 30 m
d $66\frac{2}{3}$ m and 220 m

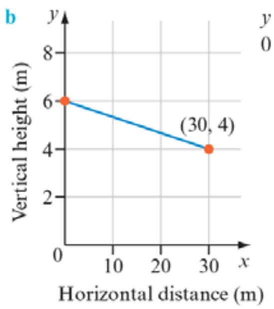
19 **a** $x - 2y = 4$ **b** $y = -23$
c $2x + 3y + 20 = 0$ **d** $12x - 17y = 0$

20 **a**



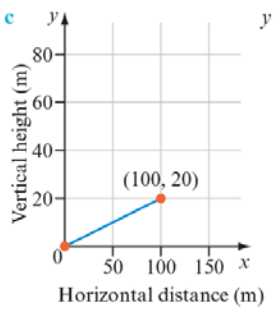
$y = \frac{1}{200}x + 50$ for $0 \le x \le 200$

b



$y = -\frac{1}{15}x + 6$ for $0 \le x \le 30$

c



$y = \frac{1}{5}x$ for $0 \le x \le 100$

21 In order of the flight:
$h = 0.3d + 20$ for $0 \le d \le 100$
$h = 50$ for $100 \le d \le 200$
$h = -0.5d + 150$ for $200 \le d \le 300$

Reflect

Possible answer: The rule for a linear graph can be found by identifying the gradient (m) and y-intercept (c) and substituting this into the rule $y = mx + c$. Alternatively, the coordinates of a point on the graph and the gradient can be substituted into the rule $y - y_1 = m(x - x_1)$.

Resources

WorkSheet

WS 4-2 Working with linear relationships

Focus: To sketch linear graphs and to determine the rule for linear relationships given relevant information

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4C–E. Q8 and Q9 relate to Exercise 4D.

Students sketch linear graphs using the gradient-intercept method and also by using the *x*- and *y*-intercept method. They perform calculations to determine the gradient of a line and to find the rule for a linear graph using $y = mx + c$ and $y - y_1 = m(x - x_1)$. Students also work with parallel and perpendicular lines.

Investigation

INV 4-2 The human body

Focus: To discover a relationship between measurements of various parts of the human body

Resources: calculator, measuring tape

Students look at measurements of various parts of the body; for example, height, waist, head length and reach. These measurements are compared and an approximate ratio found enabling a linear relationship to be written to compare different measurements.

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4E Parallel and perpendicular lines

Teaching support for pages 184–189

Teaching strategies

Learning focus

To understand the relationship between the gradients of parallel and perpendicular lines and write rules for these lines

Start thinking!

The task guides students to:

- look at the graphs of three linear relationships that are either parallel or perpendicular
- find the gradient and *y*-intercept of each linear graph
- recognise that parallel lines have equal gradients
- recognise that the product of the gradients of two perpendicular lines is –1.

Differentiated pathways

Below Level	At Level	Above Level
1–10, 12, 13a, b, 16, 18	2, 3, 5–9, 11–17, 19–22	2, 3, 5, 7–9, 11–13, 15, 17, 19–27
Students complete the **assessment** for this topic		

At Level

At Level
2, 3, 5–9, 11–17, 19–22

- You may like to provide students with copies of the BLM **Cartesian plane grids** (see Resources).
- Ensure that students are familiar with the key ideas from the previous topic.
  - $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
  - $y - y_1 = m(x - x_1)$

- Discuss the definition of gradient as being the slope of a line. Therefore, as parallel lines have the same slope, the gradients must be equal.
- Discuss the definition of perpendicular lines as being at right angles to each other. Demonstrate that if two lines are perpendicular the signs of the two gradients must be opposite. Students should also be able to see that if one gradient is steep the other must be slight. Introduce the idea of the gradients being negative reciprocals.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- In Q2, students need to read the gradient from the rule $y = mx + c$ and recognise that parallel lines have equal gradients.
- Direct students to **Example 4E-1**. It shows how to write the rule for a parallel line using the gradient and y-intercept and will help students to complete Q3 and Q4.
- For Q3, students again use the rule $y = mx + c$ to find the gradient and then use this same rule with a new value of c to write the required rule of the parallel line. Students should progress to the point where they can see they only need to change the value of c using the new y-intercept.
- **Example 4E-2** shows how to write the rule for a parallel line using the gradient and a point. This will help students to complete Q5.
- **Example 4E-3** shows how to write the rule for a perpendicular line using the gradient and y-intercept. This will help students to complete Q9.
- For Q9, students need to understand that unlike parallel lines, the coefficients of x and y will change. Therefore, it is not as simple as changing the constant term.
- **Example 4E-4** shows how to write the rule for a perpendicular line using the gradient and a point. This will help students to complete Q12.
- In Q13, guide students to write both rules in the form $y = mx + c$ and examine if the gradients are equal or negative reciprocals.
- In Q14, there will be many possible answers. Have students compare their answers and recognise that they are all of the same form, $2y – 10x = k$ (or $y = 5x + k$).
- In Q15, there will be many possible answers. Again, have students compare their answers and see they are all of the form $8x – y – k = 0$. Discuss the similarities and differences to the given line $x + 8y – 4 = 0$.
- In Q16, students use the gradient formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between two points and compare this to the gradient of the graph of $y = 4x – 7$.

- In Q20, students consider the gradients of opposite sides of a quadrilateral. Have students draw a conclusion about what type of quadrilateral ABCD would be. Extend this conversation to discuss what we would know about the gradients of sides and diagonals of common quadrilaterals.

- For Q21, guide students to see that two of the sides must be perpendicular to form a right-angled triangle and hence they need to calculate the gradients of the line segments. To show that the triangle is also isosceles, they need to think about which two sides to consider and find their lengths using the distance formula. Finding the length of the third side will enable them to calculate the perimeter of the triangle.

- In Q22, students need to remember that a rhombus is a quadrilateral with four equal side lengths and that the opposite sides of a parallelogram are equal in length.

- For additional practice, students can complete Q10–12 of **WS 4-2 Working with linear relationships** (see Resources). Additional questions similar to Exercise 4E Q2, Q3 and Q8 are provided. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.

Below Level

Below Level
1–10, 12, 13a, b, 16, 18

- For Q1, ensure that students draw their Cartesian plane quite large to enable them to see the key features of the graphs.

- In Q2, students need to read the gradient from the rule $y = mx + c$ and recognise that parallel lines have equal gradients.

- For Q3, students again use the rule $y = mx + c$ to find the gradient and then use this same rule with a new value of c to find the required rule of the parallel line.

- In Q4, students should reach the point where they see that the rules for all parallel lines only differ by the constant term.

POTENTIAL DIFFICULTY
Students need to be familiar with the key language of the topic. Define words such as perpendicular, product and reciprocal so students can progress to Q6.

- After completing Q6, students should be able to see that the gradient of a perpendicular line can be found by taking the negative reciprocal of the first gradient.

- For Q9, students need to understand that unlike parallel lines, the coefficients of x and y will change. Therefore, it is not as simple as changing the constant term.

- For students who do not progress past Q8, direct them to Q10–12 of **WS 4-2 Working with linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.

- Have students look at families of parallel lines and see that the coefficients of x and y remain the same and that only the constant term changes. This should include looking at equations in the form
  - $y = mx + c$
  - $ax + by = c$
  - $ax + by + c = 0$
- Repeat this task for perpendicular line pairs and have students look at the change in coefficients.

Above Level

Above Level
2, 3, 5, 7–9, 11–13, 15, 17, 19–27

- For Q3, students should observe that they only need to change the value of c using the new y-intercept.
- For Q9, students need to understand that unlike parallel lines, the coefficients of x and y will change. Therefore, it is not as simple as changing the constant term.
- In Q13, guide students to write both rules in the form $y = mx + c$ and examine if the gradients are equal or negative reciprocals.
- In Q17, students use the gradient formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between two points and compare this to the gradient of the graph of $y = -2x + 5$.
- In Q20, students consider the gradients of opposite sides of a quadrilateral. Have students draw a conclusion about what type of quadrilateral ABCD would be. Extend this conversation to discuss what we would know about the gradients of sides and diagonals of common quadrilaterals.
- For Q21, guide students to see that two of the sides must be perpendicular to form a right-angled triangle and hence they need to calculate the gradients of the line segments. To show that the triangle is also isosceles, they need to think about which two sides to consider and find their lengths using the distance formula. Finding the length of the third side will enable them to calculate the perimeter of the triangle.

- In Q22, students need to remember that a rhombus is a quadrilateral with four equal side lengths and that the opposite sides of a parallelogram are equal in length.
- For Q25, students may need the term ‘perpendicular bisector’ defined as a line that cuts another line exactly in half at right angles.
- For Q26, students need to know that the diagonals of a kite are perpendicular.

Extra activities

1 Quick Questions

Write each equation in the form $y = mx + c$.

a $2y = 4x + 8$ $(y = 2x + 4)$

b $2x + 5y = 10$ $(y = -\frac{2}{5}x + 2)$

c $5x - 2y = 10$ $(y = \frac{5}{2}x - 5)$

d $6x + 4y - 5 = 0$ $(y = -\frac{3}{2}x + \frac{5}{4})$

Answers

ANSWERS

4E Parallel and perpendicular lines

4E Start thinking!

1 A gradient: 2, y-intercept: 2
B gradient: 2, y-intercept: -2
C gradient: $-\frac{1}{2}$, y-intercept: 3

2 A $y = 2x + 2$ B $y = 2x - 2$ C $y = -\frac{1}{2}x + 3$

3 A and B are parallel.

4 gradient

5 90°; perpendicular

6 -1

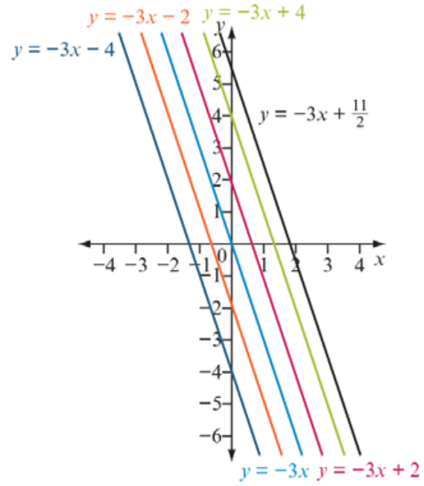
7 -1

8 Parallel lines have the same gradient. The product of the gradients of perpendicular lines is -1.

Exercise 4E Parallel and perpendicular lines

ANSWERS

1 a



b All lines are parallel and have same gradient. They have different y-intercepts.
c All have same gradient (-3).
d **i** gradient $= 3$, so not parallel
ii gradient $= -3$, so parallel
iii gradient $= 1$, so not parallel

2 a 7 **b** -1 **c** $-\frac{3}{2}$ **d** 0 **e** -4 **f** -2

3 a $y = -2x + 3$ **b** $y = 6x - 4$
c $y = x$ **d** $y = -5x + 2$

4 a $y = -3x - 5$
b One possible answer is: $y = -3x + 20$.

5 a $y = 4x - 13$ **b** $y = -3x - 6$
c $y = -x - 2$ **d** $y = \frac{1}{2}x - 3$

6 a **i** gradients: 3 and $-\frac{1}{3}$
ii gradients: $\frac{3}{4}$ and $-\frac{4}{3}$
b Both products are -1.
c The product of gradients of perpendicular lines is always -1.
d Take negative reciprocal of known gradient.

7 a $-\frac{1}{5}$ **b** $\frac{1}{8}$ **c** 1 **d** -6 **e** $-\frac{5}{4}$ **f** $\frac{2}{3}$

8 a $-\frac{1}{2}$ **b** $\frac{1}{3}$ **c** -7 **d** $\frac{5}{2}$ **e** $\frac{1}{9}$ **f** -4

9 a $y = \frac{1}{2}x + 4$ **b** $y = -\frac{1}{5}x - 1$
c $y = -x$ **d** $y = \frac{2}{7}x + 3$

10 a $y = -\frac{1}{2}x + 3$ **b** $y = x + 3$
c $y = -\frac{2}{3}x + 3$

11 a **i** $y = 2x - 4$ **ii** $y = -\frac{1}{2}x - 4$
b **i** $y = -x - 4$ **ii** $y = x - 4$
c **i** $y = \frac{3}{2}x - 4$ **ii** $y = -\frac{2}{3}x - 4$

12 a $y = -\frac{1}{5}x - 7$ **b** $y = \frac{1}{7}x + 3$
c $y = -x - 3$ **d** $y = \frac{3}{2}x + 10$

13 a parallel; same gradient of 4
b perpendicular; gradients of $\frac{2}{3}$ and $-\frac{3}{2}$
c perpendicular; gradients of $\frac{3}{8}$ and $-\frac{8}{3}$
d neither; gradients of -2 and 2
e neither; gradients of -5 and $-\frac{1}{5}$
f parallel; same gradient of $\frac{2}{7}$

14 Some possible answers are:
$y - 5x = 6$ and $4y - 20x + 1 = 0$.

15 Some possible answers are:
$y = 8x + 2$ and $8x - y = 12$.

16 Gradient of line joining $(2, -3)$ and $(4, 5)$ is 4.
Gradient of $y = 4x - 7$ is 4.
So lines are parallel.

17 Gradient of line joining $(-11, -7)$ and $(-1, -2)$ is $\frac{1}{2}$.
Gradient of $y = -2x + 5$ is -2.
Product of gradients is -1, so lines are perpendicular.

18 a $\frac{2}{3}$ **b** **i** $y = \frac{2}{3}x - 5$ **ii** $y = -\frac{3}{2}x + 2$

19 a $y = -3x$ **b** $y = \frac{1}{3}x + 9$ **c** $y = \frac{1}{3}x - 1$

20 a Yes, both have gradient of $\frac{3}{7}$.
b No. AC has gradient of $\frac{5}{4}$, while BD has gradient of $-\frac{2}{3}$.

21 a gradient of AB: 3, gradient of BC: $-\frac{1}{3}$. So line segments AB and BC are perpendicular. This makes triangle ABC right-angled.
b length of AB: 9.5 units, length of BC: 6.3 units. So triangle is not isosceles right-angled.
c 27.2 units **d** 29.9 square units

22 a AD || BC, each with gradients of $\frac{1}{2}$, and AB || DC, each with gradients of 2.
length of AD = length of BC =
length of AB = length of DC = 2.2 units
b EF || HG, each with gradients of $\frac{3}{7}$ and HE || GF, each with gradients of 2.
length of EF = length of HG = 7.6 units,
length of HE = length of GF = 8.9 units

23 gradient of KN = gradient of LM = $-\frac{1}{2}$, gradient of MN = gradient of LK = 2
So opposite sides are parallel, and adjacent sides are at right angles.
length of KL = length of LM = length of MN = length of NK = 4.5 units
perimeter: 17.9 units, area: 20.3 square units

24 gradient of PS = gradient of QR = 1, gradient of PQ = gradient of SR = −1
So opposite sides are parallel, and adjacent sides are at right angles.
length of PS = length of QR = 4.2 units, length of PQ = length of SR = 5.7 units
perimeter: 19.8 units, area: 23.9 square units

25 $y = x - 5$

26 length of YX = length of YZ = 7.1 units and length of WX = length of WZ = 20.6 units
gradient of XZ = $-\frac{3}{4}$ and gradient of WY = $\frac{4}{3}$, so diagonals are at right angles.
midpoint of XZ is (3, 5). This point also lies on the diagonal WY, which has equation $y = \frac{4}{3}x + 1$.

27 Gradients of the line segments are: AB = $\frac{1}{2}$, BC = −3, CD = $\frac{1}{2}$, DA = $\frac{6}{5}$, AC = 0, BD = $\frac{9}{11}$.
This indicates quadrilateral is a trapezium (one pair of parallel sides).

Reflect

Possible answer: Parallel lines have the same gradient while the gradients of two perpendicular lines multiply to make –1.

Resources

WorkSheet

WS 4-2 Working with linear relationships

Focus: To sketch linear graphs and to determine the rule for linear relationships given relevant information

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4C–E. Q10–12 relate to Exercise 4E.

Students sketch linear graphs using the gradient-intercept method and also by using the *x*- and *y*-intercept method. They perform calculations to determine the gradient of a line and to find the rule for a linear graph using $y = mx + c$ and $y - y_1 = m(x - x_1)$. Students also work with parallel and perpendicular lines.

BLM

Cartesian plane grids

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4F Solving simultaneous linear equations graphically

Teaching support for pages 190–195

Teaching strategies

Learning focus

To find the point of intersection of two linear graphs and recognise that this point is a solution to both equations

Start thinking!

The task guides students to:

- look at two linear graphs and identify the coordinates of points on each graph to complete a table of values
- use the table to identify which x-value has a y-value that is the same for both linear graphs
- recognise a coordinate pair that lies on both equations and see that this is the point of intersection
- find the rule or equation for each line
- see that the coordinates of the point of intersection satisfies both equations simultaneously.

Differentiated pathways

Below Level	At Level	Above Level
1–6, 9	1–3, 4e–l, 5e–l, 6–14	1e, f, 2, 4i–l, 5i–l, 6–8, 10–17
Students complete the **assessment** for this topic		

At Level

At Level
1–3, 4e–l, 5e–l, 6–14

- You may like students to use the BLM **Cartesian plane grids** (see Resources) to save them ruling up their own grids.

- Students need to understand that a single linear equation with one unknown has a single (unique) solution.
- If there are two unknowns in a single linear equation, there are an infinite number of solutions and these solutions are represented by the graph of that equation.
- If there are two linear equations and two unknowns, the point of intersection of those two graphs will be a solution of both equations.
- The values of x and y are said to be solution to the pair of simultaneous linear equations.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4F-1**. It shows how to check solutions to simultaneous linear equations and will help students to complete Q1.
- For Q1, ensure that students understand that the solution needs to satisfy both equations. Guide them to substitute the x and y values separately into each side of the equation and show that LS = RS, as demonstrated in the example.
- **Example 4F-2** shows how to identify the solution to simultaneous linear equations from graphs. This will help students to complete Q2.
- In Q2, ensure that students correctly identify the x value and the y value and don't reverse them. This will be important for their substitution in Q3.
- **Example 4F-3** shows how to solve simultaneous linear equations graphically. This will help students to complete Q4, Q6 and Q7.
- For Q4, Q6 and Q7, it may be useful to provide the BLM **Cartesian plane grids** (see Resources). Emphasise to students that graphs need to be drawn to an accurate scale if the solution is to be read from the graphs.
- In Q6, some students may need a reminder about the graphs of horizontal and vertical lines.
- In Q8, if students cannot see the reason there is no solution, have them graph both equations to recognise that they are parallel.
- Q9 is an example that can be used to demonstrate marketing concepts larger organisations use on a bigger scale. Ensure that students correctly assign n on the horizontal axis and a on the vertical axis.
- Q10 requires students to develop their own equations. For those who are unable to do this, have them use the diagram to derive $x + y = 20$ and $y = x + 4$.

- In Q11 and Q12, it will be important that students define the variables at the beginning of the question. For example in Q11, begin with 'Let Lachlan's age be *x* and Tia's age be *y*.'
- Q13 will require students to draw a diagram and label the length and the width.
- For additional practice, students can complete Q1 and Q2 of **WS 4-3 Solving simultaneous equations** (see Resources). Additional questions similar to Exercise 4F Q2 and Q4 are provided. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.

Below Level

Below Level
1–6, 9

- For Q1, ensure that students understand that the solution needs to satisfy both equations. Guide them to substitute the *x* and *y* values separately into each side of the equation and show that LS = RS, as demonstrated in the example.
- In Q2, ensure that students correctly identify the *x* value and the *y* value and don't reverse them. This will be important for their substitution in Q3.
- In Q4 and Q6, it may be useful to provide the BLM **Cartesian plane grids** (see Resources). Emphasise to students that graphs need to be drawn to an accurate scale if the solution is to be read from the graphs.
- In Q6, some students may need a reminder about the graphs of horizontal and vertical lines.
- Q9 is an example that can be used to demonstrate marketing concepts larger organisations use on a bigger scale. Ensure that students correctly assign *n* on the horizontal axis and *a* on the vertical axis.
- For students who do not progress past Q4, direct them to Q1 and Q2 of **WS 4-3 Solving simultaneous equations** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.
- Ensure that students are properly equipped to draw accurate diagrams, as many Below Level students will lack the fine motor skills to do this unaided.
- If this becomes too difficult, provide them with technology that will assist them to complete this exercise.

Above Level

Above Level
1e, f, 2, 4i–l, 5i–l, 6–8, 10–17

- It may be useful to provide the BLM **Cartesian plane grids** (see Resources). Emphasise to students that graphs need to be drawn to an accurate scale if the solution is to be read from the graphs.
- In Q8, if students cannot see the reason there is no solution, have them graph both equations to recognise that they are parallel.
- For Q11–14, it will be important that students define the variables at the beginning of the problem.
- In Q15, students need to draw the graphs by:
  - plotting the points (0, –5) and (4, –6) and ruling a line though them to obtain the first graph
  - using the gradient of –3 from the point (4, –6) to draw the second graph.
- In Q17, students will need to see that the two equations are in fact the same equation.

Extra activities

1 **Quick Questions**

Solve each equation.

a $2x + 9 = 31$ $(x = 11)$

b $9 - 3y = -15$ $(y = 8)$

c $\dfrac{z+4}{5} = 6$ $(z = 26)$

d $\dfrac{9-2a}{4} = -1$ $(a = 6.5)$

2 Give an example of a simultaneous equation pair that has:

a no solution

b one solution

c infinite solutions.

Answers

4F Solving linear simultaneous equations graphically

4F Start thinking!

1

x	−1	0	1	2	3	4	5
y_A	−6	−4	−2	0	2	4	6
y_B	6	5	4	3	2	1	0

2 $x = 3$ 3 (3, 2) 4 (3, 2)

5 This point lies on both lines, and satisfies both equations simultaneously.

6 a $y = 2x - 4$ b $y = -x + 5$

7 Line A: RS = 2 × 3 − 4 = 2 = LS
Line B: RS = −3 + 5 = 2 = LS

8 $x = 4$ 9 $x = 3, y = 2$

10 Identify coordinates of point of intersection.

Exercise 4F Solving linear simultaneous equations graphically

1 **a** solution **b** solution
c not a solution **d** solution
e not a solution **f** not a solution

2 **a** **i** (4, 3) **ii** $x = 4, y = 3$
b **i** (2, −4) **ii** $x = 2, y = -4$
c **i** (−5, −2) **ii** $x = -5, y = -2$
d **i** (−2, 3) **ii** $x = -2, y = 3$

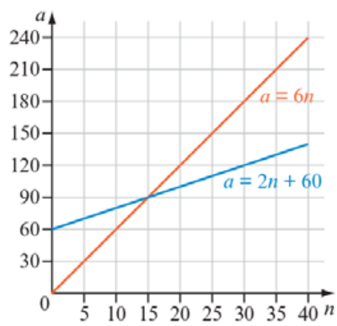
4 **a** $x = 5, y = 4$ **b** $x = 2, y = 6$
c $x = 6, y = 4$ **d** $x = -2, y = 5$
e $x = -1, y = 3$ **f** $x = -3, y = 1$
g $x = 3, y = 2$ **h** $x = 4, y = -2$
i $x = -2, y = -3$ **j** $x = -5, y = 6$
k $x = 2, y = -2$ **l** $x = 3, y = 5$

6 **a** $x = 3, y = -4$ **b** $x = 0, y = 0$
c $x = 2, y = 1$

7 **a** $x = \frac{1}{2}, y = \frac{1}{2}$ **b** $x = -3, y = -2\frac{1}{2}$
c $x = 1\frac{1}{2}, y = -4\frac{1}{2}$

8 Both graphs have gradient of 2. Lines are parallel so there is no point of intersection.

9 **a** To make n cards, cost of plain white cards is $2 \times n$ or $2n$. So, amount of money spent (in \$) is cost of plain white cards plus start-up cost of \$60.
b For selling n cards, the amount of money received (in \$) is $6 \times n$ or $6n$.
c



d (15, 90); this shows number of cards to be sold (15) so that amount of money received is same as amount of money spent (\$90).
e 15 cards **f** 16 to 40 cards

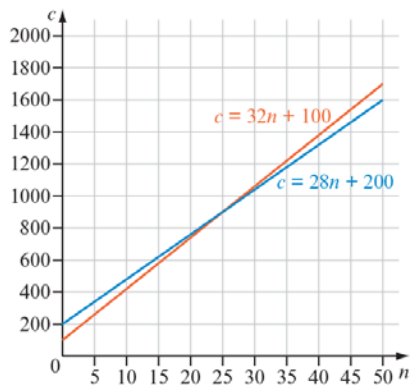
10 **a** $2x + 2y = 40$ or $x + y = 20$
b $y = x + 4$
c $x = 8, y = 12$; postcard has length of 12 cm and width of 8 cm.

11 **a** For x = Lachlan's age, y = Tia's age, equations are $y = 2x$ and $x + y = 51$.
b $x = 17, y = 34$
c Tia is 34 years old, Lachlan is 17 years old.

12 **a** For d = cost of drink, c = cost of choc top, equations are $d = c + 2$ and $5d + 7c = 70$
b Drink costs \$7, choc top costs \$5.

13 For l = length of land, w = width of land, equations are $l = w + 20$ and $2l + w = 124$. Land is 48 m long and 28 m wide.

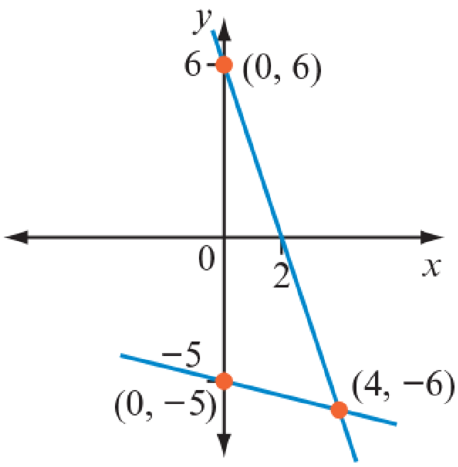
14 Let n = number of people and c = catering cost.
a $c = 28n + 200$ **b** $c = 32n + 100$
c



d Cool Food Club; at $n = 18$, the line for Cool Food Club is lower than the line for Angie's Catering. This represents a lower cost for 18 people.
e Angie's Catering **f** 25 people at cost of \$900
g When catering for less than 25 people, Cool Food Club is cheaper. When catering for more than 25 people, Angie's Catering is cheaper. Cost is same when catering for 25 people.

ANSWERS

15 **a**



b $y = -\frac{1}{4}x - 5$, $y = -3x + 6$

c $x = 4$, $y = -6$

d For $x = 4$ and $y = -6$,

For $y = -\frac{1}{4}x - 5$, RS $= -\frac{1}{4} \times 4 - 5 = -6 =$ LS.

For $y = -3x + 6$, RS $= -3 \times 4 + 6 = -6 =$ LS.

16 One possible answer is: $y = 2x - 18$ and $y = 3 - x$.

17 They are equivalent equations. When graphed, the position of the lines is identical; that is, one line lies on top of the other. There are an infinite number of points where the lines intersect so there are infinite solutions.

Reflect

Possible answer: The point of intersection shows you the only values of *x* and *y* that satisfy both of the given equations.

Resources

WorkSheet

WS 4-3 Solving simultaneous equations

Focus: To solve simultaneous equations graphically and using algebraic methods

Resources: 1-cm grid paper (BLM) or graph paper, ruler

- This WorkSheet provides a skills review of Exercises 4F and 4G. Q1 and Q2 relate to Exercise 4F.

Students solve simultaneous equations graphically and algebraically. The algebraic methods involve using substitution and elimination.

BLMs

Cartesian plane grids

1-cm grid paper

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4G Solving simultaneous linear equations algebraically

Teaching support for pages 196–201

Teaching strategies

Learning focus

To explore methods of solving a pair of simultaneous equations algebraically, making it unnecessary to draw a graph to solve the equations

Start thinking!

The task guides students to:

- consider the solution of a simultaneous equation pair presented graphically
- see that if y is the subject of one equation, this can be substituted into the other so a single equation with one unknown is formed
- solve the single linear equation for x and substitute this value into an equation to find the value of y
- recognise the algebraic solution is the same as the point of intersection found from the graphs
- consider the elimination method of solving a simultaneous equation pair by adding equations.

Differentiated pathways

Below Level	At Level	Above Level
1–8, 10a, d, g, j, 11	1–4, 5d–f, 6–9, 10a–l, 11–15	1g–l, 2f–i, 3, 4, 5e, f, 6d–f, 7–9, 10j–o, 12–18
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q11

Focus: To multiply an equation by an integer value to create an equivalent equation

- Direct students to complete **SS 4G-1 Forming equivalent linear equations** (see Resources) if they had difficulty with this question or require more practice at this skill.

- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to be able to multiply each term in the equation by an integer value.

At Level

At Level
1–4, 5d–f, 6–9, 10a–l, 11–15

- Students need to understand there are two different methods of solving simultaneous equations algebraically:
  - the substitution method
  - the elimination method.
- Explain that the substitution method is best suited to when at least one of the equations has a variable written as the subject. This enables one equation to be substituted into the other.
- Explain that the elimination method is best suited to when equations are able to be added or subtracted in such a way that one of the variables is eliminated.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4G-1**. It shows how to solve simultaneous linear equations using substitution and will help students to complete Q1 and Q2.
- **Example 4G-2** shows how to solve simultaneous linear equations using elimination. This will help students to complete Q3 and Q4.
- In Q3, each equation pair has one variable eliminated by adding the equations together. Guide students to understand that adding will eliminate a variable if the variable in each equation has the same coefficient with opposite sign.
- In Q4, each equation pair has one variable eliminated by subtracting the equations. Guide students to understand that subtracting will eliminate a variable if the variable in each equation has the same coefficient and the same sign.
- Q5 provides practice in forming an equivalent equation with a desired term. This prepares students for Q6.
- **Example 4G-3** shows how to solve simultaneous linear equations using elimination where one of the equations is multiplied by an integer to produce an equivalent equation. This will help students to complete Q6.

- After completing Q8, discuss with students how there are two ways in which they can multiply both equations to use the elimination method. One pair of equivalent equations will enable x to be eliminated while the other pair will allow y to be eliminated.
- In Q9, students identify when each method is most appropriate to be used. This is to be applied in Q10. Remind students that if:
  - one linear equation is of the form $y = mx + c$ or $x = my + c$, substitution is more appropriate
  - if the linear equations are of the form $ax + by = c$, then elimination is simpler.
- For Q11–15, students need to write their own equations. Ensure that students begin each problem by defining their variables and give a worded answer. Remind students that they need to write two different linear equations that link the two unknowns.
- For additional practice, students can complete Q3–10 of **WS 4-3 Solving simultaneous equations** (see Resources). Additional questions similar to Exercise 4G Q1–4, Q6 and Q8 are provided. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 Drawing a circumcircle around a triangle** (see Resources).

Below Level

Below Level
1–8, 10a, d, g, j, 11

- In Q3, each equation pair has one variable eliminated by adding the equations together. Guide students to understand that adding will eliminate a variable if the variable in each equation has the same coefficient with opposite sign.
- In Q4, each equation pair has one variable eliminated by subtracting the equations. Guide students to understand that subtracting will eliminate a variable if the variable in each equation has the same coefficient and the same sign.
- Q5 provides practice in forming an equivalent equation with a desired term. This prepares students for Q6. Students may need to complete **SS 4G-1 Forming equivalent linear equations** (see Resources).
- In Q10, students identify which method is most appropriate to be used. Remind students that if:
  - one linear equation is of the form $y = mx + c$ or $x = my + c$, substitution is more appropriate

- if the linear equations are of the form $ax + by = c$, then elimination is simpler.

- For Q11, students need to write their own equations. Ensure that students begin by defining the two variables and give a worded answer. Remind students that they need to write two different linear equations that link the two unknowns.

- For students who do not progress past Q8, direct them to Q3–10 of **WS 4-3 Solving simultaneous equations** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.

- Below Level students may lack the algebraic skills to manipulate equations sufficiently well. If this is the case, have them concentrate on Q1 and Q2 (substitution) and Q3 and Q4 (elimination with no multiplication of equations required)

- If students continue to struggle, have them go back to solving linear equation pairs graphically, with the help of technology if required.

Above Level

Above Level
1g–l, 2f–i, 3, 4, 5e, f, 6d–f, 7–9, 10j–o, 12–18

- For Q3 and Q4, encourage students to recognise when to add and when to subtract a pair of linear equations to eliminate a variable.

- After completing Q8, discuss with students how there are two ways in which they can multiply both equations to use the elimination method. One pair of equivalent equations will enable x to be eliminated while the other pair will allow y to be eliminated.

- In Q9, students identify when each method is most appropriate to be used. This is to be applied in Q10. Remind students that if:
  - one linear equation is of the form $y = mx + c$ or $x = my + c$, substitution is more appropriate
  - if the linear equations are of the form $ax + by = c$, then elimination is simpler.

- For Q11–15, students need to write their own equations. Ensure that students begin each problem by defining their variables and give a worded answer. Remind students that they need to write two different linear equations that link the two unknowns.

- In Q17, students should use the substitution method and will obtain the statement $11 = 5$. Explain that this is a contradiction and can never be true and so the two linear equations have no solution. This result can be checked by having students graph the two linear relationships. They should see that the lines are parallel and hence never intersect.

- In Q18, students should use the substitution method and will obtain the statement 6 = 6. Explain that this is always true and so the two linear equations will be equal for every value of x and y, hence there are infinite solutions. This result can be checked by having students graph the two linear relationships. They should see that the same line is produced for both linear relationships and hence there are infinite points where the lines intersect.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 Drawing a circumcircle around a triangle** (see Resources).

Extra activities

1 Quick Questions

Make y the subject of each equation.

a $x + y = 4$ $(y = 4 - x)$

b $2x - y + 7 = 0$ $(y = 2x + 7)$

c $3x + 2y - 10 = 0$ $\left(y = \dfrac{10 - 3x}{2}\right)$

d $x - 5y + 10 = 0$ $\left(y = \dfrac{x + 10}{5}\right)$

2 Consider the linear relationship $y = 2x - 8$ and the parabola $y = x^2 - 5x + 4$.

a Use substitution to create a single quadratic equation involving x.
$(x^2 - 7x + 12 = 0)$

b Solve the quadratic equation to find two values of x. $(x = 3, x = 4)$

c Substitute these values of x to find two corresponding values of y. $(y = -2, y = 0)$

d Explain what this means in terms of the original two graphs. [The line cuts the parabola twice, at (3, –2) and (4, 0)]

e Leighton says that a straight line will always cut a parabola twice. Is Leighton correct? Explain your answer. (Leighton is wrong. The line might cut the graph twice but it may also just touch the parabola at one point or not intersect at all.)

Answers

4G Solving linear simultaneous equations algebraically

4G Start thinking!

1 $x = 3, y = 2$

2 a $x + x - 1 = 5, 2x - 1 = 5$ b $x = 3$

c $x = 3, y = 2$

3 a $2x$ b 6 c y has been eliminated.

d $x = 3$; when $x = 3$, $x + y = 5$ becomes $3 + y = 5$ so $y = 2$

e $x = 3, y = 2$

f New equation is $2y = 4$; solution is the same.

Exercise 4G Solving linear simultaneous equations algebraically

1 **a** $x = 2, y = 7$ **b** $x = 4, y = 1$
c $x = -1, y = 3$ **d** $x = 6, y = 5$
e $x = -4, y = -2$ **f** $x = 2, y = 12$
g $x = 1, y = 3$ **h** $x = -3, y = 11$
i $x = 5, y = -5$ **j** $x = 5, y = 3$
k $x = -4, y = -1$ **l** $x = 5, y = 6$

2 **a** $x = 1, y = 8$ **b** $x = 6, y = 4$
c $x = -3, y = -5$ **d** $x = 5, y = -1$
e $x = -1, y = 4$ **f** $x = 4, y = -7$
g $x = 2, y = 1$ **h** $x = -2, y = 6$
i $x = -3, y = 19$

3 **a** $x = 2, y = 5$ **b** $x = -3, y = 1$
c $x = 1, y = -5$ **d** $x = -4, y = -2$
e $x = 5, y = 3$ **f** $x = -1, y = 2$

4 **a** $x = 5, y = 3$ **b** $x = -2, y = 9$
c $x = 3, y = -7$ **d** $x = -1, y = 5$
e $x = 6, y = 2$ **f** $x = -3, y = -4$

5 **a** $3x + 6y = 15$ **b** $-6x + 2y = 2$
c $-5x - 20y = 10$ **d** $20x + 8y = 12$
e $-4x + 6y = 14$ **f** $-12x + 6y = 9$

6 **a** $x = 3, y = 2$ **b** $x = 2, y = -1$
c $x = -4, y = 7$ **d** $x = 5, y = 3$
e $x = -3, y = -4$ **f** $x = -2, y = 6$

7 **a** No; ① + ② gives $11x + 5y = -7$ and ① − ② gives $3x - y = -9$.
b $21x + 6y = -24$ **c** $8x + 6y = 2$
d y; subtract the equations **e** $x = -2, y = 3$

8 **a** $x = 1, y = 4$ **b** $x = -3, y = 1$
c $x = 2, y = -1$ **d** $x = -4, y = 2$
e $x = 5, y = 5$ **f** $x = -3, y = -2$

9 Graphical method best when information given to draw graphs; substitution method best when one or both of the equations have a variable term as subject of the equation; elimination method best when equations are in the form $ax + by = d$.

10 **a** $x = 2, y = 0$ **b** $x = \frac{1}{3}, y = -3$
c $x = \frac{3}{2}, y = \frac{2}{5}$ or $x = 1.5, y = 0.4$
d $x = -9, y = -7$ **e** $x = 4, y = 2$
f $x = -2, y = -3$ **g** $x = 3, y = 10$
h $x = 5, y = -2$ **i** $x = 2, y = -1$
j $x = -2, y = 2$ **k** $x = 7, y = 4$
l $x = -4, y = 1$ **m** $x = -0.8, y = 2.3$
n $x = -\frac{1}{4}, y = 5$ or $x = -0.25, y = 5$
o $x = 28, y = 41$

11 **a** For two numbers x and y, equations are $x + y = 245$, $x - y = 91$. Solution is $x = 168$, $y = 77$.
b Two numbers are 77 and 168.

12 **a** $2l + 2w = 810$ or $l + w = 405$ **b** $l = 2w$
c 270 cm long and 135 cm wide

13 For x = width of pillow case, y = length of pillow case, equations are $y = 2x - 18$ and $2x + 2y = 240$ (or $x + y = 120$). Solution is $x = 46$, $y = 74$ so pillow case is 74 cm long and 46 cm wide.

14 For g = number of points for a goal, b = number of points for a behind, equations are $15g + 6b = 132$ and $13g + 12b = 128$. Solution is $g = 8$, $b = 2$ so number of points for a goal is 8 and number of points for a behind is 2.

15 For p = cost of a pie, r = cost of a sausage roll, equations are $7p + 6r = 63$ and $5p + 5r = 48$. Solution is $p = 5.4$, $r = 4.2$ so cost of a pie is \$5.40 and cost of a sausage roll is \$4.20.

16 **a** Strategy A: equation ③ is $10x - 6y = -2$.
① − ③: $-3x = -12$, $x = 4$
Substituting $x = 4$ in ②: $20 - 3y = -1$, $y = 7$
Solution is $x = 4$, $y = 7$.
Strategy B: equation ③ is $-10x + 6y = 2$.
① + ③: $-3x = -12$, $x = 4$
Substituting $x = 4$ in ②: $20 - 3y = -1$, $y = 7$
Solution is $x = 4$, $y = 7$.
Strategy C: equation ③ is $35x - 30y = -70$ and equation ④ is $35x - 21y = -7$.
③ − ④: $-9y = -63$, $y = 7$
Substituting $y = 7$ in ②: $5x - 21 = -1$, $x = 4$
Solution is $x = 4$, $y = 7$.

b Some possible answers are:
i Multiply equation ① by 5 to form equation ③ and multiply equation ② by −7 for form equation ④. Add equations ③ and ④.
ii Graph equations ① and ② and identify the coordinates of the point of intersection.
iii Rearrange equation ② to obtain $3y = 5x - 1$ and substitute into equation ①. That is, substitute $5x - 1$ for $3y$ in $7x - 2(3y) = -14$.

17 Substituting $y = 11 - 4x$ into $8x + 2y = 5$ gives:
$8x + 2(11 - 4x) = 5$
$8x + 22 - 8x = 5$
There is no solution as there is no x value that makes this equation true.

18 Substituting $y = 2x - 3$ into $4x - 2y = 6$ gives:
$4x - 2(2x - 3) = 6$
$4x - 4x + 6 = 6$
There are an infinite number of solutions as all values of x make this equation true.

Reflect

Possible answer: The aim of the first step in solving a pair of simultaneous linear equations is to reduce the two linear equations with two unknowns to a single linear equation with one unknown.

Resources

SupportSheet

SS 4G-1 Forming equivalent linear equations

Focus: To explore how equivalent linear relationships are formed

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students develop an understanding of equivalent linear equations by sketching their graphs. They form equivalent linear equations by multiplying the entire equation by an integer. This skill is needed for the elimination method when solving linear simultaneous equations.

WorkSheet

WS 4-3 Solving simultaneous equations

Focus: To solve simultaneous equations graphically and using algebraic methods

Resources: 1-cm grid paper (BLM) or graph paper, ruler

- This WorkSheet provides a skills review of Exercises 4F and 4G. Q3–10 relate to Exercise 4G.

Students solve simultaneous equations graphically and algebraically. The algebraic methods involve using substitution and elimination.

Investigation

INV 4-3 Drawing a circumcircle around a triangle

Focus: To find the circumcentre of a triangle algebraically, then draw the circumcircle

Resources: calculator, **1-cm grid paper** (BLM) or graph paper, pair of compasses

Students find the midpoints and gradients of the three sides of a triangle. They calculate the gradient of the perpendicular line through each midpoint and hence write the equation for each of the three perpendicular bisectors. Students solve these linear equations simultaneously to find the coordinates of the circumcentre of the triangle. By finding the distance of each vertex from the circumcentre, students find the radius of the circumcircle enabling them to use a pair of compasses to draw the circumcircle.

BLM

1-cm grid paper

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 202–205

Additional teaching strategies

Multiple choice

1 Answer: A. $\frac{x-4}{2} = -3$, $x - 4 = -6$, $x = -2$
B: calculated 6 – 4 instead of –6 + 4 to obtain 2.
C: calculated –6 – 4 instead of –6 + 4 to obtain –10.
D: calculated 6 + 4 instead of –6 + 4 to obtain 10.

2 Answer: C. LS = $\frac{12}{-5} = -2.4$ = RS
A: LS = 3(–5) – 7 = –15 – 7 = –22 $\neq$ RS. x = –5 is not a solution.
B: LS = $\frac{2(-5)+3}{13} = \frac{-10+3}{13} = \frac{-7}{13} \neq$ RS. x = –5 is not a solution.
D: LS = $\frac{3}{2-(-5)} = \frac{3}{7} \neq$ RS. x = –5 is not a solution.

3 Answer: C. –3.5 > –4.7, so statement of –3.5 < –4.7 is false.
A: chose a correct statement as –7 < –4 is true.
B: chose a correct statement as $\frac{2}{3} < \frac{3}{4}$ is true.
D: chose a correct statement as $\frac{8}{9} < \frac{9}{8}$ is true.

4 Answer: B. –6.3 < –6.2, so –6.3 could not be a value for x.
A: chose a value that could be x as –4.9 > –6.2.
C: chose a value that could be x as 6.2 > –6.2.
D: chose a value that could be x as 0 > –6.2.

5 Answer: C. $12x + 4y = 8$, when $x = 0$, $12(0) + 4y = 8$, $y = 2$
A: chose the constant term of 8.
B: considered $12x + 4y - 8 = 0$ and chose the constant term of –8.
D: confused the use of the negative sign and chose –2.

6 Answer: D. $12x + 4y = 8$; $4y = -12x + 8$; $y = -3x + 2$; $m = -3$, $c = 2$
A: chose the coefficient of x in the original rule for the gradient.
B: considered $4y = -12x + 8$ and taken the gradient to be the coefficient of x.
C: divided by 4 but not put the rule in the form $y = mx + c$.

7 Answer: A. $y = \frac{3}{4}x + \frac{5}{8}$, 8$y$ = 6x + 5; 6x – 8y + 5 = 0

B: considered the wrong sign for the constant term.

C: reversed the coefficients of x and y.

D: reversed the coefficients of x and y.

8 Answer: C. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6-8}{-3-1} = \dfrac{-14}{-4} = \dfrac{7}{2}$

A: added 8 and –6 in the numerator to obtain 2 and incorrectly subtracted 1 and –3 in the denominator to obtain 2.

B: added the y values in the numerator and added the x values in the denominator.

D: reversed the numerator and the denominator in the formula.

9 Answer: D. 3x + 2y = 6; 2y = –3x + 6; $y = -\frac{3}{2}x + 3$; $m = -\frac{3}{2}$

A: used the rule for perpendicular lines

B: made 2y the subject and used the coefficient of x.

C: divided the original rule by 2 and used the coefficient of x.

10 Answer: B. Gradient of perpendicular line, $m_2 = -\dfrac{1}{-\frac{3}{4}} = \dfrac{4}{3}$

A: taken the reciprocal but not changed the sign.

C: changed the sign but not taken the reciprocal.

D: used the same gradient.

11 Answer: C. x + 3 = –2x + 6; 3x + 3 = 6; 3x = 3, x = 1; y = 1 + 3 = 4; point is (1, 4)

A: subtracted 2x from x at the first step of the solution to find x.

B: added 3 to 6 instead of subtracting 3 from 6 when solving for x.

D: miscalculated y by ignoring the negative sign in the coefficient of x when substituting x = 1 into y = –2x + 6.

12 Answer: D. Graphing these two equations produces the same line so there are many solutions.

A: assumed the two lines have no point of intersection and hence the two equations have no simultaneous solutions.

B: assumed the two lines have one point of intersection and hence the two equations have one simultaneous solution.

C: assumed the two lines have two points of intersection and hence the two equations have two simultaneous solutions.

13 Answer: A. x – 2y = 3; x – 2(3x – 4) = 3; x – 6x + 8 = 3; –5x + 8 = 3

B: incorrectly multiplied –2 by –4 to obtain –8 when expanding the pair of brackets.

C: incorrectly added x and –6x to obtain 5x.

D: incorrectly added x and –6x to produce 5x and incorrectly multiplied –2 by –4 to obtain –8 when expanding the pair of brackets.

14 Answer: A. $(2x - y + 3) - (4x - y + 5) = 0$; $-2x - 2 = 0$
B: incorrectly subtracted 5 from 3 to obtain –8.
C: subtracted equation 1 from equation 2.
D: subtracted equation 1 from equation 2 and incorrectly subtracted 3 from 5 to obtain 8.

Short answer

1 **a** $3 - 2(4 - 5x) = -20$; $3 - 8 + 10x = -20$; $-5 + 10x = -20$; $10x = -15$, $x = -1.5$

b $4(3x + 5) = 3(5x - 3)$; $12x + 20 = 15x - 9$; $-3x + 20 = -9$; $-3x = -29$; $x = 9\frac{2}{3}$

2 **a** $\frac{4+x}{3} = \frac{2x-5}{3}$; $4 + x = 2x - 5$; $-x = -9$; $x = 9$

b $\frac{5-3x}{2} = \frac{2x+5}{3}$; $3(5 - 3x) = 2(2x + 5)$; $15 - 9x = 4x + 10$; $5 = 13x$; $x = \frac{5}{13}$

3 **a** $A = \frac{h(a+b)}{2}$; $2A = ha + hb$; $hb = 2A - ha$; $b = \frac{2A - ha}{h}$ or $b = \frac{2A}{h} - a$

b $A = 2[(l+w)h]$; $A = 2(l + w) \times h$; $h = \frac{A}{2(l+w)}$

4 **a** For $x > -1$, x can be $1\frac{5}{8}$, 2.6, 8.5 or $\frac{3}{4}$.

b For $x \le 0.75$, x can be –3.9, –8.5 or $\frac{3}{4}$.

c For $x \ge 2.6$, x can be 2.6 or 8.5.

5 **a** $x \ge -3$

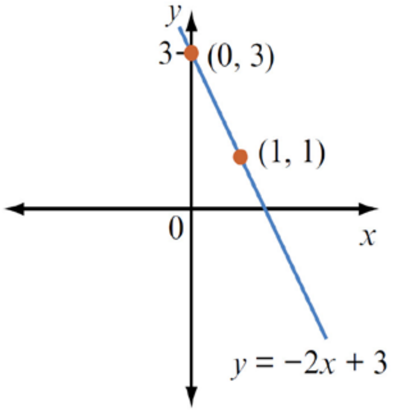
b $x < -10$

c $x \le 2.5$

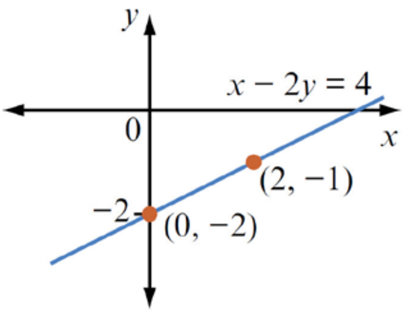
6 **a** $5 - 2x < 7$; $-2x < 2$; $x > -1$

b $\frac{2-3x}{5} > 2$; $2 - 3x > 10$; $-3x > 8$; $x < -\frac{8}{3}$ or $x < -2\frac{2}{3}$

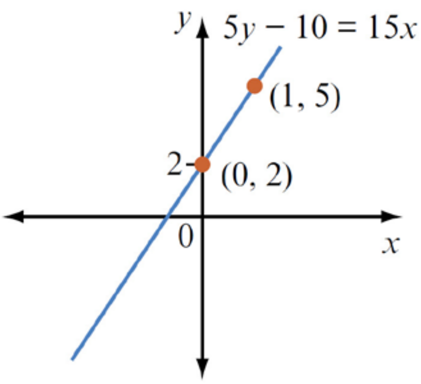
7 **a** $y = -2x + 3$; $m = -2$, $c = 3$



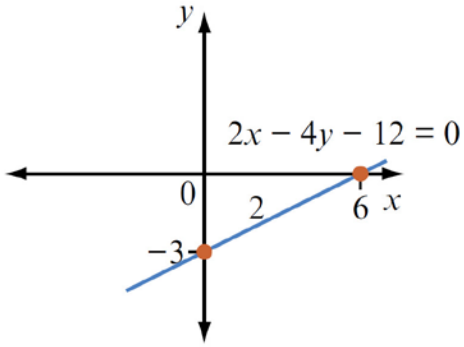
b $x - 2y = 4$; $-2y = -x + 4$; $y = \frac{1}{2}x - 2$; $m = \frac{1}{2}$, $c = -2$



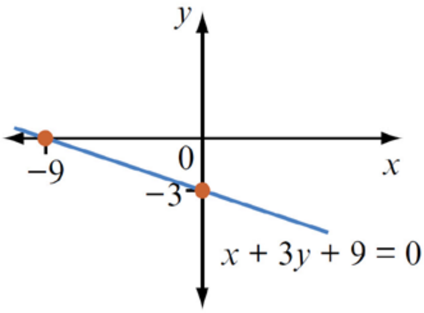
c $5y - 10 = 15x$; $5y = 15x + 10$; $y = 3x + 2$; $m = 3$, $c = 2$



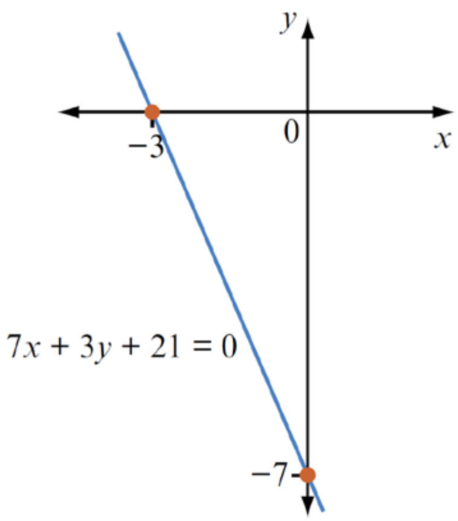
8 **a** $2x - 4y - 12 = 0$; when $x = 0$, $4y = -12$, $y = -3$; when $y = 0$, $2x = 12$; $x = 6$; intercepts are $(0, -3)$ and $(6, 0)$



b $x + 3y + 9 = 0$; when $x = 0$, $3y = -9$, $y = -3$; when $y = 0$, $x + 9 = 0$; $x = -9$; intercepts are $(0, -3)$ and $(-9, 0)$



c $7x + 3y + 21 = 0$; when $x = 0$, $3y = -21$, $y = -7$; when $y = 0$, $7x = -21$; $x = -3$; intercepts are $(0, -7)$ and $(-3, 0)$



9 **a** $m = \dfrac{6-(-4)}{5-0} = 2$; $y - (-4) = 2(x - 0)$; $y + 4 = 2x$; $y = 2x - 4$

b $m = \dfrac{6-6}{-2-3} = 0$; horizontal line; $y - 6 = 0(x - 3)$; $y - 6 = 0$; $y = 6$

c $m = \dfrac{-3-5}{-3-(-3)}$ = undefined; vertical line; $x = -3$

10 **a** $y - 5 = 2(x - 1)$; $y - 5 = 2x - 2$; $y = 2x + 3$

b $y - 5 = -2(x - 1)$; $y - 5 = -2x + 2$; $y = -2x + 7$

11 **a** $m = \dfrac{4-3}{2-1} = 1$; $y - 3 = 1(x - 1)$; $y - 3 = x - 1$; $y = x + 2$

b $m = \dfrac{-7-5}{3+1} = \dfrac{-12}{4} = -3$; $y - 5 = -3(x + 1)$; $y - 5 = -3x - 3$; $y = -3x + 2$

c $m = \dfrac{5+4}{5+4} = 1$; $y - 5 = 1(x - 5)$; $y - 5 = x - 5$; $y = x$

12 **a** $m = -4$, $c = -3$; $y = -4x - 3$

b $m = -½$, $c = -4$; $y = -½x - 4$

13 **a** $m = -\frac{1}{3}$, (3, 2); $y - 2 = -\frac{1}{3}(x - 3)$; $3y - 6 = -x + 3$; $3y = -x + 9$; $y = -\frac{1}{3}x + 3$

b $m = 3$, (–3, –2); $y + 2 = 3(x + 3)$; $y + 2 = 3x + 9$; $y = 3x + 7$

14 **a** Point of intersection on graph is (1, 2); solution is $x = 1$ and $y = 2$.

b Point of intersection on graph is (6, 17); solution is $x = 6$ and $y = 17$.

15 **a** $3x + 4y = -1$ [1]
$y = x - 2$ [2]
Substituting [2] into [1]: $3x + 4(x - 2) = -1$, $3x + 4x - 8 = -1$, $7x = 7$, $x = 1$
Substituting $x = 1$ into [2]: $y = 1 - 2 = -1$
Solution is $x = 1$, $y = -1$.

b $y = 4x + 3$ [1]
$x - 2y = 8$ [2]
Substituting [1] into [2]: $x - 2(4x + 3) = 8$, $x - 8x - 6 = 8$, $-7x = 14$, $x = -2$
Substituting $x = -2$ into [1]: $y = 4(-2) + 3 = -5$
Solution is $x = -2$, $y = -5$.

c $7x - y = 4$ [1]
$y = 4x + 2$ [2]
Substituting [2] into [1]: $7x - (4x + 2) = 4$; $7x - 4x - 2 = 4$, $3x = 6$, $x = 2$
Substituting $x = 2$ into [2]: $y = 4(2) + 2 = 10$
Solution is $x = 2$, $y = 10$.

16 **a** $2x + y = 8$ [1]
$4x - y = 4$ [2]
[1] + [2]: $6x = 12$, $x = 2$
Substituting $x = 2$ into [1]: $2(2) + y = 8$, $4 + y = 8$, $y = 4$
Solution is $x = 2$, $y = 4$.

b $x + 5y = 13$ [1]
$x + 2y = 4$ [2]
[1] – [2]: $3y = 9$, $y = 3$
Substituting $y = 3$ into [1]: $x + 5(3) = 13$, $x + 15 = 13$, $x = -2$
Solution is $x = -2$, $y = 3$.

c $2x - y = 7$ [1]
$3x + 5y = 4$ [2]
[1] × 5: $10x - 5y = 35$ [3]
[2] + [3]: $13x = 39$, $x = 3$
Substituting $x = 3$ into [1]: $2(3) - y = 7$, $6 - y = 7$, $-y = 1$, $y = -1$
Solution is $x = 3$, $y = -1$.

Mixed practice

1 $m = \dfrac{1+4}{-4-1} = -1$

a $m = -1$, $(4, 4)$; $y - 4 = -1(x - 4)$; $y - 4 = -x + 4$; $x + y - 8 = 0$ or $y = -x + 8$

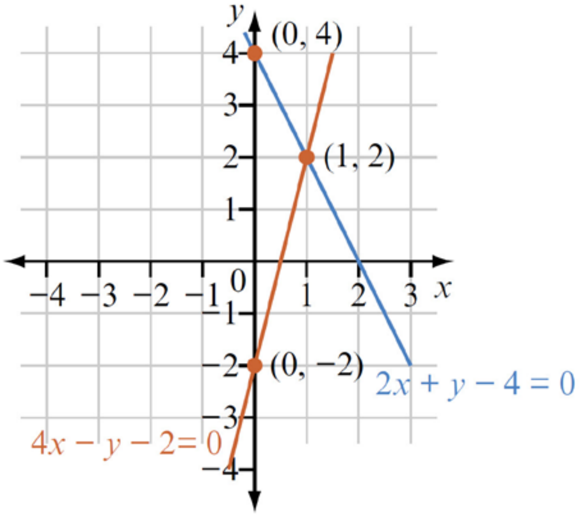
b $m = 1$, $(1, 1)$; $y - 1 = 1(x - 1)$; $y - 1 = x - 1$; $y = x$

Refer to *4E Parallel and perpendicular lines*

2 Answer: C. $2 - \dfrac{x}{4} < 1$; $-\dfrac{x}{4} < -1$; $\dfrac{x}{4} > 1$, $x > 4$

A: did not reverse the inequality after dividing both sides by –1 (or after multiplying by –4).
B: did not reverse the inequality and incorrectly divided by –1 (or incorrectly multiplied by –4).
D: incorrectly divided by –1 (or incorrectly multiplied by –4).
Refer to *4B Solving linear inequalities*

3 **a**



b Point of intersection is (1, 2).

Refer to *4F Solving simultaneous equations graphically*

4 **a** $x + 2y + 10 = 0$; $2y = -x - 10$; $y = -½x - 5$; $m_1 = -½$
$2x - y - 3 = 0$; $y = 2x - 3$; $m_2 = 2$
$m_1 \times m_2 = -½ \times 2 = -1$ $\therefore$ Lines are perpendicular.

b $x + y = 3$; $y = 3 - x$; $m_1 = -1$
$y = x$; $m_2 = 1$;
$m_1 \times m_2 = -1 \times 1 = -1$ $\therefore$ Lines are perpendicular.

c $2x + 3y = 4$; $3y = 4 - 2x$; $y = \frac{4}{3} - \frac{2}{3}x$; $m_1 = -\frac{2}{3}$
$3x + 2y = 4$; $2y = 4 - 3x$; $y = 2 - \frac{3}{2}x$; $m_2 = -\frac{3}{2}$
$m_1 \times m_2 = -\frac{2}{3} \times -\frac{3}{2} = 1$ $\therefore$ Lines are neither perpendicular or parallel.
Refer to *4E Parallel and perpendicular lines*

5 $l = w + 28$; $l = 5w$; $5w = w + 28$; $4w = 28$; $w = 7$; $l = 7 + 28 = 35$
Rectangle has length of 35 m and width of 7 m.
Refer to *4G Solving linear simultaneous equations algebraically.*

6 Answer: D. A gradient of 0 means that the line is horizontal.
A: chose a line that has a positive gradient.
B: chose a line that has a negative gradient.
C: chose a line that is vertical and has an undefined gradient.
Refer to *4C Sketching linear graphs*

7 Point of intersection of the lines has coordinates (1, –1) so the solution to the simultaneous equations $y = 2x - 3$ and $y = -5x + 4$ is $x = 1$, $y = -1$.
Refer to *4F Solving linear simultaneous equations graphically.*

8 Answer: A. rise = 9 – (–1) = 10, run = 5 – (–2) = 7
B: confused the rise and the run, and so found the difference in the x values to obtain the rise and found the difference in the y values to obtain the run.
C: mistakenly added the y values to obtain the rise and added the x values to obtain the run.
D: confused the rise and the run, and mistakenly added the x values to obtain the rise and added the y values to obtain the run.
Refer to *4C Sketching linear graphs.*

9 **a** $V = \frac{x+y+z}{3}$; $3V = x + y + z$; $z = 3V - x - y$

b **i** $z = 3(12) - 7 - 19 = 36 - 7 - 19 = 10$

ii $z = 3(42) - 5 - 12 = 126 - 5 - 12 = 109$

Refer to *4A Solving linear equations.*

10 Answer: B. At $y = 0$, $x = -2$ so coordinates of the x-intercept are (–2, 0); at $x = 0$, $y = 2$ so coordinates of the y-intercept are (0, 2).
A: confused the order of the x- and y-intercepts.
C: confused the order of the x- and y-intercepts and reversed the signs.
D: mistakenly calculated the signs.
Refer to *4C Sketching linear graphs.*

11 Coordinates of the intersection points are (1, –3), (1, 5), (–4, –3) and (–4, 5).
Refer to *4F Solving linear simultaneous equations graphically.*

12 **a** $y + 3x = 8$ [1]
$y = 3x + 2$ [2]
Substituting [2] into [1]: $3x + 2 + 3x = 8$, $6x + 2 = 8$, $6x = 6$, $x = 1$
Substituting $x = 1$ into [2]: $y = 3(1) + 2$, $y = 5$
Solution is $x = 1$, $y = 5$.

b $3x + 2y = 1$ [1]
$3x + 7y = 11$ [2]
[2] – [1]: $5y = 10$, $y = 2$
Substituting $y = 2$ into [1]: $3x + 2(2) = 1$, $3x + 4 = 1$, $3x = -3$, $x = -1$
Solution is $x = -1$, $y = 2$.

c $4x - 5y = 3$ [1]
$6x - 11y = 1$ [2]
[1] × 3: $12x - 15y = 9$ [3]
[2] × 2: $12x - 22y = 2$ [4]
[3] – [4]: $7y = 7$, $y = 1$

Substituting $y = 1$ into [1]: $4x - 5(1) = 3$, $4x - 5 = 5$, $4x = 8$, $x = 2$
Solution is $x = 2$, $y = 1$.

Refer to *4G Solving linear simultaneous equations algebraically.*

13 Let h = cost of one hamburger ($), d = cost of one drink ($)
$3h + 2d = 32$ [1]
$h + 4d = 29$ [2]
[1] × 2: $6h + 4d = 64$ [3]
[3] – [2]: $5h = 35$, $h = 7$
Substituting $h = 7$ into [1]: $3(7) + 2d = 32$, $21 + 2d = 32$, $2d = 11$, $d = 5.5$
A hamburger costs $7 and a drink costs $5.50.
Refer to *4G Solving linear simultaneous equations algebraically.*

14 **a** $5 - 4x \geq 2x - 1$; $5 \geq 6x - 1$; $6x - 1 \leq 5$; $6x \leq 6$; $x \leq 1$

b $2x + 3 < 5x + 9$; $3 < 3x + 9$; $3x + 9 > 3$; $3x > -6$; $x > -2$

c $5(2 - 3x) \leq 3(5 - 2x)$; $10 - 15x \leq 15 - 6x$; $10 \leq 15 + 9x$; $15 + 9x \geq 10$; $9x \geq -5$;
$x \geq -\frac{5}{9}$

d $-2(x - 4) > 5(3 - 2x)$; $-2x + 8 > 15 - 10x$; $8x + 8 > 15$; $8x > 7$; $x > \frac{7}{8}$

Refer to *4B Solving linear inequalities.*

15 **a** Coordinates of midpoint = $\left(\frac{5+(-7)}{2}, \frac{-6+2}{2}\right) = \left(\frac{-2}{2}, \frac{-4}{2}\right) = (-1,-2)$

b Coordinates of midpoint = $\left(\frac{-3+9}{2}, \frac{-1+(-3)}{2}\right) = \left(\frac{6}{2}, \frac{-4}{2}\right) = (3,-2)$

Refer to *4C Sketching linear graphs.*

16 **a** $d = \sqrt{(-4-5)^2 + (3--2)^2} = \sqrt{(-9)^2 + 5^2} = \sqrt{81+25} = \sqrt{106} \approx 10.3$ units

b $d = \sqrt{(1--7)^2 + (-4-3)^2} = \sqrt{8^2 + (-7)^2} = \sqrt{64+49} = \sqrt{113} \approx 10.6$ units

c $d = \sqrt{(-3-4)^2 + (-4--3)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} \approx 7.1$ units

Refer to *4C Sketching linear graphs.*

17 **a** True; $y = -2$ represents a horizontal line.

b True; (0, 0) satisfies the equation $y = 2x$.

c False; the x-axis has equation $y = 0$.

Refer to *4C Sketching linear graphs.*

18 $m = \dfrac{-2-(-4)}{4-3} = 2$

$y + 4 = 2(x - 3)$; $y + 4 = 2x - 6$; $y = 2x - 10$

a $m = 2$

b $c = -10$

Refer to *4C Sketching linear graphs*

19 Answer: D. $\dfrac{2x}{7} = 8$; $2x = 56$; $x = 28$

A: forgotten to divide by 2 as the final step.
B: multiplied 7×8 incorrectly to obtain 48.
C: multiplied 7×8 incorrectly to obtain 48 and not divided by 2 as the final step.
Refer to *4A Solving linear equations.*

20 Answer: B. $\dfrac{8-x}{3} = 4$

A: mistakenly subtracted 8 from x.
C: only divided x by three and not the result of subtracting x from 8.
D: mistakenly divided the result (4) by 3.
Refer to *4A Solving linear equations*.

21 Let x = unknown number.

a $2x - 3 = 17$, $2x = 20$, $x = 10$; so unknown number is 10

b $7(x - 1) = 21$; $7x - 7 = 21$; $7x = 28$; $x = 4$; so unknown number is 4

c $5(3x - 12) = 15$; $15x - 60 = 15$; $15x = 75$; $x = 5$; so unknown number is 5

Refer to *4A Solving linear equations.*

22 **a** $\dfrac{1-2x}{4} < \dfrac{x+2}{3}$; $3(1 - 2x) < 4(x + 2)$; $3 - 6x < 4x + 8$; $3 < 10x + 8$; $10x + 8 > 3$;

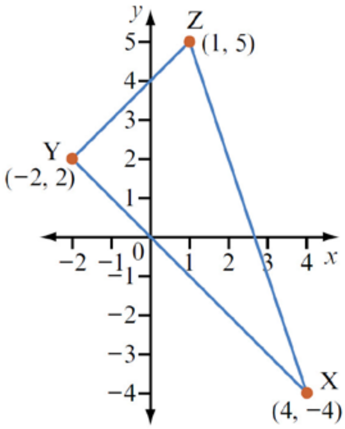
$10x > -5$; $x > -0.5$

b $\dfrac{x+1}{6} \geq \dfrac{x-1}{8}$; $8(x + 1) \geq 6(x - 1)$; $8x + 8 \geq 6x - 6$; $2x + 8 \geq -6$; $2x \geq -14$; $x \geq -7$

Refer to *4B Solving linear inequalities.*

Analysis

a



b For XY: $m = \dfrac{2 - -4}{-2 - 4} = \dfrac{6}{-6} = -1$; $y + 4 = -1(x - 4)$; $y + 4 = -x + 4$; $y = -x$

For YZ: $m = \dfrac{5 - 2}{1 - -2} = \dfrac{3}{3} = 1$; $y - 5 = 1(x - 1)$; $y - 5 = x - 1$; $y = x + 4$

For ZX: $m = \dfrac{5 - -4}{1 - 4} = \dfrac{9}{-3} = -3$; $y - 5 = -3(x - 1)$; $y - 5 = -3x + 3$; $y = -3x + 8$

c Gradient of XY = $m_1 = -1$, gradient of YZ = $m_2 = 1$.

$m_1 \times m_2 = -1$ therefore XY and YZ are perpendicular and triangle XYZ is right-angled with right angle at vertex Y.

d $x = \sqrt{(-2-1)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} \approx 4.24$ units

$y = \sqrt{(4-1)^2 + (-4-5)^2} = \sqrt{(3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} \approx 9.49$ units

$z = \sqrt{(4--2)^2 + (-4-2)^2} = \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} \approx 8.49$ units

e $c^2 = \left(\sqrt{90}\right)^2 = 90$; $a^2 + b^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{72}\right)^2 = 18 + 72 = 90$

f Midpoint of ZX, M = $\left(\dfrac{4+1}{2}, \dfrac{-4+5}{2}\right)$ = (2.5, 0.5)

g $\text{MY} = \sqrt{(2.5--2)^2 + (0.5-2)^2} = \sqrt{(4.5)^2 + (-1.5)^2} = \sqrt{20.25 + 2.25} = \sqrt{22.5} \approx 4.74$ units

h The two smaller triangles are MYZ and MYX. As MY is half the length of XZ, it means that MY = MZ = MX and the two triangles are isosceles triangles. All angles are acute, making the triangles acute-angled isosceles triangles.

i Gradient of XZ = –3 so gradient of YH = $\frac{1}{3}$

Rule for YH: $y - 2 = \frac{1}{3}(x + 2)$; $y - 2 = \frac{1}{3}x + \frac{2}{3}$; $y = \frac{1}{3}x + 2\frac{2}{3}$

j Rule for XZ: $y = -3x + 8$ [1]

Rule for YH: $y = \frac{1}{3}x + 2\frac{2}{3}$ [2]

Substituting [1] into [2]: $-3x + 8 = \frac{1}{3}x + 2\frac{2}{3}$, $-9x + 24 = x + 8$; $10x = 16$, $x = 1\frac{3}{5}$

Substituting $x = 1\frac{3}{5}$ into [1]: $y = -3(1\frac{3}{5}) + 8 = 3\frac{1}{5}$

Coordinates of H are $(1\frac{3}{5}, 3\frac{1}{5})$ or (1.6, 3.2).

k $\text{YH} = \sqrt{(1.6-(-2))^2 + (3.2-2)^2} = \sqrt{(3.6)^2 + (1.2)^2} = \sqrt{14.4} \approx 3.79$ units

l Using YX as the base and YZ as the perpendicular height:

area = ½ × 8.49 × 4.24 = 18.0 square units

Using XZ as the base and YH as the perpendicular height:

area = ½ × 9.49 × 3.79 = 18.0 square units

m $\text{YH} = \dfrac{4.24 \times 8.49}{9.49} = 3.79$ units

n With centre M, and radius MX = MY = MZ, a circle can be drawn to touch the vertices of the right-angled triangle. This is one of the geometry facts which states that the angle in a semi-circle is a right angle.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 4 Chapter test A

Chapter 4 Chapter test B

Test answers

Chapter 4 Chapter test answers

Connect

Teaching support for pages 206–207

Teaching strategies

Comparing taxi charges

Focus: To use linear relationships to model a real-life application

- Students may need to have the concept of flagfall and distance rate defined. Explain how the tariff is affected by the time and day of the week.
- They need to be guided to see that in terms of a linear relationship, the flagfall is shown by the vertical (y) intercept and that the distance rate is shown by the gradient.
- Students should then be able write a linear rule for the taxi price in each city.
- To complete the task, students will need to apply the following skills from the chapter:
  - writing a linear rule
  - substituting into an equation
  - solving linear equations
  - solving and writing linear inequalities
  - graphing a linear relationship
  - solving simultaneous linear equations.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided for the Connect task.
- An assessment rubric is available (see Resources).

Resources

Assessment rubrics

Comparing taxi changes

The path of the basketball will be curved. The two variables to be compared will be horizontal distance travelled and height of the ball.

Are you ready?

Prerequisite knowledge and skills can be tested by completing **Are you ready?**. This will give you an indication of the differentiated pathway each student can follow.

Students need to be able to:

- factorise quadratic expressions
- substitute into quadratic expressions
- simplify algebraic expressions
- complete a table of values for a relationship between *x* and *y*
- identify the rule for graphs of vertical and horizontal lines
- perform translation and reflection on a point on the Cartesian plane
- find the *x*- and *y*-intercepts for graphs of linear relationships
- find the gradient of a linear graph.

At the beginning of each topic, there is a suggested differentiated pathway that allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to select the best pathway.

Support Strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

CHAPTER 4 NON-LINEAR RELATIONSHIPS

4 Are you ready?

1 **a** $x(x + 7)$ **b** $(x - 3)(x + 3)$
c $(x + 3)(x + 2)$ **d** $-x(x + 3)$

2 **a** **i** -4 **ii** 21 **iii** 0
b **i** 0 **ii** 5 **iii** -4
c **i** 1 **ii** 16 **iii** 1
d **i** -10 **ii** 0 **iii** -12

3 **a** $4x^2 + 4x + 1$ **b** $x^2 - 5x - 5$
c $-x^2 - 5x - 6$ **d** $2x^2$

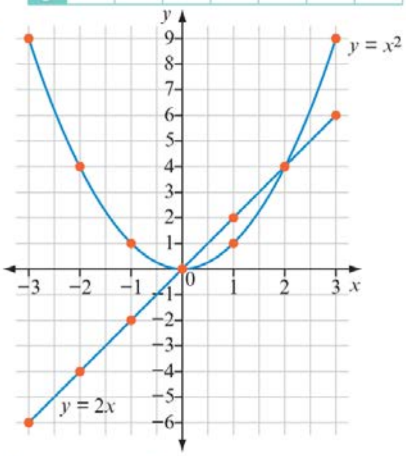
4 **a** **i**

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

ii

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b



c **i** linear **ii** non-linear

5 **a** $y = 1$ **b** $x = -3$ **c** $x = 2$ **d** $y = -2$

6 **a** $(2, -3)$ **b** $(6, 3)$ **c** $(2, -2)$
d $(2, 4)$ **e** $(-4, 3)$ **f** $(0, 6)$

7 **a** $x = 4; y = -4$ **b** $x = -3; y = 6$

8 gradient = 2

Resources

assess: assessments

Each topic of the *MyMaths 9* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students’ confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

4A Solving quadratic equations

Teaching support for pages 154–159

Teaching strategies

Learning focus

To recognise a quadratic equation and solve them using the Null Factor Law

Start thinking!

The task guides students to:

- identify both a quadratic expression and a quadratic equation
- develop the use of the Null Factor Law to solve a factorised quadratic equation.

Differentiated pathways

Below Level	At Level	Above Level
1–5, 7a–d, 10a–c, 11, 13a–f, 14–16	1–4, 5a–c, 6–12, 13a–i, 14–18	1, 3, 4, 7–9, 10d–f, 11–13, 16, 18–23
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q1–3

Focus: To develop an understanding of what constitutes a quadratic expression and revise the skills of factorising and substitution into these expressions.

- Direct students to complete **SS 4A-1 Factorising quadratic expressions** (see Resources) if they had difficulty with Q1 or require more practice at this skill.
- Direct students to complete **SS 4A-2 Substitution into quadratic expressions** (see Resources) if they had difficulty with Q2 or require more practice at this skill.
- Direct students to complete **SS 4A-3 Simplifying algebraic expressions** (see Resources) if they had difficulty with Q3 or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to remember the basic rules of factorisation for quadratic expressions.
    - Always first look for a common factor.

– If the remaining expression has three terms and is a quadratic trinomial, look for two numbers that multiply to give the constant term and add to give the coefficient of ‘x’.

– If there are only two terms, look to factorise as the difference of two squares.

At Level

At Level
1–4, 5a–c, 6–12, 13a–i, 14–18

- Demonstrate **4A eTutor** or direct students to do this independently.
- The Null Factor Law needs to be clearly understood.by students. That is, if two (or more) terms multiply to give a result of zero, at least one of those terms must equal zero.
- Factorising changes the quadratic expression into the product of two linear factors. These linear factors are solved as two separate linear equations.
- Most quadratic equations in this exercise will give two solutions. Quadratic equations that factorise as perfect squares will give one solution.
- Some quadratic equations have no solutions and these will not be able to be factorised.
- Just as with all other types of equations, remind students that they can check their solutions by substituting their solution into the equation.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- For Q1, remind students that:
  - a quadratic equation is of degree 2; that is, the term with the greatest power in the equation is x^2
  - to be an equation rather than an expression, there must be an equals sign.
- Direct students to **Example 4A-1**. It shows how to solve quadratic equations, which are already factorised, using the Null Factor Law and will help students to complete Q2 and Q3.
- In Q2, pay particular attention to part b. The single term factor of x still needs to be set equal to zero and becomes a solution. This method is used for all quadratic equations where ‘x’ is a common factor.
- **Example 4A-2** shows how to solve quadratic equations after first factorising. This will help students to complete Q4.

- In Q8, students need to understand that the only difference between $-x(x - 3) = 0$ and $x(x - 3) = 0$ is that the first equation has been produced by multiplying both sides of the second equation by –1. Similarly, in Q9, the first equation has been produced by multiplying both sides of the second equation by –2.
- **Example 4A-3** shows how to solve quadratic equations after first dividing both sides by a negative number. This will help students to complete Q10.
- In Q11, students look at the number of solutions to a quadratic equation.
  - In part d, discuss the concept that there is one solution to the equation because the quadratic is a perfect square.
  - In part e, students will be unable to factorise the expression although some may confuse it for the difference of two squares. Explain that if they cannot factorise a quadratic equation, it does not necessarily mean it has no solutions. They will learn other techniques in the future. To see that this equation (Q11e) has no solutions, students need to consider the equation $x^2 = -4$.
- For Q13, remind students that quadratic equations are solved using the Null Factor Law. This law can only be applied if the quadratic expression is equal to zero and so all equations need to be rearranged in this form first.
- Students will need a graphics calculator or similar software to complete Q14.
- From Q16 onwards, students need to consider when a negative answer has meaning and when it does not.
- For additional practice, students can complete Q1 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4A Q3 and Q4 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

Below Level

Below Level
1–5, 7a–d, 10a–c, 11, 13a–f, 14–16

- Students may need to complete **SS 4A-1 Factorising quadratic expressions** (see Resources).
- Students may need to complete **SS 4A-2 Substitution into quadratic expressions** (see Resources).
- Students may need to complete **SS 4A-3 Simplifying algebraic expressions** (see Resources).

- For Q1, remind students that:
  - a quadratic equation is of degree 2; that is, the term with the greatest power in the equation is x^2
  - to be an equation rather than an expression, there must be an equals sign.
- In Q2, pay particular attention to part b. The single term factor of x still needs to be set equal to zero and becomes a solution. This method is used for all quadratic equations where 'x' is a common factor.
- Students may wish to do Q5 simultaneously with Q4. They should check each solution by substituting the value in the left side of the equation and showing that it gives a result of zero; that is, the same as the value on the right side.

POTENTIAL DIFFICULTY
In questions such as Q10 where a numerical factor can be taken out, some students put that numerical factor equal to zero and then perceive that factor to be a solution to the equation.

- In Q11, students look at the number of solutions to a quadratic equation.
  - In part d, discuss the concept that there is one solution to the equation because the quadratic is a perfect square.
  - In part f, students will be unable to factorise the expression although some may confuse it for the difference of two squares. Explain that a quadratic equation does not factorise does not prevent it from having solutions. To see that this equation has no solutions students need to consider the equation $x^2 = –4$.
- For Q13, remind students that quadratic equations are solved using the Null Factor Law. This law can only be applied if the equation is equal to zero and so all equations need to be rearranged in this form first.
- Students will need a graphics calculator or similar software to complete Q14.
- For students who do not progress past Q4, direct them to Q1 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- This exercise will prove very difficult for students with below average algebra skills.
- If students have software that will assist with factorisation, this can be provided for support.

Above Level

Above Level

1, 3, 4, 7–9, 10d–f, 11–13, 16, 18–23

- For Q1, remind students that:
  - a quadratic equation is of degree 2; that is, the term with the greatest power in the equation is x^2
  - to be an equation rather than an expression there must be an equal to sign.
- In Q2, pay particular attention to part b. The single term factor of x still needs to be set equal to zero and becomes a solution. This method is used for all quadratic equations where ‘x’ is a common factor.
- Above Level students have not been given Q5 to do but should be routinely checking solutions by substitution.
- In Q8, students need to understand that the only difference between $-x(x - 3) = 0$ and $x(x - 3) = 0$ is that the first equation has been produced by multiplying both sides of the second equation by –1. Similarly, in Q9, the first equation has been produced by multiplying both sides of the second equation by –2.
- In Q11, students look at the number of solutions to a quadratic equation.
  - In part d, discuss the concept that there is one solution to the equation because the quadratic is a perfect square.
  - In part e, students will be unable to factorise the expression although some may confuse it for the difference of two squares. Explain that if they cannot factorise a quadratic equation, it does not necessarily mean it has no solutions. They will learn other techniques in the future. To see that this equation (Q11e) has no solutions, students need to consider the equation $x^2 = -4$.
- For Q13, remind students that quadratic equations are solved using the Null Factor Law. This law can only be applied if the quadratic expression is equal to zero and so all equations need to be rearranged in this form first.
- From Q16 onwards, students need to consider when a negative answer has meaning and when it does not.

Extra activities

1 Quick questions

a Solve $x - 9 = 0$. ($x = 9$)

b Factorise $x^2 - 9$. [$(x - 3)(x + 3)$]

c	Factorise $x^2 - 9x$.	$[x(x - 9)]$
d	Factorise $x^2 - 6x + 9$.	$[(x - 3)^2]$

2 Use the Null Factor Law to solve each equation.

a	$x(x + 1) = 0$	$(x = 0$ or $x = -1)$
b	$x(x + 1)(x - 1) = 0$	$(x = 0, x = -1$ or $x = 1)$
c	$x^2(x - 1) = 0$	$(x = 0$ or $x = 1)$
d	$(x - 1)(x + 2)(x - 3) = 0$	$(x = 1, x = -2$ or $x = 3)$
e	$x^3 - 4x = 0$	$(x = 0, x = -2$ or $x = 2)$

Answers

ANSWERS

4A Solving quadratic equations

4A Start thinking!

1 A quadratic expression has 2 as the highest power of the variable, e.g. $x^2 + 3x$, $3x^2$, $5 - x^2$.

2 a A quadratic equation has an equals sign, quadratic expression does not.

b i, iii

c Highest power in a quadratic equation is 2; highest power in a linear equation is 1.

3 a $x = -2$ or $x = 2$ b $x = 3$ or $x = 5$

c $x = 0$ or $x = -7$ d $x = -4$ or $x = 1$

4 a i 0 ii 0 iii 0 iv 0 v 0 vi 0

b i $x = 0$ ii $x = 0$ iii $x = 1$ iv $x = -5$

c zero; zero

5 a $x - 3$ and $x - 5$

b One factor or the other must equal zero.

c $(x - 3)(x - 5) = 0$
$x - 3 = 0$ or $x - 5 = 0$
$x = 3$ or $x = 5$

d Substitute each x value into the original equation $(x - 3)(x - 5) = 0$ and show that each value makes the equation a true statement.

Exercise 4A Solving quadratic equations

ANSWERS

1 a, c, e and h

2 a $(x+7)(x-4)=0$
$x+7=0$ or $x-4=0$
$x=-7$ or $x=4$

b $x(x-2)=0$
$x=0$ or $x-2=0$
$x=0$ or $x=2$

c $(x+5)(x-5)=0$
$x+5=0$ or $x-5=0$
$x=-5$ or $x=5$

3 a $x=-2$ or $x=3$ b $x=1$ or $x=7$
c $x=-4$ or $x=4$ d $x=0$ or $x=6$
e $x=-5$ or $x=-1$ f $x=-2$ or $x=0$
g $x=-8$ or $x=8$ h $x=-1$ or $x=7$
i $x=0$ or $x=11$ j $x=-3$ or $x=5$
k $x=2$ l $x=-5$

4 a $x=-2$ or $x=5$ b $x=1$ or $x=2$
c $x=0$ or $x=-5$ d $x=0$ or $x=3$
e $x=-6$ or $x=6$ f $x=-7$ or $x=-3$
g $x=-2$ or $x=4$ h $x=-1$ or $x=1$
i $x=0$ or $x=-8$ j $x=1$ or $x=3$
k $x=-3$ l $x=1$

6 a $x=-2$ or $x=2$ b $x=3$ or $x=5$
c $x=0$ or $x=-7$ d $x=-4$ or $x=1$

7 a yes b no c no d yes
e no f yes g yes h no

8 a Dividing both sides of equation by -1 gives an identical equation.
b $x=0$ or $x=3$

9 a Dividing both sides of equation by -2 gives an identical equation.
b $x=-4$ or $x=5$

10 a $x=-9$ or $x=0$ b $x=-8$ or $x=2$
c $x=1$ or $x=4$ d $x=-6$ or $x=6$
e $x=-7$ or $x=-3$ f $x=-2$ or $x=1$

11 a two b linear equation has one solution
c no
d one; the two factors produce the same solution.
e In part d the two factors are the same, whereas in part a they are different.
f Zero; not possible to factorise x^2+4.
g A quadratic equation can have zero, one or two solutions.

12 a $x=-2$ or $x=2$ b $x=-5$ or $x=2$
c $x=3$ d no solutions
e $x=-7$ or $x=0$ f $x=-8$ or $x=-4$
g $x=-8$ or $x=9$ h no solutions

13 a $x=-3$ or $x=1$ b $x=-4$ or $x=5$
c $x=-5$ or $x=5$ d $x=-2$
e $x=-8$ or $x=0$ f $x=-6$ or $x=6$
g $x=0$ or $x=3$ h $x=6$
i $x=-7$ or $x=-3$ j $x=-1$ or $x=8$
k $x=2$ or $x=4$ l $x=-2$ or $x=6$

15 a \$100 b 10 weeks
c No; if amounts were the same, relationship would be linear

16 a i 9 m ii 8 m b 8 m c 4 s
d Not possible to have negative time values.

17 a 10 b 2 c 2 and 6 d 4

18 a $x(x+8)$ cm^2 b x^2+8x cm^2
c $x^2+8x=560$ d $x^2+8x-560=0$
$(x+28)(x-20)=0$
e $x=20$ or $x=-28$. $x=20$ is the feasible solution and $x=-28$ is not, as it is not possible to have a negative length.
f width = 20 cm and length = 28 cm

19 a $x^2+2x=35$ b $x=-7$ or $x=5$
c length = 7 m, width = 5 m

20 $x(x-12)=640$
length = 32 cm, width = 20 cm

21 a $x^2-3x+2=0$ b $x^2-10x=0$
c $x^2+2x-15=0$

22 Multiplying an equation by any constant factor will result in the same solution. For example, the solution $x=1$ or $x=2$ matches the equation $(x-1)(x-2)=0$, which is equivalent to $x^2-3x+2=0$.
Multiplying by a constant (say, 3) results in an equation $3(x^2-3x+2=0)$ or $3x^2-9x+6=0$, which still has the same solutions, $x=1$ or $x=2$.

23 a $x=0$ or $x=6$ b $x=0$ or $x=1$
c $x=-3$ or $x=1$ d $x=2$ or $x=3$

Reflect

Possible answer: The Null Factor Law makes it possible to solve quadratic equations. Without setting the quadratic expression equal to zero, we would be unable to determine with certainty what any one factor of the expression is equal to.

Resources

SupportSheets

SS 4A-1 Factorising quadratic equations

Focus: To use a variety of techniques to factorise quadratic equations

Resources: ruler

Students revise what is meant by factorising and look at removing a common factor from an expression. They then look at quadratic expressions and identify the appropriate method of factorisation:

- common factors (for expressions of the form $x^2 + bx$)
- difference of two squares (for expressions of the form $x^2 – c$)
- looking for two numbers that multiply to c and add to b (for expressions of the form $x^2 + bx + c$).

SS 4A-2 Substitution into quadratic expressions

Focus: To substitute values into a variety of quadratic expressions

Students explore substitution into quadratic expressions in both expanded and factored form. Students will pay particular attention to the substitution of negative values and understanding that a negative value squared will always be positive.

SS 4A-3 Simplifying algebraic expressions

Focus: To simplify algebraic expressions by collecting like terms

Students consider like terms and in particular like terms that are used in quadratic expressions. Students will simplify expressions involving x^2, x and constant terms that will have a standard quadratic solution.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q1 relates to Exercise 4A.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x – h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

BLM

1-cm grid paper

Interactives

4A eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4B Plotting quadratic relationships

Teaching support for pages 160–165

Teaching strategies

Learning focus

To introduce the parabola as the graph of a quadratic relationship

Start thinking!

The task guides students to:

- plot a quadratic relationship from a table of values
- see that a quadratic relationship is not linear
- recognise that the graph of a quadratic relationship is called a parabola
- identify the turning point and axis of symmetry as key features of a parabola
- recognise that a parabola has a single *y*-intercept and list its coordinates
- recognise that a parabola may have two *x*-intercepts and list their coordinates.

Differentiated pathways

Below Level	At Level	Above Level
1–6, 8, 9a, b, 10, 12	1–8, 9a–c, 10–13	2, 3, 5–8, 9d–f, 11–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q4 and Q5

Focus: To revise substituting into a table of values, recognising linear and non-linear relationships and writing the rule for horizontal and vertical lines.

- Direct students to complete **SS 4B-1 Plotting relationships** (see Resources) if they had difficulty with Q4 or require more practice at this skill.
- Direct students to complete **SS 4B-2 Writing the rule for horizontal and vertical linear graphs** (see Resources) if they had difficulty with Q5 or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.

- Students need to:
  - substitute for x into an algebraic rule
  - recognise that when a negative number is squared, the result is positive
  - see that if a straight line cannot be drawn through points on a Cartesian plane, the relationship is not linear
  - recognise that a vertical line has a rule or equation of the form $x = c$, and a horizontal line has a rule or equation of the form $y = c$.

At Level

At Level
1–8, 9a–c, 10–13

- Demonstrate **4B eTutor** or direct students to do this independently.
- Provide students with copies of the BLM **Cartesian plane grids** (see Resources) to help them complete this topic.
- Students will need calculator, pencil, ruler and eraser to complete this topic.
- Define a parabola as the graph of a quadratic relationship.
- Discuss some examples of applications of parabolas:
  - the path of a projectile
  - the design of a headlight
  - satellite dishes etc.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4B-1**. It shows how to plot a parabola and will help students to complete Q1 and Q2.
- After completing Q1, ask students to discuss the most significant difference between the two graphs which is whether the graph is upright or inverted.
- After completing Q2, students should notice that upright parabolas have a positive x^2 term, while inverted parabolas have a negative x^2 term.
- **Example 4B-2** shows how to identify features of a parabola. This will help students to complete Q3–5.

- In Q7, discuss with students what features of the parabola will enable them to most quickly match the graph to the equation.
  - Positive or negative x^2 term will identify whether the parabola is upright or inverted.
  - The y-intercept will be equal to the constant term.
  - Parabolas without an 'x' term will be symmetrical about the y-axis.
  - Only then would it be necessary to find the x-intercepts.
- For additional practice, students can complete Q2 and Q3 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4B Q1–5 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Quadratics and types of numbers** (see Resources).

Below Level

Below Level
1–6, 8, 9a, b, 10, 12

- Students may need to complete **SS 4B-1 Plotting relationships** (see Resources).
- Students may need to complete **SS 4B-2 Writing the rule for horizontal and vertical linear graphs** (see Resources).
- After completing Q1, ask students to discuss the most significant difference between the two graphs which is whether the graph is upright or inverted.
- After completing Q2, students should notice that upright parabolas have a positive x^2 term, while inverted parabolas have a negative x^2 term.
- For Q4, help students to see that:
  - the axis of symmetry is an imaginary vertical line halfway between the two x-intercepts
  - the turning point lies on the axis of symmetry and the y value at this point is found by substituting the x value for this axis into the equation.
- For Q6, students will need a graphics calculator or access to graphical software such as *GeoGebra*.

- To complete Q8, students should easily see which parabolas are upright and which are inverted but they will need to be guided to see that the rule of an inverted parabola has a negative x^2 term.
- In Q10, discuss part b with students. In many practical questions, negative values of the independent variable have no meaning.
- For students who do not progress past Q5, direct them to Q2 and Q3 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- Students may lack the motor skills to draw many of these graphs. Allow as much technology as needed to support them.
- The main concepts that it is hoped Below Level students can attain from this exercise are to:
  - recognise a parabola as a non-linear relationship and as such the graph is not a straight line
  - be able to draw a parabola by plotting points
  - understand that the orientation of the parabola depends on whether the coefficient of x^2 is positive or negative.

Above Level

Above Level
2, 3, 5–8, 9d–f, 11–18

- After completing Q2, students should notice that upright parabolas have a positive x^2 term, while inverted parabolas have a negative x^2 term.
- In Q7, discuss with students what features of the parabola will enable them to most quickly match the graph to the equation.
  - Positive or negative x^2 term will identify whether the parabola is upright or inverted.
  - The y-intercept will be equal to the constant term.
  - Parabolas without an 'x' term will be symmetrical about the y-axis.
  - Only then would it be necessary to find the x-intercepts.

- In part d of Q12, students are considering whether it is possible to draw a parabola with two y-intercepts. If they say that it is not, ask them to consider a sideways parabola and what the equation would look like.
- In Q16, relate the fact that the graph has only one x-intercept to its equation. The quadratic expression will be a perfect square.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Quadratics and types of numbers** (see Resources).

Extra activities

1 Quick questions

Consider the quadratic relationship $y = x^2 + 4x - 12$.

a Find y when $x = 0$. (–12)

b Find x when $y = 0$. (–6 or 2)

c Find y when $x = -2$. (–16)

d Find y when $x = -4$. (–12)

2 A golf ball is hit and the path of the golf ball follows the relationship $h = -\frac{1}{500}d(d - 200)$, where d is the horizontal distance in metres that the ball has travelled from the point it was hit and h is the height of the ball in metres.

a What horizontal distance does the ball travel? How do you know this? (200 m, as $h = 0$)

b What is the greatest height that the ball reaches? (20 m)

c On a Cartesian plane, draw a graph to represent the path of the ball.

Answers

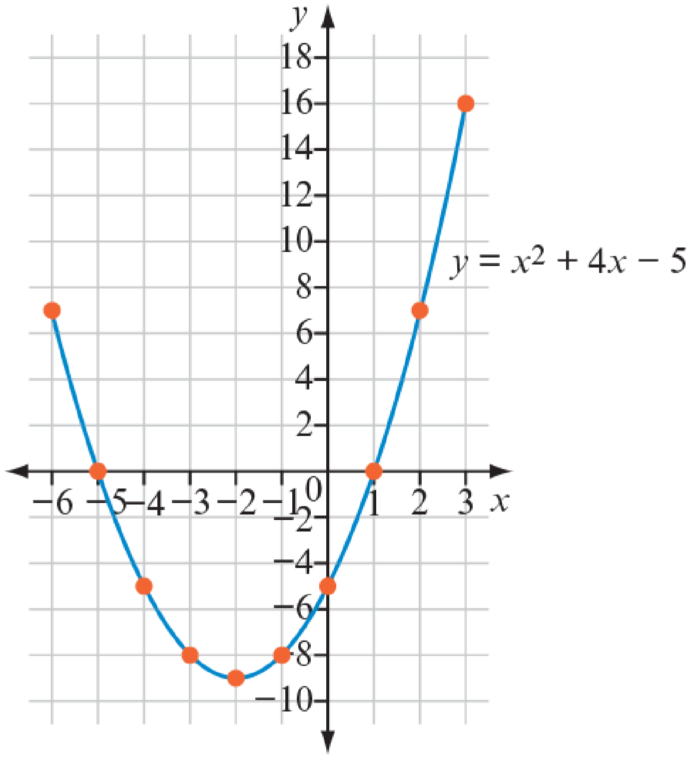
4B Plotting quadratic relationships

4B Start thinking!

1

x	−6	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7	16

2



3 The points form a symmetrical curve that changes direction at the point (−2, −9).

4 Non-linear relationship, as the points do not form a straight line.

5 a minimum b (−2, −9)

6 a It acts like a mirror line so that the left side is symmetrical to the right side.

b $x = -2$

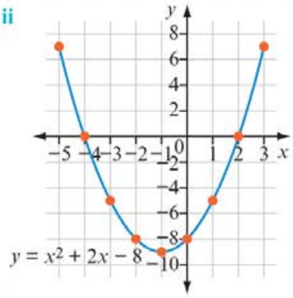
7 a one; (0, −5) b two; (−5, 0) and (1, 0)

Exercise 4B Plotting quadratic relationships

1 a i

x	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7

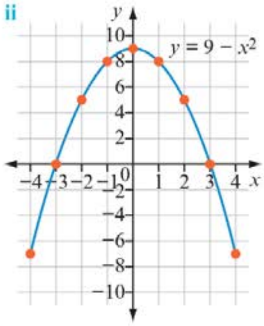
ii



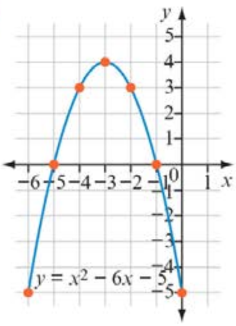
b i

x	−4	−3	−2	−1	0	1	2	3	4
y	−7	0	5	8	9	8	5	0	−7

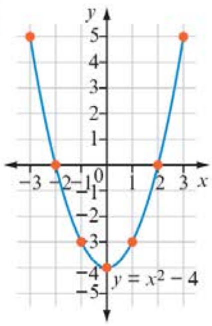
ii



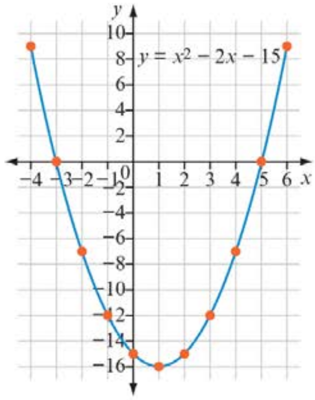
2 a



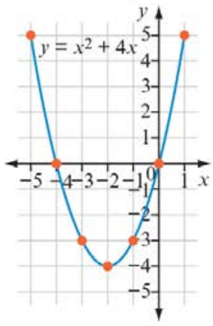
b



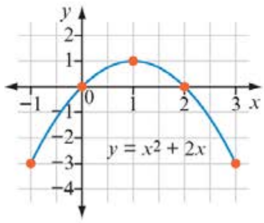
c



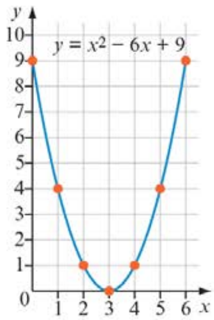
d



e



f



3 **a** minimum **b** (2, −1) **c** $x = 2$
d (0, 3) **e** (1, 0) and (3, 0)

4 **a** **i** minimum **ii** (−1, −9) **iii** $x = -1$
iv −8 **v** −4 and 2
b **i** maximum **ii** (0, 9) **iii** $x = 0$
iv 9 **v** −3 and 3

5 **a** **i** maximum **ii** (−3, 4) **iii** $x = -3$
iv −5 **v** −5 and −1
b **i** minimum **ii** (0, −4) **iii** $x = 0$
iv −4 **v** −2 and 2
c **i** minimum **ii** (1, −16) **iii** $x = 1$
iv −15 **v** −3 and 5
d **i** minimum **ii** (−2, −4) **iii** $x = -2$
iv 0 **v** −4 and 0
e **i** maximum **ii** (1, 1) **iii** $x = 1$
iv 0 **v** 0 and 2
f **i** minimum **ii** (3, 0) **iii** $x = 3$
iv 9 **v** 3

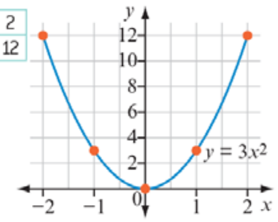
7 **a** F **b** E **c** B **d** A **e** C **f** D

8 **a** upright **b** inverted **c** upright
d upright **e** upright **f** inverted

9 a

x	−2	−1	0	1	2
y	12	3	0	3	12

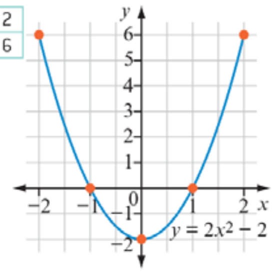
i upright
ii minimum; (0, 0)
iii 0
iv 0



b

x	−2	−1	0	1	2
y	6	0	−2	0	6

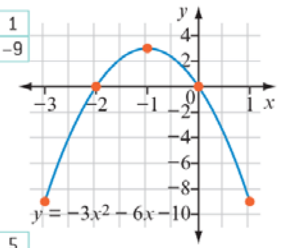
i upright
ii minimum; (0, −2)
iii −2
iv −1 and 1



c

x	−3	−2	−1	0	1
y	−9	0	3	0	−9

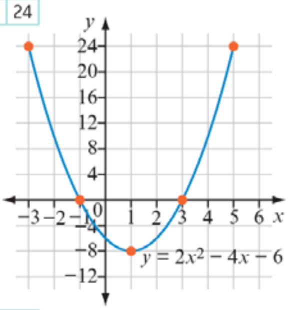
i inverted
ii maximum; (−1, 3)
iii 0
iv −2 and 0



d

x	−3	−1	1	3	5
y	24	0	−8	0	24

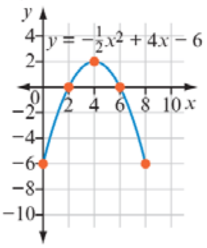
i upright
ii minimum; (1, −8)
iii −6
iv −1 and 3



e

x	0	2	4	6	8
y	−6	0	2	0	−6

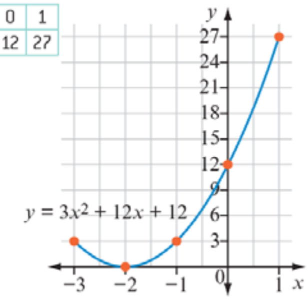
i inverted
ii maximum; (4, 2)
iii −6
iv 2 and 6



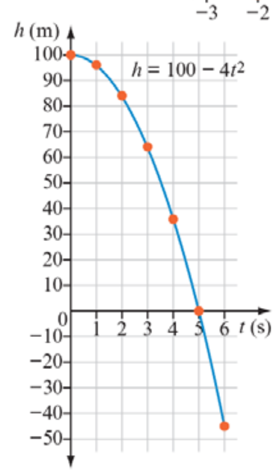
f

x	−3	−2	−1	0	1
y	3	0	3	12	27

i upright
ii minimum; (−2, 0)
iii 12
iv −2



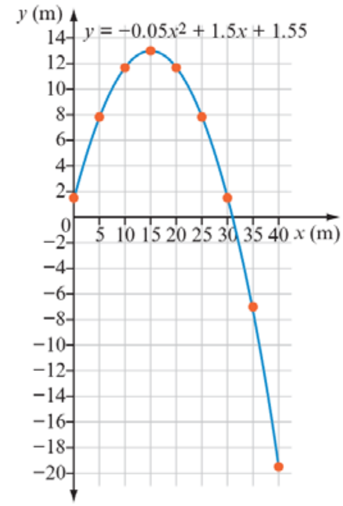
10 a



b not possible to have negative time values

c i 84 m ii 64 m **d** 100 m **e** 5 s

11 a



b 12.8 m **c** 1.55 m **d** 31 m

12 **a** **i** two **ii** one **iii** zero

b A parabola changes direction only once, so there can only be a maximum of two x-intercepts.

c one

d No; a parabola only intersects the y-axis once.

13 **a** -2 and 5 **b** $x = -2$ or $x = 5$

c They are the same.

d The x-intercepts of a parabola represent the solutions to a quadratic equation.

14 **a** $x = -5$ or $x = 1$ **b** $x = 0$ or $x = 4$

c $x = -2$ or $x = 2$ **d** $x = -5$ or $x = 1$

e $x = -2$ or $x = 2$ **f** $x = -1$ or $x = 5$

15 $x = 5$ **14** $x = -4$

17 no x-intercepts, one y-intercept

18 one x-intercept and one y-intercept

Reflect

Possible answer: A parabola can be recognised from its equation as it will be in the form $y = ax^2 + bx + c$.

Resources

SupportSheets

SS 4B-1 Plotting relationships

Focus: To plot relationships presented in a table of values and observe the patterns formed by the points on a Cartesian plane

Resources: 1-cm grid paper (BLM) or graph paper, ruler

Students use an extended table of values to first revise plotting simple linear relationships and then use similar tables of values to draw simple parabolas.

SS 4B-2 Writing the rule for horizontal and vertical linear graphs

Focus: To write the rules used to describe a horizontal line and a vertical line

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students consider rules of the form $x = b$ and recognise this as representing all points with this x value, generating a vertical line. Similarly, they consider rules of the form $y = c$ and recognise this as representing all points with this y value, generating a horizontal line.

Students practise writing the rule for different vertical and horizontal lines shown on a Cartesian plane.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q2 and Q3 relate to Exercise 4B.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

Investigation

INV 4-1 Quadratics and types of numbers

Focus: To investigate the relationship between quadratics and sets of numbers like square, triangular, pentagonal, hexagonal and octagonal numbers

Resources: 1-cm grid paper (BLM) or graph paper, calculator

Students look at square, triangular, pentagonal and hexagonal numbers and consider algebraic expressions that generate each number sequence. They see that each algebraic expression is quadratic and by solving quadratic equations for the term number, identify where particular numbers fit into the number pattern. They also draw graphs of these relationships.

As an extension, students investigate octagonal numbers.

BLMs

Cartesian plane grids

1-cm grid paper

Interactives

4B eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4C Parabolas and transformations

Teaching support for pages 166–171

Teaching strategies

Learning focus

To consider the graph of $y = x^2$ and see how other numbers in the equation change this most basic parabola

Start thinking!

The task guides students to:

- draw the graph of $y = x^2$ and identify the features of this basic parabola
- draw graphs of $y = ax^2$ for different values of a
- see the effect that different values of 'a' have on the graph of $y = x^2$.

Differentiated pathways

Below Level	At Level	Above Level
1–4, 7–9, 13	1–5, 6a–d, 7–11, 12a–d, 13–15, 16a–d, 17	1–5, 6e,f, 9–11, 12e–h, 13–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q6

Focus: To see how a point on a number plane will move under the transformations of reflection and translation

- Direct students to complete **SS 4C-1 Performing transformations on a coordinate point** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to understand that:
  - reflection means to move the object to the other side of a line of reflection inverting all its characteristics

- translation means to move an object in a given direction or directions while maintaining its orientation.

At Level

At Level
1–5, 6a–d, 7–11, 12a–d, 13–15, 16a–d, 17

- Demonstrate **4C eTutor** or direct students to do this independently.
- The concepts covered in this section are:
  - dilation of $y = x^2$ by a coefficient of x^2. If $y = ax^2$ and $a > 1$, the graph will be narrower than $y = x^2$, and if $0 < a < 1$, the graph will be wider than $y = x^2$
  - a negative coefficient of x^2 will invert the graph of $y = x^2$
  - adding or subtracting a constant from $y = x^2$ will move the graph up (adding) or down (subtracting) the y-axis
  - adding or subtracting a constant from x before squaring (creating a perfect square expression) will move the graph of $y = x^2$ horizontally along the x-axis.
- Students will need calculator, pencil, ruler and eraser to complete this topic.
- Provide students with copies of the BLM **Cartesian plane grids** (see Resources) to help them.
- For tasks that ask students to compare graphs to $y = x^2$ they can use the BLM $\boldsymbol{y = x^2}$ (see resources). This master has $y = x^2$ drawn as a starting point so students can use the graph drawn as the basis for their transformations.
- If you can use some dynamic graphic software to demonstrate the transformations, this will help with student understanding.
- Provide some graphics calculators or graphing software to assist students who find this task too difficult.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **example 4C-1**. It shows how to describe a transformation used to produce a graph from the graph of $y = x^2$ and will help students to complete Q2–4.
- After completing Q1 and Q2, ensure that students understand the concept and definition of dilation. Establish that dilation by a factor greater than 1 makes the graph *narrower*, while a factor between 0 and 1 makes the graph *wider*.

- In Q3, ensure students understand that a reflection is due to the effect of the negative x^2 term and that the reflection takes place in the y-axis.
- **Example 4C-2** shows how to describe a transformation used to produce a graph from the graph of $y = -x^2$. This will help students to complete Q5.
- **Example 4C-3** shows how to identify transformations to produce a graph from the graph of $y = x^2$. This will help students to complete Q6–8.
- In Q5–8, students will need to combine the concepts of dilation and reflection to draw the graphs and describe the transformations.
- In Q9 –11, students need to look at the graph of $y = x^2$ and see that adding or subtracting a constant moves the graph up or down the y-axis in the same direction as the addition or subtraction.
- In Q13–15, students are looking at graphs that are of the form $y = (x - h)^2$. When students graph these from a table of values they will see that there is a horizontal movement of the graph. Unlike graphs of the form $y = x^2 + k$, the movement of the graph is counter-intuitive to some students. Explain that the movement is to the point such that $x - h = 0$, hence the movement is opposite to the sign shown.
- For additional practice, students can complete Q4 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4C Q17 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

Below Level

Below Level
1–4, 7–9, 13

- Students may need to complete **SS 4C-1 Performing transformations on a coordinate point** (see Resources).
- After completing Q1 and Q2, ensure that students understand the concept and definition of dilation. Establish that dilation by a factor greater than 1 makes the graph *narrower*, while a factor between 0 and 1 makes the graph *wider*.
- In Q3, ensure students understand that a reflection is due to the effect of the negative x^2 term and that the reflection takes place in the y-axis.
- In Q7 and Q8, students will need to combine the concepts of dilation and reflection to match the graphs to the equations.

- In Q9, students need to look at the graph of $y = x^2$ and see that adding or subtracting a constant moves the graph up or down the y-axis in the same direction as the addition or subtraction.
- In Q13, students are looking at graphs that are of the form $y = (x – h)^2$. When students graph these from a table of values they will see that there is a horizontal movement of the graph. Unlike graphs of the form $y = x^2 + k$, the movement of the graph is counter-intuitive to some students. Explain that the movement is to the point such that $x – h = 0$, hence the movement is opposite to the sign shown.
- Below Level students will find the concept of transforming a graph very difficult. To support and consolidate understanding of these concepts, allow students to draw graphs by first plotting a table of values. The BLM **Quadratic table of values** (see resources) can be provided to help them do this quickly.
- Allowing students to complete questions using a graphics calculator or graphing software will also assist them to have some success with this content.

Above Level

Above Level
1–5, 6e,f, 9–11, 12e–h, 13–18

- After completing Q1 and Q2, ensure that students understand the concept and definition of dilation. Establish that dilation by a factor greater than 1 makes the graph *narrower*, while a factor between 0 and 1 makes the graph *wider*. Ask: what happens when the dilation factor is 1?
- In Q3, ensure students understand that a reflection is due to the effect of the negative x^2 term and that the reflection takes place in the y-axis.
- In Q5–8, students will need to combine the concepts of dilation and reflection to draw the graphs and describe the transformations.
- In Q10, students need to look at the graph of $y = x^2$ and see that adding or subtracting a constant moves the graph up or down the y-axis in the same direction as the addition or subtraction.
- In Q13–16, students are looking at graphs that are of the form $y = (x – h)^2$. When students graph these from a table of values they will see that there is a horizontal movement of the graph. Unlike graphs of the form $y = x^2 + k$, the movement of the graph is counter-intuitive to some students. Explain that the movement is to the point such that $x – h = 0$, hence the movement is opposite to the sign in the question.
- In Q18, students will bring all possible transformations together to look at a parabola of the form $y =a(x – h)^2 + k$. To summarise the transformations, the graph of $y = x^2$ is:

– dilated by a factor of ‘a’

– moved vertically ‘k’ units (down if k is negative)

– moved horizontally ‘h’ units in the direction of the solution to $x - h = 0$.

Extra activities

1 Quick questions

Find the value of y when $x = 0$ for each quadratic relationship.

a $y = x^2 + 4$ ($y = 4$)

b $y = x^2 - 5$ ($y = -5$)

c $y = (x + 4)^2$ ($y = 16$)

d $y = (x - 5)^2$ ($y = 25$)

2 Above Level students: Consider the graph of $x = y^2$.

a What would the graph look like? (parabola on its side)

b There are two transformations that could be performed on $y = x^2$ to produce this graph. What are they? (90° clockwise rotation or reflection in the line $y = x$)

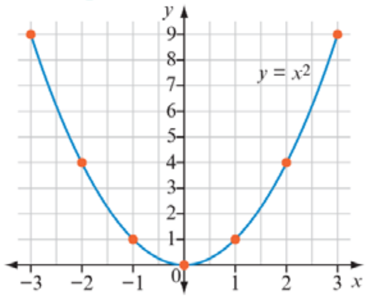
c How might the graph of $x = y^2$ differ from $y = \sqrt{x}$? (square root sign means positive square root so $y = \sqrt{x}$ is only the top half of the graph of $x = y^2$)

Answers

4C Parabolas and transformations

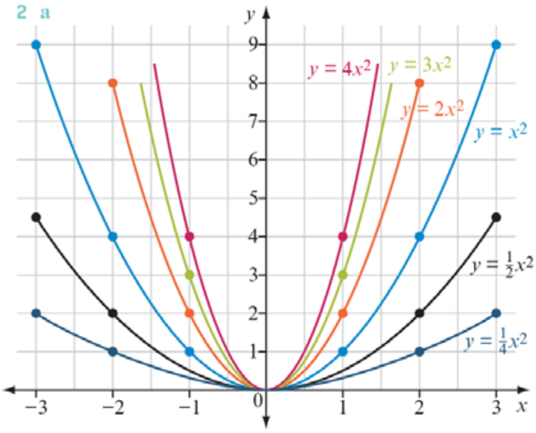
4C Start thinking!

1 a



b minimum turning point at (0, 0), x-intercept at 0 and y-intercept at 0, axis of symmetry $x = 0$

2 a



b All graphs have same minimum turning point, axis of symmetry and x- and y-intercepts but different shapes (some wider than $y = x^2$ and some narrower).

c i $y = 2x^2$, $y = 3x^2$ and $y = 4x^2$

ii $y = \frac{1}{2}x^2$ and $y = \frac{1}{4}x^2$

d For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$, dilation produces a wider graph than $y = x^2$.
For $a > 1$, dilation produces a narrower graph than $y = x^2$.

e ii dilated by factor of 3, narrower

iii dilated by factor of 4, narrower

iv dilated by factor of $\frac{1}{2}$, wider

v dilated by factor of $\frac{1}{4}$, wider

3 For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$, dilation produces a wider graph than $y = x^2$. For $a > 1$, dilation produces a narrower graph than $y = x^2$.

Exercise 4C Parabolas and transformations

1 **a** $y = \frac{1}{2}x^2$

b One possible answer is $y = \frac{1}{4}x^2$.

c $y = 2x^2$ **d** One possible answer is $y = 4x^2$.

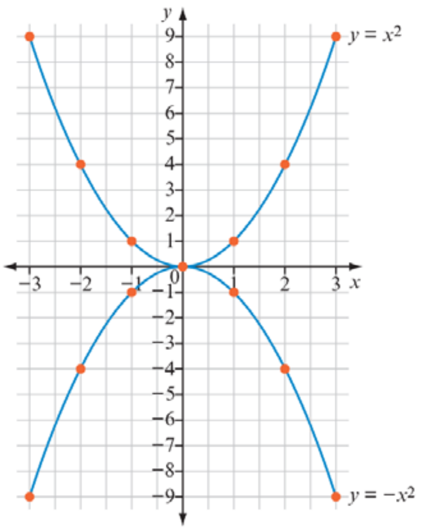
2 **a** dilate by factor of 2, narrower

b dilate by factor of $\frac{1}{2}$, wider

c dilate by factor of $\frac{1}{4}$, wider

d dilate by factor of 4, narrower

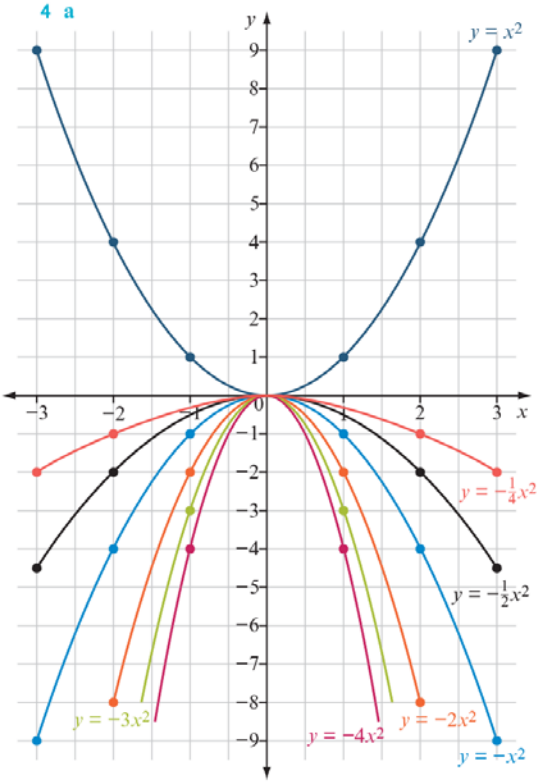
3 **a**



b Mirror image of $y = x^2$ (reflected in x-axis); no dilation; reflection has been performed.

c x-axis; exact image appears beneath x-axis, which acts as mirror line.

4 **a**



b All graphs have same maximum turning point, axis of symmetry and x- and y-intercepts, but different shapes (some wider than $y = -x^2$ and some narrower)

c **i** $y = -2x^2$, $y = -3x^2$ and $y = -4x^2$

ii $y = -\frac{1}{2}x^2$ and $y = -\frac{1}{4}x^2$

5 **a** dilate by factor of 2 (narrower)

b dilate by factor of $\frac{1}{2}$ (wider)

c dilate by factor of $\frac{1}{4}$ (wider)

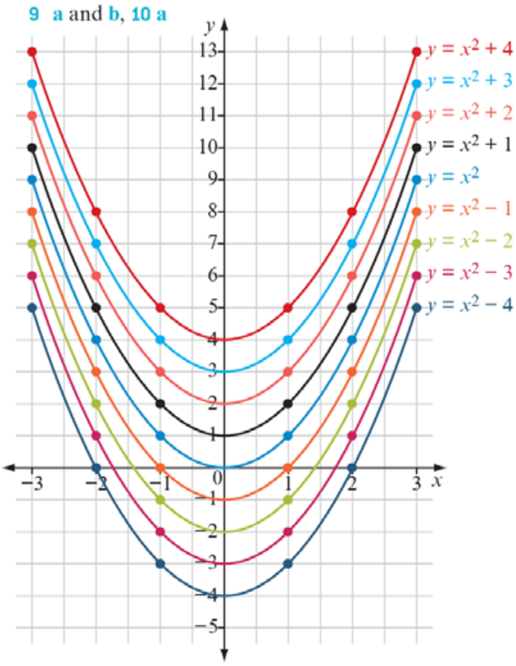
d dilate by factor of 4 (narrower)

6 a dilate by factor of 5 (narrower)
b reflect in x-axis
c dilate by factor of 4 (narrower) and reflect in x-axis
d dilate by factor of $\frac{1}{4}$ (wider)
e dilate by factor of 10 (narrower)
f dilate by factor of $\frac{1}{7}$ (wider) and reflect in x-axis
g dilate by factor of 8 (narrower) and reflect in x-axis
h dilate by factor of $\frac{2}{3}$ (wider) and reflect in x-axis

7 a dilation and reflection
b i $y = -2x^2$: dilate by factor of 2 (narrower) and reflect in x-axis
ii $y = -3x^2$: dilate by factor of 3 (narrower) and reflect in x-axis
iii $y = -4x^2$: dilate by factor of 4 (narrower) and reflect in x-axis
iv $y = -\frac{1}{2}x^2$: dilate by factor of $\frac{1}{2}$ (wider) and reflect in x-axis
v $y = -\frac{1}{4}x^2$: dilate by factor of $\frac{1}{4}$ (wider) and reflect in x-axis

8 a E b A c D d C e F f B

9 a and b, 10 a



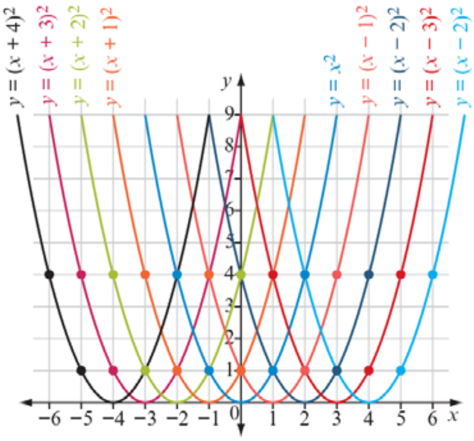
9 c i All graphs have same shape as $y = x^2$, a minimum turning point and same axis of symmetry.
ii Graphs have different turning point coordinates and different y-intercepts; graphs have no x-intercepts.
d Graph shifts up by same number of units as constant term.
e i translated 1 unit up
ii translated 2 units up
iii translated 3 units up
iv translated 4 units up

10 a see above
b i All graphs have same shape as $y = x^2$, a minimum turning point and same axis of symmetry.
ii Graphs have different turning point coordinates and different y-intercepts. Graphs all have two x-intercepts.
c Graph shifts down by same number of units as constant term.
d i translated 1 unit down
ii translated 2 units down
iii translated 3 units down
iv translated 4 units down

11 **a** shifts the graph vertically (translation)
b If k is positive, graph shifts up and if k is negative, graph shifts down.
c up, down
d Graph has no vertical movement.

12 **a** **i** $k = 6$ **ii** translated 6 units up
b **i** $k = -7$ **ii** translated 7 units down
c **i** $k = -5$ **ii** translated 5 units down
d **i** $k = 8$ **ii** translated 8 units up
e **i** $k = 9$ **ii** translated 9 units up
f **i** $k = -11$ **ii** translated 11 units down
g **i** $k = 1.5$ **ii** translated 1.5 units up
h **i** $k = -7.2$ **ii** translated 7.2 units down

13 **a, b, 14** **a**



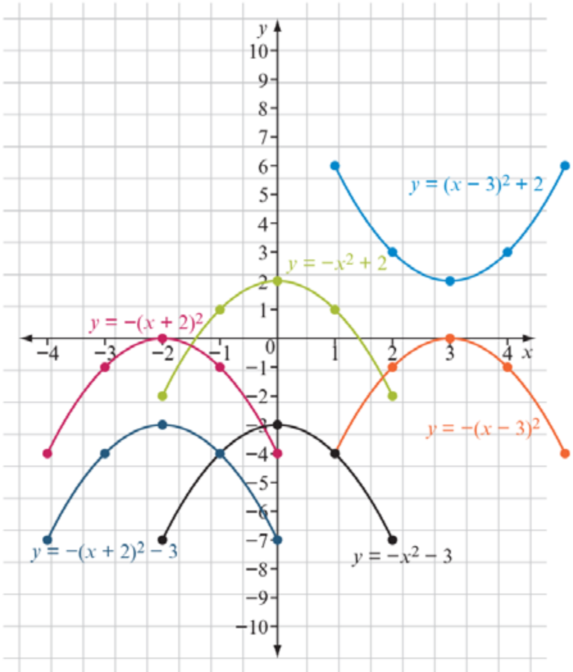
13 **c** **i** All graphs have the same shape as $y = x^2$ and a minimum turning point.
ii Graphs have different turning point coordinates, axes of symmetry and y-intercepts. Graphs all have one x-intercept (they sit on the x-axis).
d Shifts graph horizontally to right.
e **i** translated 1 unit right
ii translated 2 units right
iii translated 3 units right
iv translated 4 units right

14 **a** see above
b **i** All graphs have same shape as $y = x^2$, and a minimum turning point.
ii Graphs have different turning point coordinates, axes of symmetry and y-intercepts. Graphs all have one x-intercept (they sit on the x-axis).
c Shifts graph horizontally to left.
d **i** translated 1 unit left
ii translated 2 units left
iii translated 3 units left
iv translated 4 units left

15 a Shifts graph horizontally (translation).
b If h positive, graph moves to right; if h negative, graph moves to left.
c right, left
d Graph has no horizontal movement.

16 a i $h = 5$ ii translated 5 units right
b i $h = -7$ ii translated 7 units left
c i $h = 6$ ii translated 6 units right
d i $h = -9$ ii translated 9 units left
e i $h = -8$ ii translated 8 units left
f i $h = 12$ ii translated 12 units right
g i $h = 2.5$ ii translated 2.5 units right
h i $h = -6.7$ ii translated 6.7 units left

17 a



b i (0, 2) ii (0, −3) iii (3, 0) iv (−2, 0) v (3, 2) vi (−2, −3)
c i inverted ii inverted iii inverted iv inverted v upright vi inverted
d A iii, B i, C v, D iv, E vi, F ii

18 a i $a = -1$; $h = 0$; $k = 2$
ii $a = -1$; $h = 0$; $k = -3$
iii $a = -1$; $h = 3$; $k = 0$
iv $a = -1$; $h = -2$; $k = 0$
v $a = 1$; $h = 3$; $k = 2$
vi $a = -1$; $h = -2$; $k = -3$
b Turning point of $y = x^2$ (0, 0) shifts horizontally h units and vertically k units and end result is coordinates (h, k).
c If a greater than 0, parabola will be upright and if a less than 0, parabola will be inverted.
d Dilation: narrower when $a < -1$ or $a > 1$ and wider when $-1 < a < 0$ or $0 < a < 1$. For $a > 0$, upright parabola and for $a < 0$, inverted parabola (reflection in x-axis). Horizontal translation (h units). For $h > 0$, move right and for $h < 0$, move left. Vertical translation (k units). For $k > 0$, move up and for $k < 0$, move down.

Reflect

Possible answer: $y = x^2$ is the most basic graph because it has its turning point at the origin. All transformations can then be referenced to this most important point on the Cartesian plane.

Resources

SupportSheet

SS 4C-1 Performing transformations on a coordinate point

Focus: To perform reflections and translations on a coordinate point

Resources: ruler, **1-cm grid paper** (BLM) or graph paper

This task will help students to define key terms associated with transformations including translation, reflection and image. Students will identify the coordinates of an image point that has been translated, vertically, horizontally or both as well as reflected in a given line.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q4 relates to Exercise 4C.

Students solve a variety of quadratic equations in preparation for determining *x*-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of *x*- and *y*-intercepts and the turning point.

BLMs

Cartesian plane grids

$y = x^2$

1-cm grid paper

Interactives

4C eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4D Sketching parabolas using transformations

Teaching support for pages 172–177

Teaching strategies

Learning focus

To use the knowledge gained about transformations to sketch parabolas

Start thinking!

The task guides students to:

- consider a series of quadratic relationships
- think about how each rule is different to $y = x^2$ and what transformation that will cause
- recognise that:
  - a dilation is produced by the coefficient of x^2
  - a reflection in the x-axis is produced by a negative coefficient of x^2
  - a vertical movement is produced by the addition of a constant
  - a horizontal movement is produced by the subtraction of a constant from x before squaring the result.

Differentiated pathways

Below Level	At Level	Above Level
1–4, 7, 8, 10, 12–14	1, 2a–c, 3a–c, 4–6, 7a–c, 8a–c, 9–16	1, 2d–e, 3d–e, 4–6, 7d–e, 8d–e, 9, 11, 12e–h, 15–22
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2a–c, 3a–c, 4–6, 7a–c, 8a–c, 9–16

- Demonstrate **4D eTutor** or direct students to do this independently.
- Students will need calculator, pencil, ruler and eraser to complete this topic.

- Provide students with copies of the BLM $\boldsymbol{y = x^2}$ (see Resources). Students can then apply the transformations to this graph to obtain each of their answers.
- This topic asks students to sketch parabolas using some or all of the transformations covered in topic 4C.
- Remind students of each of the transformations individually and how they are recognised.
  - Dilation is produced by the coefficient of x^2
  - Reflection in the x-axis is produced by a negative coefficient of x^2
  - Vertical movement is produced by the addition of a constant. The vertical movement is in the direction of the constant.
  - Horizontal movement is produced by subtracting a constant from x before squaring the result. The horizontal movement is opposite to the sign in the brackets.
- Once students understand each of the individual transformations they will need to combine them using $y = a(x – h)^2 + k$. This is known as turning point form of a parabola.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4D-1**. It shows how to sketch a parabola by performing a vertical translation and will help students to complete Q2.
- **Example 4D-2** shows how to sketch a parabola by performing a horizontal translation. This will help students to complete Q3.
- After completing Q4 and Q5, students should be able to see that, when the rule for a parabola is written in the form $y = a(x – h)^2 + k$, the coordinates of the turning point of the parabola are (h, k). Hence, this is why it is called turning point form.
- **Example 4D-3** shows how to sketch a parabola by performing more than one transformation. This will help students to complete Q7–9.
- In Q8, ask students to consider whether the order in which the reflection and translation are performed makes a difference to the answer. Students should understand that if the translation is performed first the reflection that needs to occur is no longer in the x-axis.
- Q11 requires students to substitute the coordinates of the turning point into $y = a(x – h)^2 + k$. You may need to explain that since no dilation has been performed:
  - $a = 1$ if the parabola is upright

  - $a = -1$ if the parabola is inverted.

- In Q13 and Q14, students may need to draw a sketch of the graph to find the smallest and largest possible values of y and see that these are the y-coordinates of the turning point. Help students to understand that:
  - all upright parabolas have a minimum y value but no maximum
  - all inverted parabolas have a maximum y value but no minimum.
- For additional practice, students can complete Q5–7 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4D Q4, Q7 and Q12 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

Below Level

Below Level
1–4, 7, 8, 10, 12–14

- After completing Q4, students may be able to see that, when the rule for a parabola is written in the form $y = a(x - h)^2 + k$, the coordinates of the turning point of the parabola are (h, k). Hence, this is why it is called turning point form.
- The graphs in Q7 are obtained using both a horizontal and vertical translation. Remind students that the:
  - vertical translation is the value of k in $y = a(x - h)^2 + k$
  - horizontal translation is the value of h in $y = a(x - h)^2 + k$.
- Students at this level will most likely deal with the vertical translation correctly but find the direction of the horizontal translation counter-intuitive. Reinforce that the direction of the horizontal translation is such that $x - h = 0$.
- In Q8, students need to perform a reflection in the y-axis and then either a vertical or horizontal translation. As the reflection needs to be in the x-axis, students will cope better if they are taught to complete the reflection first.
- In Q13 and Q14, students may need to draw a sketch of the graph to find the smallest and largest possible values of y and see that these are the y-coordinates of the turning point. Help students to understand that:
  - all upright parabolas have a minimum y value but no maximum
  - all inverted parabolas have a maximum y value but no minimum.

- For students who do not progress past Q12, direct them to Q5–7 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- Below Level students may find it very difficult to draw graphs where they need to perform more than one transformation. Allowing them to use graphing software to obtain their answers will help them to achieve some success. If they do this, have them copy their answers onto the BLM $\boldsymbol{y = x^2}$ (see resources) and ask them to describe the transformation.
- Although the purpose of this exercise is teach students to sketch graphs without the need for a table of values, students operating at this level may need to use this method to obtain graphs.

Above Level

Above Level
1, 2d–e, 3d–e, 4–6, 7d–e, 8d–e, 9, 11, 12e–h, 15–22

- After completing Q4 and Q5, students should be able to see that, when the rule for a parabola is written in the form $y = a(x - h)^2 + k$, the coordinates of the turning point of the parabola are (h, k). Hence, this is why it is called turning point form.
- In Q8, ask students to consider whether the order in which the reflection and translation are performed makes a difference to the answer. Students should understand that if the translation is performed first, the reflection that needs to occur is no longer in the x-axis.
- Q11 requires students to substitute the coordinates of the turning point into $y = a(x - h)^2 + k$. You may need to explain that since no dilation has been performed:
  - $a = 1$ if the parabola is upright
  - $a = -1$ if the parabola is inverted.
- In Q15, students sketch the graph of a parabola that represents the height of a basketball in terms of time. Discuss with students that the graph drawn does not represent the flight path of the ball. Ask them to consider why not? (Answer: the relationship only considers the vertical position of the ball and makes no reference to the horizontal position.)

Extra activities

1 Quick Questions

State the coordinates of the turning point of each parabola with these rules.

a $y = x^2$ (0, 0)

b $y = x^2 + 2$ (0, 2)

c $y = 3x^2$ (0, 0)

d $y = (x + 4)^2$ (–4, 0)

2 The prefix ‘para’ means ‘almost’. For example, a paramedic means almost a medical practitioner but not actually a doctor. As such, a parabola is almost a bowl shape. There are many daily objects that take this shape. Identify as many objects as possible that are parabolic in shape. (Answers may include, but not be limited to, the path of a projectile, a headlight in a car, a satellite dish, a household vase.)

Answers

4D Sketching parabolas using transformations

4D Start thinking!

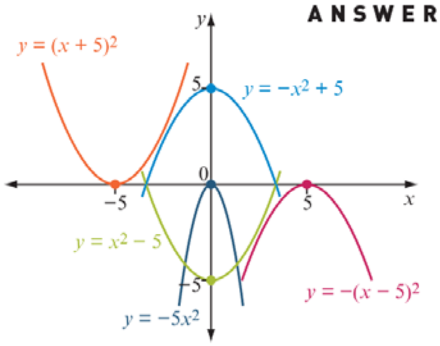
1 a i translate 5 units down
ii translate 5 units left
iii dilate by factor of 5 and reflect in x-axis
iv reflect in x-axis and translate 5 units up
v reflect in x-axis and translate 5 units right

b i upright ii upright iii inverted
iv inverted v inverted
Compare the rule to $y = a(x - h)^2 + k$. Look at sign of a. If negative, graph is inverted and if positive, graph is upright.

c Use (h, k) values.
i $(0, -5)$ ii $(-5, 0)$ iii $(0, 0)$
iv $(0, 5)$ v $(5, 0)$

ANSWERS

d



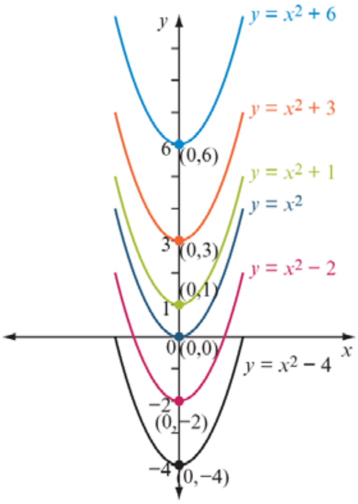
e Use values for a, h and k.

2 Use value of a to determine if there is dilation (narrower or wider) or reflection.
For $a > 0$, upright parabola and for $a < 0$, inverted parabola (reflection in the x-axis). Horizontal translation (h units). For $h > 0$, move right and for $h < 0$, move left. Vertical translation (k units). For $k > 0$, move up and for $k < 0$, move down.

Exercise 4D Sketching parabolas using transformations

1 **a** **i** upright **ii** (2, 0)
iii translated 2 units right **iv** C
b **i** inverted **ii** (0, 0)
iii dilation by factor of 2, reflection in x-axis
iv A
c **i** upright **ii** (0, 2)
iii translated 2 units up **iv** B

2



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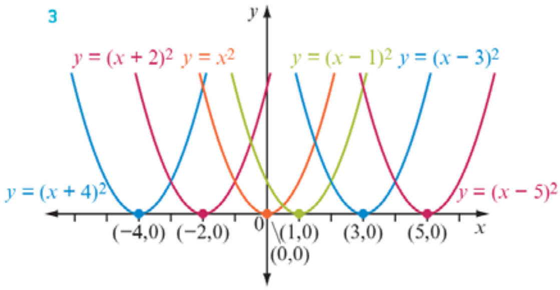
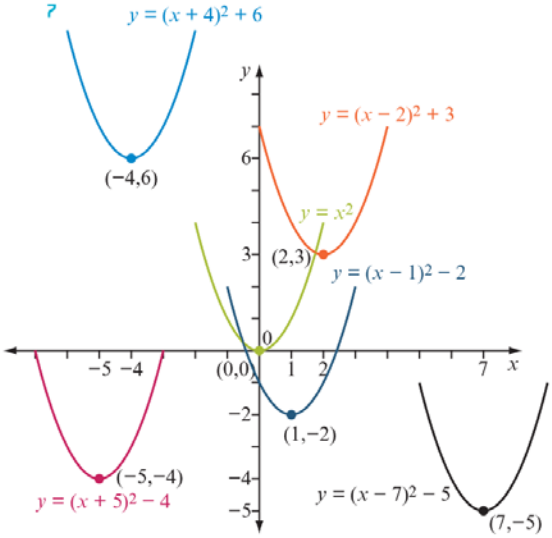


4 **a** upright **b** (4, −2)
c C. Turning point has moved right 4 units ($h = 4$) and down 2 units ($k = -2$).

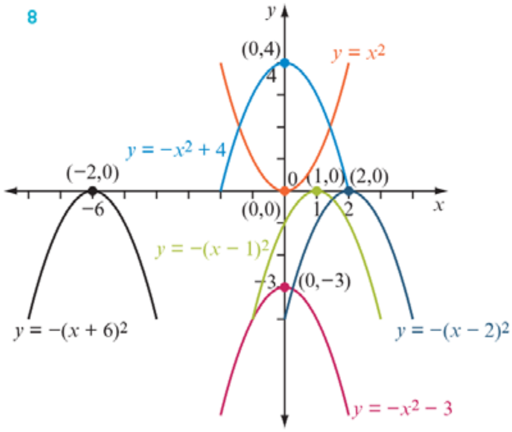
5 **a** $a = 1$, $h = 3$ and $k = 4$
b no dilation or reflection; horizontal translation (3 units right) and vertical translation (4 units up)
c (3, 4)
d Graph shifts right 3 units and up 4 units, resulting in turning point coordinates (3, 4)

6 **a** $a = 1$, $h = 4$ and $k = -2$
b no dilation or reflection; horizontal translation (4 units right) and vertical translation (2 units down)
c (4, −2)

7



8

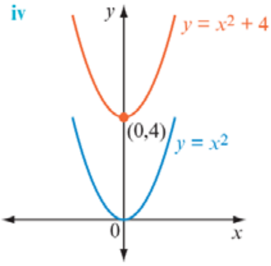


9 **a** E **b** D **c** A **d** F **e** C **f** B

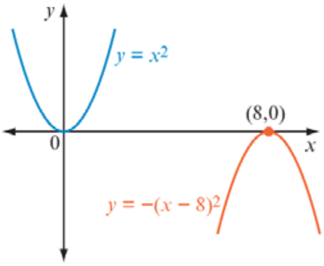
10 **a** **i** upright **ii** $(1, -4)$
iii y-intercept: $(0, -3)$; x-intercepts: $(-1, 0)$ and $(3, 0)$
b **i** inverted **ii** $(-3, 1)$
iii y-intercept: $(0, -8)$; x-intercepts: $(-4, 0)$ and $(-2, 0)$
c **i** upright **ii** $(3, -9)$
iii y-intercept: $(0, 0)$; x-intercepts: $(0, 0)$ and $(6, 0)$

11 **a** $y = (x - 3)^2 + 7$ **b** $y = (x + 2)^2 + 5$
c $y = -(x - 2)^2 + 4$ **d** $y = -(x - 6)^2 - 1$
e $y = (x - 9)^2$ **f** $y = -x^2 + 4$
g $y = -(x + 1)^2 - 2$ **h** $y = x^2 - 5$

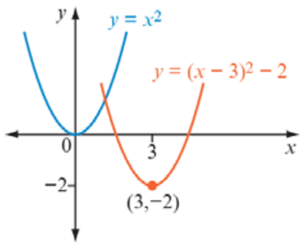
12 **a** **i** translate 4 units up
ii upright
iii $(0, 4)$
iv



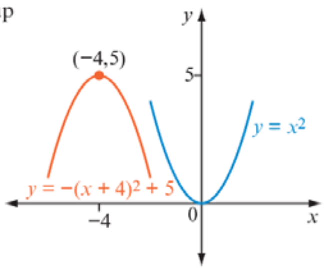
b **i** reflect in x-axis and translate 8 units right
ii inverted
iii $(8, 0)$
iv



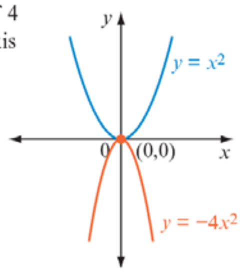
c **i** translate 3 units right and 2 units down
ii upright
iii $(3, -2)$
iv



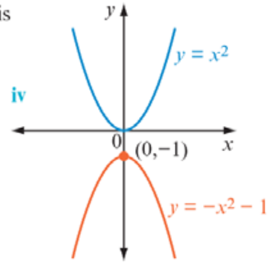
d **i** reflect in x-axis and translate 4 units left and 5 units up
ii inverted
iii $(-4, 5)$
iv



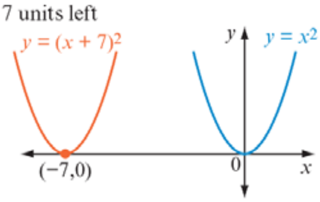
e **i** dilate by factor of 4 and reflect in x-axis
ii inverted **iv**
iii $(0, 0)$



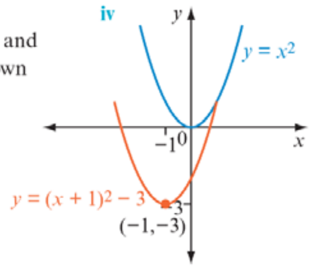
f **i** reflect in x-axis and translate 1 unit down
ii inverted **iv**
iii $(0, -1)$



g **i** translate 7 units left
ii upright
iii $(-7, 0)$
iv



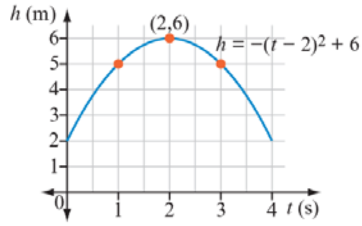
h **i** translate 1 unit left and 3 units down
ii upright
iii $(-1, -3)$
iv



13 **a** $(4, 5)$ **b** 5

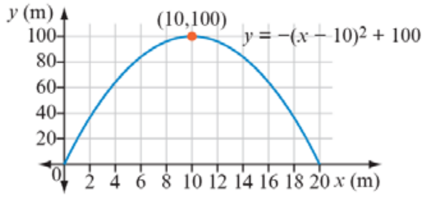
14 2. Graph is inverted so has maximum turning point. No possible larger y value than the y coordinate of the turning point, 2.

15 **a** $(2, 6)$ **b** **i** 2 m **ii** 2 m
c



d **i** 2 m **ii** 6 m

16 **a**



b 100 m **c** 20 m

17 **a** **i** narrower **ii** upright **iii** (4, −3)

b **i** narrower **ii** inverted **iii** (−1, 5)

c **i** narrower **ii** upright **iii** (−2, 0)

d **i** narrower **ii** inverted **iii** (0, −4)

e **i** the same **ii** inverted **iii** (5, 4)

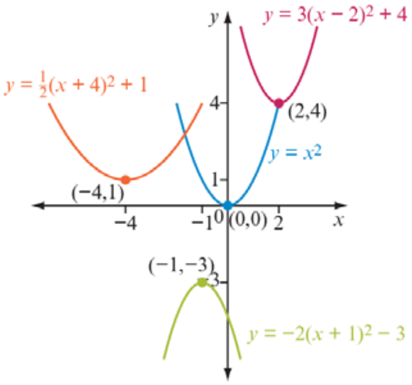
f **i** wider **ii** upright **iii** (−2, 6)

g **i** wider **ii** inverted **iii** (3, −4)

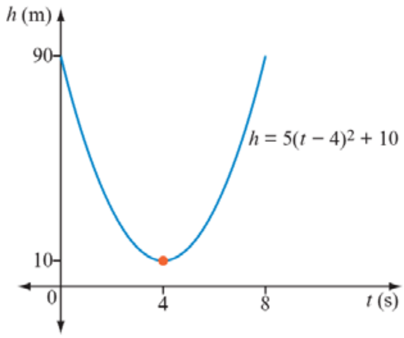
h **i** wider **ii** upright **iii** (0, 3)

i **i** wider **ii** inverted **iii** (7, 0)

18



19 **a**



b 90 m **c** 10 m

20 The graph of $y = 2x^2$ is the result of multiplying each y value in the graph of $y = x^2$ by a factor of 2. The graph of $y = -2x^2$ is the result of multiplying each y value in the graph of $y = 2x^2$ by a factor of −1. This results in an 'upside down' or inverted graph. An inverted graph is also a reflection in the x-axis.

21 **a** $y = -4x^2$ **b** $y = \frac{1}{3}x^2$

c $y = (x + 2)^2$ **d** $y = -x^2 - 5$

e $y = (x + 1)^2 + 4$ **f** $y = -2(x - 5)^2 + 3$

22 **a** $y = 3(x - 2)^2$ **b** $y = -(x + 5)^2 - 4$

c $y = -\frac{1}{2}x^2$ **d** $y = -4(x + 2)^2$

Reflect

Possible answer: Writing the quadratic relationship in turning point form allows us to find the coordinates of the turning point and identify the number of units horizontally and vertically to translate the graph of $y = x^2$.

Resources

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q5–7 relate to Exercise 4D.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

BLMs

$y = x^2$

1-cm grid paper

Interactives

4D eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4E Sketching parabolas using intercepts

Teaching support for pages 178–183

Teaching strategies

Learning focus

To be able to sketch a parabola showing the critical features of x- and y-intercepts as well as the turning point

Start thinking!

The task guides students to:

- see that a parabola whose rule is in the form $y = x^2 + bx + c$ cannot be sketched easily by using transformations
- find the y-intercept of a parabola by substituting $x = 0$
- find the x-intercepts of a parabola by substituting $y = 0$ and solving the resulting quadratic equation
- consider the coefficient of x^2 in determining whether the graph is upright or inverted
- think about using the x-intercepts of the graph to help determine the coordinates of the turning point.

Differentiated pathways

Below Level	At Level	Above Level
1–9, 10a–c, 14	1–3, 4a, c, f, 5a, c, f, 6–9, 10d–i, 11a, b, 12, 13a–c, 14, 15a–c, 16–18	1–3, 6–9, 10d–l, 11c, d, 12, 13, 15c–e, 16–20
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q7

Focus: To revise the method of finding the x- and y-intercepts of a linear function

- Direct students to complete **SS 4E-1 Finding *x*- and *y*-intercepts** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to know that:

- to find the y-intercept, they must substitute $x = 0$ into the rule
- to find the x-intercept, they must substitute $y = 0$ into the rule and solve the resulting equation.

At Level

At Level
1–3, 4a ,c, f, 5a, c, f, 6–9, 10d–i, 11a, b, 12, 13a–c, 14, 15a–c, 16–18

- Demonstrate **4E eTutor** or direct students to do this independently.
- Students will need calculator, pencil, ruler and eraser to complete this topic.
- Provide students with copies of the BLM **Cartesian plane grids** (see resources). This will assist students in drawing up a large number of Cartesian planes for their sketches.
- Students should be aware of the key features that need to be shown when sketching a parabola:
  - x-intercepts (found by solving the quadratic equation formed when $y = 0$)
  - y-intercept (found by substituting $x = 0$)
  - axis of symmetry (found by halving the distance between the x-intercepts)
  - coordinates of the turning point (y value found by substituting the x value of the axis of symmetry into the equation).
- Students may need to be reminded of what the x- and y-intercepts are and how each is found.
- In finding the x-intercepts for a parabola, students will need to solve a quadratic equation. They may need to revise that for a quadratic equation of the form:
  - $x^2 + bx = 0$ will be factorised by taking out a common factor of x
  - $x^2 – m^2 = 0$ will be factorised using the difference of two squares rule
  - $x^2 + bx + c = 0$ will be factorised using the quadratic trinomial method.

  After the quadratic is factorised, the Null Factor Law is used to find the solutions.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4E-1**. It shows how to find coordinates of the x- and y-intercepts of a quadratic relationship and will help students to complete Q1 and Q2.

- **Example 4E-2** shows how to find coordinates of the turning point using *x*-intercepts. This will help students to complete Q3 and Q4.
- **Example 4E-3** shows how to sketch a parabola using *x*- and *y*-intercepts. This will help students to complete Q6–10.
- For Q7, remind students that it is the coefficient of x^2 that determines if a parabola is upright or inverted.
- In parts f, h and i of Q10, students need to take out –1 as a common factor first before completing the factorisation.

  e.g. $y = -x^2 - 8x - 12$ when $y = 0$

  $0 = -(x^2 + 8x + 12)$

  $0 = -(x + 6)(x + 2)$

- For Q11, to find the *x*-intercepts, students will most likely need to expand and simplify the expression in order to solve the quadratic equation.
- From completing Q13, students should recognise that if the quadratic expression can be factorised as a perfect square, there will only be one *x*-intercept.
- Q15 demonstrates parabolas that have no *x*-intercepts. Students will have to draw the graphs showing the turning point and *y*-intercept only.
- In Q16, explain that the parabola drawn is a representation of the path of the arrow and questions can be answered from the graph.
- For additional practice, students can complete Q8–10 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4E Q2, Q4, Q6 and Q10 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- For more problem-solving tasks and investigations, direct students to **INV 4-2 Fencing the chicken run** (see Resources).

Below Level

Below Level
1–9, 10a–c, 14

- Students may need to complete **SS 4E-1 Finding *x*- and *y*-intercepts** (see Resources).
- For Q5, when students are sketching parabolas use the BLM **Cartesian plane grids** (see Resources). After they have found the *x*- and *y*-intercepts, assist them in working

out an appropriate scale for their axes. Remind them that the scale on the *x*- and *y*-axes need not be the same.

- For Q7, remind students that it is the coefficient of x^2 that determines if a parabola is upright or inverted.
- For students who do not progress past Q6, direct them to Q8–10 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- The idea behind this exercise is to sketch parabolas showing the *x*- and *y*-intercepts. This does not require graphs to be drawn to scale. However, students working at this level achieve more when using a scale. A scale allows students to check points on the parabola by substitution.
- Some students will still need to produce their graphs using a table of values. If students are doing this, they will easily find the *y*-intercept as $x = 0$ should be a value in their table. However, they will need more support to ensure that their graph shows any *x*-intercepts.
- A graphics calculator or graphing software may be helpful to assist students in obtaining their graphs. Use this software where appropriate to display the value of all intercepts.

Above Level

Above Level
1–3, 6–9, 10d–l, 11c, d, 12, 13, 15c–e, 16–20

- For Q7, remind students that it is the coefficient of x^2 that determines if a parabola is upright or inverted.
- In Q11, ask students to explore the *x*-intercepts in two ways.
  - Expand and simplify the expression into the form $x^2 + bx + c = 0$ and solve the equation.
  - Find the coordinates of the turning point from the turning point form that the rule is given in. Then consider the shape of $y = x^2$ and the number of units that *y* is below the *x*-axis. The square root of this number will be the number of units either side of the turning point that the *x*-intercepts will lie. For example, if the vertex is at (2, –9) the *x*-intercepts will be 3 units either side of $x = 2$ (i.e. $x = –1$ and $x = 5$).

- From completing Q13, students should recognise that if the quadratic expression can be factorised as a perfect square, there will only be one x-intercept.
- Q15 demonstrates parabolas that have no x-intercepts. Students will have to draw the graphs showing the turning point and y-intercept only.
- In Q18, explain that the parabola drawn is a representation of the path of the soccer ball and questions can be answered from the graph.
- In Q20, students can only write more quadratic rules by multiplying the entire rule by a constant.
- For more problem-solving tasks and investigations, direct students to **INV 4-2 Fencing the chicken run** (see Resources).

Extra activities

1 Quick questions

Solve each quadratic equation.

a $x^2 + 4x = 0$ $(x = 0 \text{ or } x = -4)$

b $x^2 - 16 = 0$ $(x = -4 \text{ or } x = 4)$

c $x^2 = 25$ $(x = -5 \text{ or } x = 5)$

d $x^2 + 6x + 9 = 0$ $(x = -3)$

e $x^2 - 8x - 20 = 0$ $(x = -2 \text{ or } x = 10)$

2 Some parabolas have two x-intercepts, some have one x-intercept and others have none at all. Consider the relationship $y = x^2 + 6x + 3$.

a Find the y-intercept. (3)

b Attempt to find the x-intercepts. Consider if being unable to factorise the expression on the right side means that the x-intercepts do not exist. (x-intercepts do exist but expression on RS not easy to factorise)

c The following process will convert the equation $y = x^2 + 6x + 3$ into turning point form.

Create a perfect square by halving 6 and squaring the result. Subtract 9 to compensate. The constant term should still total 3. Factorise the perfect square expression.

$y = x^2 + 6x + 3$

$= (x^2 + 6x + 9) - 9 + 3$

$= (x^2 + 6x + 9) – 6$

$= (x + 3)^2 – 6$

d What are the coordinates of the turning point? [(–3, –6)]

e Sketch the graph of $y = x^2 + 6x + 3$ showing the turning point and y-intercept.

f How many x-intercepts does the parabola have? (2)

g Using digital technology, draw the parabola. Find two whole numbers between which each x-intercept lies. (–6 and –5, –1 and 0)

h Repeat this process with the following parabolas to determine if they have two, one or no x-intercepts.

i $y = x^2 + 8x + 20$ (no x-intercepts)

ii $y = x^2 – 4x – 2$ (two x-intercepts)

iii $y = x^2 – 6x + 9$ (one x-intercept)

Answers

4E Sketching parabolas using intercepts

4E Start thinking!

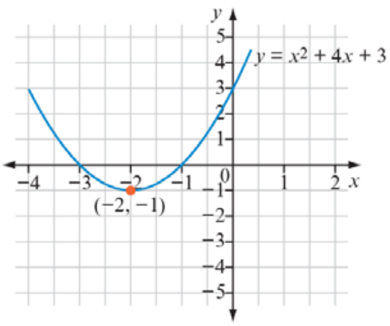
1 When a quadratic relationship is in turning point form, it is easy to identify the transformations applied to $y = x^2$ to result in the given relationship. The general form of a quadratic, that is, $y = ax^2 + bx + c$, does not follow the same rules as turning point form.

2 a $x = 0$ b $y = 3$ c 3

3 a $y = 0$ b $0 = x^2 + 4x + 3$
c $x = -3$ or $x = -1$ d -3 and -1

4 (0, 3), (−3, 0) and (−1, 0)

5 upright

6 

7 Axis of symmetry of a parabola is halfway between x-intercepts. Hence, x-coordinate of turning point is halfway between x values at x-intercepts. y-coordinate of turning point is found by substituting x-coordinate into rule and simplifying.

8 Can sketch quadratic relationships using x- and y-intercepts. x-intercept/s are found by substituting $y = 0$ into rule and solving for x. Equation may need to be factorised first so that Null Factor Law can be used. A parabola can have two, one or no x-intercepts. y-intercept is found by substituting $x = 0$ into rule and simplifying. The x-coordinate of turning point is halfway between x values at x-intercepts. y-coordinate of turning point is found by substituting x-coordinate into rule and simplifying. Orientation of parabola (upright or inverted) can be identified from coefficient of x^2 term: if $a > 0$, parabola is upright and if $a < 0$, parabola is inverted.

Exercise 4E Sketching parabolas using intercepts

1 **a** coordinates of x-intercepts (–3, 0) and (5, 0)
coordinates of y-intercept (0, –15)

c coordinates of x-intercepts (–1, 0) and (1, 0)
coordinates of y-intercepts (0, –1)

2 **a** **i** (0, 0) and (2, 0) **ii** (0, 0)

b **i** (−8, 0) and (0, 0) **ii** (0, 0)

c **i** (−4, 0) and (−2, 0) **ii** (0, 8)

d **i** (2, 0) and (6, 0) **ii** (0, 12)

e **i** (−1, 0) and (5, 0) **ii** (0, −5)

f **i** (−3, 0) and (3, 0) **ii** (0, −9)

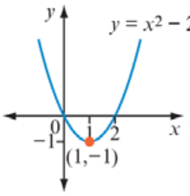
3 **a** coordinates of turning point (1, –16).

b coordinates of turning point (0, –1).

4 **a** (1, −1) **b** (−4, −16) **c** (−3, −1)

d (4, −4) **e** (2, −9) **f** (0, −9)

5 **a**



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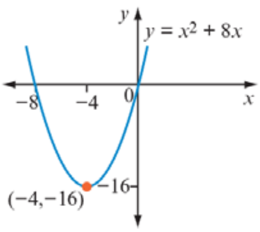

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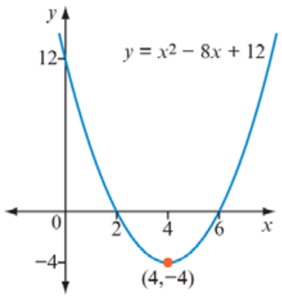

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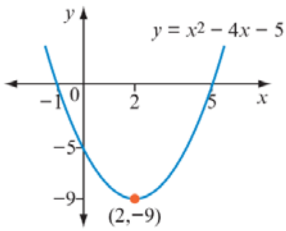

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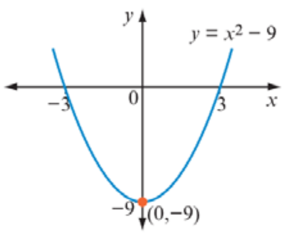
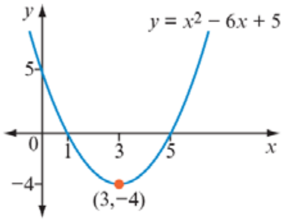


6 a


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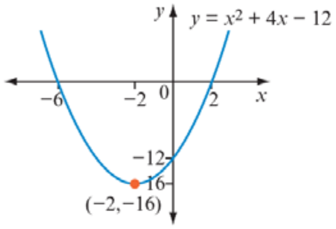

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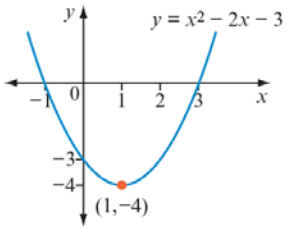

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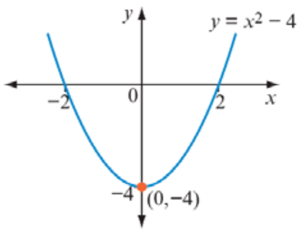

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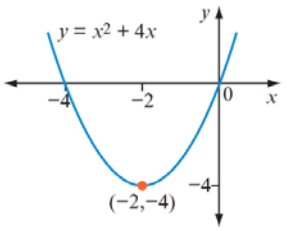

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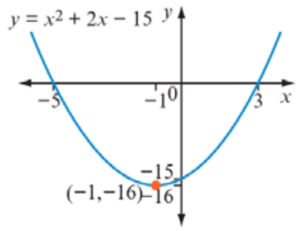

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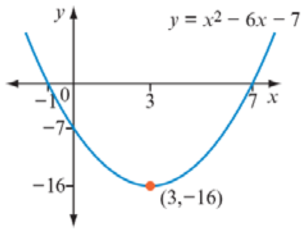

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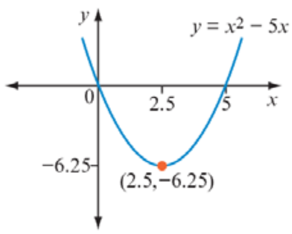


7 a C b A c B

8 Look at coefficient of x^2 term (a). If $a > 0$, parabola is upright and if $a < 0$, parabola is inverted.

9 **a** Both have x-intercepts of -3 and 2.

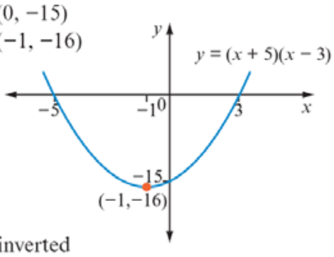
b One graph has negative x^2 coefficient and other has positive x^2 coefficient, so one will be inverted and the other upright.

10 **a** **i** upright

ii x-intercepts: $(-5, 0)$ and $(3, 0)$; y-intercept: $(0, -15)$

iii $(-1, -16)$

iv

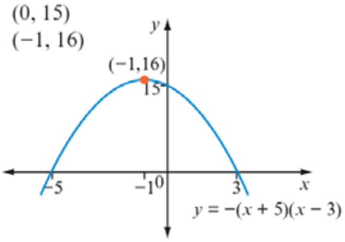


b **i** inverted

ii x-intercepts: $(-5, 0)$ and $(3, 0)$; y-intercept: $(0, 15)$

iii $(-1, 16)$

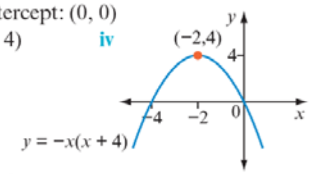
iv



c **i** inverted

ii x-intercepts: $(-4, 0)$ and $(0, 0)$; y-intercept: $(0, 0)$

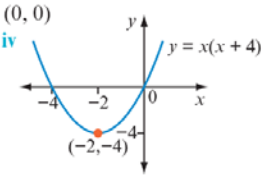
iii $(-2, 4)$ **iv**



d **i** upright

ii x-intercepts: $(-4, 0)$ and $(0, 0)$; y-intercept: $(0, 0)$

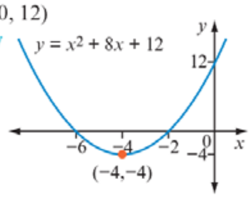
iii $(-2, -4)$ **iv**



e **i** upright

ii x-intercepts: $(-6, 0)$ and $(-2, 0)$; y-intercept: $(0, 12)$

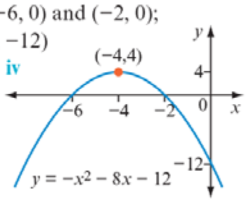
iii $(-4, -4)$ **iv**



f **i** inverted

ii x-intercepts: $(-6, 0)$ and $(-2, 0)$; y-intercept: $(0, -12)$

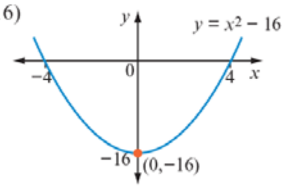
iii $(-4, 4)$ **iv**



g **i** upright

ii x-intercepts: $(-4, 0)$ and $(4, 0)$; y-intercept: $(0, -16)$

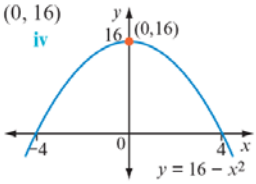
iii $(0, -16)$ **iv**



h **i** inverted

ii x-intercepts: $(-4, 0)$ and $(4, 0)$; y-intercept: $(0, 16)$

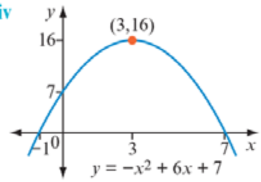
iii $(0, 16)$ **iv**



i **i** inverted

ii x-intercepts: $(-1, 0)$ and $(7, 0)$; y-intercept: $(0, 7)$

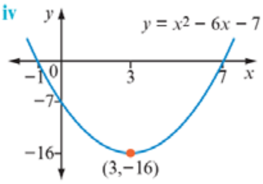
iii $(3, 16)$ **iv**



j **i** upright

ii x-intercepts: $(-1, 0)$ and $(7, 0)$; y-intercept: $(0, -7)$

iii $(3, -16)$ **iv**

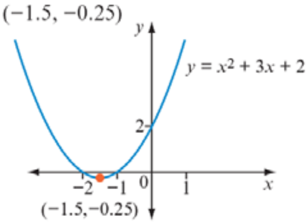


k **i** upright

ii x-intercepts: $(-2, 0)$ and $(-1, 0)$; y-intercept: $(0, 2)$

iii $(-1.5, -0.25)$

iv

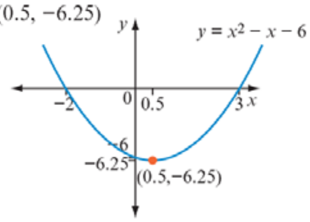


l **i** upright

ii x-intercepts: $(-2, 0)$ and $(3, 0)$; y-intercept: $(0, -6)$

iii $(0.5, -6.25)$

iv

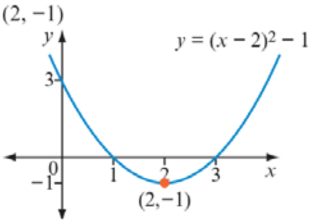


11 **a** **i** upright

ii x-intercepts: (1, 0) and (3, 0);
y-intercept: (0, 3)

iii (2, −1)

iv

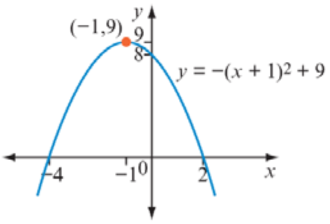


b **i** inverted

ii x-intercepts: (−4, 0) and (2, 0);
y-intercept: (0, 8)

iii (−1, 9)

iv

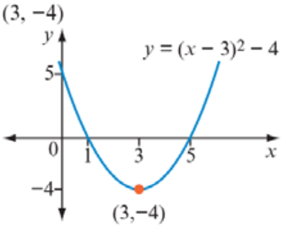


c **i** upright

ii x-intercepts: (1, 0) and (5, 0);
y-intercept: (0, 5)

iii (3, −4)

iv

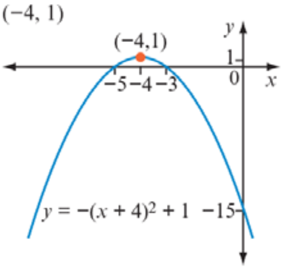


d **i** inverted

ii x-intercepts: (−5, 0) and (−3, 0);
y-intercept: (0, −15)

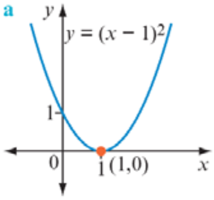
iii (−4, 1)

iv

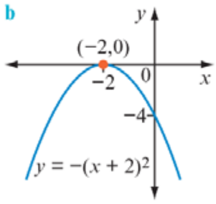


12 **a** upright **b** one (0, 9) **c** one (3, 0)
d (3, 0)

13 **a**



b



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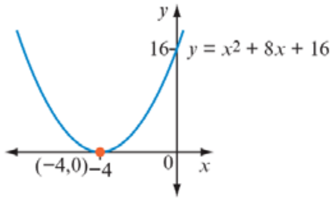

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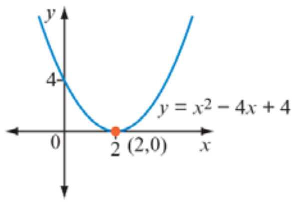

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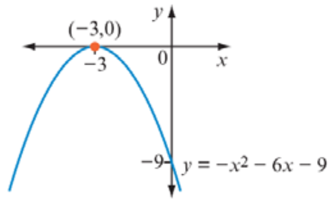
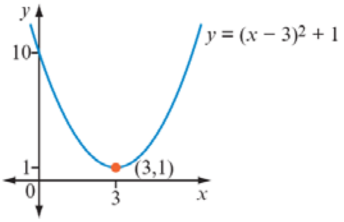


14 **a** inverted **b** one (0, −1) **c** none
d (0, −1)

15 a



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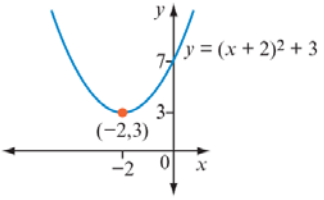

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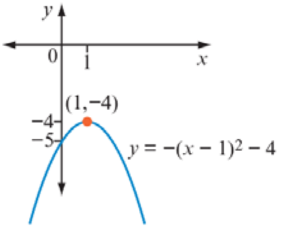

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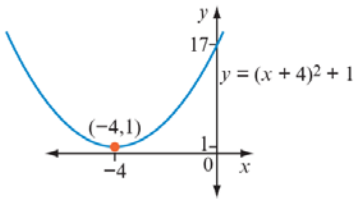

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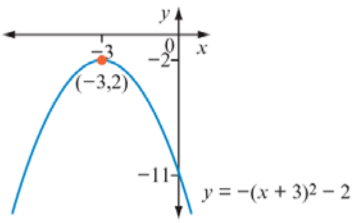
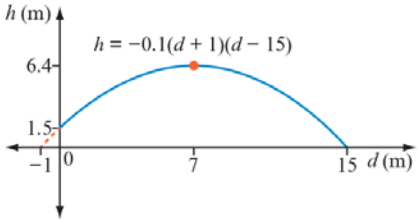
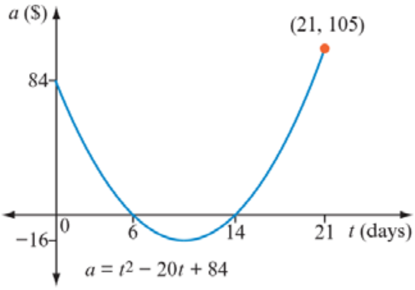


16 a



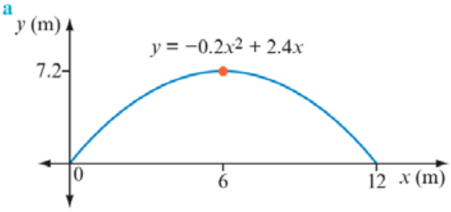
b 6.4 m **c** 1.5 m **d** 15 m

17 a



b $84 **c** $48 **d** just after sixth day
e $16 **f** after 14th day
g $105 (at end of the three weeks)

18 a



b 7.2 m **c** 6 m **d** 12 m

19, 20 Some possible answers are given.

a $y = (x - 7)(x - 2)$ or $y = -(x - 7)(x - 2)$ or $y = -6(x - 7)(x - 2)$

b $y = x^2 - 8x$ or $y = -x(x - 8)$ or $y = -4x(x - 8)$

c $y = (x + 4)(x - 5)$ or $y = -(x + 4)(x - 5)$ or $y = -3(x + 4)(x - 5)$

Reflect

Possible answer: A parabola may have two, one or no x-intercepts but will always have one y-intercept.

Resources

SupportSheet

SS 4E-1 Finding x- and y-intercepts

Focus: To explore where x- and y-intercepts occur and determine intercepts for linear relationships

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students look at linear graphs and from the graph they observe that the x-intercept occurs when $y = 0$ and the y-intercept occurs when $x = 0$. Students then obtain these values by substitution and evaluation rather than requiring the graph to be drawn.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q8–10 relate to Exercise 4E.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

Investigation

INV 4-2 Fencing the chicken run

Focus: To determine the largest enclosure that can be made with a limited length of fencing

Resources: 1-cm grid paper (BLM) or graph paper, calculator

Students consider the problem of building the fence for a rectangular chicken run. They consider the different dimensions that the chicken run could have, given a fixed length of fencing, with the aim to maximise the area contained in the chicken run.

Students explore this problem for two scenarios:

- all four sides of the chicken run need to be fenced
- three sides of the chicken run need fencing, using an existing structure as one side.

For both scenarios, students will model the problem using a quadratic relationship where the coordinates of the turning point will represent the solution to the problem.

As an extension, students draw a design for the perimeter of the enclosure which includes the positions of fence posts.

BLMs

Cartesian plane grids

1-cm grid paper

Interactives

4E eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4F Circles and other non-linear relationships

Teaching support for pages 184–189

Teaching strategies

Learning focus

To look at other non-linear relationships such as the circle, hyperbola and cubic graph

Start thinking!

The task guides students to:

- recognise the rule for a circle with its centre at the origin
- determine the radius of a circle with its centre at the origin
- consider the translations performed on a circle with its centre at the origin and, in particular, the effect on the coordinates of the centre
- recognise the form of a rule for a circle with the centre not at the origin
- determine the centre and radius of a circle with the centre not at the origin.

Differentiated pathways

Below Level	At Level	Above Level
1–7, 9–13, 15a–c, 16a–c	1, 3–6, 8–12, 14–16	1, 3–6, 8, 9, 11c, d, 12, 14–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 3–6, 8–12, 14–16

- Demonstrate **4F eTutor** or direct students to do this independently.
- To complete this topic, students will need calculator, ruler, pencil and eraser.
- To assist students with drawing many graphs, provide copies of the BLM **Cartesian plane grid** (see Resources).

- Remind students that relationships can be classified as being linear or non-linear. In a linear relationship both x and y are to a power of 1. Any other relationship is non-linear.
- Students should now recognise that when the highest power of x is 2 the graph will be a parabola.
- Most of this topic is about the graph of a circle. A circle has a rule where both x and y are squared.
- Discuss that the most basic form of a circle is $x^2 + y^2 = r^2$. This graph has:
  - its centre at (0, 0)
  - a radius of r units.
- Explain that a circle with its centre not at (0, 0) is of the form $(x - h)^2 + (y - k)^2 = r^2$. This graph has:
  - its centre at the point with coordinates (h, k)
  - a radius of r units.
- To draw this graph the graph of $x^2 + y^2 = r^2$ is translated h units horizontally and k units vertically.
- Other graphs that students should be able to recognise after completing this topic are:
  - simple cubic relationships of the form $y = x^3 + k$
  - square root relationships of the form $y = \sqrt{x - h}$

    Optional:
  - reciprocal relationships (hyperbolas) of the form $y = \dfrac{1}{x - h}$.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- In Q8, students will need to explain the translation that is made in both the horizontal and vertical directions. With the negative sign in the general form of a circle, the direction of the translation may be counter-intuitive to some students.
- Direct students to **Example 4F-1**. It shows how to sketch a circle from its rule and will help students to complete Q9.
- In Q9, an alternative method for students to find the centre of a circle is to consider the expression in each pair of brackets equal to zero and solve each linear equation to find x and y. This method ensures they have the correct sign.

- You will need to ensure students have access to graphics calculators or graphing software such as *GeoGebra* to complete Q10.
- **Example 4F-2** shows how to write the rule for a circle. This will help students to complete Q11–13.
- In Q15, students plot the graph of $y = x^3$. Discuss with students why this graph has negative values of y but $y = x^2$ does not. Students are guided to see that performing a translation on the curve is best done by translating the point of inflection.
- In Q16, students plot the graph of $y = \sqrt{x}$ and identify its key feature which is the point where y is a minimum. Again, students use their understanding of performing translations to produce sketch graphs of other square root relationships.
- You may like some students to work on part (say, a–c) or all of Q17 which relates to plotting a hyperbola and using transformations to sketch other hyperbolas. This is a more difficult question and introduces the idea of asymptotes.
- For additional practice, students can complete Q1–4 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4F Q1, Q9, Q11 and Q16 are provided. If students have completed Ex 4F Q17 relating to hyperbolas, they can also complete Q5 of WS 4-2. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Below Level

Below Level
1–7, 9–13, 15a–c, 16a–c

- Consistent reminders may be needed for students to take the square root of the constant to find the radius.
- Use the BLM **Cartesian plane grids** (see Resources) to save students time.
- Provide pairs of compasses for students to draw circles.
- In Q1–4, students need to be reminded that all circles with rule of the form $x^2 + y^2 = r^2$ have centre at (0, 0). They only need to concentrate on the relationship between the constant term and the radius.
- In Q5 and Q6, emphasise the connection between the relationship for a circle written in the general form of $(x - h)^2 + (y - k)^2 = r^2$ and the coordinates (h, k) for the centre of this circle.
- You will need to ensure students have access to graphics calculators or graphing software such as *GeoGebra* to complete Q10.

- For Q11, have students write the rule $(x - h)^2 + (y - k)^2 = r^2$ and identify the values for h and k before carefully substituting into the rule.

  e.g. Q11b $(x - h)^2 + (y - k)^2 = r^2$, $h = -2$ and $k = 4$

  $(x - -2)^2 + (y - 4)^2 = 5^2$

  $(x + 2)^2 + (y - 4)^2 = 25$

- For Q12, ensure students write the radius and the coordinates of the centre of the circle first before using the method of Q11.

- For additional practice, students can complete Q1–4 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4F Q1, Q9, Q11 and Q16 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Above Level

Above Level
1, 3–6, 8, 9, 11c, d, 12, 14–18

- In Q8, students will need to explain the translation that is made in both the horizontal and vertical directions. With the negative sign in the general form of a circle, the direction of the translation may be counter-intuitive to some students.

- In Q9, an alternative method for students to find the centre of a circle is to consider the expression in each pair of brackets equal to zero and solve each linear equation to find x and y. This method ensures they have the correct sign.

- You will need to ensure students have access to graphics calculators or graphing software such as *GeoGebra* to complete Q10.

- In Q15, students plot the graph of $y = x^3$. Discuss with students why this graph has negative values of y but $y = x^2$ does not. Students are guided to see that performing a translation on the curve is best done by translating the point of inflection.

- In Q16, students plot the graph of $y = \sqrt{x}$ and identify its key feature which is the point where y is a minimum. Again, students use their understanding of performing translations to produce sketch graphs of other square root relationships. Ask them to compare the graph of this relationship to that of $y^2 = x$. Explain that as the square root only refers to the positive square root, the graph of $y = \sqrt{x}$ is only half of the full 'sideways' parabola.

- In Q17, students plot the graph of the hyperbola $y = \dfrac{1}{x}$. Discussion points include:

  - why the graph does not exist at $x = 0$
  - the concept of a limit. To what value does y approach as x gets larger?
  - why the value of y can never equal 0.

- Q18 develops some important concepts that have been established with parabolas but can be applied to all graphs.
- For additional practice, students can complete Q1–5 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4F Q1, Q9, Q11, Q16 and Q17 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Extra activities

1 Quick questions

Calculate each value.

a $\sqrt{64}$ (8)

b $\sqrt{1}$ (1)

c 9^2 (81)

d 12^2 (144)

2 In Q16, you looked at the graph of $y = \sqrt{x}$. This graph could be described as 'half of a sideways parabola'.

a Why is the graph only half of a parabola? (square root means positive)

b If we consider this to be the top half of a parabola, what rule would create the bottom half of the graph? ($y = -\sqrt{x}$)

3 Consider the circle given by the rule $x^2 + y^2 = 4$.

a Identify the radius and coordinates of the centre of the circle. [radius of 2 units, centre at (0, 0)]

b Pepper's teacher asks her to make y the subject of this equation. Pepper's working is shown below.

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

There is an error (very small) in Pepper's working. What is the error? (last line should read $y = \pm\sqrt{4 - x^2}$)

c Try to draw the graph of $y = \sqrt{4 - x^2}$ and explain how it differs from $x^2 + y^2 = 4$. (semicircle; it is the top half of the graph of $x^2 + y^2 = 4$)

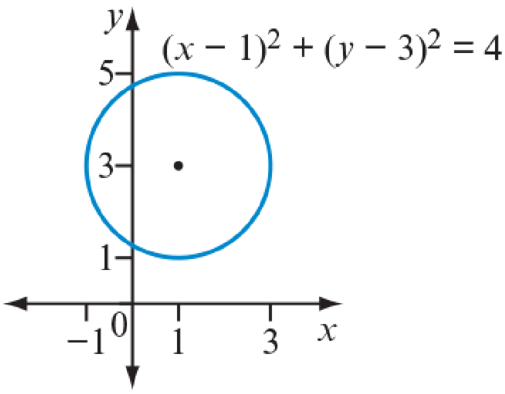
Answers

4F Circles and other non-linear relationships

4F Start thinking!

1 a i (0, 0) ii (0, 0) iii (0, 0)
b i 2 units ii 5 units iii 9 units
c Each has centre at (0, 0) and $x^2 + y^2$ written on left side of rule. Radius is equal to square root of number on right side of rule.

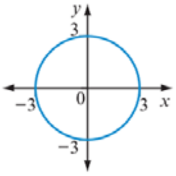
2 a i (1, 0) ii (0, 3)
b i translated 1 unit right
ii translated 3 units up
c Each circle has radius 2 units. Values that describe the translations help to determine coordinates of centre of circle.
d Graph of $x^2 + y^2 = 4$ translated 1 unit right and 3 units up.



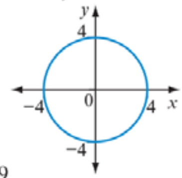
Exercise 4F Circles and other non-linear relationships

1 **a** centre (0, 0), radius = 6 units
b centre (0, 0), radius = 8 units
c centre (0, 0), radius = 1 unit
d centre (0, 0), radius = 10 units

2 **a** (0, 0) **b** 3 units
c centre and points on circle directly above, below, left and right of centre
d, e



3 $x^2 + y^2 = 16$



4 $x^2 + y^2 = 7^2$ or $x^2 + y^2 = 49$

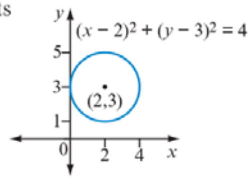
5 **a** $h = 2$, $k = 5$, $r = 3$
b (2, 5) **c** 3 units

6 **i** **a** $h = 1$, $k = 4$, $r = 4$
b (1, 4) **c** 4 units
ii **a** $h = 6$, $k = -3$, $r = 5$
b (6, −3) **c** 5 units
iii **a** $h = -4$, $k = 2$, $r = 1$
b (−4, 2) **c** 1 unit
iv **a** $h = -7$, $k = -2$, $r = 6$
b (−7, −2) **c** 6 units

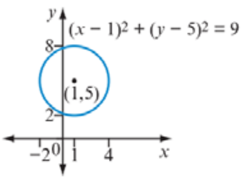
7 **a** (−2, 0) **b** 7 units
c Since $h = -2$ and $k = 0$, centre coordinates (h, k) are (−2, 0). Since $r = 7$, the radius is 7 units.
d translated 2 units left

8 **a** **i** (3, −1) **ii** (−4, −2)
b **i** 5 units **ii** 4 units
c **i** translated 3 units right and 1 unit down
ii translated 4 units left and 2 units down

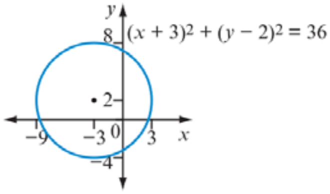
9 **a** centre (2, 3); radius = 2 units



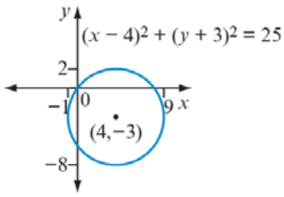
b centre (1, 5); radius = 3 units



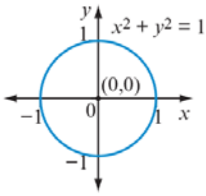
c centre (−3, 2); radius = 6 units



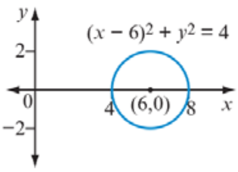
d centre (4, −3); radius = 5 units



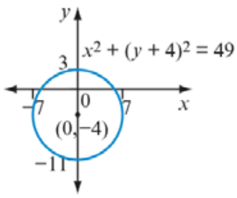
e centre (0, 0); radius = 1 units



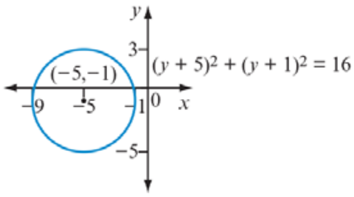
f centre (6, 0); radius = 2 units



g centre (0, −4); radius = 7 units



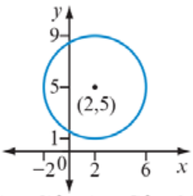
h centre (−5, −1); radius = 4 units



11 a $(x-3)^2+(y-5)^2=16$
b $(x+2)^2+(y-4)^2=25$
c $(x+7)^2+(y+6)^2=81$
d $(x-4)^2+(y+8)^2=121$

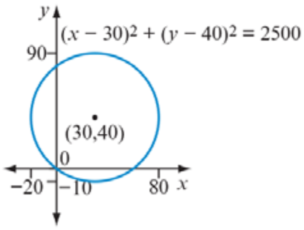
12 a centre (4, 2); radius = 2 units
$(x-4)^2+(y-2)^2=4$
b centre (−3, 3); radius = 1 units
$(x+3)^2+(y-3)^2=1$
c centre (−5, −2); radius = 3 units
$(x+5)^2+(y+2)^2=9$

13



$(x-2)^2+(y-5)^2=16$

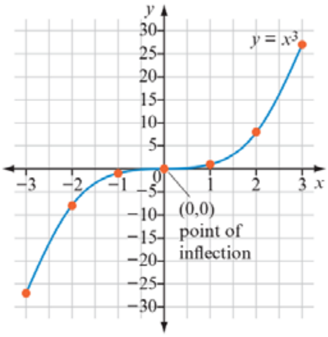
14 a



b 314 m c 7854 m^2

15 a, c

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

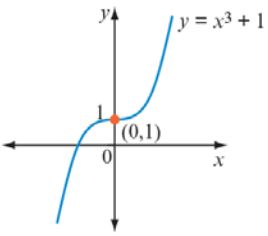


b As x increases, y increases. Graph appears to level out at $x = 0$ before rising more steeply again.

d **i** (0, 2) **ii** translate graph 2 units up

e **i** translate 1 unit up
ii translate 3 units down
iii translate 2 units right
iv translate 4 units left
v translate 1 unit right and 2 units up

f **i**



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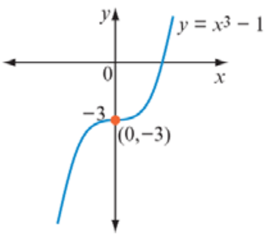

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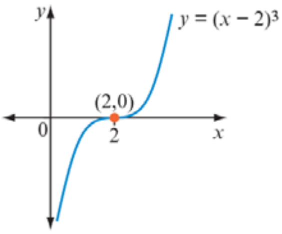

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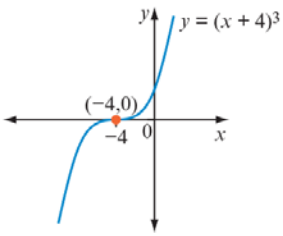

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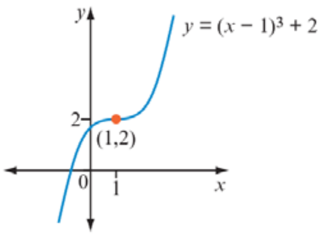
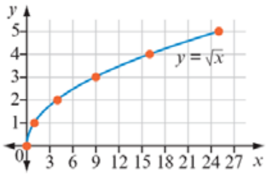


16 a, c

x	0	1	4	9	16	25
y	0	1	2	3	4	5

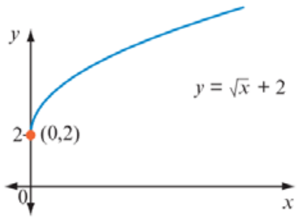


b No; not possible to take square root of negative number.

d **i** (3, 0)
ii translate 3 units to right

e **i** translate 2 units up
ii translate 1 unit down
iii translate 1 unit right
iv translate 4 units left
v translate 2 units right and 3 units up

f **i**



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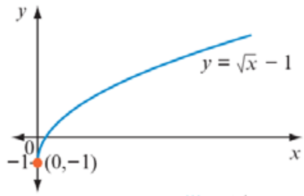

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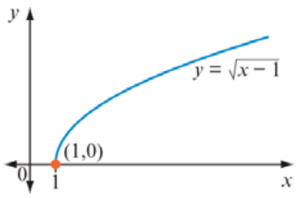

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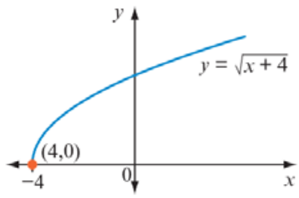

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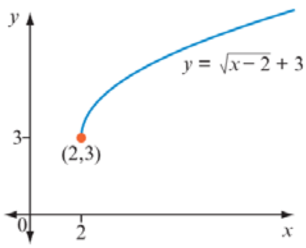
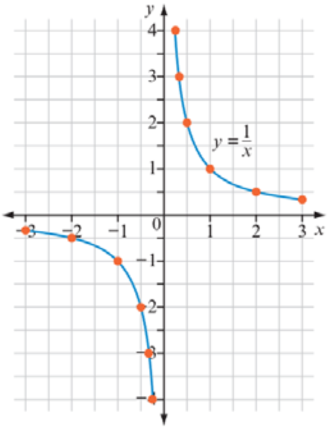


17 **a**

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	-3	-4	4	3	2	1	$\frac{1}{2}$	$\frac{1}{3}$



b y is undefined when $x = 0$. As x approaches 0 from left, graph approaches $-\infty$ (very small y values) and as x approaches 0 from right, graph approaches $+\infty$ (very large y values).

c **i** $y = 0$ **ii** $x = 0$

d **i** $y = 0$ (horizontal) and $x = 2$ (vertical)

ii translate 2 units right

e **i** translate horizontal asymptote 2 units up

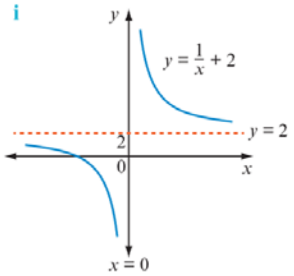
ii translate horizontal asymptote 3 units down

iii translate vertical asymptote 4 units right

iv translate vertical asymptote 1 unit left

v translate vertical asymptote 3 units right and horizontal asymptote 2 units up

f **i**



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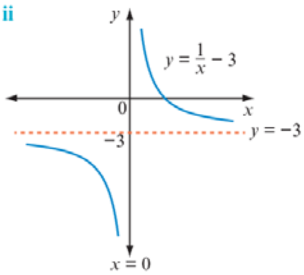

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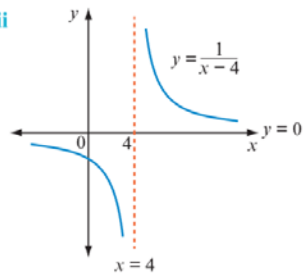

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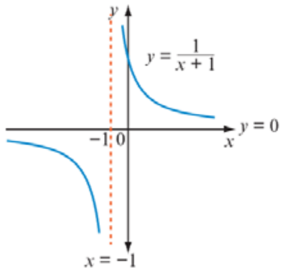

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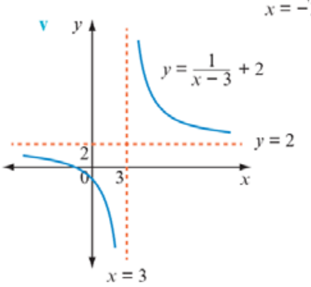
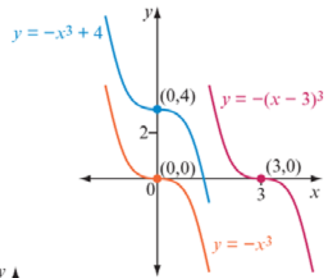


18 **a**



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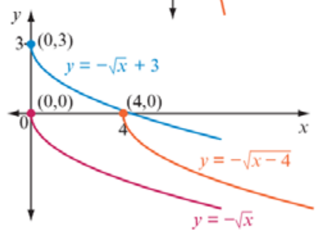

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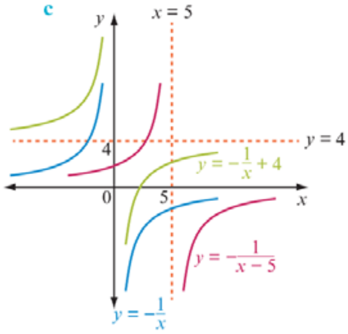


Reflect

Possible answer: The radius of a circle is found by taking the square root of the constant term. The centre can be found by setting the expression in each pair of brackets equal to zero and solving the equations to find x and y.

Resources

WorkSheet

WS 4-2 More non-linear relationships

Focus: To understand the key features of circles, square root functions and hyperbolas and to develop rules for linear and non-linear relationships using direct proportion

Resources: ruler, sketching aid (optional), **1-cm grid paper** (BLM) or graph paper (optional)

- This WorkSheet provides a skills review for Exercises 4F–4G. Q1–5 relate to Exercise 4F.

Students work with circles to determine the radius and the coordinates of the centre based on the form of the relationship and hence sketch its graph. They describe and perform transformations on other non-linear relationships including the square root function and the hyperbola and sketch their graphs. They also test for linear and non-linear relationships between two variables involving direct proportion.

BLMs

Cartesian plane grids

1-cm grid paper

Interactives

4F eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4G Relationships and direct proportion

Teaching support for pages 190–195

Teaching strategies

Learning focus

To consider relationships between quantities that vary in proportion to each other

Start thinking!

The task guides students to:

- recognise a direct relationship between two quantities or variables
- see how in a direct relationship, one quantity increases (or decreases) as the other increases (or decreases)
- define and identify the constant of proportionality and see that it is the gradient of the linear graph that represents the relationship between the two quantities
- define the general form of the relationship between x and y as $y = kx$ where y is directly proportional to x and k is the constant of proportionality
- see that $y = kx$ is treated the same way as the linear relationship $y = mx$ where m is the gradient.

Differentiated pathways

Below Level	At Level	Above Level
1–4, 6–8, 9a, b, 10a, b, 11–13	1–6, 7c–e, 8, 9a–d, 10a–d, 11–14	1–6, 8c–f, 9c–f, 10c–f, 11c, 12c, 14, 15
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q8

Focus: To revise linear relationships of the form $y = mx$.

- Direct students to complete **SS 4G-1 Finding the gradient of a linear graph** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.

- Students need to recognise that the coefficient of x is the gradient of a linear relationship in the form $y = mx$.

At Level

At Level
1–6, 7c–e, 8, 9a–d, 10a–d, 11–14

- Demonstrate **4G eTutor** or direct students to do this independently.
- To complete this topic, students will need a calculator, ruler, pencil, and eraser.
- Introduce the idea of direct proportion as having the following features.
  - As one quantity increases or decreases, the other does as well.
  - One quantity can be multiplied by a constant to obtain the value of the other. Define this constant as the *constant of proportionality*.
- Explain that price is a simple example of direct proportion (or direct variation). If one can of soft drink costs $3 then what is the cost of 5 cans? Explain that, in this case:
  - the constant of proportionality is 3
  - the graph will be a linear relationship starting at (0, 0) with a gradient of 3.
- Explain that as students work through the exercise they will meet relationships where one quantity varies with the square or square root of the other.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4G-1**. It shows how to identify whether a relationship shows direct proportion and will help students to complete Q1.
- **Example 4G-2** shows how to find the rule for a linear relationship using direct proportion. This will help students to complete Q3.
- **Example 4G-3** shows how to find the rule for a non-linear relationship using direct proportion. This will help students to complete Q5.
- In Q6, students need to see that only the graphs of straight lines that pass through (0, 0) represent a direct proportion.
- **Example 4G-4** shows how to find the constant of proportionality from given information. This will help students to complete Q8 and Q9.
- In Q9, explain that the symbol α means ‘proportional to’. Hence, y α x can become $y = kx$. Then the method of Q8 can be used to finish the question.

- For additional practice, students can complete Q6–10 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4G Q1, Q3 and Q5 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 How much gold is actually in your gold jewellery?** (see Resources).

Below Level

Below Level
1–4, 6–8, 9a, b, 10a, b, 11–13

- Students will only need a brief exposure to proportions that vary with anything other than *x*.
- Concentrate on defining and finding the constant of proportionality by solving an equation.
- Explain what is meant by the proportionality statement $y \alpha x$ and how it is equivalent to $y = kx$.
- Students may need to complete **SS 4G-1 Finding the gradient of a linear graph** (see Resources).
- In Q1, students will need reminding they are looking for the value of $\frac{y}{x}$ to be the same.
- In Q7, students need to be reminded they are looking for the number by which the independent variable is being multiplied.
- For Q11, some students will need a reminder that gradient = $\frac{\text{rise}}{\text{run}}$ and this is equal to the constant of proportionality. Explain that for Q12, the proportionality sign α is replaced by an equals sign together with the constant of proportionality to create the rule for the relationship.
- For students who do not progress past Q5, direct them to Q6–10 of **WS 4-2 More non-linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Above Level

Above Level

1–6, 8c–f, 9c–f, 10c–f, 11c, 12c, 14, 15

- In Q6, students need to see that only the graphs of straight lines that pass through (0, 0) represent a direct proportion.
- In Q9, explain that the symbol α means 'proportional to'. Hence, $y \alpha x$ can become $y = kx$. Then the method of Q8 can be used to finish the question.
- For additional practice, students can complete Q6–10 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4G Q1, Q3 and Q5 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 How much gold is actually in your gold jewellery?** (see Resources).

Extra activities

1 Solve each equation.

a $4x = 44$ $(x = 11)$

b $\frac{50}{x} = 12.5$ $(x = 4)$

c $9x = 6$ $(x = \frac{2}{3})$

d $\frac{6}{x} = 4$ $(x = 1.5)$

2 An example where one quantity varies in proportion with the square of another is the area of a circle.

a We can say $A \alpha r^2$ and hence $A = kr^2$. In this case, what is the value of the constant of proportionality? $(k = \pi)$

b Can you think of another example where one quantity varies as the square of another?

c Give an example where one quantity varies as the:

i cube of another quantity (for a cube, $V \propto l^3$; for a sphere, $V \propto r^3$)

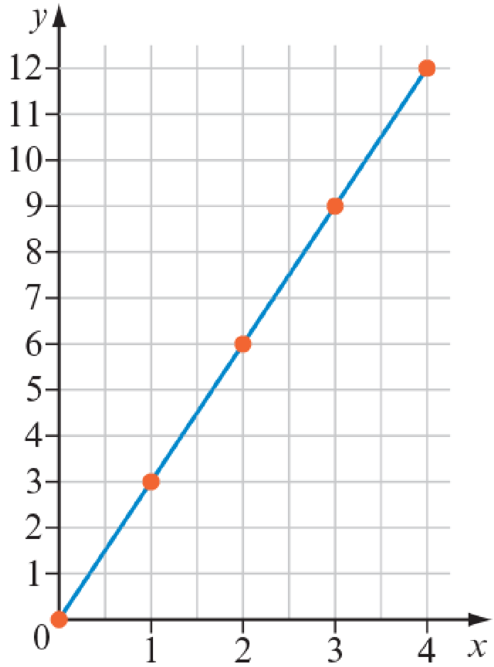
ii square root of another quantity.

Answers

4G Relationships and direct proportion

4G Start thinking!

1 a



b linear

c Increase; as increase is by same amount each time, change is constant.

d $\frac{y}{x} = 3$

e Gradient is 3. Rate of change is same as gradient and is constant for a linear relationship.

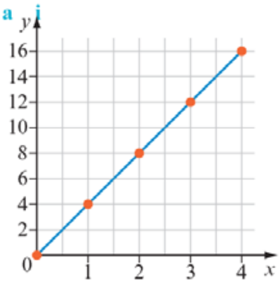
2 Graph is linear and passes through the origin (0, 0).

3 $y = 3x$

Exercise 4G Relationships and direct proportion

1 a and c show direct proportion

2 a i

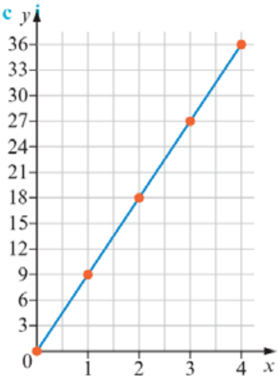


ii 4

iii They are the same.

iv $y = 4x$

c i



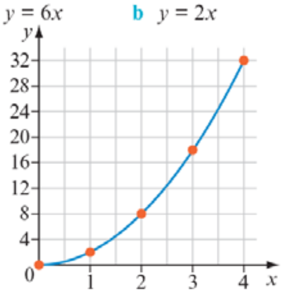
ii 9

iii They are the same.

iv $y = 9x$

3 a $y = 6x$ b $y = 2x$ c $y = \frac{1}{2}x$

4 a



b non-linear

c Increase; as increase is not by same amount each time, change is not constant.

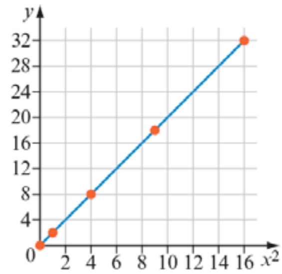
d 2, 4, 6, 8. Value for $\frac{y}{x}$ is different for each set of values.

e Not in direct proportion as $\frac{y}{x}$ is not constant.

f

x^2	0	1	4	9	16
y	0	2	8	18	32

g



h Yes, as it forms a straight line passing through (0, 0).

i–k $y = 2x^2$

5 a $y = 3x^2$ b $y = 7x^2$ c $y = 4x^2$

6 a no direct proportion (not a straight line passing through (0, 0))

b direct proportion (straight line passing through (0, 0))

c no direct proportion (straight line but does not pass through (0, 0))

d direct proportion (straight line passing through (0, 0))

e no direct proportion (not a straight line passing through (0, 0))

f no direct proportion (straight line but does not pass through (0, 0); also, y decreases as x increases)

7 a 4 b 3.5 c 6 d 10 e 2

8 a 10 b 8 c 4 d 7 e 44 f $\frac{1}{2}$

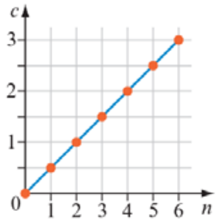
9 a 5 b 1.25 c 2 d 7 e 27 f 2.5

10 a $y = 5x$ b $p = 1.25q^2$ c $d = 2c^3$
d $h = 7\sqrt{g}$ e $a = \frac{27}{m}$ f $w = 2.5v$

11 a 3 b 4 c 8

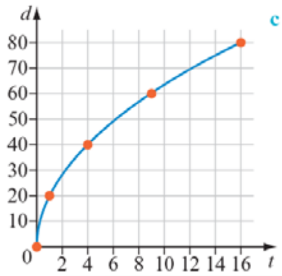
12 a $y = 3x^2$ b $y = 4\sqrt{x}$ c $y = \frac{8}{x}$

13 a



b linear
c yes, as forms a straight-line graph that passes through (0, 0)
d $c \propto n$; $c = kn$
e gradient $= \frac{1}{2}$, $c = \frac{1}{2}n$
f $10

14 a

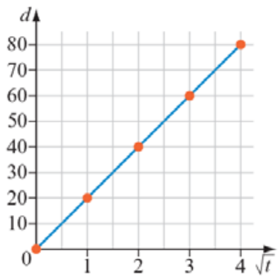


b non-linear
c no, as graph is not a straight line passing through (0, 0)

d

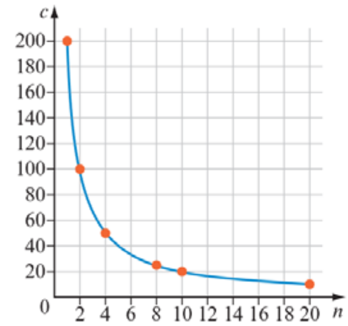
$\sqrt{t}$	0	1	2	3	4
d	0	20	40	60	80

e



f Yes; it forms a straight-line graph that passes through (0, 0).
g $d \propto \sqrt{t}$; $d = k\sqrt{t}$
h gradient = 20, $d = 20\sqrt{t}$ i 120 km

15 a

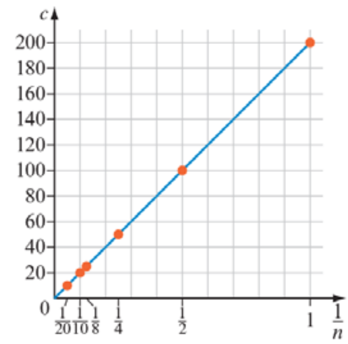


b non-linear
c No; graph is not a straight line passing through (0, 0).

d

$\frac{1}{n}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{20}$
c	200	100	50	25	20	10

e



f Yes; it forms a straight-line graph that passes through (0, 0).
g $c \propto \frac{1}{n}$; $c = \frac{k}{n}$
h gradient = 200, $c = \frac{200}{n}$
i $200 j $16.67

Reflect

Possible answer: Direct proportion can be used to work out the rule for a non-linear relationship by exploring other powers of the independent variable. For example, compare one quantity with the square, cube or square root of the other.

Resources

SupportSheet

SS 4G-1 Finding the gradient of a linear graph

Focus: To review the method for calculating the gradient of a linear graph

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students look at linear graphs and revise the concept that gradient is the measure of the slope of a line. They consider that a positive gradient slopes upwards to the right while a negative gradient slopes downwards to the right. Students will also calculate the gradient of a linear graph using both gradient = $\frac{\text{rise}}{\text{run}}$ and gradient = $\frac{y_2 - y_1}{x_2 - x_1}$.

WorkSheet

WS 4-2 More non-linear relationships

Focus: To understand the key features of circles, square root functions and hyperbolas and to develop rules for linear and non-linear relationships using direct proportion

Resources: ruler, sketching aid (optional), **1-cm grid paper** (BLM) or graph paper (optional)

- This WorkSheet provides a skills review for Exercises 4F–4G. Q6–10 relate to Exercise 4G.

Students work with circles to determine the radius and the coordinates of the centre based on the form of the relationship and hence sketch its graph. They describe and perform transformations on other non-linear relationships including the square root function and the hyperbola and sketch their graphs. They also test for linear and non-linear relationships between two variables involving direct proportion.

Investigation

INV 4-3 How much gold is actually in your gold jewellery?

Focus: To explore a practical example of direct proportion

Resources: 1-cm grid paper (BLM) or graph paper, calculator

Students consider the measurement of gold purity, the karat. Different grades of gold are looked at and students graph a direct proportion to find the amount of pure gold in any jewellery given the mass of the jewellery and the number of karats of the alloy.

As an extension, students investigate the amount of gold in $1 and $2 coins.

BLM

1-cm grid paper

Interactives

4G eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 196–199

Additional teaching strategies

Multiple choice

1 Answer: C. The highest power of x in the expression $x^2 - 4x$ is 2 (expression has an x^2 term) and the expression does not contain an equals sign.
A: chose $y = x^2 + 2$ which is a quadratic equation as it contains an equals sign.
B: chose $x = 34$ which is a linear relationship.
D: chose $y = 12x$ which is a linear relationship.

2 Answer: A. The highest power of a variable in a quadratic equation is 2 and it contains an equals sign.
B: chose $x = 34$ which is a linear relationship.
C: chose $x^2 - 4x$ which is a quadratic expression not a quadratic equation.
D: chose $y = 12x$ which is a linear relationship.

3 Answer: D. $y = x^2 - 4$ is not a linear relationship as it has a variable to the power of 2.
A: chose a linear relationship where the highest power of x and y is 1.
B: chose a linear relationship where the highest power of x and y is 1.
C: chose a linear relationship where the highest power of x and y is 1.

4 Answer: C. Subtracting 5 from x^2 will move the parabola 5 units down.
A: chose the rule where adding 5 will move the graph 5 units up.
B: chose the rule where multiplying by 5 will make the graph narrower.
D: chose the rule where dividing by 5 will make the graph wider.

5 Answer: D. See below why all other answers are true.
A: chose statement where the factor of 4 dilates the original graph, making it narrower.
B: chose statement where the coefficient of x^2 is negative which means that the original graph is reflected in the x-axis.
C: chose statement where adding 1 translates the original graph vertically.

6 Answer: A. The $(x - 4)$ factor translates the graph of $y = x^2$ and hence its turning point 4 units to the right. So (0, 0) becomes (4, 0).
B: assumed graph is translated 4 units to the left.
C: assumed graph is translated 4 units upwards.
D: assumed graph is translated 4 units downwards.

7 Answer: D. At the x-intercepts, $y = 0$; so $x^2 - 4x - 12 = 0$, $(x - 6)(x + 2) = 0$, $x = 6$ or $x = -2$. The coordinates of the x-intercepts are (6, 0) and (–2, 0).
A: confused signs of the solutions to the quadratic equation and placed them on the y-

axis.
B: found the correct solutions to the quadratic equation but placed them on the y-axis.
C: confused the signs of the solutions to the quadratic equation.

8 Answer: B. At the y-intercept, $x = 0$; so $y = (0 – 5)(0 + 2) = –5 \times 2 = –10$.
A: only considered the expression in the second pair of brackets and solved $x + 2 = 0$.
C: only considered the expression in the first pair of brackets and solved $x – 5 = 0$.
D: assumed value of expression in first pair of brackets is 5 (not –5) so calculated $5 \times 2 = 10$.

9 Answer: B: $\sqrt{9} = 3$ so radius is 3 units.
A: only looked at the first pair of brackets containing the 'x' term.
C: only looked at the second pair of brackets containing the 'y' term.
D: did not take the square root of the constant term.

10 Answer: A. Using the general form of a circle, $h = 2$ and $y = –4$ means that the coordinates of the centre of the circle are (2, –4).
B: has reversed the signs of the x- and y-coordinates or simply used the values directly from each expression in the pairs of brackets.
C: used the values directly from each expression in the pairs of brackets but swapped the order.
D: swapped the order of the x- and y-coordinates.

11 Answer: C. If y is directly proportional to x, the relationship is linear.
A: incorrectly chose a parabola which indicates a squared relationship.
B: incorrectly chose a circle which does not represent a proportional relationship.
D: incorrectly chose a hyperbola which represents an inverse relationship.

Short answer

1 **a** $x^2 – 5x + 6 = 0$, $(x – 3)(x – 2) = 0$, $x = 3$ or $x = 2$

b $x^2 + x – 30 = 0$, $(x – 5)(x + 6) = 0$, $x = 5$ or $x = –6$

c $x^2 + 9 = 0$, $x^2 = –9$ has no solution

d $x^2 – 12x = 0$, $x(x – 12) = 0$, $x = 0$ or $x = 12$

2 **a** **i** upright

ii minimum (0, –4)

iii $x = 1$, $x = –1$, $y = –4$

b **i** inverted

ii maximum (–0.5, 1)

iii $x = -1$, $x = 0$, $y = 0$

c **i** upright

ii minimum (2, 0)

iii $x = 2$, $y = 4$

3 **a** **i** upright

ii narrower

iii minimum

b **i** upright

ii wider

iii minimum

c **i** inverted

ii narrower

iii maximum

d **i** upright

ii same width

iii minimum

e **i** inverted

ii same width

iii maximum

f **i** inverted

ii wider

iii maximum

4 **a** Dilation by a factor of 5.

b Dilation by a factor of $\frac{1}{5}$.

c Dilation by a factor of 5 and reflection in the x-axis

d Translation of 5 units upwards

e Reflection in the x-axis and translation 5 units downwards

f Dilation by a factor of $\frac{1}{5}$ and reflection in the x-axis

5 **a** $y = -5(x + 2)^2 - 1$; parabola is inverted; graph of $y = x^2$ has been dilated by a factor of 5, reflected in the x-axis, translated 2 units left and 1 unit down.

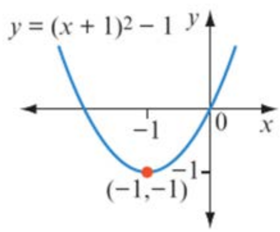
b $y = 5x^2 + 4$; parabola is upright; graph of $y = x^2$ has been dilated by a factor of 5, translated 4 units upwards.

c $y = -\frac{1}{4}(x - 5)^2$; parabola is inverted; graph of $y = x^2$ has been dilated by a factor of $\frac{1}{4}$, reflected in the x-axis, translated 5 units right.

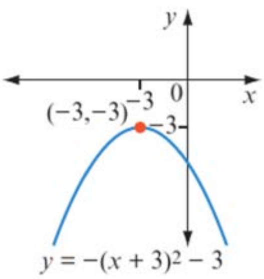
d $y = -3x^2$; parabola is inverted; graph of $y = x^2$ has been dilated by a factor of 3, reflected in the x-axis.

6

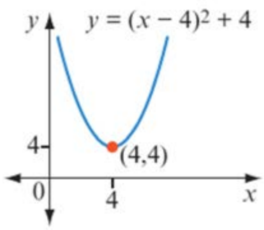
a



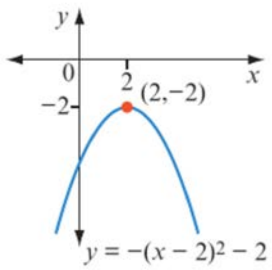
b



c



d



7 **a** At $x = 0$, $y = -0^2 - 4(0) = 0$; coordinates of y-intercept is (0, 0)
At $y = 0$, $-x^2 - 4x = 0$, $-x(x + 4) = 0$, $x = 0$ or $x = -4$; coordinates of x-intercepts are (0, 0) and (–4, 0).

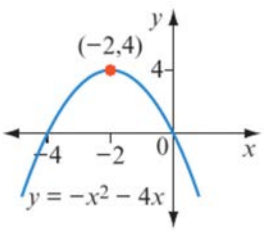
b At $x = 0$, $y = 0^2 - 0 - 12 = -12$; coordinates of y-intercept is (0, –12)
At $y = 0$, $x^2 - x - 12 = 0$, $(x + 3)(x - 4) = 0$, $x = -3$ or $x = 4$; coordinates of x-intercepts are (–3, 0) and (4, 0).

c At $x = 0$, $y = (0 + 5)(0 - 4) = 5 \times -4 = -20$; coordinates of y-intercept is (0, –20)
At $y = 0$, $(x + 5)(x - 4) = 0$, $x = -5$ or $x = 4$; coordinates of x-intercepts are (–5, 0) and (4, 0).

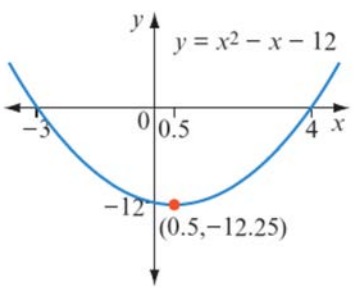
d At $x = 0$, $y = -(0 + 2)(0 + 1) = -2 \times 1 = -2$; coordinates of y-intercept is (0, –2)
At $y = 0$, $-(x + 2)(x + 1) = 0$, $x = -2$ or $x = -1$; coordinates of x-intercepts are (–2, 0) and (–1, 0).

8

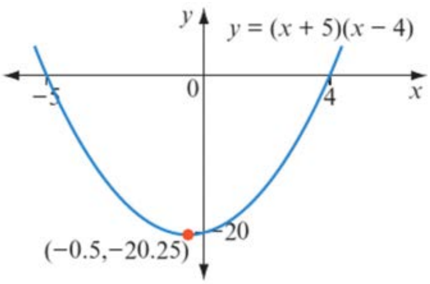
a



b



c



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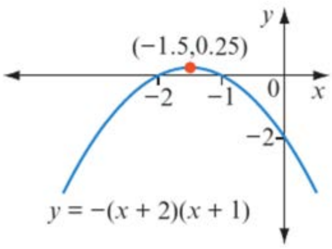
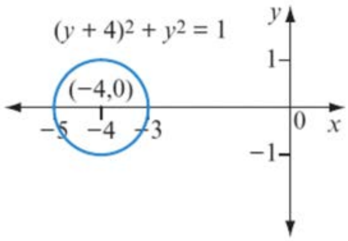
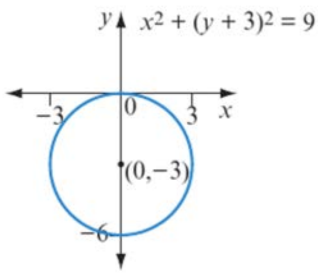


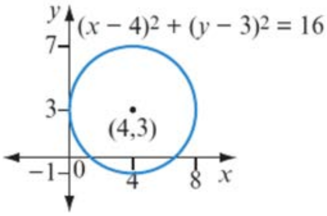
9 **a** centre (–4, 0); radius = 1 unit



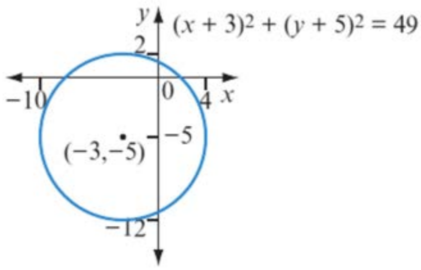
b centre (0, –3); radius = 3 units



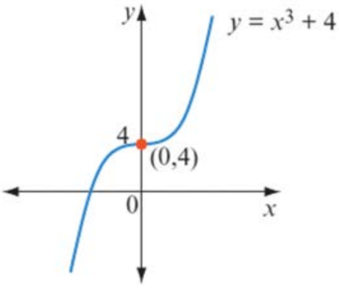
c centre (4, 3); radius = 4 units



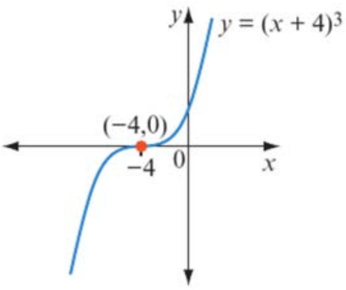
d centre (–3, –5); radius = 7 units



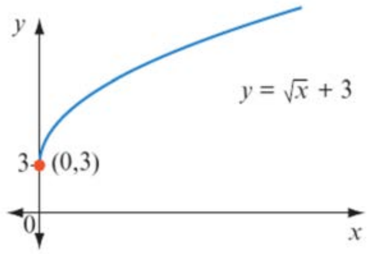
10 **a** $y = x^3$ translated 4 units vertically upwards



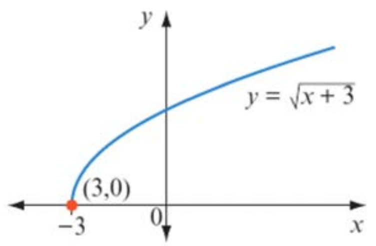
b $y = x^3$ translated 4 units left



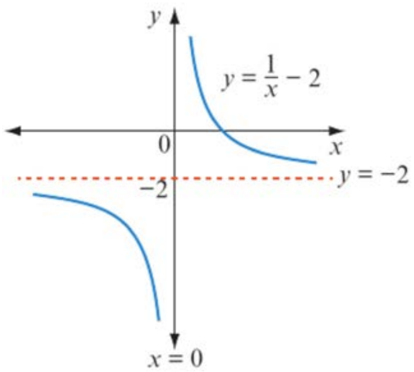
c $y = \sqrt{x}$ translated 3 units vertically upwards



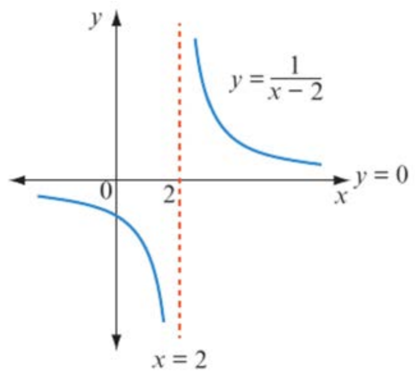
d $y = \sqrt{x}$ translated 3 units left



e $y = \dfrac{1}{x}$ translated 2 units vertically downwards



f $y = \dfrac{1}{x}$ translated 2 units right



11 **a** $y = kx^2$, $4 = k \times 2^2$, $4 = k \times 4$, $k = 1$

b $y = kx^3$, $2 = k \times \left(\frac{1}{2}\right)^3$, $2 = k \times \frac{1}{8}$, $k = 16$

c $y = k\sqrt{x}$, $2 = k \times \sqrt{16}$, $2 = k \times 4$, $k = \frac{1}{2}$

d $y = \frac{k}{x}$, $8 = \frac{k}{\frac{1}{2}}$, $k = 4$

12 **a** Each value of x has been multiplied by 5 to get y, so rule is $y = 5x$.

b There is no constant multiplier for x to get y. Consider x^2.

x^2	0	1	4	9	16
y	0	0.5	2	4.5	8

Each value of x^2 has been multiplied by $\frac{1}{2}$ to get y, so rule is $y = \frac{1}{2}x^2$.

c There is no constant multiplier for x to get y. Consider x^2.

x^2	0	1	4	9	16
y	0	3	24	81	192

There is no constant multiplier for x^2 to get y. Consider x^3.

x^3	0	1	8	27	64
y	0	3	24	81	192

Each value of x^3 has been multiplied by 3 to get y, so rule is $y = 3x^3$

d There is no constant multiplier for x to get y. Consider $\sqrt{x}$.

$\sqrt{x}$	0	1	2	5	6
y	0	3	6	15	18

Each value of $\sqrt{x}$ has been multiplied by 3 to get y, so rule is $y = 3\sqrt{x}$.

NAPLAN-style practice

Multiple-choice options have been listed as A, B, C and D for ease of reference.

1 Answer: B. $x - 6 = 0$, $x = 6$ or $x + 1 = 0$, $x = -1$
A: incorrectly solved both equations.
C: incorrectly solved the first equation.
D: incorrectly solved the second equation.
Refer to *4A Solving quadratic equations*.

2 Answer B: $(x - 8)(x + 2) = 0$,
$x(x - 8) + 2(x - 8) = 0$
$x^2 - 8x + 2x - 16 = 0$

$x^2 - 6x - 16 = 0$
A: calculated $8x - 2x = 6x$ to get the middle term.
C: calculated $-8x - 2x = -10x$ to get the middle term.
D: calculated $8x + 2x = 10x$ to get the middle term.
Refer to *4A Solving quadratic equations*.

3 $x^2 - 11x + 10 = 0$
$(x - 10)(x - 1) = 0$
$x - 10 = 0$ or $x - 1 = 0$
$x = 10$ or $x = 1$Refer to *4A Solving quadratic equations*.

4 $y = x^2 + 3x - 2$ when $x = -1$
$= (-1)^2 + 3(-1) - 2$
$= 1 - 3 - 2$
$= -4$
Refer to *4B Plotting quadratic relationships*.

5 $y = x^2 + x - 20$
x-intercepts occur when $y = 0$.
$(x + 5)(x - 4) = 0$
$x + 5 = 0$ or $x - 4 = 0$
$x = -5$ or $x = 4$
x intercepts at $(-5, 0)$ and $(4, 0)$.
Refer to *4E Sketching parabolas using intercepts*.

6 Answer: B: x-intercepts: $x^2 + 2x - 15 = 0$, $(x + 5)(x - 3) = 0$, $x = -5, 3$.
Vertical axis of symmetry is halfway between the x-intercepts and has the rule $x = -1$.
A: chose rule for vertical line halfway between –3 and 5.
C: chose rule for horizontal line halfway between –3 and 5.
D: chose rule for horizontal line hallway between –5 and 3.
Refer to *4E Sketching parabolas using intercepts*.

7 $y = x^2 - 8x + 15$
x-intercepts occur when $y = 0$.
$x^2 - 8x + 15 = 0$
$(x - 5)(x - 3) = 0$
$x - 5 = 0$ or $x - 3 = 0$
$x = 5$ or $x =$
So x-intercepts are 3 and 5.
Turning point will be at $x = 4$, $y = 4^2 - 8(4) + 15 = 16 - 32 + 15 = -1$ so coordinates are $(4, -1)$
Refer to *4E Sketching parabolas using intercepts*.

8 Answer D: Adding a constant produces a vertical translation.
A: assumed transformation produces a dilation. This would be $y = 3x^2$.
B: assumed transformation produces a horizontal translation. This would be $y = (x \pm 3)^2$.
C: assumed transformation produces a reflection in the x-axis. This would be $y = -x^2$.
Refer to *4C Parabolas and transformations.*

9 Answer: B: The narrowest parabola will be produced when the size of the coefficient of x^2 is the largest (furthest from 0). The dilation factor is 4.
A: chose relationship with dilation factor of 1 so its graph is the same width as the graph of $y = x^2$.
C: chose relationship with dilation factor of $\frac{1}{2}$ so its graph is wider than the graph of $y = x^2$.
D: chose relationship with dilation factor of 2 so its graph is narrower than the graph of $y = x^2$ but not as narrow as $y = -4x^2 - 5$.

Refer to *4C Parabolas and transformations*.

10 $y = -3(x - 2)^2 - 4$
y-intercept occurs when $x = 0$.
$y = -3(0 - 2)^2 - 4$
$= -3 \times 4 - 4$
$= -16$
Coordinates of y-intercept are (0, –16).
Refer to *4E Sketching parabolas using intercepts.*

11 Using the turning point form of a parabola, $h = 2$ and $k = -4$ so the coordinates of the turning point are (2, –4)
Refer to *4D Sketching parabolas using transformations.*

12 Answer: C. The turning point is located at (–3, 4) so $h = -3$ and $k = 4$. Substituting these values into $y = a(x - h)^2 + k$ gives the correct option of $y = (x + 3)^2 + 4$. When $x = 0$, $y = (0 + 3)^2 + 4 = 13$. This matches the y-intercept shown on the graph.
A: used correct value for k but incorrectly substituted for h.
B: incorrectly substituted for both h and k.
D: incorrectly substituted a value.
Refer to *4D Sketching parabolas using transformations.*

13 Answer B: x-intercepts occur when $y = 0$; $x - 3 = 0$, $x = 3$. There is only one solution so there is only one x-intercept.
A: not recognised the perfect square.
C: assumed all parabolas have two x-intercepts.

D: given the value of the x-intercept not the number of x-intercepts.
Refer to *4E Sketching parabolas using intercepts*.

14 Answer: B: Graph is inverted and $y = -(x^2 - 2x - 3) = -(x - 3)(x + 1)$, x-intercepts at $x = 3$ and $x = -1$.
A: chose rule for graph that is not inverted.
C: chose rule for graph that is not inverted.
D: chose rule for graph that is inverted but x-intercepts are at $x = -3$ and $x = 1$.
Refer to *4E Sketching parabolas using intercepts*.

15 Answer A: Substituting (2, –3) and $r = 5$ into the general form of a circle:
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 2)^2 + (y + 3)^2 = 25$
B: correctly calculated the value of r^2 but incorrectly substituted values for h and k.
C: correctly substituted values for h and k but value of r has not been squared.
D: incorrectly substituted values for h and k and value of r has not been squared.
Refer to *4F Circles and other non-linear relationships.*

16 Answer B: From general form of a circle $(x - h)^2 + (y - k)^2 = r^2$, $h = 0$, $k = 8$ and $r = \sqrt{4} = 2$. Centre has coordinates (0, 8) and radius is 2 units.
A: incorrectly interpreted the value for k hence y-coordinate of centre is incorrect.
C: did not take square root of constant term.
D: did not take square root of constant term and swapped values for h and k so coordinates for centre are incorrect.
Refer to *4F Circles and other non-linear relationships.*

17 $(x - 2)^2 + (y + 5)^2 = 36$. Radius = 6 units
$A = \pi r^2$, $A = \pi \times 6^2 \approx 113$ square units
Refer to *4F Circles and other non-linear relationships.*

18 Answer: D $y = \sqrt{x}$ is the top half of a sideways parabola.
A: chose rule that produces an upright parabola.
B: chose rule that produces a hyperbola.
C: chose rule that produces a cubic function (upright curve steeper than a parabola).
Refer to *4F Circles and other non-linear relationships.*

19 Answer: A. $y = kx$. At $y = 2$, $x = 10$ so $2 = k \times 10$, $k = 0.2$. Rule is $y = 0.2x$. At $y = 10$, $10 = 0.2x$ so $x = 50$.
B: chose the original y value.
C: divided the two values of y given in the question.
D: swapped the original values for x and y to obtain $k = 5$ and hence solved $10 = 5x$.
Refer to *4G Relationships and direct proportion.*

20 Answer: C. $y = \frac{1}{4}x^2$. At $x = 8$, $y = \frac{1}{4} \times 8^2$, $y = \frac{1}{4} \times 64 = 16$.

A: squared $\frac{1}{4}$ as well as the x value of 8 to obtain 4.
B: incorrectly linked x^2 and y.
D: squared 8 and not accounted for the constant of proportionality.
Refer to *4G Relationships and direct proportion.*

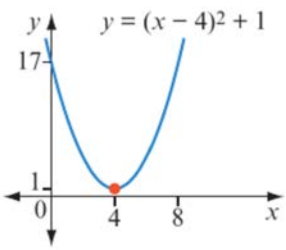
21 At $y = 15$, $x = 25$ so $y = k\sqrt{x}$ becomes $15 = k\sqrt{25}$, $15 = 5k$, $k = 3$.
Refer to *4G Relationships and direct proportion.*

22 Answer D: Direct proportions increase from left to right not decrease.
A: chose correct statement as direct proportions do pass through the origin.
B: chose correct statement as direct proportions are linear.
C: chose correct statement as direct proportions are linear and hence have constant gradient.
Refer to *4G Relationships and direct proportion.*

23 There is a direct relationship between y and x^2 so gradient of line is the constant of proportionality. Gradient is 4 so $k = 4$. Rule is $y = 4x^2$.
Refer to *4G Relationships and direct proportion.*

Analysis

1 a

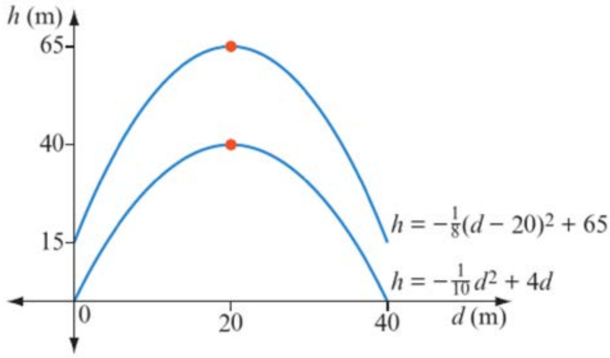


b At $x = 0$, $y = (0 – 4)^2 + 1 = 16 + 1 = 17$
At $y = 17$, $17 = (x – 4)^2 + 1$, $(x – 4)^2 = 16$, $x – 4 = \pm 4$, $x = 0$ or $x = 8$
The two points are (0, 17) and (8, 17).

c Diameter = 8 cm

d At $x = 4$, $y = (4 – 4)^2 + 1 = 1$. The bowl is 1 cm thick.

2 a



b Both have same axis of symmetry (d = 20) and are inverted. Lower arch has maximum turning point at (20, 40) and upper arch has maximum turning point at (20, 65). Lower arch has wider shape than upper arch. End points of upper arch are 15 m above end points of lower arch.

c Bridge ends at d = 0 and d = 40. End points of lower arch at h = 0. For upper arch:

At d = 0, $h = -\frac{1}{8}(0-20)^2 + 65 = 15$.

At d = 40, $h = -\frac{1}{8}(40-20)^2 + 65 = 15$.

So height of upper arch above the lower arch at the ends of the bridge is 15 m.

d **i** 40 m by looking at the x-intercepts of the lower arch graph

ii For lower arch bridge, $h = -\frac{1}{10}d^2 + 4d$. For d-intercepts, h = 0 so $-\frac{1}{10}d(d-40) = 0$, d = 0 or d = 40. d-intercepts are 0 and 40 so span of bridge is 40 m.

e Max of lower arch at d = 20, $h = -\frac{1}{10} \times 20^2 + 4 \times 20 = 40$ m

Max of upper arch at d = 20, $-\frac{1}{8}(20-20)^2 + 65 = 65$ m

f Difference = 65 – 40 = 25 m

g For lower arch at d = 10, $h = -\frac{1}{10} \times 10^2 + 4 \times 10 = 30$ m

For upper arch at d = 10, $h = -\frac{1}{8}(10-20)^2 + 65 = 52.5$ m

Difference = 52.5 – 30 = 22.5 m

h The two arches are 15 m apart at the start. Approaching the highest points on both arches, the distance between the two arches increases until it is a maximum of 25 m apart at the top of the arch. Approaching the far right side of the bridge, the distance between the two arches decreases until they are a distance of 15 m apart at the end of the bridge.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 4 Chapter test A

Chapter 4 Chapter test B

Test answers

Chapter 4 Chapter test answers

Connect

Teaching support for pages 200-201

Teaching strategies

Path of a soccer ball

Focus: To relate the path of a projectile to a quadratic relationship

- Begin by taking students out onto the football field to kick a soccer ball. Have some of them kick a ball from the ground as well as from a point in the air and others viewing from the sidelines to observe the path of the ball.
- Have students measure the length of kicks and estimate the maximum height of the ball.
- Introduce the two relationships in the task as similar examples.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided for the Connect task.
- An assessment rubric is available (see Resources).

Resources

Assessment rubric

Path of a soccer ball

- Ask students about each of the relationships that they identify. Is one component of the relationship dependent on the other? (Possible answer: All of the relationships are dependent on time. Time is the independent variable.)
- Ask students if they know a mathematical term to describe each component of a relationship. (Each component of a relationship is a *variable*.)
- Classify the variables in the relationships discussed as dependent or independent. (In each case, time is independent. Distance fallen, height from the ground and speed will all depend on the time from the jump, the time of which could be considered as zero.)
- How could the relationships be represented? (These relationships can be represented as tables or graphs.)
- Could a rule be formed for the relationships? Explain, using one relationship as an example. (Yes, a rule can be used to describe each relationship. Each variable would need to be assigned a pronumeral. For example, distance fallen could be represented by d, and time could be represented by t. If the skydiver falls 10 metres per second, the rule could be $d = 10t$.)
- Could the relationship be used to model what would be likely to happen if the skydiver were to jump again? (If all of the conditions of the jump were exactly the same [wind, temperature, size of the skydiver], yes, this relationship could be used to model the expected time taken to fall in future jumps.)
- *4B Evaluating expressions*, Q14 asks students to consider a scenario in which a skydiver jumps from a plane, and could be used to consolidate ideas following this class discussion.

Essential question

Algebra helps us to identify and describe patterns in information and to visualise these patterns using graphs. Graphs of patterns can help us to predict and compare other events.

Are you ready?

Prerequisite knowledge and skills can be tested by completing **Are you ready?** This will give you an indication of the differentiated pathway each student can follow.

Students need to be able to:

- represent numbers using pronumerals
- define algebraic terms
- perform all four operations with integers

- perform operations according to BIDMAS conventions
- calculate the basic numeral for squared and cubed whole numbers
- reorganise numerical sentences
- calculate the perimeter and area of a rectangle
- convert between index form and expanded notation
- calculate the value of the basic numeral for numbers in index form
- identify the highest common factor for two or more whole numbers
- simplify fractions.

At the beginning of each topic there is a suggested differentiated pathway which allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to select the best pathway.

Support Strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

ANSWERS

CHAPTER 4 ALGEBRA

4 Are you ready?

1 **a** 2	**b** C	**c** D	**d** B
2 C	**3** B	**4** C	
5 **a** −4	**b** −8		**6** A
7 **a** 10	**b** −3		
8 A	**9** **a** 7		**b** 15
10 **a** D	**b** D		**11** 24
12 B	**13** 22 cm		**14** 28 cm^2
15 **a** B	**b** 3^5		
16 **a** A	**b** D		
17 **a** D	**b** $\frac{5}{8}$		

Resources

assess: assessments

Each topic of the *MyMaths 8* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students’ confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

4A Using pronumerals

Teaching support for pages 178–83

Teaching strategies

Learning focus

To develop understanding of algebraic definitions when identifying and writing expressions and equations.

To apply understanding of algebraic definitions to writing expressions and equations that represent scenarios.

Start thinking!

The task guides students to:

- review algebraic definitions for number of terms, variable, pronumeral, expression, coefficient, constant, equation and formula within the context of an online purchase
- write an expression; and then an equation, both of which can have values substituted into them to calculate a total cost
- consider the differences between expressions, equations and formulas.

Differentiated pathways

Below Level	At Level	Above Level
2a–c, 4a–f, 7a, 8a, 13a, 15	1–3, 4g–l, 5–17	1–3, 5c–f, 6d–f, j–l, 9, 12–14, 16–22
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q1–3

Focus: To understand algebraic language such as terms, variables, coefficients, constants, expressions and equations

- Direct students to complete **SS 4A-1 Understanding algebraic language** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand the meaning of *term*, *coefficient*, *variable*, *constant*, *expression* and *equation*. Students could write

these terms on cards with a class definition, and keep for future activities.

- Use the BLM **Algebra definitions** (see Resources) as the basis for a whole class activity.

- Write each of the following, in turn, on the whiteboard. Ask students to come forward and identify whether each example is an expression or an equation and also to label components of expressions and equations, by attaching their cards at the relevant position, using Blu-Tack® or magnets.

a $5r + 10$
Expression or equation? (expression)
Number of terms? (2)
Variable? (r)
Coefficient of r? (5)
Constant? (10)

b $13y - x^2 + 15x = 12$
Expression or equation? (equation)
Number of terms? (4)
Variables? (x, y)
Coefficient of x^2? (–1)
Coefficient of x? (15)
Coefficient of y? (13)
Constant? (12)

- Students would benefit from including more definitions and examples in their glossary.

- Explicitly remind students that, when a term has only a pronumeral, without a number in front, that the coefficient is 1. A good strategy to use is to write a 1 in front of any term with no coefficient shown, using a different colour to highlight this. For example: $-4x + y = -4x + 1y$

At Level

At Level
1–3, 4g–l, 5–17

- Demonstrate **4A eTutor** or direct students to do this independently.

- During this exercise, students are introduced to the concept of expansion of brackets. Guide students to see that the multiple outside the first bracket tells us how many equivalent terms (inside the brackets) we have.

For example, $4(x + 3)$ is equivalent to $(x + 3) + (x + 3) + (x + 3) + (x + 3)$

When solving, have students collect like terms first and then perform the addition.

$x + x + x + x + 3 + 3 + 3 + 3$

$= 4x + 12$

Provide students with additional questions which involve expanding brackets until they become more familiar with this concept. Encourage students to find the following shortcut themselves.

$4(x + 3) = 4 \times x + 4 \times 3$

- For additional support using algebra language, refer students to **WS 4A-2 Writing algebraic expressions** (see Resources). Students review each of the terms *expression*, *term*, *pronumeral*, *coefficient*, *constant* and *equation* and consider descriptions of the four operations and write algebraic expressions.
- For more problem-solving and investigations, refer students to **INV 4A-3 Language 'mix and match'** (see Resources). Students match definitions and terms.
- For an additional investigation, refer students to **INV 4A-4 Expressions memory game** (see Resources). As an extension, students can replace the expressions in words with descriptions of real-life scenarios.

Below Level

Below Level
2a–c, 4a–f, 7a, 8a, 13a, 15

- Demonstrate **4A eTutor** or direct students to do this independently.
- Direct students to complete **SS 4A-1 Understanding algebraic language** (see Resources) if they have difficulty with algebraic language or require more practice at recognising algebraic terms.
- The use of pronumerals can be difficult for students to comprehend. Initially some benefit is gained from using a box in place of pronumerals. Ask them to think of the value which is *hidden* inside the box which will make the equation true. Once they become familiar with the idea of finding an unknown, then introduce pronumerals.
- When writing equations, support students' conceptual understanding by encouraging them to talk through a specific situation and then lead them to generate a generalised rule. For example, when writing an equation for '*when 4 is added to a number, the*

result is' ask the student to describe the mathematics they are using to work out the answer. It is not sufficient for students to say 'I just know it'; they are operating with automaticity. Encourage these students to identify the process. Direct students to write in words what they are doing, then ask them to substitute numbers and symbols into the written sentence.

- For additional support using algebra language, refer students to **WS 4A-2 Writing algebraic expressions** (see Resources). Students review each of the terms *expression*, *term*, *pronumeral*, *coefficient*, *constant* and *equation* and consider descriptions of the four operations and write algebraic expressions.
- Some students at this level may be able to complete the investigation **INV 4A-3 Language 'mix and match'** (simplified version) (see Resources). Students match definitions and terms.
- For an additional task matching expressions, refer students to **INV 4A-4 Expressions memory game** (simplified version) (see Resources). As an extension, students can replace the expressions in words with descriptions of real-life scenarios.

Above Level

Above Level
1–3, 5c–f, 6d–f, j–l, 9, 12–14, 16–22

- Demonstrate **4A eTutor** or direct students to do this independently.
- Direct students to the **examples**. **Example 4A-1** shows how to write expressions where one operation has been performed on the start number. **Example 4A-2** demonstrates how to write an expression where two operations have been performed on the start number.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- As an additional activity, have students write an algebraic expression that describes a relationship within their family. For example, *Jodie is 5 years younger than her sister Tania*. $J = T - 5$. It is important to define what each pronumeral represents. Remind them that pronumerals represent numbers not objects. Have them share with another class member.
- For more problem-solving tasks and investigations, refer students to **INV 4A-3 Language 'mix and match'** (see Resources). Students match definitions and terms.
- For an additional task matching expressions, refer students to **INV 4A-4 Expressions memory game** (see Resources). As an extension, students can replace the expressions

in words with descriptions of real-life scenarios.

Extra activities

1 Identify each of the following as an expression or an equation. List the number of terms, the variables, the coefficients of each variable and any constant terms.

a $15 + 8xy - 6y - 2x$

b $32 - 17x + 16y + 12z - 10$

c $x^2 + 6x - 10 = 0$

d $23 - 4r + 7t - 2s$

e $100 = 4e - 7f$

2 Write expressions to represent each of the following.

a 7 more than a number $(c + 7)$

b 5 divided by a number $(\frac{5}{a})$

c Double a number and then subtract 6 from the result $(2d - 6)$

d Three lots of a number and the result is divided by 2. $(\frac{3x}{2})$

e A number is divided by 7 and 8 is added to the result. $(\frac{y}{7} + 8)$

f Double a number, add 7 to the result and then divide by 9. $(\frac{2x+7}{9})$

Answers

4A Using pronumerals

4A Start thinking!

1 a i $10 \times 8 + 12 = \$92$
ii $25 \times 8 + 12 = \$212$
b $8 for each brush and $12 for delivery charge
2 2; number of brushes and total cost
3 Cost of brushes is $8 \times n$ or $8n$. Delivery cost is $12. Total cost is cost of brushes plus delivery; that is, $8n + 12$.
4 a 2 b 8
c yes, 12; it is the term without a variable (pronumeral)
5 equation

Exercise 4A Using pronumerals

1 a true b false; it is an expression
c false; the constant is 7 d true
e false; the coefficient is 6 f true
2 a expression b equation c equation
d equation e expression f equation
3 a $x + y + z = 9$, $k = 4m + 1$
b $2a + 3$ c $3a + b - 7$
4 a $a + 5$ b $\frac{a}{3}$ c $a - 4$ d $3a$
e $a - 8$ f $a + 2$ g $10 - a$ h $\frac{6}{a}$
i $50a$ j $a + 19$ k $2a$ l $a + x + 8$
5 a $7x + 2$ b $\frac{x}{3} - 6$ c $2x - 5$
d $\frac{x}{10} + 1$ e $15x + 28$ f $\frac{x}{20} - 11$
6 a $3(x + 1)$ b $5(x - 9)$ c $7(x + 2)$
d $4(x - 6)$ e $11(x + 8)$ f $9(4 - x)$
g $\frac{x + 5}{2}$ h $\frac{x - 9}{8}$ i $\frac{x + 1}{4}$
j $\frac{7 + x}{12}$ k $\frac{3 - x}{5}$ l $\frac{x - 10}{21}$
7 a $x + 2$ b $(x + 2) \times 5$ or $5 \times (x + 2)$
c $5(x + 2)$
8 a $m - 3$ b $(m - 3) \div 4$ c $\frac{m - 3}{4}$
9 a $2(x + 5)$ b $3(x - 9)$ c $\frac{x + 2}{4}$
d $\frac{x - 10}{7}$ e $5(20 - x)$ f $\frac{8 - x}{6}$
10 An equation has an equals sign (=).
11 a $a + 6 = 18$ b $a - 2 = 9$ c $3a = 24$
d $\frac{a}{5} = 2$
12 a $4x + 3 = 11$ b $\frac{x}{2} + 1 = 6$ c $7x - 5 = 12$
d $\frac{x}{3} - 6 = 21$ e $3(x + 6) = 24$
f $\frac{x + 11}{2} = 9$ g $10(x - 2) = 60$
h $\frac{x - 5}{8} = 100$
13 a $n + 40$
b i 300 more than initial number of bees
ii 200 less than initial number of bees
iii twice initial number of bees
c less (500 less bees than Mac)
d Some possible answers are:
$n + 1$, $3n$, $n + 100$, $5n + 55$.
14 a i $y - 6$ ii $y - 10$ b i $y + 5$ ii $y + 23$
c $y - x$ d $2y$
15 a $35m$ b $25k$ c $35m + 25k$
16 a $c + p$ b $p - 1$ c $c + p - 1$
17 a the amount she spends
b the amount she saves
c Some possible answers are:
$x = 50, y = 250$; $x = 100, y = 200$; $x = 150, y = 150$; $x = 250, y = 50$; $x = 300, y = 0$.
d Both x and y can vary.
18 Expressions, equations and formulas all have pronumerals. Equations and formulas both have equals signs; expressions do not. A formula is an equation that has more than one variable.
19 The relationship can be described more simply and the pronumeral can represent many different values or an unknown value.
20 $t = s + 18$, where s is original number of songs and t is number he has now
22 a $a + 6 = 20$ b $b - 1 = 20$ c $\frac{c}{4} = 20$
d $2d = 20$ e $50 - e = 20$ f $3f + 2 = 20$

Reflect

Possible answer: A pronumeral is used to represent a variable to make it easier to represent information or relationships with a letter or symbol rather than words.

Resources

SupportSheet

SS 4A-1 Understanding algebraic language

Focus: To review the key words used to describe the different parts of algebraic expressions and equations

Resources: coloured pencils or highlighters

Students consider definitions used in algebraic language and identify each defined part of an algebraic expression. Students also consider the difference between an expression and an equation.

WorkSheet

WS 4A-2 Writing algebraic expressions

Focus: To review some key terms from the language of algebra and use pronumerals to write expressions

Students review each of the terms *expression*, *term*, *pronumeral*, *coefficient*, *constant* and *equation*. They consider descriptions of the four operations and write algebraic expressions.

Investigations

INV 4A-3 Language 'mix and match'

Focus: To review the language of algebra and identify examples which describe different words used

Resources: white card, scissors, poster paper, glue, partner

Students make three types of cards from given information:

- Type 1 each consist of a word used in algebra
- Type 2 each consist of a definition of a word used in algebra
- Type 3 each consist of mathematical notation showing a specific example for a word used in algebra

Students match the cards into groups of three and create a poster for classroom display and reference purposes. As an extension, students create their own sets of cards and swap with a classmate.

INV 4A-3 Language 'mix and match' (simplified version)

Resources: Type 1 and Type 2 cards as above

Students try to match the word with the definition.

INV 4A-4 Expressions memory game

Focus: To match written expressions with the equivalent expressions written algebraically

Resources: white card, scissors, partner

Students make playing cards containing expressions in words and equivalent algebraic expressions and play a game of 'Memory'. As an extension, students can replace the expressions in words with descriptions of real-life scenarios.

INV 4A-4 Expressions memory game (simplified version)

Resources: white card, scissors, partner

Students only use the following playing cards:

$m + 7$, $p – 4$, $4x$, $2w$, $6 – u$, $12 + r$ and their matching worded descriptions

BLM

Algebra definitions

Interactives

4A eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4B Evaluating expressions

Teaching support for pages 184–189

Teaching strategies

Learning focus

To apply understanding of substitution to evaluate expressions, equations and formulas in context.

Start thinking!

The task guides students to:

- build on student understanding of writing expressions and equations to represent scenarios; to discover how patterns can be used to complete a table of values; and to substitute values into expressions to calculate numerical answers
- consider the real-life context in which a certain number of chairs can be placed around a table
- complete a table of results using the information found in their pattern, write a formula to describe the relationship, and use the formula to calculate the number of chairs required for different numbers of tables
- discover that they can use the formula to calculate the number of chairs required for a specified number of tables.

Differentiated pathways

Below Level	At Level	Above Level
1a–e, 2a–e, 4a–d, 5a,b	1, 2, 4a–j, 5, 6a–d, 7a,bi, 8a,bi, 9a–f, 10a–f, 11a,b, 12–14	3, 4, 5e–h, 6e–h, 7, 8, 9g–t, 10e–l, 11c,d, 12–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q4–11

Focus: To review addition, subtraction, multiplication and division of integers, the order of operations, powers (squares and cubes) and expanded form

- Direct students to complete **SS 4B-1 Adding and subtracting integers** (see Resources) if they had difficulty with Q4 and Q5, or require more practice at this skill.

- You may need to undertake some explicit teaching so students are reminded of strategies that can be used to add or subtract integers. Some students will prefer to use the counter model (for which a blue counter represents +1 and a red counter represents –1) and others will prefer to use a number line. Explicitly demonstrate both models using the **Are you ready?** questions. It can be seen that the answer will be the same, regardless of the strategy implemented.
- Students may refer to *3B Adding integers*, *3C Subtracting integers* and *3D Simplifying addition and subtraction of integers*.
- Direct students to complete **SS 4B-2 Multiplying and dividing integers** (see Resources) if they had difficulty with Q6 and Q7, or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand the patterns to use when they multiply and divide directed numbers.
- Students can highlight the signs to help them identify the pattern. Explicitly demonstrate the pattern using the preview questions. For example, for Q7a: –2 × –5 = +10 (Same signs means a positive result.)
  For Q7b: 18 ÷ (–6) = +18 ÷ (–6) = –3 (Different signs means a negative result.)
- Students may refer to *3E Multiplying and dividing integers*.
- Direct students to complete **SS 4B-3 Understanding order of operations** if they had difficulty with Q8 and Q9, or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how **BIDMAS** operates.
- Students may refer to *1B Order of operations* and *3F Operations with directed numbers*.
- Direct students to complete **SS 4B-4 Finding squares and cubes of numbers** (see Resources) if they had difficulty with Q10 and Q11, or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded that a power indicates the number of times that a base is written in a repeated multiplication. Students may refer to *1H Powers and roots.*

At Level

At Level
1, 2, 4a–j, 5, 6a–d, 7a,bi, 8a,bi, 9a–f, 10a–f, 11a,b, 12–14

- Demonstrate **4B eTutor** or direct students to do this independently.
- It may be beneficial to review the strategies discussed in the Year 7 text *1G Order of operations* or in the Year 8 text *1H Powers and roots* and *1I Index laws*.
- When substituting negative integers, encourage students to place brackets around the number and the sign, otherwise students may *lose* the negative sign.

> **POTENTIAL DIFFICULTY**
> When substituting into an equation, students may not be aware that $4mp$ is equivalent to $4 \times m \times p$, therefore when substituting values, they do not do the appropriate multiplication. For example, when $m = 3$ and $p = 2$, $4mp$ may become 432 and not $4 \times 3 \times 2$.

- Direct students to complete **SS 4B-2 Multiplying and dividing integers** (see Resources) if they have difficulty with these concepts, or require more practice at this skill.
- Direct students to complete **SS 4B-3 Understanding order of operations** if they have difficulty with these concepts, or require more practice at this skill.
- Direct students to complete **SS 4B-4 Finding squares and cubes of numbers** (see Resources) if they have difficulty with these concepts, or require more practice at this skill.
- When developing a formula to calculate a value, encourage students to speculate how many they need for one, then two, then five, then 10 etc. It may be beneficial for the students to write these equations down, one underneath another, or in a table so that they can easily identify the values which change and the values which remain the same.
- For additional practice at evaluating expressions by substitution, refer students to **WS 4B-5 Substituting for pronumerals** (see Resources). Students substitute given values into expressions and evaluate.
- For more problem-solving tasks and investigations, refer students to **INV 4B-7 Cake time!** (see Resources). Students use their knowledge of area formulas (squares and rectangles) to assist a baker with calculations related to decorating a cake.

Below Level

Below Level
1a–e, 2a–e, 4a–d, 5a,b

- Demonstrate **4B eTutor** or direct students to do this independently.
- Direct students to **Example 4B-1**, **Example 4B-2** and **Example 4B-4 (part a)**. These examples show how to substitute values into an expression.

- Direct students to the **Key ideas**. You may like them to copy this summary. This will revise the order convention used for directed numbers.
- Direct students to complete **SS 4B-1 Adding and subtracting integers** (see Resources) if they have difficulty with adding and subtracting integers, or require more practice at this skill.
- Direct students to complete **SS 4B-2 Multiplying and dividing integers** (see Resources) if they have difficulty with multiplying and dividing integers, or require more practice at this skill.
- Direct students to complete **SS 4B-3 Understanding order of operations** if they have difficulty with understanding the order convention, or require more practice at this skill.
- When substituting into an equation, it is beneficial to change the colour of the substituted number compared to the remainder of the equation. This allows students to better see the substituted number and track it.

POTENTIAL DIFFICULTY
Encourage students to show all working so that errors are not introduced by taking shortcuts. They should include all mathematical operations and substituted values and then any steps taken using BIDMAS.

- The BLM **Substitution card game** (see Resources) can be used to make the cards for a substitution activity which will reinforce the concept that the value substituted into an expression, equation or formula replaces the pronumeral. Provide two sets of cards, each printed on a different colour. Using the first set of cards, students can represent expressions. Using the second set of cards, the students can substitute a number and calculate the numerical answer.
- For additional practice at evaluating expressions by substitution, refer students to **WS 4B-5 Substituting for pronumerals** (Q1a–e, 2) (see Resources). Students substitute given values into expressions and evaluate.
- For problem-solving tasks and investigations, refer students to **INV 4B-7 Cake time!** (Q1, 2) (see Resources).

Above Level

Above Level
3, 4, 5e–h, 6e–h, 7, 8, 9g–t, 10e–l, 11c,d, 12–18

- Demonstrate **4B eTutor** or direct students to do this independently.
- For additional practice at evaluating expressions by substitution, refer students to **WS**

4B-5 Substituting for pronumerals (see Resources). Students substitute given values into expressions and evaluate.

- For Q10 and Q11, remind students that they may need to include brackets around the negative number when raising it to a power.

It may be useful to demonstrate the patterns occurring when negative numbers are repeatedly multiplied.
For example:
$(-2)^2 = (-2) \times (-2) = +4$(Even powers give a positive result.)
$(-2)^3 = (-2) \times (-2) \times (-2)$
$= +4 \times (-2)$
$= -8$ (Odd powers give a negative result.)

- For more problem-solving and investigations, refer students to the following four tasks:

To use a real-life formula, and substitute values to calculate results, refer students to **INV 4B-6 Interesting times** (see Resources). Students use both the simple interest formula and the compound interest formula to compare the amount of interest earned by bank accounts offering different rates, and different periods of investment.

To investigate arithmetic sequences using substitution, refer students to **INV 4B-8 Arithmetic sequences** (see Resources). Students use a given formula to calculate terms within the sequence. They then apply an arithmetic sequence to a real-life problem.

To explore geometric sequences using substitution, refer students to **INV 4B-9 Geometric sequences** (see Resources). Students explore geometric sequences and use a given formula to calculate terms.

To write expressions that represent scoring systems, refer students to **INV 4B-10 Algebra in sport** (see Resources). Students write both an expression and an equation that can be used to calculate the score in AFL and identify winners. They consider the impact of the introduction of another scoring method.

Extra activities

1 Evaluate the following expressions by substituting $m = -6$ and $n = 4$.

a $4m + 4n$ (–8)

b $8m + 3n + 16$ (–20)

c $52 - 2(m + 7n)$ (8)

2 Find the y value for each of these x values, using the formula $y = 4x - 3$.

a $x = 3$ (9)

b $x = 0$ (–3)

c $x = –5$ (–23)

Answers

4B Evaluating expressions

4B Start thinking!

1 6; one side of each table is not used as these two sides are joined

2 a 8 b 10 c 12

3

Number of tables	1	2	3	4	5
Number of chairs	4	6	8	10	12

4 For each table there are two chairs along the 'long sides', one opposite the other. This means there are $2t$ chairs along the 'long sides'. There is always a chair at each end, so there are two extra chairs. This means that for t tables there is a total of $(2t + 2)$ chairs, so the formula is $c = 2t + 2$.

5 $c = 2 \times 5 + 2$ 6 12

7 a 52 b 102 c 236 d 326

8 substituting into a formula is much easier and quicker than drawing diagrams

Exercise 4B Evaluating expressions

1 a 17 b 7 c 56 d 4 e 29 f 10 g 18 h 4

2 a 11 b 26 c 8 d 5 e 2 f −4 g 6 h 3

3 a 4 b 16 c −8 d −26 e −2 f 1 g 0 h 13

4 a 9 b 6 c 43 d 0 e 80 f −2 g 10 h −9 i 2 j −8 k −5 l 0

5 a 13 b 23 c −5 d 5 e 16 f 7 g 8 h 4

6 a 1 b 21 c −11 d 197 e 3 f −2 g −3 h −6

7 a $a \times a$ b i 25 ii 75 iii 32

8 a $p \times p \times p$ b i 64 ii 128 iii 52

9 a 16 b 9 c 23 d 4 e 48 f 18 g 78 h 83 i 134 j 107 k 90 l 93 m 8 n 2 o 5 p 1 q $\frac{7}{8}$ r $\frac{1}{2}$ s $\frac{1}{91}$ t $\frac{3}{5}$

10 a −8 b 40 c −84 d −34 e 72 f −24 g −64 b −188 i 73 j −126 k 29 l −15

11 a

x	0	1	2	3	4
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	−11	−7	−3	1	5

c

x	0	1	2	3	4
y	1	4	13	28	49

d

x	−2	−1	0	1	2
y	−23	−9	−7	−5	9

12 a $l = 6n$
b i 18 ii 42 iii 300 iv 606
c 5700
d any whole number from 84 to 99; must be between $\frac{500}{6}$ and $\frac{600}{6}$
e $e = 5n$, where n is number of grasshoppers and e is number of eyes
f i 2280 ii 10 175

13 a i 4 ii 8 iii 1.3 iv 20.5
b 3 m/s c 94 km/h

14 a 4.9 m b i 44.1 m ii 122.5 m
c increasing; average speed in first second is 4.9 m/s, in first 3 seconds is $\frac{44.1}{3}$ or 14.7 m/s, in first 5 seconds is $\frac{122.5}{5}$ or 24.5 m/s

15 449 tables and 900 chairs 16 38.9 cm^3

17 a i 25°C ii −20°C iii 5°C iv 38°C v −18°C vi 17°C
b i 86°F ii 14°F iii 32°F iv 81°F v 18°F vi 108°F
c −5°C d 86°F, as 32°C = 89.6°F

Reflect

Possible answer: Substitution is useful to evaluate expressions, equations and formulas in specific scenarios.

Resources

SupportSheets

SS 4B-1 Adding and subtracting integers

Focus: To understand how to add and subtract positive and negative whole numbers
Resources: ruler

Students work through adding and subtracting integers, using the visual model of a number line. Students may prefer to use counters and the zero pair model.

SS 4B-2 Multiplying and dividing integers

Focus: To understand how to multiply and divide positive and negative whole numbers
Resources: calculator

Students review and summarise the patterns occurring when they multiply and divide integers. Students then predict the sign of answers and complete practice problems independently.

SS 4B-3 Understanding order of operations

Focus: To review the rules associated with the order of operations

Resources: coloured pencils (optional), calculator

Students review **BIDMAS** and perform calculations using the order of operations.

SS 4B-4 Finding squares and cubes of numbers

Focus: To review square and cube numbers and perform calculations

Resources: coloured pencils

Students review index notation and complete calculations involving squares and cubes.

WorkSheet

WS 4B-5 Substituting for pronumerals

Focus: To evaluate expressions by substituting values into them

Students substitute given values to replace pronumerals in expressions and calculate the result.

Investigations

INV 4B-6 Interesting times

Focus: To use a real-life formula, and substitute values to calculate results

Resources: calculator

Students use both the simple interest formula and the compound interest formula to compare the amount of interest earned by bank accounts offering different rates, and different periods of investment.

INV 4B-7 Cake time!

Focus: To use a real-life formula, and substitute values to calculate results

Resources: calculator (optional)

Students use their knowledge of area formulas (squares and rectangles) to assist a baker with calculations related to decorating a cake. They write formulas and evaluate them by substituting values.

INV 4B-8 Arithmetic sequences

Focus: To explore arithmetic sequences using substitution

Resources: calculator (optional)

Students use a given formula to calculate terms within the sequence. They then apply an arithmetic sequence to a real-life problem. As an extension, students consider other arithmetic sequences.

INV 4B-9 Geometric sequences

Focus: To explore geometric sequences using substitution

Resources: calculator (optional)

Students explore geometric sequences and use a given formula to calculate terms. They then apply a geometric sequence to a real-life problem. As an extension, students consider other geometric sequences.

INV 4B-10 Algebra in sport

Focus: To write expressions that represent scoring systems within different sports and to evaluate the scores achieved

Resources: calculator (optional)

Students write both an expression and an equation that can be used to calculate the score in AFL and identify winners. They consider the impact of the introduction of another scoring method. As an extension, students consider basketball.

BLM

Substitution card game

Interactives

4B eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

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4C Simplifying expressions containing like terms

Teaching support for pages 190–5

Teaching strategies

Learning focus

To simplify expressions by adding and subtracting like terms.

To apply understanding of simplifying expressions containing like terms.

Start thinking!

The task guides students to:

- discover adding like items to simplify an expression
- consider how $a + a + a$ is the same as $3 \times a$ and $3a$, in that each represents '3 lots of a'
- use similar notation to express a second situation and then consider combining expressions. The combined expression will have two variables.
- discover the meaning of *like terms* and that the terms in their combined expression are not *like*. Students also consider that expressions containing like terms can be simplified, whereas expressions which contain non-like terms cannot.

Differentiated pathways

Below Level	At Level	Above Level
1a–d, 2a–d, 3a–c, 4a–d, 5a–c, 10a,b, 12a–fi	1–7, 8a–e, 9, 10a–c, 11–15, 16a–d, 17a–d, 18a,b, 19a,b	2e–h, 4, 5, 7–21
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q12 and Q13

Focus: To reorder terms within an expression, and to review substitution

- Direct students to complete **SS 4C-1 Adding integers in a different order** (see Resources) if they had difficulty with Q12, or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand that

problems can be completed in small steps, as shown. For example, in Q12a:

3 + 5 – 9 + 2 – 7 = **8** – 9 + 2 – 7

8 – 9 + 2 – 7 = **–1** + 2 – 7

–1 + 2 – 7 = **1** – 7

1 – 7 = –6

- Students need to understand that the sign directly in front of a number 'belongs' to it, and that they can reorder the terms in a problem. When reordering, the sign in front of a number must remain the same. When reordering terms, students may like to highlight all additions and reorder these first and then highlight the subtractions and reorder those, to be done second.

  8 – 4 + 6 – 5 + 1 = 8 + 6 + 1 – 4 – 5

  This can then be calculated in steps, as shown above, if required.

- Direct students to complete **SS 4C-2 Understanding perimeter** if they had difficulty with Q13 or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand that values can be substituted into a formula to calculate perimeter. Students may need to be reminded to write any missing measurements on the diagram before adding all of the side lengths.

At Level

At Level
1–7, 8a–e, 9, 10a–c, 11–15, 16a–d, 17a–d, 18a,b, 19a,b

- Demonstrate **4C eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
When calculating the perimeter from a shape drawn on a grid, students mistakenly count the boxes and not the lines around the outside of the shape. This means that they lose a unit for each corner of the shape.

- Some students may not understand how combinations of pronumerals can be considered to be *like*. This can be demonstrated with informal use of the commutative law. Ask students to consider two simple multiplication problems: 3 × 4 and 4 × 3. Is the order important? Link this to combinations of pronumerals. Remind students that there are multiplication signs implied between the pronumerals and that, as shown in the numerical example, the order is not important, although alphabetical order is preferable.

- When substituting values into an equation, students may benefit from initially changing the colour of the substituted value so they can track the substituted value.
- Some students may find it difficult to reorder the terms in expressions. Using highlighters to colour the like terms can assist students in making sure that the correct sign stays with each term.
- Alternatively, encourage students to place brackets around each term with a negative coefficient *before* moving the term. This will reduce the chance of a negative sign being lost.
- Students may also need to be reminded that, when there is no coefficient shown, the value of the coefficient is 1.
- For students who have difficulty with *index laws* refer to the teaching strategies contained within exercise *1H Powers and roots* and *1I Index laws*.
- For additional support simplifying algebraic expressions, refer students to **WS 4C-3 Adding like terms** (see Resources). Students simplify expressions by adding like terms.
- For further problem-solving tasks and investigations, refer students to the following three investigations.

  To simplify expressions by collecting like terms, refer students to **INV 4C-4 Like terms answer search** (see Resources). During this activity, students simplify expressions by collecting like terms and locate the answers in a search grid. Also refer students to **INV 4C-5 Like terms Snap** (see Resources). Students are given question and answer cards. Each question card contains an un-simplified expression and each answer card contains a simplified expression. Students include their own question and answer cards in the playing deck and then play a game of ‘Snap’.

  To use simulation tools to create equations to be simplified, refer students to **INV 4C-7 Make your own questions** (see Resources). Students construct their own expressions and then simplify them by collecting the like terms.

Below Level

Below Level
1a–d, 2a–d, 3a–c, 4a–d, 5a–c, 10a,b, 12a–fi

- Demonstrate **4C eTutor** or direct students to do this independently.

- Direct students to complete **SS 4C-1 Adding integers in a different order** (see Resources) if they had difficulty with this concept, or require more practice at this skill.
- Direct students to complete **SS 4C-2 Understanding perimeter** if they had difficulty understanding perimeter (Q12), or require more practice at this skill.
- When teaching students how to collect like terms, use counters or other items to demonstrate the difference. It is a good idea to use completely different items and not just different coloured or similar items as this can be misleading.
  For example, $2a + 4c + 3a + 2c$ could be represented using
  2 popsticks + 4 oranges + 3 popsticks + 2 oranges.
- For additional support and practice simplifying algebraic expressions, refer students to **WS 4C-3 Adding like terms** (see Resources). Students simplify expressions by adding like terms.
- To experience some problem-solving, refer students to **INV 4C-4 Like terms answer search** (Q1) (see Resources). During this activity, students simplify expressions by collecting like terms and locate the answers in a search grid.
- For further problem-solving, refer students to **INV 4C-7 Make your own questions** (see Resources). Students construct their own expressions and then simplify them by collecting the like terms.

Above Level

Above Level
2e–h, 4, 5, 7–21

- Demonstrate **4C eTutor** or direct students to do this independently.
- Q11 provides an opportunity to differentiate based on the student's demonstrated understanding. Direct students to either use a specific number of operations or specific operations. You may also ask some students to include negative integers or fractions.
- For further problem-solving tasks and investigations, refer students to the following four investigations.

  To simplify expressions by collecting like terms, refer students to **INV 4C-4 Like terms answer search** (see Resources). During this activity, students simplify expressions by collecting like terms and locate the answers in a search grid. Alternatively, refer students **to INV 4C-5 Like terms Snap** (see Resources). Students are given question and answer cards. Each question card contains an un-simplified expression and each answer card contains a simplified expression. Students include their own question and answer cards in the playing deck and then play a game of 'Snap'.

To apply simplification of expressions to real life scenarios, refer students to **INV 4C-6 Help the animals!** (see Resources). Students calculate perimeter and area of a new enclosure to be built by the local animal rescue centre. As an extension, students consider a triangular design and compare this to the rectangular enclosure.

To use simulation tools to create equations to be simplified, refer students to **INV 4C-7 Make your own questions** (see Resources). Students construct their own expressions and then simplify them by collecting the like terms.

Extra activities

1 Write the following lists of terms on the board and highlight like terms using the same colour.

a $4x$, $7y$, $– x$, $9y$, $3xy$, $6y$, $6tyx$
($4x$, $7y$, $–5x$, $9y$, $3xy$, $6y$, $–9yx$)

b $12ab$, $13m$, ba, $8b$, 83ea, $6b$, $14m$, $3a$
($12ab$, $13m$, ba, $8b$, $–10a$, $6b$, $14m$, $3a$)

2 Simplify each of the following expressions by reordering and collecting like terms.

a $9t + 6v + 5t + 2v$
($9t + 5t + 6v + 2v = 14t + 8v$)

b $+4abc$ bc2b $+ 4abc$ bc6b
($–4abc + 4abc$ bc2b bc6b $= –8b$)

Answers

4C Simplifying expressions containing like terms

4C Start thinking!

1 $a + a + a$ means 3 lots of a or $3 \times a$. This can be simplified to 3a.

2 $a + a + a + a + a$ or $5 \times a$ or $5a$. 3 $5a$

4 a $3a + 5a$

b yes; $8a$. In total, this is $(a + a + a) + (a + a + a + a + a)$ or $8a$.

5 $2b$ 6 a $8a + 2b$ b no

7 No, as the pronumerals are not exactly the same.

8 Expressions containing like terms can be simplified.

Exercise 4C Simplifying expressions containing like terms

1 a true b true

c false; pronumerals not the same in each term

d true

e false; pronumerals not exactly the same in each term

f true

2 Some possible answers are given.

a $5y, 3y, y$ b $a, 4a, -6a$

c $n, 8n, -2n$ d $4ab, 2ba, -ab$

e $gh, 2gh, -5gh$ f $12kmn, -5mkn, 4mnk$

g $a^2b, -7a^2b, 2ba^2$ h $9x, -0.2x, 5x$

3 a true

b false; $3p + 6q$ (does not simplify further)

c true d false; $10k$

e false; $9ab + 3a$ f true

g true h false; $11p + 5q$

4 a $9x$ b $5y$ c $15mn$ d $8kp$

e $-4a$ f $xy + 4$ g $4c$ h $4g$

i $2xy + 4y$ j $5ab$ k $-7m$ l $-abc$

5 a $15k$ b $6ab$ c $12x + 3y$

d $13m + 3n$ e $2d + 1$ f $3x + 6xy$

g $11x + 11y$ h $2a + 9$ i 0

j $4dy + 5y + 8$ k gh l $6a + 2ab + 3b$

6 C

7 a true

b false; pronumerals not exactly the same in each term (first term has one factor of b while second term has two factors of b)

c true

d false; pronumerals not exactly the same in each term (first term has one factor of a while second term has two factors of a)

e false; pronumerals not exactly the same in each term (b only in second term)

f true

g false; pronumerals not exactly the same in each term (c and d not in both terms)

h false; pronumerals not exactly the same in each term (first term has one factor of a and two factors of b, the second term has two factors of a and one factor of b)

8 a $14x^2y$ b $2ab^2$ c $7k^2$

d $4mn + 2m^2n$ e $-2c^2$ f $pq + 8p^2q$

g $4a^2b + 4ab^2$ h $11x^2y$ i $-2g^3h + 5g^2h$

j $m^2n^4 + 6m^2n$

9 a 63 b $14m + 5k$ c yes

d simplified expression; only had to substitute values into two terms

10 a $6a + 14b$; 72 b $9a + 14b$; 87

c $3a + 9b$; 42 d $5ab + 5b + 2a$; 100

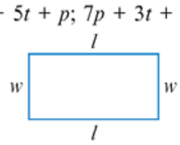
e $8ab + a$; 125 f $3b + 7a$; 44

g $5a^2 - 3b^2$; 98 h $2ab^2 + 2a^2b$; 240

11 Some possible answers are:

$3p + 5t + p$; $7p + 3t + 2t - 3p$; $2p + 3p + 5t - p$.

12 a, b



c $l + w + l + w$

d $2l + 2w$

e $P = 2l + 2w$

f i 14 m ii 18.4 m iii 46 m

g Some possible answers are: length 8 m, width 2 m; length 6 m, width 4 m; length 8.1 m, width 1.9 m.

13 a $7a - 2a + 4a + 5b + 3b - 6b + c$

b $6b$ also has a minus sign that has moved with it.

c yes d $9a + 2b + c$

e For example, if $a = 1$, $b = 2$ and $c = 3$, original expression is $7 + 10 - 2 + 6 + 4 - 12 + 3 = 16$, simplified expression is $9 + 4 + 3 = 16$; so expressions are equivalent.

f simplified expression, as there are fewer terms

14 a $6x - 3x + 5x + 4y + 7y - 2y + y$
$= 8x + 10y$

b $2mn + 8mn - 6mn + 5m - 9m + 2m + 4n$
$= 4mn - 2m + 4n$

15 a She didn't keep minus sign with $2y$ and addition sign with $7x$.

b $3x + 7x - 2y - 4y = 10x - 6y$

16 a $11x + 2y$ b $5y + 4$ c $2x - y - 3$

d $-2xy + 8x$ e $11x^2 - 5$ f $-9x + 6xy + 6y$

17 a 12 b -21 c 6 d 36 e 39 f -108

18 a $8x + 4$ b $18x + 6$ c $36x + 4$ d $40x + 2$

19 a 36 cm b 78 cm c 148 cm d 162 cm

20 a any value between 24.5 and 43.25

b any value between 10.77 and 19.11

c any value between 5.44 and 9.61

d any value between 4.95 and 8.70

21 a $11a^2b + 11ab^2$ b $4e^2fg + e^2fh - 5efgh$

c $x^2y + 4xy^2 + 7xy$

Reflect

Possible answer: Like terms have exactly the same pronumerals.

Resources

SupportSheets

SS 4C-1 Adding integers in a different order

Focus: To add and subtract integers and to understand the importance of signs involved in the calculations

Students compare the results when an expression is rearranged so that all additions are completed before the subtractions, to when all additions and subtractions are calculated from left to right. Extra practice problems are provided and can be completed independently.

SS 4C-2 Understanding perimeter

Focus: To review the meaning of perimeter and to calculate the perimeter of some simple 2D shapes

Resources: ruler, **1-cm grid paper** (BLM)

Students consider the meaning of the term *perimeter*. They calculate the perimeter of a rectangle drawn on 1-cm grid paper and discover two rules which they apply to calculate the perimeter of other rectangles.

WorkSheet

WS 4C-3 Adding like terms

Focus: To simplify algebraic expressions by adding like terms

Resources: coloured pencils or highlighters

Students are reminded of the definition for *like terms* and collect like terms in an expression

Investigations

INV 4C-4 Like terms search

Focus: To simplify expressions by collecting like terms

Resources: coloured pencils or highlighters

Students simplify expressions by collecting like terms and locate the answers in a search grid. As an extension, students construct their own search grid.

INV 4C-5 Like terms Snap

Focus: To simplify expressions by collecting like terms

Resources: white card, scissors, partner

Students are given question and answer cards. Each question card contains an un-simplified expression and each answer card contains a simplified expression. Students include their own question and answer cards in the playing deck and then play a game of 'Snap', matching simplified and un-simplified expressions. As an extension, students design their own game.

INV 4C-6 Help the animals!

Focus: To apply simplification of expressions to a real-life problem

Resources: calculator (optional)

Students calculate perimeter and area of a new enclosure to be built by the local animal rescue centre. As an extension, students consider a triangular design and compare this to the rectangular enclosure.

INV 4C-7 Make your own questions

Focus: To use simulation tools to create problems which can then be simplified

Resources: one ten-sided die, two regular dice, coloured sticky dots, white cardboard, a paper clip or paper fastener, coloured pencils or highlighters, calculator (optional)

Students construct their own expressions and then simplify them by collecting the like terms. As an extension, they construct expressions containing more complex terms, involving multiple pronumerals and squares.

BLM

1-cm grid paper

Interactives

4C eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4D Multiplying algebraic terms

Teaching support for pages 196–201

Teaching strategies

Learning focus

To convert between the simplified and expanded form; and to use multiplication to simplify algebraic terms.

To apply understanding of multiplication of algebraic terms to application questions.

Start thinking!

The task guides students to:

- discover how algebraic terms can be multiplied
- write a term in expanded form (with the multiplication symbol included) and in simplified form (with the multiplication symbol implied)
- apply this conversion to a more complex scenario

Differentiated pathways

Below Level	At Level	Above Level
1a,c,e,g, 2, 3, 4a–d	1, 3f–l, 4, 5, 6a, 7–9, 10a–d, 13a–f, 14, 15, 16a–c	1, 4–6, 9–12, 13e–i, 14–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q14 and Q15

Focus: To review substitution into formulas; and to convert between index form and expanded form

- Direct students to complete **SS 4D-1 Area of a rectangle** (see Resources) if they had difficulty with Q14 or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand the difference between calculating the perimeter and the area of a rectangle. Students need to be able to recall the formula required to calculate the area of a rectangle ($A = l \times w$) to complete this question. They must also substitute values into a formula.
  For example:

$A = l \times w$
length = 7 cm, width = 4 cm
$A = 7 \times 4 = 28$ cm^2

- Direct students to complete **SS 4D-2 Index notation** if they had difficulty with Q15, or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded that a power indicates the number of times that a base is written in a repeated multiplication. Students need to be able to convert between expanded form and index form with ease.

At Level

At Level
1, 3f–l, 4, 5, 6a, 7–9, 10a–d, 13a–f, 14, 15, 16a–c

- Demonstrate **4D eTutor** or direct students to do this independently.
- At the beginning of this lesson, undertake a whole class warm-up which revises the index laws learnt during Exercise 1I. Ensure students understand the difference between adding and multiplying terms containing indices.
- Ensure students understand the difference between adding like terms and multiplying like terms. Terms do not need to be like terms to be multiplied together.
- For students initially struggling with multiplying two algebraic terms, encourage them to write the terms in fully-expanded form.
- Students can be provided with the BLM **Algebraic multiplication tables** (see Resources) to complete Q6.
- Q7 and Q8 guide students to develop a shortcut for multiplying numbers in index form with the same base. It may be necessary to complete a large number of examples in which students multiply numbers raised to powers as well as pronumerals raised to powers before they can identify the shortcut. Encourage students to write the fully-expanded version of any multiplication and to count the number of times the base is written to find the power. Remind students that any base with no power shown is raised to the power of 1.

  Guide students towards the index law:

  $x^a \times x^b = x^{a+b}$

- Direct students to complete **SS 4D-1 Area of a rectangle** (see Resources) if they have difficulty calculating the area of a rectangle, or require more practice at this skill. This skill is needed in Q15 and Q16.

- When completing Q15, students may benefit from using the BLM **1-cm grid paper** (see Resources). Have students draw different quadrilaterals which have an area of 24. Guide students to identify that the length and width values for each quadrilateral are related to the factors of 24. This learning can be extended further by asking students to compare the perimeter of each shape with its area. Are these also the same? If not, why not?

- For further practice writing algebraic terms in expanded form, refer students to **WS 4D-3 Writing terms in expanded form** (see Resources). Students identify the coefficient and the pronumeral in algebraic terms. They convert a simplified term into its expanded form, and vice versa.

- For further practice simplifying algebraic expressions by multiplying terms, refer students to **WS 4D-4 Multiplying algebraic terms** (see Resources). Students are guided through examples in which two terms, each containing a coefficient and pronumerals, are multiplied.

- For more problem-solving tasks and investigations, refer students to **INV 4D-5 Make me colourful!** (see Resources). Students simplify expressions involving multiplication of numbers and pronumerals to crack a colouring code for a gingerbread house. They trace and colour their own house.

Below Level

Below Level
1a,c,e,g, 2, 3, 4a–d

- Demonstrate **4D eTutor** or direct students to do this independently.

- Development of multiplicative thinking is achieved when students think of the number of groups and handle the actual group size as a single unit. To help some students with this critical concept, emphasise the number of groups by using the numeral and write the size of the group in words: 2 groups of four.

- Direct students to **Example 4D-1, Example 4D-2** and **Example 4D-3.** These show how to multiply numeric and algebraic terms.

- Direct students to complete **SS 4D-2 Index notation** if they have difficulty understanding the representation of indices, or require more practice at this skill.

- For a problem-solving task and additional practice simplifying algebraic expressions involving multiplication, refer students to **INV 4D-5 Make me colourful!** (see Resources). Students simplify expressions involving multiplication of numbers and pronumerals to crack a colouring code for a gingerbread house. They trace and colour their own house.

Above Level

Above Level
1, 4–6, 9–12, 13e–i, 14–18

- Demonstrate **4D eTutor** or direct students to do this independently.
- Students can be provided with the BLM **Algebraic multiplication tables** (see Resources) to complete Q6.
- When completing Q17, ensure students use correct conventions.
- For further practice simplifying algebraic expressions by multiplying terms, refer students to **WS 4D-4 Multiplying algebraic terms** (see Resources). Students are guided through examples in which two terms, each containing a coefficient and pronumerals, are multiplied.
- For more problem-solving tasks and investigations, refer students to **INV 4D-6 Viral multiplication** (see Resources). Students explore the multiplication of two different viruses and the times taken to infect populations.

Extra activities

1 Expand each of the following:

a $7p$ $(7 \times p)$

b $13a^2bc$ $(13 \times a \times a \times b \times c)$

2 Simplify each of the following:

a $-5 \times b$ $(-5b)$

b $4 \times r \times t$ $(4rt)$

c $3de \times -4de$ $(-12d^2e^2)$

d $5r \times 6t$ $(30rt)$

e $7 \times -2w \times 4w$ $(-56w^2)$

f $-4a \times 2b \times 3c$ $(-24abc)$

g $5r \times 6t \times 2rtp$ $(60pr^2t^2)$

Answers

4D Multiplying algebraic terms

4D Start thinking!

1 a i $30 ii $105
b multiplied (×) number of DVDs by cost
c $15k$ d $15 \times k = 15k$
e multiplication sign not shown

2 a i $2p$ ii $7p$ b kp

3 a Numbers can be multiplied together to produce a single number.
b Pronumerals are written beside each other without multiplication signs. If any pronumerals are the same, they can be simplified using index notation.

4 Multiply numbers together and write result at start of term (coefficient). List pronumerals after coefficient.

Exercise 4D Multiplying algebraic terms

1 a $5 \times m$ b $-2 \times a$
c $8 \times x \times y$ d $-6 \times c \times d$
e $7 \times a \times b \times c$ f $-10 \times g \times k \times m \times n$
g $3 \times p \times p \times q$
h $-4 \times x \times x \times x \times y \times y \times z$

2 a and B; b and A; c and C

3 a $4x$ b $6k$ c $7d$ d $5m$
e p f $-3h$ g $-9a$ h $-10y$
i $-8c$ j $-e$ k $2f$ l $3m$

4 a $3ab$ b $4mn$ c $2pt$ d $-8cd$
e $9xy$ f $5fk$ g abc h ehy
i $8q$ j $-18m$ k $7a^2$ l $2n^2$

5 a $21ac$ b $-30p$ c $24n^2$
d $10efgh$ e $-18x^2yz$ f $24abdf$
g $-70bghk$ h $48m^2nq$ i $24a^3bc$
j $18m^3np$ k $-21fg^3h^3$ l $40u^2w^2x^2y$

6 a

×	$4a$	3	$6m$	$2abc$
$3d$	$12ad$	$9d$	$18dm$	$6abcd$
ab^2	$4a^2b^2$	$3ab^2$	$6ab^2m$	$2a^2b^3c$
$5mn$	$20amn$	$15mn$	$30m^2n$	$10abcmn$
$2ac$	$8a^2c$	$6ac$	$12acm$	$4a^2bc^2$

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$	$-6x^2$	$-8x^2yz$	$10xy$	$-4x^3$
xy^2	$3x^2y^2$	$4x^2y^3z$	$-5xy^3$	$2x^3y^2$
$6xy$	$18x^2y$	$24x^2y^2z$	$-30xy^2$	$12x^3y$
$-7yz$	$-21xyz$	$-28xy^2z^2$	$35y^2z$	$-14x^2yz$

7 a $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$
b $a \times a \times a \times a \times a \times a \times a = a^7$
c $m \times m \times m \times m \times m \times m \times m \times m = m^8$
d $x \times x \times x \times x \times x \times x \times x \times x \times x \times x = x^{10}$

8 When multiplying terms that have the same base, write the base and add the indices (powers).

9 a a^{16} b x^{13} c m^{17} d k^5 e y^{20} f e^6
g c^{15} h p^{12} i g^{14} j n^{15} k x^{16} l h^{13}

10 a $3y^9$ b $7g^7$ c $6b^{11}$
d $12k^{13}$ e $40w^{17}$ f $80g^{11}$
g $27c^{14}$ h $15p^{10}$ i $14h^{12}$

11 No; the base of each term is not the same so the powers cannot be added.

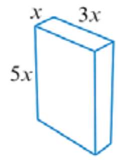
12 This index law only works when multiplying terms that have the same base. Write the base and add the indices (powers). A general rule could be $a^m \times a^n = a^{m+n}$.

13 a a^2h^3 b $12x^4y^5$ c $20t^3b^8$
d m^8q^7 e x^8y^7 f $30g^{10}h^3$
g a^8b^6 h $15x^8y^{10}$ i $54w^4x^{13}y^4$

14 One possible answer is given.
a $abc \times d = abcd$ b $12x \times xyz = 12x^2yz$
c $5mnp \times 4n^2p = 20mn^3p^2$

15 Some possible answers are:
$4x \times 6x$; $12x \times 2x$; $24x \times x$; $48x \times 0.5x$.

16 a $3x$ b $3x^2$ c 192 cm²
d When $x = 8$, width $= x = 8$ cm and length $= 3x = 24$ cm. So area = length × width = $24 \times 8 = 192$ cm². This gives the same result as substituting $x = 8$ directly into $3x^2$.
e Quicker and easier to substitute into simplest form of expression ($3x^2$).
f $5x$
g



h

	area
top	$3x \times x = 3x^2$
bottom	$3x \times x = 3x^2$
front	$3x \times 5x = 15x^2$
back	$3x \times 5x = 15x^2$
left side	$x \times 5x = 5x^2$
right side	$x \times 5x = 5x^2$

i $3x^2 + 3x^2 + 15x^2 + 15x^2 + 5x^2 + 5x^2 = 46x^2$
j Use formula ($46x^2$) or calculate area of each face individually and find sum. surface area = 736 cm²
k i 1150 cm² ii 6624 cm² iii 11.50 cm²
iv 202.86 cm²
l $15x^3$
m i 1875 cm³ ii 25 920 cm³
iii 1.875 cm³ iv 138.915 cm³

18 $a^{m+n}b^{x+y}$

Reflect

Possible answer: When multiplying algebraic terms it is important to remember that the coefficients are multiplied together, then the pronumerals are simplified, paying attention to any which are the same. When writing the answer, the coefficient is written first and the pronumerals follow, in alphabetical order.

Resources

SupportSheets

SS 4D-1 Area of a rectangle

Focus: To discover and apply the relationship between the length, width and area of a rectangle

Resources: ruler, **1-cm grid paper** (BLM)

Students find the area of rectangles and discover the relationship between area, length and width of a rectangle. They discover and apply the rule used to calculate the area of a rectangle.

SS 4D-2 Index notation

Focus: To review the key terms associated with index notation and move between index form and expanded form

Resources: coloured pencils or highlighters

Students review index notation, convert between expanded and index form, and calculate the value of the basic numeral.

WorkSheets

WS 4D-3 Writing terms in expanded form

Focus: To write algebraic terms in expanded form

Students identify the coefficient and the pronumeral in algebraic terms. They convert a simplified term into its expanded form, and vice versa.

WS 4D-4 Multiplying algebraic terms

Focus: To simplify algebraic expressions by multiplying terms

Students are guided through examples in which two terms, each containing a coefficient and pronumerals, are multiplied.

Investigations

INV 4D-5 Make me colourful!

Focus: To simplify algebraic expressions involving multiplication, and to use a code to colour a diagram

Resources: tracing paper, coloured pencils, Internet access (optional)

Students simplify expressions involving multiplication of numbers and pronumerals to crack a colouring code for a gingerbread house. They trace and colour their own house. As an extension, students design their own colouring code.

INV 4D-6 Viral multiplication

Focus: To explore indices in the context of a real-life scenario

Resources: calculator, Internet access (optional)

Students explore the multiplication of two different viruses and the times taken to infect populations. As an extension, they explore the possibility of infecting the world's population, presenting their findings to the class.

BLMs

1-cm grid paper

Algebraic multiplication tables

Interactives

4D eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4E Dividing algebraic terms

Teaching support for pages 202–7

Teaching strategies

Learning focus

To understand the strategy of writing terms in expanded form to complete algebraic division.

To apply understanding of division of algebraic terms to application questions.

Start thinking!

The task guides students to:

- discover how algebraic terms can be divided
- discover that dividing algebraic terms is similar to dividing numbers, only set out as a fraction
- consider the similar divisions of $\frac{3}{3}$ and $\frac{a}{a}$. Both are equal to 1
- complete simple divisions with numbers and pronumerals
- discover that division can be completed by writing each term in expanded form and cancelling numbers or pronumerals in the same way as when fractions are cancelled.

Students review knowledge of division and fractions.

Encourage students to identify the highest common factor.

Differentiated pathways

Below Level	At Level	Above Level
See Extra Activities for Below Level then: 1a–d	1a–f, 2a–d, 3a–f, 4a–d, 5–7, 9, 10a–c, 11a–j, 12a–j, 13, 14	1, 2, 3g–r, 4, 7–16
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q16 and Q17

Focus: To find factors, including the highest common factor, and to simplify fractions by cancelling

- Direct students to complete **SS 4E-1 Finding highest common factors** (see Resources) if they had difficulty with Q16 or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how to find factors for a given number.
- Counters can be used to consolidate understanding of factors.
- Some students may also find a times tables grid useful. The BLM **Multiplication facts** (see Resources) can be printed and laminated and given to students who need to refer to this to calculate factors. When finding the highest common factor (HCF), students can use counters to find all factors and list the result. They can compare lists to find common factors and can also identify the highest common factor.
- Direct students to complete **SS 4E-2 Simplifying fractions** (see Resources) if they had difficulty with Q17, or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how to simplify fractions. Students may need to be reassured that, if they do not use the highest common factor, they can complete a sequence of simplifications to complete a problem. For example, in Q17a: $\frac{18}{24}=\frac{9}{12}=\frac{3}{4}$
- Students may refer to *1C Understanding fractions.*

At Level

At Level
1a–f, 2a–d, 3a–f, 4a–d, 5–7, 9, 10a–c, 11a–j, 12a–j, 13, 14

- Demonstrate **4E eTutor** or direct students to do this independently.
- When simplifying an algebraic fraction, ensure students first write the numerator and the denominator in expanded form. Then they look for common factors. In order to identify the factors, students must first understand the area model of multiplication. Give students counters and ask them to create as many different rectangles (arrays) as they can with that number of counters. Give students 12 counters first and use the language 3 groups of 4 and 4 groups of 3; 12 groups of 1 and 1 group of 12 etc., to describe the arrays. Once students are creating arrays, help them to define the number and size of groups as factors of the original number. Consolidate their understanding by using 7 or 11 counters. Students may say that there are no arrays for these last two numbers because 1 group of 7 or 7 groups of 1 does not look like a rectangle but emphasise that they are. This model also helps to describe the difference between composite (more than 2 arrays), prime numbers (only 2 arrays) and square numbers

(one array which is square). Once students have identified the common factors, have them cancel out with a different coloured pen and then re-write the expression. This re-writing of the expression provides an opportunity for you and the student to spot any mistakes.

- Direct students to complete **SS 4E-1 Finding highest common factors** (see Resources) if they have difficulty identifying highest common factors, or require more practice at this skill.
- Direct students to complete **SS 4E-2 Simplifying fractions** (see Resources) if they have difficulty simplifying fractions using the method of cancelling highest common factors, or require more practice at this skill.
- For additional practice at the skill of simplifying algebraic terms expressed as a fraction, refer students to **WS 4E-3 Dividing algebraic terms** (see Resources). Students review the link between division and fraction notation and simplify the fraction.
- For more problem-solving tasks and investigations, refer students to **INV 4E-4 Mystery word** (see Resources). Students simplify algebraic expressions involving division. They match the question to the answer by ruling a line from question box to answer box. Each line will go through a number and a letter, and a mystery word is spelt out.

Below Level

Below Level
See *Extra activities* for Below Level then: 1a–d

- Direct students to complete **SS 4E-1 Finding highest common factors** (see Resources) if they have difficulty identifying highest common factors, or require more practice at this skill.
- Direct students to complete **SS 4E-2 Simplifying fractions** (see Resources) if they have difficulty simplifying fractions using the method of cancelling highest common factors, or require more practice at this skill.
- Refer to *Extra activities* for Below Level activities.
- For those students who may be able to progress to simple algebraic fractions, direct them to **part a** of **Example 4E-1** which shows how to write the numerator of an algebraic fraction in expanded form before cancelling common factors. They can use this as a guide to completing Q1a–d.

- Students at this level could benefit from a copy of the BLM **Multiplication facts** (see Resources). It can be printed, laminated and given to students for reference.

Above Level

Above Level
1, 2, 3g–r, 4, 7–16

- Demonstrate **4E eTutor** or direct students to do this independently.
- Throughout this chapter we have dealt with positive indices. Provide students with expressions that result in negative integers and have them express the answer as a positive index.

For example:

$$\frac{a^4}{a^6} =$$

Students who are familiar with the appropriate index law will complete this division as:

$$\frac{a^4}{a^6} = a^{4-6}$$

And then simplify to:

a^{-2}

However, we want to encourage students to write the answer with a positive index. If students expand the expression they are more likely to understand how to write the answer with a positive index.

$$\frac{a^4}{a^6} = \frac{\cancel{a}^1 \times \cancel{a}^1 \times \cancel{a}^1 \times \cancel{a}^1}{\cancel{a}_1 \times \cancel{a}_1 \times \cancel{a}_1 \times \cancel{a}_1 \times a \times a}$$

The answer is simplified to:

$$\frac{a^4}{a^6} = \frac{1}{a \times a} = \frac{1}{a^2}$$

Have students compare both answers and identify that they are equivalent but that one is written with a positive index and one with a negative index.

$$\frac{a^4}{a^6} = a^{-2} \text{ or } \frac{a^4}{a^6} = \frac{1}{a^2}$$

- To extend student understanding and knowledge, have students investigate the index law $a^{-n} = \frac{1}{a^n}$ and how writing in the above form supports this law.

- For additional practice at the division of algebraic terms represented as a fraction, refer students to **WS 4E-3 Dividing algebraic terms** (see Resources). Students review the link between division and fraction notation and simplify the fraction.

- For more problem-solving tasks and investigations, refer students to **INV 4E-4 Mystery word** (see Resources). Students simplify algebraic expressions involving division. They match the question to the answer by ruling a line from question box to answer box. Each line will go through a number and a letter, and a mystery word is spelt out.

- For further problem-solving tasks and investigations, refer student to **INV 4E-5 How many times?** (see Resources). Students explore the division of algebraic terms using a concrete model. They count 'how many times' one representation of an algebraic expression 'goes into' another.

Extra activities

Below Level

1 Complete these whole number divisions.

a $12 \div 3$

b $12 \div 4$

c $32 \div 8$

d $24 \div 6$

e $18 \div 9$

2 Simplify each fraction.

a $\frac{25}{5}$

b $\frac{100}{10}$

c $\frac{16}{4}$

d $\frac{22}{2}$

e $\frac{7}{14}$

At/Above Level

Simplify each of the following:

a $\frac{20a}{14ac}$ $\left(\frac{10}{7c}\right)$

b $\frac{15b^2m^3}{45bm}$ $\left(\frac{bm^2}{3}\right)$

c $\frac{14xyz}{21x}$ $\left(\frac{2yz}{3}\right)$

d $\frac{18p^2qrst}{30t}$ $\left(\frac{3p^2qrs}{5}\right)$

Answers

4E Dividing algebraic terms

4E Start thinking!

1 a 1 b 1

2 HCF is 3 so $\frac{3}{3} = \frac{3^{\div 3}}{3_{\div 3}} = \frac{1}{1} = 1$;

HCF is a so $\frac{a}{a} = \frac{a^{\div a}}{a_{\div a}} = \frac{1}{1} = 1$.

3 HCF is 2. Divide both numerator and denominator by 2.
$\frac{10}{2} = \frac{10^{\div 2}}{2_{\div 2}}$ (or $\frac{\cancel{10}^5}{\cancel{2}_1}$) $= \frac{5}{1} = 5$

4 HCF is 2. Divide both numerator and denominator by 2.
$\frac{10a}{2} = \frac{10^{\div 2} \times a}{2_{\div 2}}$ (or $\frac{\cancel{10}^5 \times a}{\cancel{2}_1}$) $= \frac{5 \times a}{1} = \frac{5a}{1} = 5a$

5 a $\frac{10 \times a}{2 \times a}$ b 2 and a

c $\frac{10^{\div 2} \times a^{\div a}}{2_{\div 2} \times a_{\div a}}$ (or $\frac{\cancel{10}^5 \times \cancel{a}^1}{\cancel{2}_1 \times \cancel{a}_1}$) $= \frac{5 \times 1}{1 \times 1} = \frac{5}{1} = 5$

6 a $\frac{10 \times a \times b}{2 \times a}$ b 2 and a

c $\frac{10^{\div 2} \times a^{\div a}}{2_{\div 2} \times a_{\div a}} \times b$ (or $\frac{\cancel{10}^5 \times \cancel{a}^1 \times b}{\cancel{2}_1 \times \cancel{a}_1}$) $= \frac{5 \times 1 \times b}{1 \times 1}$
$= \frac{5b}{1} = 5b$

7 Write division problem as a fraction and look for common factors in numerator and denominator. The factors can be numbers or pronumerals. Divide numerator and denominator by common factors and write result in simplified form.

Exercise 4E Dividing algebraic terms

1 a i $5 \times a$, 5 ii 5 iii a
b i $9 \times d$, d ii d iii 9
c i $4 \times m$, m ii m iii 4
d i $18 \times c$, 6 ii 6 iii $3c$
e i $10 \times b$, 15 ii 5 iii $\frac{2b}{3}$
f i $4 \times x$, $4 \times y$ ii 4 iii $\frac{x}{y}$
g i $6 \times a \times b$, a ii a iii $6b$
h i $5 \times m$, $m \times n$ ii m iii $\frac{5}{n}$
i i $c \times d$, $3 \times c$ ii c iii $\frac{d}{3}$
j i $14 \times s \times t$, $7 \times t$ ii 7 and t iii $2s$
k i $8 \times h \times k$, $10 \times k$ ii 2 and k iii $\frac{4h}{5}$
l i $15 \times a \times b$, $12 \times a$ ii 3 and a iii $\frac{5b}{4}$

2 a x b d c $2k$ d $\frac{p}{6}$
e $\frac{7f}{6}$ f $4gh$ g $18a$ h $9y$
i $\frac{mn}{4}$ j abd k $\frac{7ef}{g}$ l $4kn$

3 a $\frac{4a}{b}$ b $\frac{5x}{y}$ c $\frac{12c}{5d}$ d 3
e $\frac{3}{8}$ f $\frac{1}{3}$ g $4b$ h $\frac{d}{2}$
i $3b$ j $\frac{3t}{4}$ k $\frac{5}{7n}$ l $\frac{rt}{2}$
m $\frac{5}{3y}$ n 7 o $\frac{5}{2}$ p $4b$
q $\frac{p}{6}$ r $\frac{3w}{7}$

4 a a b $6c$ c $2w$ d $\frac{m}{3}$
e $\frac{2f}{5}$ f $\frac{2}{r}$ g $\frac{4tx}{5y}$ h ac
i $\frac{6bd}{5}$ j $\frac{2m^2b}{3}$ k $2k$ l $\frac{c}{b}$

5 a $a^6 \div a^4$
$= \frac{a^6}{a^4}$
$= \frac{a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$
$= \frac{{}^1\cancel{a} \times {}^1\cancel{a} \times {}^1\cancel{a} \times {}^1\cancel{a} \times a \times a}{{}_1\cancel{a} \times {}_1\cancel{a} \times {}_1\cancel{a} \times {}_1\cancel{a}}$
$= \frac{1 \times 1 \times 1 \times 1 \times a \times a}{1 \times 1 \times 1 \times 1}$
$= \frac{a^2}{1}$
$= a^2$

b $m^5 \div m^2$
$= \frac{m^5}{m^2}$
$= \frac{m \times m \times m \times m \times m}{m \times m}$
$= \frac{{}^1\cancel{m} \times {}^1\cancel{m} \times m \times m \times m}{{}_1\cancel{m} \times {}_1\cancel{m}}$
$= \frac{1 \times 1 \times m \times m \times m}{1 \times 1}$
$= \frac{m^3}{1}$
$= m^3$

6 When dividing terms that have the same base, write the base and subtract the indices (powers).

7 a p^3 b a^5 c n^3 d r^8 e $8x^{11}$ f $6m^6$

8 When dividing terms that are in index form and have the same base, write the base and subtract the indices (powers). A general rule could be $a^m \div a^n = a^{m-n}$.

9 a $a^3 \div a^3 = \frac{a^3}{a^3} = \frac{a \times a \times a}{a \times a \times a} = 1$

b $a^3 \div a^3 = a^{3-3} = a^0$

c The answers to parts a and b should be the same so $a^0 = 1$

10 a 1 b 1 c 1 d 2 e 5 f 8

11 a b^2 b q^4 c $5c^4$ d $\frac{5y^7}{2}$ e x^9 f a^6b

g n h b^6 i x^5 j m^4 k $6a^6$ l n^5

m 5 n 4 o 2 p 1

12 a $\frac{5}{4n}$ b $2jk^7$ c $3x$ d $\frac{m^7}{p^3}$

e b^2c^2 f $\frac{2k^8}{a^5}$ g $\frac{9x^4}{11y^4}$ h $\frac{3a^3b}{c^3}$

i $\frac{xy}{4}$ j $4n^2$ k m^8n^5 l x^4

m $\frac{6w^4x^3}{y^4}$ n $\frac{4}{5p}$ o $\frac{abd}{c}$ p $\frac{3a^4ce^2}{4}$

13 a total cost = 10×4 = \$40; cost of one apple = $40 \div 75$ = \$0.55 (or 55 cents)

b total cost = 10×5 = \$50; cost of one apple = $50 \div 75$ = \$0.65 (or 65 cents)

c $10 \times x \div 75 = \frac{2x}{15}$

d i \$0.80 ii \$1.05 iii \$0.95 iv \$0.40

14 a length = $2x$ cm; area = length × width = $2x \times x = 2x^2$ cm²

b $100x^2$ cm² c 50

d i 40 cm ii 80 cm iii 3200 cm²
iv 400 cm v 160 000 cm²

e 50; same

f No; the expression used to calculate the number of pavers simplifies to 50 in each case as the pronumeral x is cancelled in the fraction.

16 $a^{m-n}b^{x-y}$

Reflect

Possible answer: When dividing algebraic terms it is important to remember that the division problem should be written as a fraction. Numbers (or coefficients) and pronumerals are cancelled if possible. When writing the answer, any numbers and pronumerals which are part of the numerator remain part of the numerator; and any numbers and pronumerals which are part of the denominator remain part of the denominator.

Resources

SupportSheets

SS 4E-1 Finding highest common factors

Focus: To determine the highest common factor of a pair of numbers after first finding the factors of each number

Resources: coloured pencils or highlighters

Students consider the definition of the term *factors* and are guided to find the highest common factor (HCF) of two numbers. Extra practice questions are provided.

SS 4E-2 Simplifying fractions

Focus: To express a fraction in its simplest form

Students review fractions and review the meaning of the terms *numerator* and *denominator*. They find the HCF of the numerator and the denominator and are taken through the steps of the process in which the numerator and the denominator are each divided by the HCF in order to find a simplified fraction. Extra practice questions are provided.

WorkSheet

WS 4E-3 Dividing algebraic terms

Focus: To represent the division of algebraic terms as a fraction and express the answer in simplest form

Students review the link between division and fraction notation. They simplify a fraction, for which the HCF is a term with a coefficient (which may be 1) and a pronumeral. Extra practice questions are provided.

Investigations

INV 4E-4 Mystery word

Focus: To simplify algebraic expressions and find the matching answer

Resources: ruler

Students simplify algebraic expressions involving division. They match the question to the answer by ruling a line from question box to answer box. Each line will go through a number and a letter, and a mystery word is spelt out. As an extension, students create their own puzzle.

INV 4E-5 How many times?

Focus: To explore the division of algebraic terms using a concrete model

Resources: 1-cm grid paper (BLM), ruler, scissors

Students explore the division of algebraic terms using a concrete model. They count 'how many times' one representation of an algebraic expression 'goes into' another.

BLMs

Multiplication facts

1-cm grid paper

Interactives

4E eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4F Working with brackets

Teaching support for pages 208–213

Teaching strategies

Learning focus

To use the distributive law to expand expressions involving brackets.

To apply the distributive law to application questions.

Start thinking!

The task guides students to:

- discover the distributive law and develop an understanding of how expressions involving brackets are expanded
- discover that, when there are brackets, the number outside the brackets is multiplied by each numerical term inside the brackets.
- discover that the same is true if there are pronumerals inside the brackets.
- discover that any expansion should be simplified as part of finding the final answer.

Students review knowledge of expansion of brackets and multiplication.

Differentiated pathways

Below Level	At Level	Above Level
1a,b, 2a,b, 3a–f, 4a,b, 6a, 7a, 11a	1–4, 5a–c, 6–10, 12a, 13a, 14, 15a	3g–p, 4, 5, 7–17
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1–4, 5a–c, 6–10, 12a, 13a, 14, 15a

- Demonstrate **4F eTutor** or direct students to do this independently.
- Encourage students to develop their own definition of the distributive law and include diagrams where necessary. As a class, discuss student suggestions for the definition and

come up with a definition as a whole class so students who were unable to do it on their own have a definition they can copy down.

- When expanding equations with more than one set of brackets, have students expand one set at a time.
- When completing the problem-solving and reasoning questions, ensure students copy or produce a sketch that they can label appropriately.
- For further practice at using the distributive law to expand algebraic expressions, refer students to **WS 4F-1 Using the distributive law to expand algebraic expressions** (see Resources). Students review the order of operations and discover the reasoning behind the distributive law.
- For more problem-solving and investigating, refer students to **INV 4F-2 Andrew's vegie patch** (see Resources). Students explore the use of the area model to assist with the expansion of expressions involving brackets. To complete this activity, students should be familiar with the calculation of the perimeter and area of a rectangle.
- For an additional investigation, refer students to **INV 4F-3 It's a mystery** (see Resources). Students explore patterns which occur when adding two even numbers, two odd numbers and also, one even and one odd number. They explore the common factors occurring when expressions involving brackets are expanded and are guided to recognise the number in front of the first bracket as a factor of each number in the expanded expression. This task could be completed as a whole class activity.

Below Level

Below Level
1a,b, 2a,b, 3a–f, 4a,b, 6a, 7a, 11a

- Demonstrate **4F eTutor** or direct students to do this independently.
- Q1 involves evaluating an expression containing a set of brackets and then evaluating another expression in expanded notation. Students should see that the values calculated are the same. Students could be guided to complete this question on their calculators if they are struggling with mental calculations.
- Direct students **Example 4F-1**. This shows students how to apply the distributive law when a numeral appears outside a set of brackets.
- Q2 provides students with scaffolding for expansion of brackets. Students copy and complete the missing values to finalise the expansion.

- When calculating the area of the lawn to be mown in Q11a, students may benefit from measuring out the area using 1-cm grid paper.
- For a problem-solving task and investigation, refer students to **INV 4F-2 Andrew's vegie patch** (see Resources). Students explore the use of the area model to assist with the expansion of expressions involving brackets. To complete this activity, students should be familiar with the calculation of the perimeter and area of a rectangle.

Above Level

Above Level
3g–p, 4, 5, 7–17

- Demonstrate **4F eTutor** or direct students to do this independently.
- Q7 involves the expansion of two pairs of brackets and the gathering of like terms to find the final answer. Students are provided with scaffolding in Q6 if they feel they need it before commencing Q7.
- In Q11–15, students use an area model to assist with expanding expressions involving brackets. To complete these tasks, students need to know the formula for calculating the area of a rectangle and substitute values for length and width, as shown on the diagram.

  For example, Q11: Students break the larger rectangle into two parts.

  The area of part 1 is:

  $A = l \times w$

  $= 15 \times 8$

  $= 120$ m^2

  The area of part 2 is:

  $A = l \times w$

  $= y \times 8$

  $= 8y$ m^2

  Total area in m^2 = area of part 1 + area of part 2 = $8y + 120$

  The area of the larger rectangle (the whole lawn) can also be found by:

  $A = l \times w$

$= 8 \times (y + 15)$

$= 8(y + 15)\ \text{m}^2$

As an additional task, ask students to explain how this shows that the two expressions are equivalent.

- For more problem-solving and investigating, refer students to **INV 4F-2 Andrew's vegie patch** (see Resources). Students explore the use of the area model to assist with the expansion of expressions involving brackets. In order to complete this activity, students should be familiar with the calculation of the perimeter and area of a rectangle.

- For an additional investigation, refer students to **INV 4F-3 It's a mystery** (see Resources). Students explore patterns which occur when adding two even numbers, two odd numbers and also, one even and one odd number. They explore the common factors occurring when expressions involving brackets are expanded and are guided to recognise the number in front of the first bracket as a factor of each number in the expanded expression.

Extra activities

1 Describe the distributive law in your own words and provide two examples: the first using numbers only, and the second using pronumerals only.
Possible answer: The distributive law means that each term inside a set of brackets is multiplied by the term outside the brackets.

$2(9 + 10) = 2 \times 9 + 2 \times 10$

$= 18 + 20$

$= 38$

$x(y - z) \quad = x \times y + x \times (-z)$

$= xy - xz$

2 In your own words describe the different combinations and outcomes for combining signs.
(Possible answer: When multiplying a pair of terms, like signs result in a positive answer, unlike signs result in a negative answer.)

3 Expand and simplify the following expressions:

a $4(p - 3)$ $(4p - 12)$

b $-4(2a - 9b + 3c)$ $(-4a + 36b - 12c)$

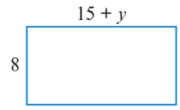
Answers

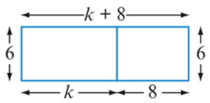
4F Working with brackets

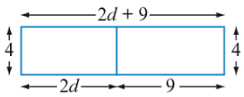
4F Start thinking!

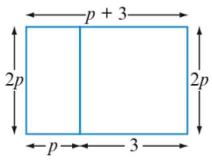
1 a i 11 ii 21
 b No, as a different order was required to perform the operations in each problem.
2 3 lots of 2 + 3 lots of 5; $3 \times (2 + 5)$
 $= 3 \times 2 + 3 \times 5$
3 3 lots of a + 3 lots of 2; $3 \times (a + 2)$
 $= 3 \times a + 3 \times 2$
4 Each term inside the pair of brackets is multiplied by the term outside the brackets.
5 No, as the terms are not like terms.

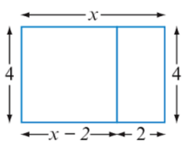
Exercise 4F Working with brackets

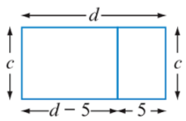
1 Each pair has same value.
 a 16 b 54 c 20 d 15 e −27 f −2
2 a $4(m + 3) = 4 \times m + 4 \times 3 = 4m + 12$
 b $a(c + 5b) = a \times c + a \times 5b = ac + 5ab$
 c $7(2p - 3) = 7 \times 2p + 7 \times (-3) = 14p - 21$
 d $-2(3a + 1) = (-2) \times 3a + (-2) \times 1 = -6a - 2$
3 a $3y + 6$ b $5k + 35$ c $10 + 2a$
 d $12 + 6d$ e $8a + 8b$ f $3p + 3t$
 g $8x + 24$ h $2 + 16m$ i $20a + 5b$
 j $6m + 18n$ k $35a + 21b$ l $6x + 27y$
 m $3m - 6$ n $12 - 2x$ o $4 - 4p$
 p $48c - 30d$
4 a $ab + 4a$ b $3x + wx$ c $cd - de$
 d $2mp + pq$ e $bk - 4ck$ f $3st + 5rt$
 g $8xy + 6wx$ h $6ab - 12ac$ i $27pq + 45kp$
 j $-16mn - 56mw$ k $-20at + 8bt$
 l $15xy + 20yz$ m $12a^2 + 8a$
 n $12k^2 - 18k$ o $3x - 12x^2$ p $-8e^2 - 10e$
 q $-7n^2 + 42n$ r $-4ab + 6b^2$ s $10a^3 - 15a^2$
 t $-11x^2y - 11y^3$
5 a $6x + 2y + 14$ b $10ab + 20d - 15$
 c $14c - 7d + 35ef$ d $-18g + 12h - 6gh$
 e $-20k - 8km + 36$ f $-6xy - 12 + 21w$
6 a $5 \times a + 5 \times 4 + 2 \times a + 2 \times 7$
 $= 5a + 20 + 2a + 14 = 7a + 34$
 b $3b \times c + 3b \times 2 + 2c \times a + 2c \times 5b$
 $= 3bc + 6b + 2ac + 10bc = 13bc + 6b + 2ac$
 c $d \times d + d \times 8 + 6 \times d + 6 \times (-3)$
 $= d^2 + 8d + 6d - 18 = d^2 + 14d - 18$
 d $4k \times 2k + 4k \times 3 + (-2) \times k + (-2) \times (-5)$
 $= 8k^2 + 12k - 2k + 10$
 $= 8k^2 + 10k + 10$
7 a $7a + 27$ b $10b$
 c $11mn + 20m + 3nk$ d $21x + xy + 16y$
 e $k^2 + 4k + 20$ f $c^2 - 3c + 35$
 g $11rst + 5rst^2 + 36rt$ h $h^3 - 4h^2 - 14h$
8 a $4 \times m + 4 \times (-2) + 1 \times m + 1 \times 6$
 $= 4m - 8 + m + 6 = 5m - 2$
 b $2x \times y + 2x \times 3 + (-1) \times 5 + (-1) \times xy$
 $= 2xy + 6x - 5 - xy = xy + 6x - 5$
 c $k \times k + k \times (-1) + (-1) \times k + (-1) \times (-7)$
 $= k^2 - k - k + 7 = k^2 - 2k + 7$
 d $3w \times w + 3w \times (-2) + 1 \times 4w + 1 \times (-1)$
 $= 3w^2 - 6w + 4w - 1$
 $= 3w^2 - 2w - 1$
9 a $7p + 20$ b $2d - 14$
 c $x^2 + 3x + 5$ d $k^2 + k - 1$
 e $2e^2 + 7e + 3$ f $3ab + 4a - 6$
 g $4wy - 15w - 2$ h $mnp - 4m^2p + n$
10 a Let Chanelle's current age = c. In three year's time, Chanelle's age = $c + 3$.
 b $2(c + 3)$ c $2c + 6$
 d i 34 ii 42 iii 56
11 a 120 m^2
 b $8y$ m^2
 c $(120 + 8y)$ m^2
 d


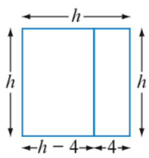
 e length = $(15 + y)$ m, width = 8 m, area = length × width = $(15 + y) \times 8$ or $8 \times (15 + y)$ or $8(15 + y)$.
 f The expressions represent same area of lawn and should be equivalent.
12 area of large rectangle = sum of areas of two small rectangles
 a area of large rectangle = $(a + 2) \times 3$ or $3(a + 2)$
 sum of areas of small rectangles =
 $3 \times a + 3 \times 2 = 3a + 6$
 b area of large rectangle = $(x + 4) \times x$ or $x(x + 4)$
 sum of areas of small rectangles =
 $x \times x + x \times 4 = x^2 + 4x$
13 a


 b


 c


14 a $a - 3$, 5 b $5(a - 3)$
 c i a, 5 ii $5a$ iii 3, 5 iv 15
 v $5a - 15$
 d $5(a - 3)$ = area of rectangle 1. Another way of working out this same area is to subtract area of rectangle 2 from area of large rectangle; that is, $5a - 15$.
15 a


 b


 c


16 a $7x^5 - 10x^4 - 21x^2$ b $4a^3c$
 c $6m^3np^2 + 4m^2n - 6m^2np^2 + 24n$ d $3wy$
17 a −20 b −432 c −256 d 90

Reflect

Possible answer: The distributive law describes the expansion of expressions involving brackets, with each term inside the brackets being multiplied by the term outside the brackets.

Resources

WorkSheet

WS 4F-1 Using the distributive law to expand expressions

Focus: To use the distributive law to expand algebraic expressions

Resources: coloured pencils (optional)

Students review the order of operations and discover the reasoning behind the distributive law. They are taken through the steps for numerical and algebraic examples.

Investigations

INV 4F-2 Andrew's vegie patch

Focus: To apply the area model to the expansion of brackets

Resources: calculator (optional)

Students explore the use of the area model to assist with the expansion of expressions involving brackets. They need to be familiar with the calculation of the perimeter and area of a rectangle. As an extension, students consider the division of the vegie patch into triangles.

INV 4F-3 It's a mystery

Focus: To explore patterns occurring in different types of simple additions and to relate these to the distributive law through an informal use of factorising

Resources: calculator (optional)

Students explore patterns which occur when adding two even numbers, two odd numbers and also, one even and one odd number. They explore the common factors occurring when expressions involving brackets are expanded and are guided to recognise the number in front of the first bracket as a factor of each number in the expanded expression. This task could be used as a whole class activity.

Interactives

4F eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4G Factorising expressions

Teaching support for pages 214–19

Teaching strategies

Learning focus

To identify the HCF of pairs of terms and to use the distributive law to factorise expressions.

To apply understanding of factorising as an opposite operation of expanding.

Start thinking!

The task guides students to:

- discover how to factorise expressions, and consider how expanding and factorising an expression are related
- develop an understanding of the relationship between expanded form and factor form
- factorise expressions containing numbers only and then factorise an expression containing pronumerals
- discover that expanding and factorising are opposite operations.

Students review knowledge of factors.

Encourage students to identify all the factors of a pair of numbers then select the highest.

Differentiated pathways

Below Level	At Level	Above Level
See Extra Activities for Below Level then: 1a–h	1–4, 5a–l, 11–15, 16a, 17a, 18a–c, 19a–c, 20a–e, 21, 22a–c, 23	1f–l, 2g–l, 5–10, 12d–i, 13–19, 20d–l, 21–26
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1–4, 5a–l, 11–15, 16a, 17a, 18a–c, 19a–c, 20a–e, 21, 22a–c, 23

- Demonstrate **4G eTutor** or direct students to do this independently.
- In order to factorise efficiently, students must be able to multiply and find the HCF. Students who are unable to multiply efficiently may find this task difficult. Refer to *At Level Teaching Strategies 4D* for strategies to support multiplication.
- Have students list the factors of each pair side by side so that they can easily identify the HCF.
- After factorising an expression, have students develop the habit of expanding the brackets to check their answers.
- Students may find factorising an expression where the HCF is a negative term difficult if they do not have a good understanding of positive and negative numbers. Refer them to Chapter 3 if required.
- For additional practice factorising algebraic expressions by taking out the highest common factor, refer students to **WS 4G-1 Factorising expressions** (see Resources). Students consider equivalent representations of the same expression, expanded form and factor form. They are reminded of how to find the HCF of two terms and that the HCF may be a term containing a coefficient and a pronumeral. Students are guided through the process of factorisation.
- For a problem-solving task and investigation, refer students to **INV 4G-2 Factorising memory game** (see Resources). Students create playing cards and play a game of 'Factorising memory'.
- For an additional problem-solving task and investigation, refer students to **INV 4G-3 Surface area of a soft drink can** (Q1–6) (see Resources). Students explore the difference between the non-factorised form and the factorised form of the formula used to calculate the total surface area of a cylinder.

Below Level

Below Level
See *Extra activities* for Below Level then: 1a–h

- Refer to *Extra activities* for Below Level activities.
- Q1 provides students with scaffolding and asks them to find factors for different terms. For those students who have difficulty recalling their times tables, they could be provided with a copy of the BLM **Multiplication facts** (see Resources). This could be laminated for longer lasting.

Above Level

Above Level
1f–l, 2g–l, 5–10, 12d–i, 13–19, 20d–l, 21–26

- Demonstrate **4G eTutor** or direct students to do this independently.
- For students who can factorise with minimal effort, introduce them to factorising expressions with three terms. Have students work in small groups and investigate the *area model* and apply this knowledge to factorise trinomials. Once they become familiar with the factorisation technique and no longer need to use the *area model*, have students write the 4 or 5 steps method they have developed and share it with another group. Both groups then work together to determine if an improved idea can be developed after viewing the others' suggestions.
- An alternative method of factorising an expression where the HCF is a negative term can be used to assist students. Explicitly remind students of patterns which need to be considered when dividing positive and negative numbers. For example:

  Q18a: $-4a - 12$

  Term 1 = $-4 \times a$

  Term 2 = -4×3

  HCF = –4

  $$\frac{-4 \times a}{-4} = \frac{a}{1} = a$$

  $$\frac{-12}{-4} = \frac{3}{1} = +3$$

  So:

  $-4a - 12 = \mathbf{-4}(\_\_ + \_\_)$

  $= \mathbf{-4(a + 3)}$

- For a problem-solving task and investigation, refer students to **INV 4G-2 Factorising memory game** (see Resources). Students create playing cards and play a game of 'Factorising memory'.
- For an additional problem-solving task and investigation, refer students **to INV 4G-3 Surface area of a soft drink can** (see Resources). Students explore the difference

between the non-factorised form and the factorised form of the formula used to calculate the total surface area of a cylinder. As an extension, they calculate the height of a cylindrical water tank given the total surface area and the radius.

Extra activities

Below Level

1 Find the HCF of the following numbers.

a	24 and 12	(12)
b	10 and 30	(10)
c	14 and 49	(7)
d	48 and 32	(16)
e	27 and 81	(27)
f	100 and 150	(50)

At/Above Level

1 How is factorising related to expanding? (opposite process)

2 Identify the HCF for the following pairs of terms.

i	$6b$, $9c$	(3)
ii	$24xy$, $18y$	($6y$)

3 Factorise each of the following expressions.

i	$5z + 10$	[$5(z + 2)$]
ii	$16m – 24$	[$8(2m – 3)$]

Answers

4G Factorising expressions

4G Start thinking!

1 a 16 cm **c** 2
d Common factor is placed at front of brackets. Remaining factors are written inside brackets.
e Write HCF (4) in front of brackets and write remaining factors as a sum inside brackets. That is, $4 \times 7 + 4 \times 11 = 4(7 + 11)$.

2 a $2 \times l + 2 \times w$ **b** common factor is 2
c $2 \times (l + w)$ **d i** 2 and $l + w$ **ii** 2

3 Expanding is multiplying terms to remove brackets. Factorising is writing the expression in factor form. Expanding and factorising are opposite or reverse processes.

Exercise 4G Factorising expressions

1 a 3 **b** -2 **c** 6 **d** x **e** -7 **f** b
g d **h** 2 **i** 5 **j** $-2f$ **k** $5p$ **l** -1

2 a 4 **b** 5 **c** 2 **d** 3 **e** 9 **f** 7
g $2y$ **h** $3c$ **i** $6g$ **j** $4b$ **k** m **l** $5s$

3 a $4 \times x, 4 \times 3$ **b** $5 \times 2d, 5 \times (-1)$
c $2 \times 3, 2 \times 4k$ **d** $3 \times a, 3 \times (-1)$
e $9 \times c, 9 \times e$ **f** $7 \times x, 7 \times 2y$
g $2y \times 3x, 2y \times 1$ **h** $3c \times 1, 3c \times 3b$
i $6g \times 5h, 6g \times (-4)$ **j** $4b \times a, 4b \times 2c$
k $m \times 2n, m \times 3p$ **l** $5s \times 3r, 5s \times (-4t)$

4 a $4 \times x + 4 \times 3$
$= 4 \times (x + 3)$
$= 4(x + 3)$

b $5 \times 2a + 5 \times (-7)$
$= 5 \times (2a - 7)$
$= 5(2a - 7)$

c $8k \times 3m + 8k \times 2$
$= 8k \times (3m + 2)$
$= 8k(3m + 2)$

d $6d \times 3c + 6d \times (-2e)$
$= 6d \times (3c - 2e)$
$= 6d(3c - 2e)$

5 a $4(a + 2)$ **b** $7(e + 3)$ **c** $5(9 + k)$
d $3(1 + 4y)$ **e** $2(d - 3)$ **f** $9(3 - p)$
g $4(2m + 5)$ **h** $6(h - 5)$ **i** $6(4b + 3)$
j $2(6t - 7)$ **k** $5a(b + 2)$ **l** $3n(3 + m)$
m $2x(11y - 2)$ **n** $3b(a + c)$ **o** $8m(2n - p)$
p $9x(3y + 2w)$ **q** $3(4k + 5m)$ **r** $3(7ef - 8gh)$

6 a bc **b** x **c** 9mn **d** $3d$ **e** $2pq$
f $6ak$ **g** 4 **h** $2bcd$ **i** f

7 a $bc \times a, bc \times d$ **b** $x \times wy, x \times 1$
c $9mn \times 1, 9mn \times k$ **d** $3d \times bc, 3d \times 2ef$
e $2pq \times 4, 2pq \times (-r)$ **f** $6ak \times 2g, 6ak \times 3$
g $4 \times 4gh, 4 \times (-7mn)$ **h** $2bcd \times 5a, 2bcd \times 7e$
i $f \times 6egh, f \times (-7abk)$

8 a $bc \times a + bc \times d$
$= bc \times (a + d)$
$= bc(a + d)$

b $2pq \times 4 + 2pq \times (-r)$
$= 2pq \times (4 - r)$
$= 2pq(4 - r)$

c $6ak \times 2g + 6ak \times 3$
$= 6ak \times (2g + 3)$
$= 6ak(2g + 3)$

d $f \times 6egh + f \times 7abk$
$= f \times (6egh - 7abk)$
$= f(6egh - 7abk)$

9 a $mn(p + k)$ **b** $3ab(1 - 3c)$
c $7rt(2s + 3)$ **d** $de(4c + 5f)$
e $2x(wyz - rst)$ **f** $8ay(3p + 5q)$

10 Expand each answer. Result should match original expression.

11 a a **b** m **c** $7x$ **d** $2b$ **e** $3k$
f $7y$ **g** $-4p$ **h** 3 **i** $5y$

12 a x **b** a **c** $7k$ **d** $2y$ **e** $2m$
f $3p$ **g** $7t$ **h** a **i** $3n$

13 a $x \times x + x \times 8$
$= x \times (x + 8)$
$= x(x + 8)$

b $5m \times m + 5m \times (-1)$
$= 5m \times (m - 1)$
$= 5m(m - 1)$

c $3b \times 1 + 3b \times 2b$
$= 3b \times (1 + 2b)$
$= 3b(1 + 2b)$

d $3k \times 6k + 3k \times (-7)$
$= 3k \times (6k - 7)$
$= 3k(6k - 7)$

14 a $c(c + 4)$ **b** $k(6 + k)$ **c** $a(a - 2)$
d $3m(m + 1)$ **e** $p(10p + 11)$ **f** $h(4 - 7h)$
g $3x(2x + 1)$ **h** $5b(b - 2)$ **i** $4n(2n + 9)$
j $3y(9y - 5)$ **k** $2a(7 + 8a)$ **l** $e(e + 1)$

16 a -5 b $-2p$ c $-3k$ d $-7b$
e $-4y$ f $-4m$ g $-xy$ h $-4bc$

17 a $-5 \times (a+2)$
$= -5(a+2)$
b $(-2p) \times q + (-2p) \times 3$
$= -2p \times (q+3)$
$= -2p(q+3)$
c $(-4y) \times y + (-4y) \times (-1)$
$= -4y \times (y-1)$
$= -4y(y-1)$
d $(-4m) \times 3 + (-4m) \times (-5np)$
$= -4m \times (3-5np)$
$= -4m(3-5np)$

18 a $-4(a+3)$ b $-2(3+4k)$ c $-7(m-2)$
d $-5(3b-7)$ e $-3t(s+3)$ f $-6c(3+4d)$
g $-3m(4x-7y)$ h $-h(h+2)$
i $-8x(2x-1)$ j $-5b(9+2ac)$
k $-x(1+x)$ l $-ab(11a-9)$

19 a $2(x+3)$ b $4(2a-9)$ c $h(h+7)$
d $5n(5m+6)$ e $mn(k-1)$ f $6f(2e-3d)$
g $-3(4+3y)$ h $-2(13b+12c)$
i $-ac(b-d)$ j $2xy(3+2x)$
k $mn(n+5)$ l $8c(3d-2ab)$

20 a $6(g+1)$ b $2(5+9x)$ c $k(p+q)$
d $m(m-1)$ e $7(2-ab)$ f $3h(e+g)$
g $4abd(2c+e)$ h $-5a(a+3)$ i $-12(2k-p)$
j $x(3x-y)$ k $4e(4cd+5e)$ l $-d(a+6)$

21 a $x(x+6)$ b x and $x+6$ c $x+6$
d $(x+6) \times x = x \times (x+6) = x \times x + x \times 6$
$= x^2 + 6x$
e using $x+6$, length $= 3+6 = 9$; width $= 3$ so
area $=$ length $\times$ width $= 9 \times 3 = 27$
using $x^2 + 6x$, area $= 3^2 + 6 \times 3 = 9 + 18 = 27$

22 a $(x+2)$ cm b $(5x+7)$ m
c $(k+20)$ cm d $2m$ cm

23 Work out area by multiplying length and width together and writing in expanded form.
a $8(x+2) = 8 \times x + 8 \times 2 = 8x + 16$
b $2(5x+7) = 2 \times 5x + 2 \times 7 = 10x + 14$
c $k(k+20) = k \times k + k \times 20 = k^2 + 20k$
d $2m(3m+5) = 2m \times 3m + 2m \times 5 = 6m^2 + 10m$

24 Some possible answers are given.
a $12xy$ and $14x$, $-2x^2$ and $4x$
b $-3ab$ and $-9a^2b^2$, $-12ab^2$ and $-15a^2b$

25 a $x(8-x)$ b length $= x$, width $= 8-x$
c Some possible answers are: length 5 cm, width 3 cm; length 4.5 cm, width 3.5 cm.

26 a $5x \times 3y + \frac{1}{2} \times 5x \times 2x = 15xy + 5x^2$
b $5x(3y+x)$

Reflect

Possible answer: Expanding and factorising are opposite operations. When expanding you are removing pairs of brackets, when factorising you are using brackets to write an expression as a product of factors.

Resources

WorkSheet

WS 4G-1 Factorising expressions

Focus: To factorise algebraic expressions by taking out the highest common factor

Resources: coloured pencils or highlighters

Students consider equivalent representations of the same expression, expanded form and factor form. They are reminded of how to find the HCF of two terms and that the HCF may be a term containing a coefficient and a pronumeral. Students are guided through the process of factorisation. Extra practice questions are provided.

Investigations

INV 4G-2 Factorising memory game

Focus: To identify the highest common factor (HCF) for terms in an expression, and match the non-factorised expression with the factorised expression

Resources: white card, scissors, partner

Students create playing cards and play a game of 'Factorising memory'. An extension is provided, in which students remove the HCF cards and add extra cards of their own, before playing another game.

INV 4G-3 Surface area of a soft drink can

Focus: To investigate the use of the surface area formula in both non-factorised and factorised form

Resources: soft drink can, calculator

Students explore the difference between the non-factorised form and the factorised form of the formula used to calculate the total surface area of a cylinder. As an extension, they calculate the height of a cylindrical water tank given the total surface area and the radius.

BLM

Multiplication facts

Interactives

4G eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 220–3

Additional teaching strategies

Multiple-choice

1 Answer: C. The four terms are $2x^2$, $-3xy$, $4y^2$ and -1

2 Answer: C. The coefficient is the number in front of the pronumerals (–6).
A: may have incorrectly identified one of the pronumerals as the coefficient.
B: correctly identified the number 6, but have forgotten the negative sign, which is part of the coefficient.
D: incorrectly identified the pronumerals, rather than the coefficient.

3 Answer: A. $2xy - 9y = 2 \times 4 \times -3 - 9 \times -3 = -24 + 27 = 3$.
B: may have incorrectly substituted 3 rather than –3: $2 \times 4 \times 3 - 9 \times 3 = -3$
C: may have incorrectly substituted –4 rather than 4: $2 \times -4 \times -3 - 9 \times -3 = 51$
D: may have incorrectly substituted –4 rather than 4, and may also have made errors within the directed numbers calculations.

4 Answer: B. Like terms must have the same combination of pronumerals.
A: $3a$ and $-5a$ are like terms, but ab is not a like term.
C: There are no like terms.
D: There are no like terms because each term has a different combination of pronumerals.

5 Answer: A. Add like terms only. $7m + 2n - 5m - 2n = (7m - 5m) + (2n - 2n) = 2m$

6 Answer: D. These are like terms. $3x^2 - 2x^2 = 1x^2 = x^2$

7 Answer: B. $-5a^2bc = -5 \times a \times a \times b \times c$
A: may have forgotten that the 5 is negative and also that $a^2 = a \times a$
C: may have expanded the pronumerals correctly, but inadvertently left the negative sign out
D: may have forgotten that $a^2 = a \times a$ when expanding the pronumerals

8 Answer: C. Multiply the coefficients and add the indices, remembering $a = a^1$.
$3a^4c^5 \times 4a^2b \times 2ab^2 = 24a^7b^3c^5$
A: $2a^2b^2 \times 3a^2c^3 \times 4a^2bc^2 = 24a^6b^3c^5$

B: $3a^4b \times 8b^2c \times a^2c^4 = 24a^6b^3c^5$
D: $bc^4 \times 6a^3 \times 4a^3b^2c = 24a^6b^3c^5$

9 Answer: D. $\frac{4ab^2}{8ab} = \frac{4 \times a \times b \times b}{8 \times a \times b} = \frac{b}{2}$

A: may have cancelled and forgotten to keep the two as part of the denominator
B: may have incorrectly collected the remaining terms on the denominator of the fraction
C: may have cancelled and switched the numerator and the denominator

10 Answer: A. Multiply every term in the bracket by $5m$.
$5m(3m + 2n - 6) = 5m \times 3m + 5m \times 2n - 5m \times 6 = 15m^2 + 10mn - 30m$
B: may have incorrectly added $5m$ to each term, rather than multiplying each term by $5m$
C: may have multiplied each term correctly, but have incorrectly changed the last operation sign from + to –
D: may have added rather than multiplied in the first calculation, then multiplied correctly for the second calculation and forgotten to multiply the last term by $5m$

11 Answer: C. $m(2n - 3) + 2m(5n + 4) = 2mn - 3m + 10mn + 8m = 12mn + 5m$
A: $m(3n - 4) + 2m(5n + 4) = 3mn - 4m + 10mn + 8m = 13mn + 4m$
B: $4n(2m - 3) + 2m(2n + 4) = 8mn - 12n + 4mn + 8m = 12mn - 12n + 8m$
D: $3m(3n - 3) + m(3n + 4) = 9mn - 9m + 3mn + 4m = 12mn - 5m$

12 Answer: C. $24ab^2c = 6 \times 4 \times a \times b \times b \times c$ and $18bc = 6 \times 3 \times b \times c$
HCF = $6bc$
A: $2bc$ is a factor of both terms; however it is not the HCF
B: $6c$ is a factor of both terms; however it is not the HCF
D: 8 can be a factor of 24, but it is not a factor of 18, therefore it cannot be used as a common factor. In addition, the pronumeral a is not common to both terms.

Short answer

1 **a** $m \times 4 + 3 = 4m + 3$

b $m \div 5 - 8 = \frac{m}{5} - 8$

c $(m + 3) \div 7 = \frac{m+3}{7}$

d $(m - 6) \times 2 = 2(m - 6)$

2 **a** $\$25 \times x = \$25x$

b $\$13 \times y = \$13y$

c $\$20 \times p = \$20p$

d $\$(25x + 13y + 20p)$

3 $a = 2$, $b = 5$ and $c = -3$

a $7 \times 2 + 2 \times 5 = 14 + 10 = 24$

b $5 \times 2^2 + 4 \times -3 = 20 - 12 = 8$

c $6(3 \times 2 \times 5 - 2) = 6(30 - 2) = 6 \times 28 = 168$

d $\frac{5^2 - 2^2}{-3} = \frac{25 - 4}{-3} = \frac{21}{-3} = -7$

4 **a** Any letters can be used but it is convenient to use d = number of days and w = number of weeks.

b $d = 7w$

c **i** $d = 7 \times 13 = 91$ days

ii $d = 7 \times 29 = 203$ days

iii $d = 7 \times 145 = 1015$ days

iv $d = 7 \times 7.5 = 52.5$ days

5 **a** $6a + 7a = 13a$

b $4b - b = 4b - 1b = 3b$

c $3c - 8c = -5c$

d $5de + 1de + 9de = 15de$

e $7g + 2g + 4h = 9g + 4h$

f $3x^2 + 8x + 2x^2 = 5x^2 + 8x$

6 **a** $4m + 5n + 8m + 3n = 4m + 8m + 5n + 3n = 12m + 8n$

b $2k^2 + 6 + 9k^2 + 1 = 2k^2 + 9k^2 + 6 + 1 = 11k^2 + 7$

c $3cd + 2c + 5dc + 4 = 3cd + 5dc + 2c + 4 = 8cd + 2c + 4$

d $7x + 5y - 2x + y + 3x - 2y = 7x - 2x + 3x + 5y + 1y - 2y = 8x + 4y$

e $5ab^2 - 2a^2b + 4a^2b - 3ab^2 = 5ab^2 - 3ab^2 - 2a^2b + 4a^2b = 2ab^2 + 2a^2b$

f $6pq + 7p + 8q - p^2 - 5q = 6pq + 7p + 8q - 5q - p^2 = 6pq + 7p + 3q - p^2$

7 **a** $5d \times 4a = 20ad$

b $3ab \times 8bc = 24ab^2c$

c $-6gk \times 2gh = -12g^2hk$

d $7ef \times 1bf \times 9cf = 63bcef^3$

8 **a** $x^6 \times x^8 = x^{6+8} = x^{14}$

b $4a^5 \times 7a^4 = 28a^{5+4} = 28a^9$

c $h^7 \times 5h^1 \times 2h^3 = 10h^{7+1+3} = 10h^{11}$

d $2m^5n^8 \times 3m^7n^4 = 6m^{5+7}n^{8+4} = 6m^{12}n^{12}$

9 **a** $\frac{15a}{20} = \frac{3a}{4}$

b $\frac{6cd}{2d} = \frac{3c}{1} = 3c$

c $\frac{4xy}{x} = \frac{4y}{1} = 4y$

d $\frac{6abcd}{10bc} = \frac{3ad}{5}$

e $\frac{18m^2n}{12mn} = \frac{18mmn}{12mn} = \frac{3m}{2}$

f $\frac{2e^2hk^2}{5ek} = \frac{2eehkk}{5ek} = \frac{2ehk}{5}$

10 **a** $k^{11} \div k^7 = x^{11-7} = k^4$

b $8c^9 \div c^1 = 8c^{9-1} = 8c^8$

c $6a^8b \div a^5 = 6a^{8-5}b = 6a^3b$

d $\frac{16x^{13-6}y^{6-1}}{28x^6y^1} = \frac{4x^{13-6}y^{6-1}}{7} = \frac{4x^7y^5}{7}$

e $\frac{3w^6 \times 2w^3}{4w^7} = \frac{6w^{6+3}}{4w^7} = \frac{6w^9}{4w^7} = \frac{3w^2}{2}$

f $\frac{2t^4 \times 4t^5}{8t^9} = \frac{t^{4+5}}{t^9} = \frac{t^9}{t^9} = \frac{1}{1} = 1$

11 **a** $5(a + 3) = 5 \times a + 5 \times 3 = 5a + 15$

b $4(7b - 2) = 4 \times 7b - 4 \times 2 = 28b - 8$

c $c(3d + 10a) = c \times 3d + c \times 10a = 3cd + 10ac$

d $-6m(4m + 1) = -6m \times 4m + -6m \times 1 = -24m^2 - 6m$

12 **a** $3(k + 7) + 5(3k - 2) = 3k + 21 + 15k - 10 = 18k + 11$

b $2y(y - 6) - 1(y + 4) = 2y^2 - 12y - 1y - 4 = 2y^2 - 13y - 4$

13 **a** HCF = 2
$14h - 18 = 2(7h - 9)$

b HCF = $10a$
$20ab + 10ac = 10a(2b + c)$

c HCF = 2
$6a - 4b = 2(3a - 2b)$

d HCF = $4c$
$8c^2 + 12c = 4c(2c + 3)$

e HCF = $-5f$
$-5ef - 5f = -5f(e + 1)$

f HCF = $-n$
$-kmn + 7np = -n(km - 7p)$

g HCF = $2ab$
$4a^2b - 14ab = 2ab(2a - 7)$

h HCF = xy
$xy^2 + wx^2y = xy(y + wx)$

NAPLAN-style practice

Multiple-choice options have been listed as A, B, C and D for ease of reference.

Q1–3 refer to *4A Using pronumerals*.

1 The coefficient of $-8mnp$ is -8 (the number).

2 Answer: D.
The cost of x pens = \$$2x$; The cost of y pencils = \$$1y$ = \$$y$.
The total cost is \$$2x$ + \$$y$ or \$$(2x + y)$.
A: may have forgotten to multiply the variable representing pens (x) by 2

B: may have incorrectly multiplied the variables and the prices together to form one term ($2 \times 1 \times x \times y$)
C: may have inadvertently swapped the variables for pens and pencils

3 Answer: A. $5ab = 5 \times a \times b$
B: may have incorrectly used addition signs rather than multiplication signs
C: may have correctly multiplied the number and the first variable, but incorrectly used an addition sign between the two variables
D: may have incorrectly used an addition sign between the number and the first variable, but correctly used a multiplication sign between the two variables

4 Answer: B.
$7a + 2b = 7 \times 4 + 2 \times -3 = 28 - 6 = 22$
A: May have inadvertently swapped the variables and used $a = -3$ and $b = 4$: $7 \times -3 + 2 \times 4 = -21 + 8 = -13$
C: May have incorrectly substituted in $b = 3$ rather than $b = -3$: $7 \times 4 + 2 \times 3 = 28 + 6 = 34$
D: May have incorrectly substituted in $b = 3$ rather than $b = 3$, but forgotten to multiply: $74 + 23 = 97$
Refer to *4B Evaluating expressions*.

5 Answer: D. $-3m^2n = -3 \times m \times m \times n$
A: may have forgotten that the 3 is negative and also that $m^2 = m \times m$
B: may have forgotten that $m^2 = m \times m$ when expanding the pronumerals
C: may have expanded the pronumerals correctly, but inadvertently left the negative sign out
Refer to *4C Simplifying expressions containing like terms*.

6 $5x^2 - 3x + x^2 + 2x - 4 = 5x^2 + x^2 - 3x + 2x - 4 = 6x^2 - x - 4$
Refer to *4C Simplifying expressions containing like terms*.

7 Answer: B.
$4ab \times 3bc = 4 \times a \times b \times 3 \times b \times c = 12ab^2c$
A: may have added 4 and 3 rather than multiplying and have forgotten that $b \times b = b^2$
C: may have added 4 and 3 rather than multiplying, the pronumerals have been multiplied correctly
D: may have forgotten that $b \times b = b^2$
Refer to *4D Multiplying algebraic terms*.

8 Answer: C. $h^4 \times h^1 \times h^7 = h^{4+1+7} = h^{12}$
A: may have simply counted the number of times that the variable h is written
B: may have forgotten to add 1, that is; $h = h^1$
D: may have multiplied the powers 4 and 7
Refer to *4D Multiplying algebraic terms*.

9 Answer: B.

$$\frac{24ef}{20e}=\frac{\cancel{4}\times 6\times \cancel{e}\times f}{\cancel{4}\times 5\times \cancel{e}}=\frac{6f}{5}$$

A and C: The fraction has not been fully simplified.
D: The pronumeral *e* has not been cancelled.
Refer to *4E Dividing algebraic terms*.

10 $b^8 \div b^2 = b^{8-2} = b^6$
Refer to *4E Dividing algebraic terms*.

11 $3mn \times 2m^2n \times n = 3 \times 2 \times m^{1+2}n^{1+1+1} = 6m^3n^3$
Refer to *4D Multiplying algebraic terms*.

12 $$\frac{2x^3\times 3x^2y^4}{8x^4y^2}=\frac{6x^5y^4}{8x^4y^2}=\frac{3xy^2}{4}$$
Refer to *4E Dividing algebraic terms*.

13 Length = $3w$, width = w
Area = length × width = $3w \times w = 3w^2$
Refer to *4D Multiplying algebraic terms*.

14 Answer: C. Multiply each term in the brackets by *a*.
$a(b + c) = a \times b + a \times c$
A: may have incorrectly written an addition sign in between the pronumeral and the bracket. This should be a multiplication sign, that is; $a \times (b + c)$
B: may have correctly multiplied the first term from the bracket, but not the second
D: may have incorrectly multiplied all of the pronumerals together, rather than using the distributive law
Refer to *4F Working with brackets.*

15 Answer: D. $5x(x - 2y) = 5x \times x - 5x \times 2y = 5x^2 - 10xy$
A: may have correctly multiplied the first term from the bracket, but not the second
B: may have forgotten that $x \times x = x^2$ when multiplying $5x$ by x
C: may have correctly multiplied the first term from the bracket, but have incorrectly multiplied the numbers for the second term
Refer to *4F Working with brackets.*

16 $$\frac{2m^2-5n}{4mp}=\frac{2\times 5^2-5\times -2}{4\times 5\times 3}=\frac{50+10}{60}=\frac{60}{60}=1$$
Refer to *4B Evaluating expressions*.

Q17–22 refer to *4F Working with brackets.*

17 Answer: B. $3a(b + 2) = 3a \times b + 3a \times 2 = 3ab + 6a$

A: $3(ab + 2) = 3 \times ab + 3 \times 2 = 3ab + 6$
C: $3b(a + 2) = 3b \times a + 3b \times 2 = 3ab + 6b$
D: $3a(b + 6) = 3a \times b + 3a \times 6 = 3ab + 18a$

18 Answer: B.
$d(6 - d) + 2d(3d - 1) = d \times 6 - d \times d + 2d \times 3d - 2d \times 1 = 6d - d^2 + 6d^2 - 2d = 4d + 5d^2$
A: may have forgotten that $d \times d = d^2$ when multiplying terms that is: $6d - 2d + 6d - 2d = 8d$
C: may have incorrectly completed the calculation using addition for all signs: that is: $6d + d^2 + 6d^2 + 2d = 8d + 7d^2$
D: may have incorrectly completed the calculation using subtraction signs: that is: $6d - d^2 - 6d^2 - 2d = 4d - 7d^2$

19 Answer: A.
$2xy(2x - 3y^2) = 2xy \times 2x + 2xy \times (-3y^2)$
$= 4x^2y - 6xy^3$
B: $2xy(2x - 6y^2) = 2xy \times 2x + 2xy \times (-6y^2) = 4x^2y + (-12xy^3) = 4x^2y - 12xy^3$
C: $-xy(4x - 6y^2) = -4xy \times 2x + (-xy) \times (-6y^2) = -4x^2y + 6xy^3$
D: $xy(4x + 6y) = xy \times 4x + xy \times 6y) = 4x^2y + 6xy^2$

20 Answer: C.
$5(x - y) + 3(2y - x) = 5 \times x - 5 \times y + 3 \times 2y - 3 \times x = 5x - 5y + 6y - 3x = 2x + y$
A: $\dfrac{8x^2 - 12xy}{4x} = \dfrac{4x(2x - 3y)}{4x} = 2x - 3y$
B: $5x + 3x(y - 1) - 3y(x + 1) = 5x + 3xy - 3x - 3xy - 3y = 2x - 3y$
D: $2(x - 3y) + 3y = 2x - 6y + 3y = 2x - 3y$

21 Answer: A. $2p(2p - 5q) = 2p \times 2p - 2p \times 5q = 4p^2 - 10pq$
B: $4p(p - 10q) = 4p \times p - 4p \times 10q = 4p^2 - 40pq$
C: $2(2p - 5pq) = 2 \times 2p - 2 \times 5pq = 4p - 10pq$
D: $2p^2(2 - 5q) = 2p^2 \times 2p - 2p^2 \times 5q = 4p^3 - 10p^2q$

22 Answer: D.
$4(m + 3n) - 1(5m - 2n) = 4m + 12n - 5m + 2n = 14n - m$
A: may have forgotten to multiply $3n$ by 4: that is, $4m + 3n -5m + 2n = -m + 5n$ or $5n - m$
B: may have forgotten that a negative × negative = positive: that is, $4m + 12n -5m - 2n = -m + 10n$ or $10n - m$
C: may have forgotten to include the second term from the second bracket in the calculation: that is, $4m + 12n - 5m = 1m + 12n$ or $12n$ or$m$

23 $c^2 + 5c = c(c + 5)$ so the unknown side length would be the other factor, $c + 5$.
Refer to *4G Factorising expressions*.

24 Answer: D. $(3x)(2x + 4) = 3x \times 2x + 3x \times 4 = 6x^2 + 12x$
A: $(2x)(3x + 4) = 2x \times 3x + 2x \times 4 = 6x^2 + 8x$
B: $(2x)(4x + 6) = 2x \times 4x + 2x \times 6 = 8x^2 + 12x$
C: $(x)(6x + 10) = x \times 6x + x \times 10 = 6x^2 + 10x$
Refer to *4F Working with brackets.*

25 Answer: C. Length $= 2 \times 2 + 4 = 8$ m, width $= 3 \times 2 = 6$ m, area $= 8 \times 6 = 48$ m^2
Refer to *4B Evaluating expressions*.

Analysis

All costs are in dollars. Note the prices shown on the photos.

a **i** $3 \times 10 = 30$

ii $30 \div 5 = 6$

b **i** $10x$

ii $\frac{10x}{5} = 2x$

c **i** $\frac{6x}{4} = \frac{3x}{2}$

ii $\frac{6x}{5}$

d **i** $10x + 6x = 16x$

ii $x = 1, 2$ or 3. Treat one small and one large bag as a unit costing $16.

$50 \div 16 = 3$ remainder 2, so you could but no more than three of these.

Some students may include $x = 0$.

e **i** $10x + 6y$

ii $2(5x + 3y)$

f **i** $\frac{2(5x+3y)}{6} = \frac{5x+3y}{3}$

ii $\frac{2(5x+3y)}{p}$

g $\frac{2\times(5\times3+3\times2)}{6} = \frac{2\times(15+5)}{6} = \frac{2\times21}{6} = \frac{42}{6} = 7$

Each person would spend $7.

h For six people, $p = 6$ and the cost per person is $\frac{2(5x+3y)}{6} = \frac{5x+3y}{3}$.

For this to be $5, $5x + 3y = 5 \times 3 = 15$, and $5x + 3y$ must not be greater than 15.

$x = 0$: $5x = 0$, $3y < 15$ so $y = 0, 1, 2, 3, 4$ or 5

$x = 1$: $5x = 5$, $3y < 10$ so $y = 0, 1, 2$ or 3

$x = 2$: $5x = 10$, $3y < 5$ so $y = 0$ or 1

$x = 3$: $5x = 15$, so $y = 0$

This lists all possibilities. More capable students should be encouraged to find a methodical solution such as the above.

i For this to be no more than $8, $5x + 3y$ must not be greater than $8 \times 3 = 24$.

$x = 0$: $5x = 0$, $3y < 24$ so $y = 0, 1, 2, 3, 4, 5, 6, 7$ or 8

$x = 1$: $5x = 5$, $3y < 19$ so $y = 0, 1, 2, 3, 4, 5$ or 6

$x = 2$: $5x = 10$, $3y < 14$ so $y = 0, 1, 2, 3$ or 4

$x = 3$: $5x = 15$, $3y < 9$ so $y = 0, 1, 2$ or 3

$x = 4$: $5x = 20$, $3y < 4$ so $y = 0$ or 1

This lists all possibilities. More capable students should be encouraged to find a methodical solution such as the above. Some students may discard solutions which include $x = 0$ or $y = 0$.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 4 Chapter test A

Chapter 4 Chapter test B

Test answers

Chapter 4 Chapter test answers

Connect

Teaching support for pages 224–5

Teaching strategies

The magnificent mind reader!

Focus: To use a familiar context to connect the key ideas of algebra

- Students investigate how maths number tricks work.
- This is an open-ended task in which more capable students should be encouraged to try their own number tricks on family and friends. Students will need to show how their trick works each time.
- The task requirements are expressed using everyday language so that students need to recognise the type of solution required to solve each problem.
- You may like to have students discuss the task requirements in small groups to identify the:
  - operations involved in each of the tricks shown in this task
  - notation required to show the algebraic solution for each trick.
- Students may need to be reminded that each operation is performed on the total result obtained in the previous step.
- Direct students to complete the matching **Connect worksheet**. This section provides scaffolding for the task, to guide students through the problem-solving process. Students can use this as a foundation for presenting their findings in a report. Their report could take a number of forms because this task lends itself to a creative approach by students.
- Encourage students to be creative in presenting their report, but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided to the **Connect worksheet**.
- An assessment rubric is available (see Resources).
- Students can undertake an alternative or extra connect task using the investigations provided. Direct them to **CI 4-1 A safe journey home** and/or **CI 4-2 More complex expansions** (see Resources).

Additional Connect investigations

CI 4-1 A safe journey home

Focus: To link the algebraic skills learned throughout this chapter to a practical, real-life scenario

Resources: calculator

Students write expressions and equations representing real-life scenarios.

They consider:

- the cost of different types of train tickets
- the cost of snacks at the movies
- the cost of a taxi fare.

Students write expressions and equations and substitute values into their equations to calculate costs.

There are different costs involved for different friends within the group and, as an extension, students can calculate the cost for each member of the group. They will need to consider their algebra carefully and may need to be reminded to clearly define each variable.

An assessment rubric is available (see Resources).

CI 4-2 More complex expansions

Focus: To consider the use of the area model in more complex expansion problems

Resources: coloured pencils, **1-cm grid paper** (BLM, see Resources)

Students compare the use of **BIDMAS** and the area model to expand binomial products.

This is a challenging task which will require lateral thinking. More able students should be encouraged to complete this task in preparation for expanding and factorising quadratics in Year 9. Students consider:

- expansions using numbers only
- expansions using pronumerals, but with only positive signs
- the patterns that exist within these calculations.

As an extension, students consider expansions which include pairs of brackets that both hold a negative term and expansions in which one set of brackets holds a positive term and the other set of brackets holds a negative term. They consider whether the pattern found in all positive expansions applies to these different types of expansions.

An assessment rubric is available (see Resources).

Resources

Connect worksheet

CW 4 The magnificent mind reader!

Additional Connect investigations

CI 4-1 A safe journey home

CI 4-2 More complex expansions

Assessment rubrics

The magnificent mind reader!

A safe journey home

More complex expansions

BLM

1-cm grid paper

integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Direct students to examine the opening photo for this chapter on pages 182 and 183 of their Student Book.
- Ask students what they think the photograph is about. (An iceberg)
- There are two parts of an iceberg – the part above the water which is visible, and the part below the water which is not visible. Ask students to use their knowledge of icebergs to compare these using fractions. (Approximately $\frac{1}{3}$ of the iceberg is above sea level and approximately $\frac{2}{3}$ of the iceberg is below sea level)
- If the top of the iceberg in the photo is 40 m above sea level, what distance from sea level would the bottom of the iceberg be? (Approximately 80 m)
- Discuss how sea level is a reference point that helps us describe the positions of the top and bottom of the iceberg.
- Have students think of a number to represent sea level in this case. (0)
- Using sea level as the reference point (or 0), what numbers could be used to describe 40 m above sea level and 80 m below sea level? (40 m above sea level: +40 or 40; 80 m below sea level: –80)
- Discuss why it can be useful to use positive and negative numbers.
- Brainstorm ideas about other examples where positive and negative numbers are used in everyday life. Discuss what would be considered the reference point in each case.

  Some examples are buildings with lifts that have aboveground floors and belowground car parking, temperature, deposits and withdrawals on bank accounts.
- Have more able students consider the numbers used for the positions of the top and bottom of the iceberg in the photo if the reference is not at sea level; for example, if the reference point is considered to be at –10 (10 m below sea level).
- Ask students if they can recall what happened to the Titanic. (Sank after hitting an iceberg)
- As a homework task, students can research the sinking of the Titanic and the size of the iceberg (early newspaper reports describe the iceberg as having a height above sea level

of between 50 and 100 feet or between 15.24 and 30.48 m). Have them write numbers for the positions of the top of the iceberg and the bottom of the iceberg. Ask them to explain their choice of positive and negative numbers and the reference they used.

Essential question

How can we describe numbers that are lower or less than zero?

Sometimes we have to describe an amount that is less than zero like very low temperatures or using money you do not have. For some, this is an incredibly difficult concept to understand and to be able to operate with negative numbers, students must first understand the concept that these numbers exist.

Are you ready?

Prior knowledge and skills can be tested by completing **Are you ready?** This will give you an indication of the differentiated pathway each student should follow.

Students will demonstrate their ability to:

- order Numbers on a number line
- identify the difference between consecutive numbers on a scaled number line to determine the scale
- recognise and use the symbols for more than (>) and less than (<)
- identify the direction of movement on the number line which is equivalent to more positive and less positive
- identify the direction of movement on the number line which is equivalent to more negative and less negative
- locate a whole number on the number line
- locate a fractional or decimal number on the number line
- understand that speed is a measure of how distance is related to time
- calculate speed from a given distance and time.

At the beginning of each topic there is a suggested differentiated pathway which allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to help students select the best pathway.

Support strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

ANSWERS

CHAPTER 4 INTEGERS AND THE CARTESIAN PLANE

4 Are you ready?

1 6 2 B

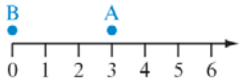
3 A is at 10; B is at 0; C is at 25.

4 a 5 > 3 b 0 < 9

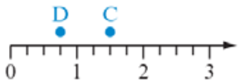
5 A 6 B 7 D

8 D 9 2 10 0

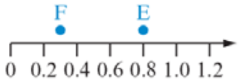
11



12



13



14 D 15 50 km/h

Resources

assess: assessments

Each topic of the *MyMaths 7* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students' confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

4A Understanding negative numbers

Teaching support for pages 184–9

Teaching strategies

Learning focus

To understand numbers that are less than zero and apply understanding of integers to a range of contexts

Start thinking!

In this task, students:

- consider how negative numbers are written
- consider the order of negative numbers – numbers more negative or less negative
- are introduced to the term ‘integer’.

Discuss the occurrence of negative numbers in real life. Encourage students to share their current understanding.

An area for potential misconception for students is that the numbers they have being operating with up until now are now being called positive numbers and annotated with a + sign. Explain that this is only done to differentiate between positive and negative numbers.

Highlight how we annotate negative numbers (–).

Take time to make the distinction between numbers that are greater than others and numbers that are more positive than others. Students instinctively feel that –7 is greater than –5 but what they are recognising is that –7 is more negative than –5. This is a useful idea when adding and subtracting negatives, so rather than discourage its use; highlight the distinction that *more than* is accepted as meaning *more positive than*.

Differentiated pathways

Below Level	At Level	Above Level
1a–c, 2a–c, 3a–d, 5a, 7, 11a, 12a–c, 13	1d, e, 2d–i, 3c–i, 5a–d, 6a–c, 7, 8, 10–14, 16	2–6, 8, 9, 11c,d, 14–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q1–7

Focus: To recognise that different number lines may represent different scales, describe the position of a point on a number line, and to compare numbers and list them in ascending and descending order according to size

- Direct students to complete **SS 4A-1 Understanding scales on number lines** (see Resources) if they had difficulty with Q1 and Q2 or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand these skills. The BLM **Number lines 0 to 10** (see Resources) can be used.
- To emphasise that each number line needs to be looked at carefully in terms of scale, it can be beneficial to take a variety of items into the classroom for comparison. Some examples are protractor, measuring cylinder, ruler, thermometer.
- The protractor has a scale in degrees. Each major interval represents 10° and each minor interval represents 1°.
- A measuring cylinder has a scale in mL. Each major interval represents 50 mL and each minor interval represents 10 mL.
- The ruler has a scale in cm/mm. Each major interval represents 1 cm (or 10 mm) and each minor interval represents 1 mm.
- Some thermometers have two scales: one in degrees Celsius and the other in degrees Fahrenheit.
- For degrees Celsius, each major interval represents 10°C and each minor interval represents 1°C. For degrees Fahrenheit, each major interval represents 20°F and each minor interval represents 2°F.
- Direct students to complete **SS 4A-2 Describing points on a number line** (see Resources) if they had difficulty with Q3 or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand these skills. The BLM **Number lines 0 to 10** (see Resources) can be used.
- You may like students to copy the scale from a measuring cylinder into their workbooks as a number line, starting at zero and ending at the last number on the cylinder. Have them fill in the scale and identify the major intervals and the minor intervals.
- Fill their cylinder to a specific level; for example, 30 mL. Ask them to represent this on their number line model so that they can see the link. This can be repeated as many times as required.
- Direct students to complete **SS 4A-3 Comparing the size of two numbers** (see Resources) if they had difficulty with Q4–7 or require more practice at this skill.

- You may need to undertake some explicit teaching so students understand how to compare the size of two numbers.
- Have students draw a number line from 0 to 20 in their workbooks. Ask them to make piles of counters beside different pairs of numbers; for example, 4 and 16. Complete a statement such as 4 _ 16, using the signs *greater than* or *less than*. Give students several such questions to answer and fill in.
- Students can be guided to discover that the further a number is to the left on a number line, the smaller it is, and the further a number is to the right on a number line, the larger it is.
- To ensure that students have a good grasp of this concept, a larger number line can be drawn on the board and the task repeated, using larger numbers. As a final component of this task, students could complete similar questions, using larger numbers, without a number line for reference.

At Level

At Level
1d,e, 2d–i, 3c–i, 5a–d, 6a–c, 7, 8, 10–14, 16

- Demonstrate **4A eTutor** or direct students to do this independently.
- Be consistent in your description of negative numbers; do not flip between a negative amount and a negative position.
- Students need to understand a negative amount before a negative position.
- Do not use the number line until students are confident with negative amounts.

POTENTIAL DIFFICULTY
Consider the following before introducing negative integers:
Jack lost $1 000 000 (–$1 000 000) and Anh has lost $1 (–$1). Using the number-line model we encourage students to see that –1 is bigger than –1 000 000, so logically it follows that Anh must have lost more!! When using real-life examples be sure you understand the difference between the bigger number and the bigger loss. In real life, students are thinking about the bigger loss.

- Avoid saying that +3 is bigger than –3. Instead help to consolidate the mental models of the position of negative numbers by describing +3 as 3 more than zero and –3 as 3 less than zero. To build understanding of the comparative size of numbers, use the language more positive and less positive until student has a sound grasp of the concept. Then connect these terms to their equivalent inverse: more positive = less negative, and less positive = more negative.
- Where possible avoid ‘*moving left or right*’ on the number line but rather use *more than* or *less than* to support conceptual understanding.

- For additional practice, students can complete **WS 4A-4 Integers and number lines** (see Resources), where they compare the size of positive and negative integers using position on a number line.

- For problem-solving and investigating, refer students to **INV 4A-6 Weather at Thredbo** (see Resources). Students work with real-life data collected about temperature at Thredbo and place these temperatures on a number line. The raw data has been rounded so students work with whole integers to compare temperatures. As an extension, students analyse the raw data that contains decimal numbers.

- For an additional investigation, refer students to **INV 4A-7 Lots of different ways** (see Resources). Students use counters to represent positive and negative integers. They work with zero pairs and explore different representations of the same value. The opportunity to extend the model to represent larger values is provided.

POTENTIAL DIFFICULTY
We often use bank accounts as a model for negative numbers, but very few students can use this example to develop their understanding. For example, if a student has $10 and buys a DVD for $20 by borrowing $10 from a friend, when asked how much money they have they will say $0 they do not see the amount they have as –$10. Use of the verb *have* for many people can only describe a positive entity and this thinking can make it extremely difficult for students to accept the concept of negativity.

Below Level

Below Level
1a–c, 2a–c, 3a–d, 5a, 7, 11a, 12a–c, 13

- Demonstrate **4A eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
We do not use a sign to distinguish positive numbers except when teaching integers where they appear for the first time.

- Some students may only be able to visualise negative numbers as an actual amount which means that –3 is the same size as 3 but in the other direction. This can be described as a space for three. This means that the idea that –1 is larger than –4 is nonsensical to these students as the magnitude of the negative is what they ‘see’. Integers can still be ordered if students have this thinking but 0 is a more pivotal point under these circumstances. They can be asked which negative number is closer to zero rather than which number is bigger.

- Numbers which are too far away from zero are too difficult to model mentally for some students; zero may always need to be seen.

POTENTIAL DIFFICULTY
Negative integers are an abstract concept and the number line in particular is not a concrete scaffold for pre-abstract thinkers.

- Scales are an unnecessary distraction from the desirable conceptual learning outcome for some students as it requires an ability to recognise patterns and determine differences. Allow students to use a 1-to-1 scale when using the number line.

- Some students will have trouble visualising the value of each number on the number line. Teachers often model jumping from one number to the next, which only some students will interpret as an amount.
- Students who are unable to order positive integers should not attempt to order negative integers.
- Students of this level may find it helpful to draw a number line with the negative portion red in colour, and the positive portion blue in colour.
- For Q12, students can be provided with 20 coloured counters (10 each of two colours) or a copy of the BLM **Counters** (see Resources) so they can cut out a set for themselves.
- For additional practice, students can complete **WS 4A-4 Integers and number lines** (see Resources**),** where they compare the size of positive and negative integers using position on a number line.
- For additional practice at moving along a number line, refer students to **INV 4A-5 Wilderness adventure** (see Resources). Students play a board game to practise moving along a number line in both positive and negative directions.

POTENTIAL DIFFICULTY
The number line does not relate to the place-value structure which students have been using previously. In the number line, the zero is the place holder whereas the decimal point is the place holder on a place value mat.

Above Level

Above Level
2–6, 8, 9, 11c,d, 14–18

- Demonstrate **4A eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
In Australia, students rarely see a sliding scale of anything which includes negative numbers and often on a freezer or in the car the temperature will be a digital reading. Few students have any real experience of reading scales so these become another example of an abstract number line.

- Students who already have a sound grasp of negative integers should be challenged to use numbers which are far from the zero reference point.
- Negative decimals or, for greater degree of difficulty, negative fractions can be ordered instead of whole numbers.
- Introduce the vertical number line as early as possible for these students.
- For Q15, students may need the terms ‘deposit’ and ‘withdrawal’ explained.

- For problem-solving and investigating, refer students to **INV 4A-6 Weather at Thredbo** (see Resources). Students work with real-life data collected about temperature at Thredbo and place these temperatures on a number line. The raw data has been rounded so students work with whole integers to compare temperatures. As an extension, students analyse the raw data that contains decimal numbers

- For an additional investigation, refer students to **INV 4A-7 Lots of different ways** (see Resources). Students use counters to represent positive and negative integers. They work with zero pairs and explore different representations of the same value. The opportunity to extend the model to represent larger values is provided.

Extra activities

1 Students can use the numbered playing cards from an ordinary deck to generate integers. Black cards represent positive and red cards represent negative integers. Students share out the cards evenly among 3 or 4 players. Each player then makes as many combinations of zero as they can. The winner is the player who can demonstrate the most zeros.

2 This task can be undertaken as a class activity.

Provide 20 coloured counters (10 each of two colours) to each student or pair of students and have them work through Q12 in Exercise 4A as a practical task. If counters are not available, provide a copy of the BLM **Counters** (see Resources) so students can cut out a set for themselves.

Answers

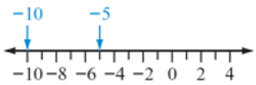
4A Understanding negative numbers

4A Start thinking!

1 Negative numbers have a negative (minus) sign in front.
2 −27 is larger than −60.
3 a Some examples are: +5, 29, +102.
b Some examples are: −8, −26, −354.
4 Some examples are: +19, 0, −83

Exercise 4A Understanding negative numbers

1 a Some examples given.
i −2, −4, −5 ii 2, 3, 4 b 0
c


d i 4 ii 2 iii 0 iv 4
v 2 vi −2 vii 4 viii −1
e i 1 ii 0 iii −4 iv −1
v −2 vi −3 vii −1 viii −4
2 a −4 < 6 b −5 > −9 c 7 > 0
d 0 > −3 e −2 < −1 f −8 > −9
g 6 > −6 h −7 < −1 i −4 < −3
3 a −25 b −27 c −75
d 0 e −81 f −300
g −853 h −2000 i −4444
4 a −40 < 30 b 0 > −22 c −65 < −55
d 78 > −87 e 127 > −134
f −500 < 0 g −120 > −170
h 248 > −300 i −362 < −326
5 a −6, −2, 0, 4, 6 b −15, −13, −3, 4, 10
c −12, −11, −2, −1, 1
d −18, −9, −5, 5, 10, 16
e −45, −20, −1, 0, 3, 35
f −77, −37, −17, −7, 7, 57
6 a −10°C
b i higher ii higher
iii lower iv higher
c A larger number is higher on the scale.
d i Some examples are: −8°C, −2°C, 12°C.
ii Some examples are: −18°C, −12°C, −20°C.
7 −4, −3, −2, −1, 0, +1, +2, +3, +4, +5, +6
8 One possible answer is: −39, −33, −27.
9 One possible answer is: +22, +3, 0, −9, −15.
10 a −3 b +360 c −2 d −15
e +2230 f −28
11 a i 6 units to the left of 0
ii 8 units to the right of 0
iii 2 units to the right of 0
iv 2 units to the left of 0
v 4 units to the left of 0
vi 4 units to the right of 0
b +2 and −2, −4 and +4
c i +9 ii −7 iii +25 iv −10
v −99 vi +53
d Some examples are: −24 and +24, −77 and +77, −3 and +3.
12 a two
b i +3 ii −4 iii −5 iv +6
c i four blue counters
ii three red counters
iii seven blue counters
iv eight red counters
v two red counters vi five blue counters
vii nine red counters viii ten blue counters
d Since +1 and −1 are opposite integers (same distance from zero), one blue counter and one red counter 'cancel' each other out and you are left with zero.
e Two blue counters and two red counters form two zero pairs. These counters 'cancel' each other out. The three red counters left over represent −3.
f i +2 ii −1 iii +1 iv −4
v 0 vi 0
13 a


b −10°C c −10°C, −7°C, −5°C
d cooler
14 a nine levels b Level 5 c Level −3
d You go up three levels.
e You go down two levels.
f Higher; Level 1 is located higher on the scale than Level −1.
g 3 and −3, 2 and −2, 1 and −1
h So that you can see how many levels you are above or below ground level.
15 a +100 b $20 c −20
16 a +5 b −30 c 35 m d −3800
17 a Paulo: −85; Kamilla: +98; Amad: +135; Jessica: −172
b Amad c Jessica
18 One example is given in each case.
a five blue counters and two red counters
b five blue counters and seven red counters
c one blue counters and seven red counters
d eight blue counters and four red counters
e three blue counters and three red counters
f six blue counters and eleven red counters

Reflect

Possible answer: We need negative numbers to describe values below zero; for example, debt and temperatures.

Resources

SupportSheets

SS 4A-1 Understanding scales on number lines

Focus: To understand how a scale works on a number line

Resources: ruler

Students are guided through questions regarding the size of intervals on number lines.

SS 4A-2 Describing points on a number line

Focus: To use a number line scale to describe the position of points

Resources: ruler

Students are guided to identify the scale on different number lines and then name points located along the number line.

SS 4A-3 Comparing the size of two numbers

Focus: To compare the size of pairs of numbers using < and >.

Students are guided through specific questions about the comparison of whole numbers. They then complete questions independently using > and < symbols.

WorkSheet

WS 4A-4 Integers and number lines

Focus: To use a number line to help compare the size of integers

Students compare the size of positive and negative integers using position on a number line. Additional practice questions are also provided.

Investigations

INV 4A-5 Wilderness adventure

Focus: To move along a number line in both positive and negative directions.

Resources: two dice, dot stickers, **Counters** (BLM), partner

Students play this board game to practise moving along the number line in both positive and negative directions.

INV 4A-6 Weather at Thredbo

Focus: To place values that represent different temperatures at a mountain resort along a number line.

Resources: coloured pencils, ruler.

Students work with real-life data collected about temperature at Thredbo and place these temperatures on a number line. The raw data has been rounded so students work with whole integers to compare temperatures. As an extension, students analyse the raw data that contains decimal numbers.

INV 4A-7 Lots of different ways

Focus: To use counters and zero pairs to represent the same integers in different ways.

Resources: coloured pencils, 20 counters (10 each of two colours)

Students use counters to represent positive and negative integers. They work with zero pairs and explore different representations of the same value. The opportunity to extend the model to represent larger values is provided.

BLM

Number lines 0 to 10

Counters

Interactives

4A eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4B Adding integers

Teaching support for pages 190–5

Teaching strategies

Learning focus

To develop an understanding of how positive and negative numbers are added together and apply understanding of addition of integers

Start thinking!

In this task, students:

- use counters to model the addition of two integers
- work towards understanding the concept of a zero pair
- model (+6) + (+2) using 6 blue counters and another 2 blue counters
- model (+6) + (–2) using 6 blue counters and 2 red counters

  experiment with other combinations of the counters to understand the result of adding positive and negative integers
- look for patterns and are guided to generalise the results they obtain.

This activity allows students to develop their ability to model integer operations using two different coloured counters, blue to represent positive and red to represent negative integers. If counters are not available, provide students a copy of the BLM **Counters** (see Resources) so they can cut out a set for themselves.

Consolidate addition with positive integers before progressing to additions including negative integers. The assertion that the overall net value of 1 blue and 1 red being zero (one zero pair) has to be accepted by students if they are to successfully use this model.

Using zero pairs is particularly helpful for students who require a concrete model of negativity. Initially, students need to create as many models of one number as possible.

Differentiated pathways

Below Level	At Level	Above Level
1, 2a–f, 7, 8a–f, 13	1d–f, 2–4, 5a–f, 6, 7, 8g–o, 9a–f, 10, 11a–f, 12–14	2–6, 7, 9–11, 14–18

Students complete the **assessment**, **eTutor** and **Guided example** for this topic

Support strategies for Are you ready? Q8–10

Focus: To recognise that the number line can be used for the addition of integers

- Direct students to complete **SS 4B-1 Position of points on a number line** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how to move along a number line.
- It may be helpful for students to physically move a counter or the point of a pencil left or right along a number line from the given point. They can count the number of units to be moved as the number of 'jumps' to be performed.

At Level

At Level
1d–f, 2–4, 5a–f, 6, 7, 8g–o, 9a–f, 10, 11a–f, 12–14

- Demonstrate **4B eTutor** or direct students to do this independently.
- To emphasise the net value of the blue (positive) counters and the red (negative) counters, students can cover up equivalent numbers of each counter to see how many and what colour are left over.
- When using the net value of counters, the order in which they are assembled is immaterial. However, to facilitate the progression towards using a number-line model, students can arrange the first number above the second number.
- When using a number line to add, teachers often demonstrate the count by drawing a hump above each place added along the number line. Some students will not see the spaces as representing the objects counted and will count the lines instead. Modelling this can lead to confusion with the students. Ensure you explicitly explain the count model you are using.
- For Q8, some students may benefit from a copy of the BLM **Number lines –10 to +10** (see Resources).
- For additional practice creating zero pairs, refer students to **WS 4B-2 Adding integers using counters** (see Resources). Students are guided through a task where different coloured counters are used to represent positive and negative integers, and the zero pair. Additional practice questions are also provided.

- For problem-solving and investigating, refer students to **INV 4B-4 Wheelie clever addition** (see Resources). Students work with a magic wheel in which all of the spokes need to add to a specific integer. The opportunity to create a magic wheel is provided.

Below Level

Below Level
1, 2a–f, 7, 8a–f, 13

- Demonstrate **4B eTutor** or direct students to do this independently.
- Encourage students to organise their counters as they would in a tens frame (two rows of five counters).

POTENTIAL DIFFICULTY
When using counters, students can forget which colour represents which type of integer. Therefore some students may benefit from labelling the counters with a permanent marker.

- When using counters, keep the counters of the same colour in one row.
- Organise the counters so that the first number lies above the second number to help students to visualise which counters form zero pairs (those which cancel each other out).

POTENTIAL DIFFICULTY
Students can have difficulty building a mental model of a negative number when they have pre-abstract processing skills. It may help these students to think of negative numbers as a space for that number. So –3 is a space for 3.

- When using a number line to add integers, provide students with a copy of the BLM **Number lines –10 to +10** (see Resources). They may need some explicit teaching to show them the correct procedure. If the integer being added is positive, move right (positive direction). If the integer being added is negative, move left (negative direction).
- It may be useful for students of this level to produce the answer to Q7 as a poster which could be displayed in the classroom for easy reference.
- For additional practice creating zero pairs, refer students to **WS 4B-2 Adding integers using counters** (see Resources). Students are guided through a task where different coloured counters are used to represent positive and negative integers, and the zero pair. Additional practice questions are also provided.
- Students can also practise some problem-solving and investigate zero pairs by completing **INV 4B-3 It all adds up!** (see Resources). Students use a concrete model to represent positive and negative integers. They work with the creation of zero pairs and explore addition of integers.

Above Level

Above Level
2–6, 7, 9–11, 14–18

- Demonstrate **4B eTutor** or direct students to do this independently.
- Student who can easily operate with negative integers can be challenged by placing negative fractions and decimals on a number-line.
- Use negative fractions and decimals when completing additions.
- For Q16 and Q17, encourage students to be creative in their answers, and not provide simple, straightforward examples.

POTENTIAL DIFFICULTY
For many younger teenagers, if they owed someone $5, and you asked them how much money they have, they will tell you that they have no money, not negative $5. Using owing the bank money may not help students to visualise a negative amount.

- For further problem-solving and investigating, refer students to **INV 4AB-5 Slide rule addition** (see Resources). Students follow instructions to create their own slide rule that can be used for the addition of positive and negative integers.

Extra activities

1 As described in in Extra Activities 4A, students can use the numbered playing cards from an ordinary deck to generate integers.

The black cards represent positive integers and the red cards represent negative integers.

a In pairs, play a game of 21. Using only the numbered cards from the deck, students deal out five cards each. Students then add the cards in their hand (black are positive and red are negative). Students have the option to discard one card. Closest to 21 wins.

b Zero Snap can be played in small groups. Students deal out all of their cards and place their pile face down in front of them. Students take turns to play a card and ‘snap’ if two (or more for a greater challenge) cards add to make zero.

2 A small group activity can be performed with those students who find the number line model confusing. Draw a number line on the ground and mark a scale from –10 to +10. Model problems such as (–3) + (+4) by having the student start by standing on –3. Ask the student which way the movement will be to add (+4). What will the result be after this movement? Many combinations of addition of integers can be modelled in this manner.

Answers

4B Adding integers

4B Start thinking!

1 a eight blue counters b +8
c $(+6) + (+2) = +8$
2 a +4 b $(+6) + (-2) = +4$
3 a $(-4) + (+3) = -1$ b $(-4) + (-3) = -7$
c $(-3) + (+5) = +2$ d $(-5) + (+5) = 0$
4 a positive b negative
c positive, negative (or negative, positive)

Exercise 4B Adding integers

1 a $(-6) + (-1) = -7$ b $(-8) + (+3) = -5$
c $(+7) + (-4) = +3$ d $(-3) + (-6) = -9$
e $(+2) + (+4) = +6$ f $(-5) + (+3) = -2$
2 a +5 b −9 c +2 d −5
e +3 f −2 g −3 h −6
i 0 j +1 k 0 l 0
3 a +8 b +5 c −8 d +10
e −2 f −8 g −4 h +5
i 0 j 0 k 0 l 0
4 If the positive number is further from zero than the negative number, the sum will be positive.
If the negative number is further from zero than the positive number, the sum will be negative.
If the positive number and the negative number are the same distance from zero (opposite numbers), the sum will be zero.
5 a negative b zero c positive
d negative e positive f negative
g zero h negative i positive
j negative k positive l zero
6 a $(-8) + (+3) = -5$ b $(+7) + (-4) = +3$
c $(-6) + (-1) = -7$ d $(-1) + (+4) = +3$
e $(+5) + (+3) = +8$ f $(-2) + (-7) = -9$
7 a right, positive b left, negative
8 a +1 b +7 c −2 d −7
e −9 f +3 g −3 h +2
i 0 j +1 k 0 l +1
m 0 n +4 o −5
9 a +8 b +5 c −8 d +10
e −2 f −8 g −4 h +5
i 0 j 0 k 0 l 0
11 a −5 b +13 c −13 d −23
e −29 f +12 g −59 h −96
i −27 j −22 k +2 l +108
12 a $(-18) + (+98) = +80$ b 80°C
13 a $(-8) + (+5) = -3$ b −3°C
14 a −26 b $(-26) + (+40) = +14$ c $14
15 a −50 b $(-50) + (+35) = -15$ c $15
16 Some examples are: −6 and +2, −18 and −3, 0 and −4.
17 Some examples are: −9, −13, −7, −45.
18 a +4 b −7 c −1 d +10

Reflect

Possible answer: Counters can help you to visualise the amount that you have, especially if you are able to pair off two opposite counters to make zero.

A number line can help you to locate the position of your number relative to other numbers and move the correct amount which you are adding on. The number line helps you to visualise what happens when you add a positive number and that the opposite action happens when you add a negative number.

Resources

SupportSheet

SS 4B-1 Position of points on a number line

Focus: To use a number line to locate a point that is a given number of units to the right or to the left of a number

Students are guided through a series of questions in which they locate the position of a point on a number line and then count a number of steps to the left or the right of the original number.

WorkSheet

WS 4B-2 Adding integers using counters

Focus: To use counters (of two different colours) to model the addition of integers

Resources: 10 blue counters and 10 red counters (or any two colours)

Students are guided through a task where different coloured counters are used to represent positive and negative integers, and the zero pair. Additional practice questions are also provided.

Investigations

INV 4B-3 It all adds up!

Focus: To model the addition of positive and negative integers using counters

Resources: coloured pencils, 20 counters (10 each of two colours)

Students use a concrete model to represent positive and negative integers. They work with the creation of zero pairs and explore addition of integers.

INV 4B-4 Wheelie clever addition

Focus: To add positive and negative integers

Resources: Counters (BLM) (optional)

Students work with a magic wheel in which all of the spokes need to add to a specific integer. The opportunity to create a magic wheel is provided.

INV 4B-5 Slide rule addition

Focus: To add positive and negative integers using a number line and relate this to modelling with counters

Resources: counters, cardboard, large paper clip, coloured marker pens, ruler

Students follow instructions to create their own slide rule that can be used for the addition of positive and negative integers.

BLMs

Counters

Number lines –10 to +10

Interactives

4B eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4C Subtracting integers

Teaching support for pages 196–201

Teaching strategies

Learning focus

To consolidate and apply student understanding of subtraction of integers

Start thinking!

This is a practical task where students use counters to model the subtraction of two integers. Each student or small group will need 20 counters (10 each of two colours). If counters are not available, provide a copy of the BLM **Counters** (see Resources) so students can cut out a set for themselves.

In this task, students:

- model the expression (+6) – (+2) using blue and red counters
- model subtracting a negative integer from a negative integer using the counters
- use zero pairs to find the result of a subtraction of two integers.

It is important that students understand the concept of a zero pair and can confidently use counters to model integers.

Students are guided to reflect on when zero pairs are needed, and how many are needed, to perform a subtraction calculation using counters

Differentiated pathways

Below Level	At Level	Above Level
1, 2a–f, 3–5, 8, 13a–d, 14, 15a–f, 20	1, 2e–i, 3–10, 12a–h, 13, 14, 15a–h, 16a–h, 17, 19–21	7–11, 12g–o,13d–f, 14, 15, 16g–o, 17–19, 21–24
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2e–i, 3–10, 12a–h, 13, 14, 15a–h, 16a–h, 17, 19–21

- Demonstrate **4C eTutor** or direct students to do this independently.

- Ensure students can:
  - understand how to subtract two integers using counters and zero pairs
  - also use a number line to subtract integers.
- Encourage students to identify their starting position on the number-line for each calculation.
- Students who struggle to understand zero pairs should avoid using the counter model.
- When using number lines with a scale other than a unit scale, support students to think about the difference between the two numbers that are marked to identify the scale.
- Having a permanent copy of a number line in the students' workbook will help students to build a mental model of the number line.
- Colour coding of the number line can help with retention; draw the positive section in blue (or black) and the negative section in red. This mimics the colours of the counters that are used for the zero pair (or the colours of banking – 'in the black' or 'in the red').
- For support in subtracting integers using counters, refer students to **WS 4C-1 Subtracting integers using counters** (see Resources). Students are guided through a task in which different coloured counters are used to represent positive and negative integers.
- For more problem-solving and investigating, refer students to **INV 4C-2 Modelling subtraction** (see Resources). Students model subtraction of negative integers with the net value of counters.
- To investigate the subtraction of positive and negative integers, refer students to **INV 4C-3 Wheelie clever subtraction** (see Resources). Students create mathematical subtraction equations using a magic square method of identifying the missing numbers.

POTENTIAL DIFFICULTY
Up until this point in their learning, the negative sign (–) has described an action; to take away. Now we are introducing the sign as a description of a number but we still use the same sign (–). This can be a source of confusion for some students.

Below Level

Below Level
1, 2a–f, 3–5, 8, 13a–d, 14, 15a–f, 20

- Demonstrate **4C eTutor** or direct students to do this independently.
- Students need to be able to subtract positive integers before attempting to subtract negative ones.

- If using the coloured counters model, it is easy to show subtraction of a smaller number of negative counters from a larger number of negative counters (and positives from positives) but this model becomes very confusing for some students when asked to subtract a negative number from a positive number or vice versa. This is because the students only 'see' the types of counters which are on the table. For example, –2 (2 red counters on the table) subtract +3 (no blue counters on the table for the student to physically take away).

POTENTIAL DIFFICULTY
Some students find subtraction difficult even when modelled with positive numbers. In general, competent mathematicians will solve a subtraction mentally by adding on.

- As described in Example 4C-3, this problem is overcome by using zero pairs so students can place the counters that are needed to complete the subtraction on the table. However, each of these additional counters are paired with its opposite colour so that the overall effect of adding these counters is zero. The ability to add zero pairs requires a level of cognitive understanding that may be beyond the capability of some students, as it almost means that the students have calculated the answer before constructing the counters model. For these students it would be better to arrange about 10 zero pairs initially and then model the first integer, so that counters of the correct colour can be 'seen' before subtraction takes place.

- Q14 describes the procedure for subtracting positive and negative numbers. These statements could be made into posters and displayed within the classroom for easy reference.

- Q15 uses a number line to subtract integers. You may like to provide some students with the BLM **Number lines –10 to +10** (see Resources).

- For students who find it difficult to understand subtraction, avoid number lines that do not have a unit scale (for example, go up in twos).

- For students who require additional support, refer them to **WS 4C-1 Subtracting integers using counters** (see Resources). This is an extremely challenging concept for some students so they may benefit from completing this worksheet in a simplified format. They should be presented with all required zero pairs already available, so ask students to place 10 zero pairs on the table and then construct the number. There should then be ample counters to take away.

Above Level

Above Level
7–11, 12g–o,13d–f, 14, 15, 16g–o, 17–19, 21–24

- Demonstrate **4C eTutor** or direct students to do this independently.

- Students should develop their ability to visualise the number line and the starting position of their calculation without referring to an actual number line as much as possible.

- Students could use decks of cards instead of counters (black is positive as with banking) to practise doing quick mental calculations, dealing 2 cards and always subtracting the second from the first (not the smallest from the biggest as they have been exposed to up until this point).
- Although students who are operating at this level are capable of memorising the shortcut + (+) = + and – (–) = +, they will develop a deeper understanding of the concept if they are allowed to investigate the different models (counters, horizontal and vertical number lines, playing cards) for as long as is required.
- To investigate the subtraction of positive and negative integers, refer students to **INV 4C-3 Wheelie clever subtraction** (see Resources). Students create mathematical subtraction equations using a magic square method of identifying the missing numbers.
- To explore the use of slide rules, which can be used for subtraction of positive and negative integers, refer students **to INV 4C-4 Slide rule subtraction** (see Resources). Students follow instructions to create their own slide rule for this investigation.

Extra activities

1 **Bingo**: Whole Class

Ask students to create a 9-box Bingo card and place any numbers in it from –12 to +12. Create flash cards (or a PowerPoint) representing each of the numbers shown using the counter model and zero pairs.

2 **Lucky 8**: Above Level

As in Extra Activities section for 4A, students can use the numbered playing cards from an ordinary deck to generate integers. Black cards represent positive integers and the red cards represent negative integers.

Deal out all the cards among small groups of players. Starting from zero, players place a card in the middle and subtract from the total. The player who makes the total 8 takes all the cards in the pile and play resumes from zero.

Answers

4C Subtracting integers

ANSWERS

4C Start thinking!

1 a four blue counters b $+4$
c $(+6) - (+2) = +4$

2 a one red counter b -1
c $(-4) - (-3) = -1$

Exercise 4C Subtracting integers

1 a $+2$ b -3 c -6 d $+3$

2 a $+4$ b -3 c -7
d $+1$ e -3 f $+5$
g 0 h 0 i 0

3 no, $(-2) + (0) = -2$

4 a $+4$ b -3 c -1 d $+2$

5 a -5 b $+4$

6 By adding zero pairs we can introduce counters of the second colour so that they can be used for the subtraction. The number of zero pairs to be added is the same as the number of counters of the second colour that are to be removed.

7 a $+6$ b -7 c -6
d $+9$ e -5 f $+4$
g $+8$ h -8 i $+10$

8 a -4 b $+3$

9 The number of zero pairs to be added is the same as the number of extra counters of the same colour that are to be removed.

10 a -3 b $+5$ c -1
d $+4$ e -2 f $+5$
g $+1$ h $+4$ i -2

11 You need to add zero pairs when there are no counters of the correct colour to take away.

12 a -3 b $+5$ c -1 d 0
e $+4$ f -9 g $+7$ h -9
i -1 j $+2$ k -5 l $+2$
m 0 n -6 o $+4$

13 a $(-6) - (-4) = -2$ b $(+7) - (+3) = +4$
c $(+2) - (-8) = +10$ d $(+2) - (+6) = -4$
e $(-3) - (-4) = +1$ f $(-1) - (+5) = -6$

14 a left, negative b right, positive

15 a $+5$ b $+1$ c -10 d $+6$
e -8 f -2 g -3 h 0
i -4 j $+5$ k $+10$ l -6
m -1 n $+3$ o -6

16 a -3 b $+5$ c -1 d 0
e $+4$ f -9 g $+7$ h -9
i -1 j $+2$ k -5 l $+2$
m 0 n -6 o $+4$

17 They are the same.

18 a -5 b -11 c -49
d $+9$ e -80 f $+65$
g -10 h -71 i $+300$

19 a $(-15) - (+4) = -19$ or $(+4) - (-15) = +19$
b -19°C or 19°C

20 a $(+14) - (+20) = -6$ b -6°C

21 a $+74$ b $(-31) - (+74)$ c $105

22 Some examples are: $(-1) - (+2) = -3$, $(+4) - (+6) = -2$, $(-3) - (-2) = -1$.

23 Some examples are: $+16$, $+21$, $+32$.

24 a -5 b $+2$ c -8 d 7

Reflect

Possible answer: Different students will give different responses; some will prefer the counters model and others will prefer the number line model. Ask a selection of students to explain their preferred model to the rest of the class.

Resources

WorkSheet

WS 4C-1 Subtracting integers using counters

Focus: To use counters (of two different colours) to model the subtraction of integers

Resources: 10 blue and 10 red counters

Students are guided through a task where different coloured counters are used to represent positive and negative integers, and the zero pair. Additional practice questions are also provided.

This can be an extremely challenging concept for some students and could be presented with all required zero pairs already available. Ask students to place 10 zero pairs on the table and then construct the number so there are ample counters to take away.

Investigations

INV 4C-2 Modelling subtraction

Focus: To model the subtraction of positive and negative integers using counters

Resources: coloured pencils, 20 counters (10 each of two colours)

Students use a concrete model to represent positive and negative integers. Students work with the creation of zero pairs and explore subtraction of integers.

INV 4C-3 Wheelie clever subtraction

Focus: To subtract positive and negative integers

Resources: counters (optional)

Students work with a magic wheel in which both numbers on the spoke of the wheel need to be subtracted from the central number to reach a specific value; for example, –5. The opportunity to create a magic wheel is provided.

INV 4C-4 Slide rule subtraction

Focus: To subtract positive and negative integers using a number line and relate this to modelling with counters

Resources: counters, cardboard, large paper clip, coloured marker pens, ruler

Students follow instructions to create their own slide rule that can be used for the subtraction of positive and negative integers.

BLMs

Counters

Number lines –10 to +10

Interactives

4C eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4D Simplifying addition and subtraction of integers

Teaching support for pages 202–7

Teaching strategies

Learning focus

To consolidate and apply student understanding of simplifying addition and subtraction of integers

Start thinking!

In this task, students are guided to:

- discover a simpler, equivalent way of writing an addition or subtraction calculation
- represent subtracting a positive number and adding a negative number on a number line
- observe that they can replace two adjacent signs with one equivalent sign to produce a simpler expression
- generalise their results to come up with a shortcut for adding and subtracting integers.

Students discover the equivalence between:

- subtracting a positive number and adding a negative number
- adding a positive number and subtracting a negative number.

Students who are able to identify this equivalence will be able to use the shortcuts for adding and subtracting negative integers.

It may be helpful to use the BLMs **Number lines –10 to +10** and **Number lines –20 to +20** (see Resources) to project on to a board during class discussion of using number lines to perform addition and subtraction calculations.

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 3d–f, 4a–f, 5, 6d–f, 7a–c, 8a, 13, 16, 18	1–7, 8b, 9a–c, 10, 11, 12a, b, 13–17	1, 2, 4, 5, 8b, 9–12, 15, 17, 19–22
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1–7, 8b, 9a–c, 10, 11, 12a, b, 13–17

- Demonstrate **4D eTutor** or direct students to do this independently.
- If possible, avoid teaching the concept that the further right a number is on the number line, the bigger the number is. When students are first beginning to understand the concept of negative integers they will see that the further left a number is away from zero the more negative it is and the further right, the more positive. It is extremely difficult for students to understand that –3 is more positive than –5 and equally that +7 is more negative than +10.

POTENTIAL DIFFICULTY
Learning tricks and shortcuts in any area of mathematics is only possible for students who have good powers of retention, and often hinders the acquisition of conceptual knowledge. Minimise the amount of time spent on learning tricks and maximise the time spent on building conceptual understanding.

- Adding a negative number produces a result that is more negative (moves further left) and subtracting a negative number produces a result that is less negative (moves further right). This model holds for any starting number, but again this can be an extremely challenging concept for some students.
- For additional support and practice writing addition and subtraction calculations, refer students to **WS 4D-1 Writing simpler equivalent calculations** (see Resources). Students are guided to simplify addition and subtraction problems involving integers by writing equivalent expressions. Additional practice questions are also provided.
- To practise solving mathematical calculations whilst playing a game, refer students to **INV 4D-3 Roll the dice** (see Resources). Students play a game in which they create their own addition and subtraction problems by rolling dice. Once written, they can simplify their problem by combining signs. A calculator can be used to check answers.
- For an additional problem-solving activity and investigation, refer students to **INV 4D-5 The toy library** (see Resources). Students complete a problem-solving task where they calculate the number of toys available at a toy library, and the number of toys on loan, over a week.

Below Level

Below Level
1, 2, 3d–f, 4a–f, 5, 6d–f, 7a–c, 8a, 13, 16, 18

- Demonstrate **4D eTutor** or direct students to do this independently.
- For guidance and support in practising the addition and subtraction of integers, direct

students to **WS 4D-2 Adding and subtracting integers using a number line** (see Resources).

- Direct students to **Example 4D-1** and **Example 4D-2**. They show how to write a simpler equivalent of a calculation, and then obtain the result using a number line.
- Do not introduce subtraction of negative integers too quickly (if at all) for some students. A sound understanding of what happens when negative numbers are added is preferable to a confused understanding of two operations.
- Encourage students to identify the starting point of the calculation first so, if the problem is –3 + 4, students need to be competent at locating –3 on the number line to be able to perform the required calculation.
- Build student confidence with negative numbers by having a negative number as the start point initially then adding positive numbers. This is a more concrete concept for students who are struggling to visualise negative numbers. The next stage is to add negative numbers from a positive starting position (4 + (–2)) before attempting addition of any number.
- To practise solving mathematical calculations whilst playing a game, refer students to **INV 4D-3 Roll the dice** (see Resources). The activity can be simplified for students at this level. Students can use 6-sided dice to generate positive and negative symbols and addition or subtraction signs, and a 10-sided die to generate random numbers. Students generate only two digits and determine whether these digits are positive or negative. Students can then be instructed to add the two digits together.
- For an additional problem-solving activity, refer students to **INV 4D-5 The toy library** (see Resources). Students should complete Q1–3 of this investigation where the concept of a negative number is modelled as a toy on loan from a toy library.

Above Level

Above Level
1, 2, 4, 5, 8b, 9–12, 15, 17, 19–22

- Demonstrate **4D eTutor** or direct students to do this independently.
- Once students demonstrate a sound understanding of the ‘rules’ for addition and subtraction of integers, they should be challenged cognitively with more difficult numbers like negative fractions and decimals.
- Some students could be given the opportunity to investigate multiplication of negative integers, starting with problems like 2 groups of –6 (2 × –6) before introducing the concept of –2 groups of 6 (–2 × 6) which can only be justified through the Commutative Law.

POTENTIAL DIFFICULTY

A significant source of error in these types of calculations is where students look at the sign of the first number and then the operation sign and combine them. When writing each problem, encourage students to highlight the signs which are adjacent and then combine the signs in the next step. For example: (+5) – (–2) would be rewritten as 5 + 2 (not 5 – 2).

- A way for students to remember the combination of adjacent signs is:
  - if the adjacent signs are the same, a positive sign results
  - if the adjacent signs are different, a negative sign results.
- Q21 and Q22 involve calculations with three terms. As only addition and subtraction of numbers are involved, advise students to work from left to right as the order convention dictates.
- To practise solving mathematical calculations whilst playing a game, refer students to **INV 4D-3 Roll the dice** (see Resources). Students play a game in which they create their own addition and subtraction problems by rolling dice. Once written, they can simplify their problem by combining signs. A calculator can be used to check answers.
- To provide students with an additional problem-solving and investigation opportunity to model negative numbers, refer students to **INV 4D-4 Saving up!** (see Resources). Students complete a problem-solving task in which they calculate the number of coins and the dollar value in a savings jar. Using positive and negative integers, students calculate a final total after a period of time. The opportunity to think about opening a bank account and the reading of a bank statement is provided.
- For further problem-solving and investigating, refer students to **INV 4D-6 The credit card trap** (see Resources). Students will get the opportunity to practise reading a credit card statement and investigate the concept of a negative amount of money.

Extra activities

First simplify and then calculate each of these.

a (+7) + (+1) (7 + 1 = 8)

b (–4) + (+2) (–4 + 2 = –2)

c (+5) + (–6) (5 – 6 = –1)

d (–7) – (+2) (–7 – 2 = –9)

e (–2) – (–9) (–2 + 9 = 7)

f (+5) – (–4) (5 + 4 = 9)

Answers

4D Simplifying addition and subtraction of integers

4D Start thinking!

1 a i +8 ii +8 b right
c You move in the same direction along the number line (to the right) when adding a positive number or subtracting a negative number.
d They both produce the same result.

2 a i +4 ii +4 b left
c You move in the same direction along the number line (to the left) when subtracting a positive number or adding a negative number.
d They both produce the same result.

3 a positive b –
c – d negative, +

Exercise 4D Simplifying addition and subtraction of integers

1 the same; different

2 a – b + c – d +
e – f +

3 a –3 + 5 b 6 – 5 c –2 – 4
d 5 + 7 e 7 + 1 f 4 – 9

4 a 2 b 4 c –4 d –5
e –1 f 5 g 8 h –1
i +11 j –8 k –2 l –7

5 a –9 b 9 c –9 d 4
e –1 f 2

6 a 2 b 1 c –6 d 12
e 8 f –5

7 a 5 b –3 c 6 d –7
e –4 f –4 g –5 h –6
i 0 j –3 k –14 l 5

8 a

+	–2	–1	0	+1	+2
–2	–4	–3	–2	–1	0
–1	–3	–2	–1	0	+1
0	–2	–1	0	+1	+2
+1	–1	0	+1	+2	+3
+2	0	+1	+2	+3	+4

b

+	–5	–3	–2	2	4
7	2	4	5	9	11
5	0	2	3	7	9
0	–5	–3	–2	2	4
–2	–7	–5	–4	0	2
–4	–9	–7	–6	–2	0

9 a 2 b –74 c –4
d –21 e –10 f –191
g 348 h –72 i –3000

11 a i –6 ii –6 iii –6
b i –6 ii –6 iii –6
c –6, –6 d the same e –6

12 a

4	–3	2
–1	1	3
0	5	–2

b

1	0	–4
–6	–1	4
2	–2	–3

c

6	–6	–9
–18	–3	12
3	0	–12

13 a 2 b –5 c 0 d –9 e –1 f –2

14 a 6°C b 5°C
c Tue 9°C, Wed 5°C, Thu 4°C, Fri 6°C, Sat 5°C
d Tue

15 a +182 b (+182) – (+200) or 182 – 200
c –18 d Raj owes $18.

16 third level below ground level; 7 – 10 = –3

17 62 m; (+50) – (–12) = 50 + 12

18 1 m; (–2) + (+3) = –2 + 3

19 a deposit b withdrawal
c Lily has money in her account.
d Lily owes money to the bank.
e +$28 f withdrawal of $9

20 Some possible answers are: –3 and 3, 5 and –5, 20 and –20.

21 a 7 b 3 c –4
d –10 e –1 f –10
g –3 h –15 i –14

22 a –8 b 9 c 4 d 14
e –15 f –5 g –11 h 3

Reflect

Answers will vary, and may include: highlight the signs to be combined; the same adjacent signs can be replaced with + and different adjacent signs can be replaced with –.

Another possible answer is: Always locate the first number of the problem on the number line as your starting point. Then think:

- when adding positive numbers, the result will be more positive and therefore you are moving right.
- when adding negative numbers, the result will be more negative and therefore you are moving left.
- when subtracting positive numbers, the result will be less positive and therefore you are

moving left.

- when subtracting negative numbers, the result will be less negative and therefore you are moving right.

Resources

WorkSheets

WS 4D-1 Writing simpler equivalent calculations

Focus: To write addition and subtraction calculations involving integers in a simpler form

Students are guided to simplify addition and subtraction problems involving integers by writing equivalent expressions. Additional practice questions are also provided.

WS 4D-2 Adding and subtracting integers using a number line

Focus: To add and subtract integers using a number line

Resources: ruler

Students simplify and then calculate addition and subtraction problems involving integers by writing equivalent expressions. Additional practice questions similar to those in Exercise 4D are also provided.

Investigations

INV 4D-3 Roll the dice

Focus: To create addition and subtraction problems in a game format

Resources: 6-sided die, 10-sided die, dot stickers, pencils, counters (optional), slide rule (optional), calculator (optional)

Students play a game in which they create their own addition and subtraction problems by rolling dice. Once written, they can simplify their problem by combining signs. A calculator can be used to check answers.

A simpler version of this activity can be provided to Below Level students.

Students can use 6-sided dice to generate positive and negative symbols and addition or subtraction signs and a 10-sided die to generate random numbers. Students generate only two digits and determine whether these digits are positive or negative. Students can then be instructed to add the two digits together. This will give them the opportunity to practise solving mathematical calculations whilst playing a game.

INV 4D-4 Saving up!

Focus: To apply knowledge of adding and subtracting integers in a real-life context.

Resources: pencils, counters (optional), slide rule (optional), calculator (optional)

Students complete a problem-solving task in which they calculate the number of coins and the dollar value in a savings jar. Using positive and negative integers, students calculate a final total after a period of time. The opportunity to think about opening a bank account and the reading of a bank statement is provided.

INV 4D-5 The toy library

Focus: To apply knowledge of adding and subtracting integers to a real-life context

Resources: pencils, counters (optional), slide rule (optional), calculator (optional)

Students complete a problem-solving task where they calculate the number of toys available at a toy library, and the number of toys on loan, over a week.

INV 4D-6 The credit card trap

Focus: To apply knowledge of adding and subtracting integers in a real-life context.

Resources: pencils, counters (optional), slide rule (optional), calculator (optional)

Students complete a problem-solving task in which they explore a credit card statement. They answer a range of questions, examining debt, interest and the use of positive and negative signs in a real-life context.

BLMs

Number lines –10 to +10

Number lines –20 to +20

Interactives

4D eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4E Introducing the Cartesian plane

Teaching support for pages 208–13

Teaching strategies

Learning focus

To plot and describe points using the first quadrant of the Cartesian plane, and apply understanding of plotting points to a range of tasks.

Start thinking!

In this task, students:

- demonstrate that they understand how to describe the position of a point on a number line
- are introduced to the first quadrant of the Cartesian plane and the terms *set of axes*, *Cartesian plane* and *Cartesian coordinates*
- consolidate their understanding of the terms *x-axis*, *y-axis* and *origin*
- learn how to label and locate a point in the first quadrant of the Cartesian plane.

The questions in this section allow the teacher to identify student competency levels when plotting coordinates on the Cartesian plane. You may like to provide students with copies of the BLM **1-cm grid paper** (see Resources) to assist them in plotting points on a Cartesian plane.

Students demonstrate their understanding of labelling and numbering axes, the numerical value of the origin and how to write and plot coordinates in the positive quadrant.

Students need to recognise that points can be identified with a letter and that the order of the coordinates is important.

Differentiated pathways

Below Level	At Level	Above Level
1–3, 5ii, 7, 13	1–5, 7–10, 12, 13	1–3, 5, 6, 10, 11, 13–15
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q11

Focus: To demonstrate an ability to plot points on a number line

- Direct students to complete **SS 4E-1 Plotting points on a number line** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how to show the plotted point on a number line.
- It is possible to demonstrate plotting of points on a number line with a simple outdoor activity. Draw a number line on the ground. Start at zero and extend the number line to 30. Discuss the scale of the number line with students. Initially, mark in 10, 20 and 30, and ask students to mark smaller intervals. This is a good opportunity to discuss the need to have the intervals as equal parts.
- An alternative is a number line represented by string. Students can be given pegs and a number card. They then take turns pegging their number into position.

At Level

At Level
1–5, 7–10, 12, 13

- Demonstrate **4E eTutor** or direct students to do this independently.
- Ensure students:
  - understand how to plot a point on a number line and that a point can be referenced by the use of a letter
  - are introduced to the first quadrant of the Cartesian plane and the *x*- and *y*-axes
  - define 'origin' and can use the *x*- and *y*-coordinates to describe the position of a point
  - know which coordinate is always written first in a pair of coordinates.
- A practical activity can assist students' understanding of plotting points. Draw a Cartesian plane (first quadrant only) on the ground, with both the *x*- and *y*-axes extending to 10. To plot (4, 6), ask a student to start by standing at zero, and then walk to 4 along the *x*-axis, turn left and walk to a position in line with 6 on the *y*-axis. Discuss how this relates to the coordinates (4, 6). Repeat with a range of points.
- Still using the Cartesian plane drawn on the ground, stand at the point (5, 8) and ask students to give the coordinates. Ask a student to demonstrate how these coordinates work. Compare with the position of a point at (8, 5).
- Ask students to give you the coordinates of a number of points and, for some of the points, stand at a position where the coordinates have been swapped so as to create confusion. Guide discussion of which coordinate we say first and obtain class

agreement that the *x*-axis coordinate should come first.

- One way for students to remember the order of coordinates is ‘run before you jump’, or *x* comes before *y* in the alphabet.
- It can be difficult for students to understand that they can plot points on the axes. Test students understanding of this by giving coordinates such as (5, 0) or (0, 2).
- You may like to provide students with copies of the BLM **1-cm grid paper** (see Resources) to assist them in plotting points on a Cartesian plane.
- For tables, such as those in Q12, encourage students to include an additional row in the table showing the *x*- and *y*-values written as a pair of coordinates. They could highlight this row when they start to plot each point.
- Provide students with an additional opportunity to demonstrate their understanding of how to construct the Cartesian plane and plot coordinates by completing the problem-solving investigation **INV 4E-4 Help, I’m only half done!** (see Resources). Students plot a sequence of points to complete an image, which is only half shown. The points include positive whole numbers and fractions.
- For further problem-solving and investigating, refer students to **INV 4E-5 Saving up** (see Resources). During this investigation, students will be expected to extrapolate from a linear graph after constructing a graph with a scaled *y*-axis. Students work through a savings program, where a specific amount is added to a bank account each week. They represent this linear relationship on a Cartesian plane, using the first quadrant only. Students answer a range of questions and make predictions from the graph.

Below Level

Below Level
1–3, 5ii, 7, 13

- Demonstrate **4E eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
Students who have shown difficulty understanding the number line may struggle to number their axes correctly.

- Restrict coordinate mapping to the positive quadrant only until competent.
- There are three main challenges for students when plotting coordinates on Cartesian planes: remembering which axis is which, which coordinate is which and which coordinate is located first.

  It can help some students to just focus on one axis and one coordinate instead of the two, so focus only on the *x*-axis and the *x* coordinate because that is the one which is located first.

POTENTIAL DIFFICULTY
Some students may confuse the order of the *x*- and *y*-coordinates. Have them write their own instructions for how to plot a point from a given pair of coordinates. They could think of a fun slogan to help them remember the order.

- You may like to provide students with copies of the BLM **1-cm grid paper** (see Resources) to assist them in plotting points on a Cartesian plane.

- For additional practice labelling the positive quadrant of the Cartesian plane including correctly naming the *x*- and *y*-axes and naming coordinates in the correct order, refer students to **WS 4E-2 The Cartesian plane (positive numbers)** (see Resources). Students are guided through a sequence of steps to reproduce a Cartesian plane (first quadrant only). They then practise plotting points and giving the coordinates of points.

- To investigate how much money students can save over time, refer them to **INV 4E-5 Saving up** (see Resources). During this investigation, students will be expected to extrapolate from a linear graph after constructing a graph with a scaled *y*-axis. They should be supported to construct a graph with a scaled *y*-axis where 1 box equals $10.

Above Level

Above Level
1–3, 5, 6, 10, 11, 13–15

- Demonstrate **4E eTutor** or direct students to do this independently.

- You may like to provide students with copies of the BLM **1-cm grid paper** (see Resources) to assist them in plotting points on a Cartesian plane.

- Students can be extended to demonstrate their level of understanding of the numbering of a Cartesian plane by asking students to reflect any given set of coordinates in the *x*-axis and predict what the new coordinates will be.

- Q9 could be completed in pairs, each student completing one half of the instructions. This will provide them with the opportunity to understand why it is important to have a universal system of sharing information (such as why the *x*-coordinate is always first and the y coordinate is always second).

- Students will have the opportunity to undertake more problem-solving and investigating by referring them to **INV 4E-3 What am I?** (see Resources). Students plot a sequence of points to create an image. The points include positive whole numbers and fractions.

- Provide students with an additional opportunity to demonstrate their understanding of how to construct the Cartesian plane and plot coordinates by completing the problem-solving investigation **INV 4E-4 Help, I'm only half done!** (see Resources). Students plot a sequence of points to complete an image, which is only half shown. The points include positive whole numbers and fractions.

- To investigate how much money students can save over time, refer them to **INV 4E-5 Saving up** (see Resources). During this investigation, students will be expected to extrapolate from a linear graph after constructing a graph with a scaled *y*-axis. Students work through a savings program, where a specific amount is added to a bank account each week. They represent this linear relationship on a Cartesian plane, using the first quadrant only. Students answer a range of questions and make predictions from the graph.

Extra activities

Whole class

As an introduction to the convention of giving the *x* or horizontal coordinate first, play a game of Noughts and Crosses. Draw up a 3 × 3 grid on the board and be careful to label the lines and not the spaces of the grid. Each nought or cross should be placed on the intersection of two lines.

Divide the class into two teams and ask for students to suggest the coordinates of the position where you should put the nought or cross. Reinforce the idea of why it is important for us all to agree on which coordinate we locate first.

Battleships

Students draw a Cartesian plane and should play in pairs of similar ability to discover the ships their partner has hidden by giving coordinates. All coordinates are recorded in the student books along with the word hit or miss.

(Below Level: draw axes from 0 to +5 and locate 5 ships, At Level: draw axes from 0 to +10 and locate 10 ships, Above Level: draw axes from –7 to +7 and locate 15 ships).

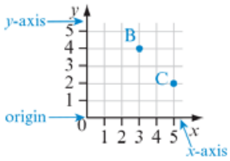
Answers

4E Introducing the Cartesian plane

4E Start thinking!

1 3 units

2, 3



5 Move 5 units along the x-axis from the origin and then 2 units up parallel to the y-axis.

Exercise 4E Introducing the Cartesian plane

1 The x-coordinate comes first. One possible answer is: x comes before y in the alphabet.

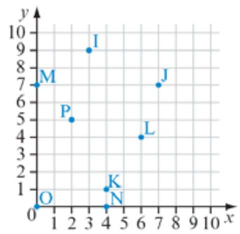
2 A(3, 5), B(6, 8), C(1, 2), D(9, 6), E(8, 3), F(2, 4), G(2, 7), H(5, 3)

3 (0, 0)

4 a F and G b E and H
c B d D

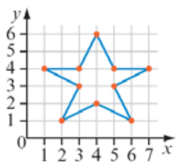
5 i a 3 units right and 9 units up
b 7 units right and 7 units up
c 4 units right and 1 unit up
d 6 units right and 4 units up
e 0 units right and 7 units up
f 4 units right and 0 units up
g 0 units right and 0 units up (at the origin)
h 2 units right and 5 units up

ii



6 a K and N; M and O
b J and M; N and O
c M and O d N and O
e O f L

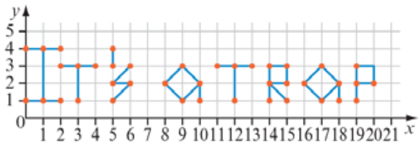
7 star



8 One possible answer is: eyes at (3, 5) and (5, 5); nose at (4, 4); mouth to start at (2, 3) then join to (4, 2) and then to (6, 3).

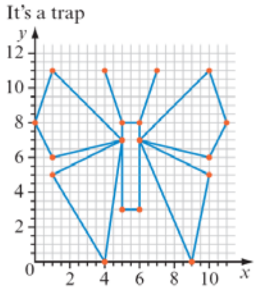
9 a The email password is TICKLES.
b (6, 4) (2, 1) (5, 4) (1, 2) (2, 4) (6, 4) (5, 4) (2, 2) (2, 5) (4, 1) (6, 1) (5, 4) (3, 3)

10



It's a trap

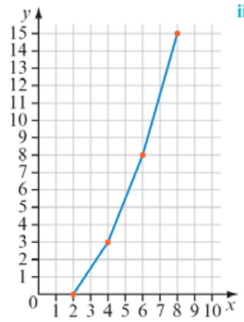
11



butterfly

12 a i (2, 0), (4, 3), (6, 8), (8, 15)

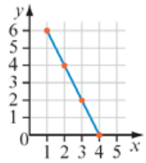
ii



iii curved line

b i (1, 6), (2, 4), (3, 2), (4, 0)

ii

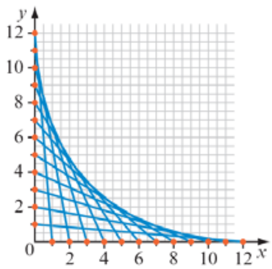


iii straight line

13 a (12, 6) (10, 7) (6, 6) (4, 5) (3, 5) (3, 7) (1, 11) (1, 13) (4, 14) (5, 16) (7, 16) (7, 17) (10, 17) (10, 16) (12, 15)

c One possible answer is: Perth (3, 7), Adelaide (14, 5), Melbourne (16, 3), Sydney (19, 6), Brisbane (21, 10), Darwin (7, 17), Canberra (18, 5).

15



Reflect

Possible answer: By convention, the horizontal coordinate is always written first.

Resources

SupportSheet

SS 4E-1 Plotting points on a number line

Focus: To plot points on a number line containing positive whole numbers and zero

Resources: ruler

Students are guided through a series of questions in which they plot points on horizontal and vertical number lines.

WorkSheet

WS 4E-2 The Cartesian plane (positive numbers)

Focus: To read the coordinates of points and plot points on a Cartesian plane using only the first quadrant

Resources: ruler, **1-cm grid paper** (BLM) (optional)

Students are guided through a sequence of steps to reproduce a Cartesian plane (first quadrant only). They then practise plotting points and giving the coordinates of points.

Investigations

INV 4E-3 What am I?

Focus: To plot points in the first quadrant of the Cartesian plane to create an image

Resources: graph paper or **1-cm grid paper** (BLM), ruler

Students plot a sequence of points to create an image. The points include positive whole numbers and fractions.

INV 4E-4 Help, I'm only half done!

Focus: To plot points in the first quadrant of the Cartesian plane to finish an image

Resources: graph paper or **1-cm grid paper** (BLM), tracing paper, ruler

Students plot a sequence of points to complete an image, which is only half shown. The points include positive whole numbers and fractions.

INV 4E-5 Saving up

Focus: To predict savings made over time after listing points as coordinates and plotting them onto a graph.

Resources: graph paper or **1-cm grid paper** (BLM), ruler

Students work through a savings program, where a specific amount is added to a bank account each week. They represent this linear relationship on a Cartesian plane, using the first quadrant only. Students answer a range of questions and make predictions from the graph.

Below Level students may need to be supported to construct a graph with a scaled *y*-axis where 1 box equals $10.

BLM

1 cm grid paper

Interactives

4E eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4F Negative numbers and the Cartesian plane

Teaching support for pages 214–19

Teaching strategies

Learning focus

To extend student knowledge of the Cartesian plane to include all four quadrants

Start thinking!

In this task, students are guided to:

- build on their knowledge of plotting points in the first quadrant of the Cartesian plane to plot points in all four quadrants of the Cartesian plane
- plot points that have coordinates that are integers; that is, positive and negative whole numbers and zero
- state coordinate points correctly, with the *x*-coordinate first and the *y*-coordinate second.

Students can use the BLM **Cartesian plane (four quadrants)** (see Resources) for this task. (The second Cartesian plane can be used for Exercise 4F Q3.)

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 3ii, 5a, 10a	1–6, 9–11	1, 2, 5–13
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q12 and Q13

Focus: To demonstrate an ability to plot fractional and decimal values on a number line

- Direct students to complete **SS 4F-1 Fractions and decimals on a number line** (see Resources) if they had difficulty with these questions. This activity will support student understanding of reading scale and the numbers between whole numbers.
- You may need to undertake some explicit teaching so students understand how to represent fractional and decimal values on a number line.

- Draw a number line. Start at zero and extend the number line to ten. Discuss the scale for the number line with students. Initially, mark in 5 and 10, and ask students to mark in smaller intervals.
- Ask a student to indicate where $1\frac{1}{2}$ would be on the number line. Discuss where this would lie between 1 and 2. Ask a different student to identify where 1.75 would be. Would this be closer to 1 or 2?
- Ask students to indicate where other fractions and decimals would be located.
- To extend this task, number lines with different scales could be drawn. For example, instead of numbering the line from 0 to 10, use decimal increments of 0.1. Ask students to identify where 0.5 and 0.75 would be located.

At Level

At Level
1–6, 9–11

- Demonstrate **4F eTutor** or direct students to do this independently.
- Remind students that all counting starts from the origin and that the *x*-axis coordinate is located first.
- Axes are numbered on the grid lines not in the boxes between lines as with column graphs.
- For Q3, students can plot points I to P on the BLM **Cartesian plane (four quadrants)** (see Resources). If this BLM was provided for *4F Start thinking!*, students can use the second Cartesian plane provided on the sheet.

POTENTIAL DIFFICULTY
Students don't 'see' the zero coordinate and will struggle positioning coordinates that lie on either axis.

- One way for students to remember the order of coordinates is 'run left or right before you jump up or down'.
- For students having difficulty plotting fractional values in Q9, direct them to complete **SS 4F-1 Fractions and decimals on a number line** (see Resources).

POTENTIAL DIFFICULTY
Fractional coordinates are incredibly difficult for students to locate and some students should use only whole number coordinates.

- To plot data representing real-life information, refer students to **INV 4F-4 Phones and charges** (see Resources). This problem-solving investigation allows students to explore some of the costs involved with mobile phones. They plot the linear relationships and use the graphs to answer questions and predict outcomes.

- For more problem-solving tasks and investigations, refer students to **INV 4F-5 Ice-cream maths** (see Resources). Students plot the temperature at which an ice-cream mixture chills over a period of time. This is a linear relationship and students answer a series of questions related to this graph. They think about the variables and which axis best suits each one.

Below Level

Below Level
1, 2, 3ii, 5a, 10a

- Demonstrate **4F eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
Students often wrongly number axes or number the boxes instead of the grid lines. It is worth spending some time ensuring all axes are properly constructed before introducing coordinates. Students forget which axis is which, and which coordinate represents which axis.

- Some students may benefit from plotting their graphs on the BLMs **1-cm grid paper** or **Cartesian plane (four quadrants)** (see Resources) as opposed to actual graph paper.
- It can help some students to draw only the *x*-axis, find the starting point (*x*-coordinate) and then move a number of steps up or down (*y*-coordinate). Think: *x*-coordinate = position, *y*-coordinate = move.
- Avoid using scaled axes and only plot whole numbered coordinates.
- If you are planning to expose these students to simple negative coordinates it can be useful to colour code the axis exactly as they were coloured for number lines – red is the negative end and blue (or black) is the positive end – as this can help some students to store the positions in long-term memory.
- The coordinates can also be colour coded – red for negative and blue (or black) for positive.
- To practise plotting points on all four quadrants, refer students to **WS 4F-2 The Cartesian plane (four quadrants)** (see Resources). Students construct a four quadrant plane and name and plot some coordinates.

Above level

Above Level
1, 2, 5–13

- Demonstrate **4F eTutor** or direct students to do this independently.
- Students working to locate fractional coordinates in Q8 will probably plot the coordinates on graph paper with 10 divisions between each whole number. Using 3 divisions to represent a third will produce a good enough outcome, but it may be an

opportunity to have a discussion with the students about accuracy and where 1 third or 2 thirds would really lie.

- For more problem-solving and investigating, refer students to **INV 4F-3 Fishy business** (see Resources). Students plot a sequence of points to create an image on a Cartesian plane (all four quadrants). The points to be plotted include positive and negative whole numbers and fractions.
- To plot data representing real-life information, refer students to **INV 4F-4 Phones and charges** (see Resources). This problem-solving investigation allows students to explore some of the costs involved with mobile phones. They plot the linear relationships and use the graphs to answer questions and predict outcomes.
- For further problem-solving and investigating, refer students to **INV 4F-5 Ice-cream maths** (see Resources). Students plot the temperature at which an ice-cream mixture chills over a period of time. This is a linear relationship and students answer a series of questions related to this graph. They think about the variables and which axis best suits each one.

Extra activities

- As an outdoor activity, draw a Cartesian plane (all four quadrants) on the ground, without numbering the scale on the axes. Explain to students that, just like the first quadrant, you number the axes from zero. Start at zero, number the positive x-axis and then number the positive y-axis. Number the negative x-axis and the negative y-axis.

  Walk to the point (–2, 5). Ensure that you start at the origin. Walk to –2 on the x-axis, turn right and walk to a position in line with 5 on the y-axis. Discuss how this point has coordinates (–2, 5). Repeat for a number of different points.

  Now stand at the point (–7, –3) and ask students to give the coordinates. Ask a student to demonstrate how these coordinates work to describe the position of the point. Compare with the position of a point at (–3, –7).

  Once students have an understanding of this strategy, ask students to stand on the Cartesian plane at different points. Ask students to give the coordinates for their position.

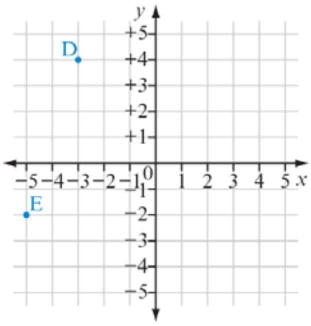
Answers

4F Negative numbers and the Cartesian plane

4F Start thinking!

2 a 3 units left and 4 units up
  b (–3, 4) c quadrant 2

3 a



  c quadrant 3

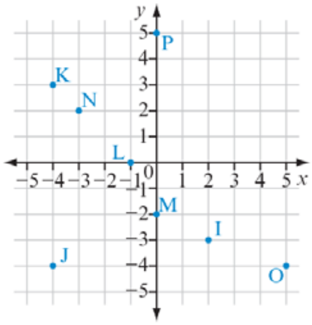
Exercise 4F Negative numbers and the Cartesian plane

1 A(2, 5), B(3, –2), C(4, 0), D(–5, –4), E(–3, 3), F(0, 2), G(2, –4), H(0, –3)

2 A is in quadrant 1; B is in quadrant 4; C is on the x-axis; D is in quadrant 3; E is in quadrant 2; F is on the y-axis; G is in quadrant 4; H is on the y-axis.

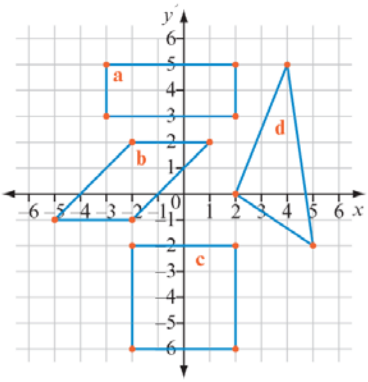
3 i a 2 units right and 3 units down
  b 4 units left and 4 units down
  c 4 units left and 3 units up
  d 1 unit left and 0 units up or down
  e 0 units right or left and 2 units down
  f 3 units left and 2 units up
  g 5 units right and 4 units down
  h 0 units right or left and 5 units up

 ii



4 I is in quadrant 4; J is in quadrant 3; K is in quadrant 2; L is on the x-axis; M is on the y-axis; N is in quadrant 2; O is in quadrant 4; P is on the y-axis.

5

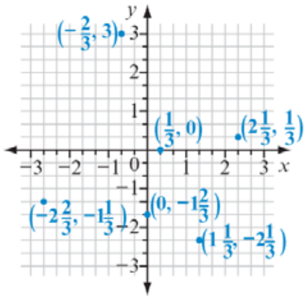


  a rectangle b parallelogram
  c square d triangle

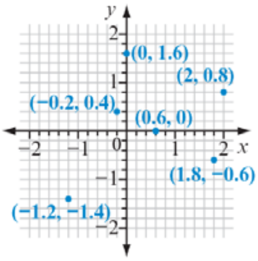
6 a i quadrant 3 ii quadrant 1
  b quadrants 2 and 4

7 a $A(1\frac{1}{2}, 2)$, $B(-2, -3\frac{1}{2})$, $C(-3\frac{1}{2}, -1\frac{1}{2})$, $D(-4\frac{1}{2}, 2\frac{1}{2})$, $E(-2\frac{1}{2}, 3\frac{1}{2})$, $F(4\frac{1}{2}, 3\frac{1}{2})$, $G(\frac{1}{2}, 0)$, $H(0, 4\frac{1}{2})$, $I(3\frac{1}{2}, -2)$, $J(4\frac{1}{2}, -\frac{1}{2})$
  b A(–0.4, 0.6), B(0.4, 0.4), C(–0.8, –0.6), D(0.8, 0), E(0.5, –0.7), F(0, –0.5), G(–0.9, 0.3), H(0.2, –0.2), I(0.3, –0.9), J(–0.5, –0.3)

8 a



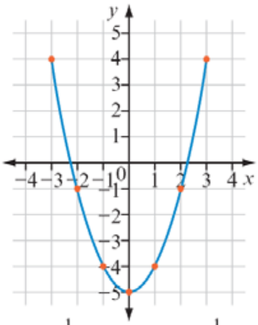
  b



9 Compare your answer with those of your classmates.

10 a i (–3, 4), (–2, –1), (–1, –4), (0, –5), (1, –4), (2, –1), (3, 4)

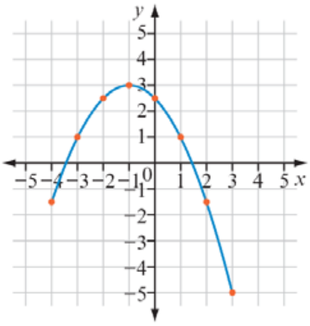
ii



iii curved line

b i $(-4, -1\frac{1}{2})$, $(-3, 1)$, $(-2, 2\frac{1}{2})$, $(-1, 3)$, $(0, 2\frac{1}{2})$, $(1, 1)$, $(2, -1\frac{1}{2})$, $(3, -5)$

ii



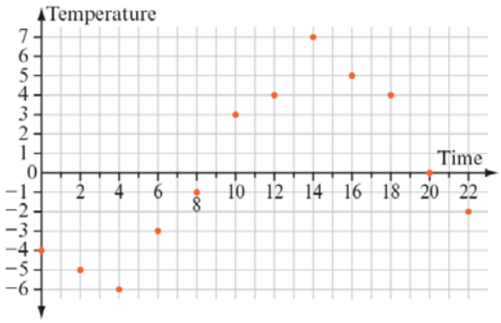
iii curved line

11 b Negative numbers not needed. Scale should be from 0 to 22 on the horizontal axis.

c Scale should go from –6 to 7 on the vertical axis.

d (0, –4), (2, –5), (4, –6), (6, –3), (8, –1), (10, 3), (12, 4), (14, 7), (16, 5), (18, 4), (20, 0), (22, –2)

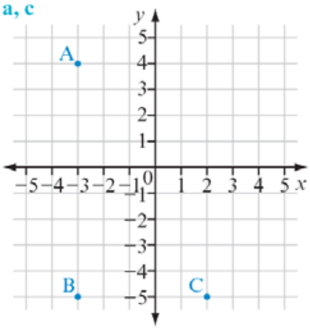
a, e



f –6°C at 4 am

g 7°C at 2 pm

12 a, c



b 9 units

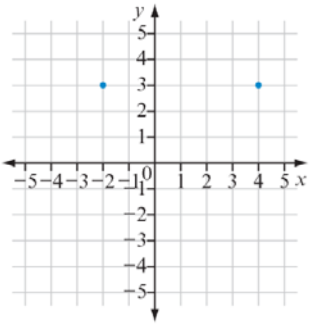
d 5 units

e D(2, 4)

13 a 6 units

b One possible answer is: (4, –3) and (–2, –3).

c No; other possible answers are: (4, 9) and (–2, 9); (1, 6) and (1, 0).



Reflect

Possible answer: A way to work out a suitable scale for the axes is to look initially at the *x*-coordinates and determine the largest and the smallest values. Repeat this process for the *y*-coordinates. The Cartesian plane that is drawn needs to be able to cater for all points in these ranges.

Resources

SupportSheet

SS 4F-1 Fractions and decimals on a number line

Focus: To plot fractional and decimal values on a number line

Resources: ruler

Students initially label the intervals of a number line. They then plot points representing fractional values on a horizontal number line. This activity will support student understanding of reading scale and the numbers between whole numbers.

WorkSheet

WS 4F-2 The Cartesian plane (four quadrants)

Focus: To plot and read points in all four quadrants of the Cartesian plane

Resources: graph paper or **1-cm grid paper** (BLM)

Students are guided through a sequence of steps to reproduce a Cartesian plane showing all four quadrants. Students then practise plotting points and giving the coordinates of points.

Investigations

INV 4F-3 Fishy business

Focus: To plot points on a Cartesian plane where all four quadrants are shown

Resources: pencils, graph paper or **1-cm grid paper** (BLM), ruler

Students plot a sequence of points to create an image on a Cartesian plane (all four quadrants). The points to be plotted include positive and negative whole numbers and fractions.

INV 4F-4 Phones and charges

Focus: To plot a graph representing results calculated from real-life data and analyse the results

Resources: pencils, graph paper or **1-cm grid paper** (BLM), ruler

Students explore some of the costs involved with mobile phones. They plot the linear relationships and use the graphs to answer questions and predict outcomes.

INV 4F-5 Ice-cream maths

Focus: To plot a graph representing results calculated from real-life data and analyse the results

Resources: pencils, graph paper or **1-cm grid paper** (BLM), ruler

Students plot the temperature at which an ice-cream mixture chills over a period of time. This is a linear relationship and students answer a series of questions related to this graph. They think about the variables and which axis best suits each one.

BLMs

Cartesian plane (four quadrants)

1-cm grid paper

Interactives

4F eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4G Interpreting graphs

Teaching support for pages 220–5

Teaching strategies

Learning focus

To discover the features of a travel graph and interpret various graphs

Start thinking!

In this task, students:

- are introduced to a travel graph, which examines distance travelled over a period of time
- learn that the variable *Time* is placed on the horizontal axis and the variable *Distance* is placed on the vertical axis
- examine specific sections of a graph, analysing what each different segment represents
- start to develop an ability to read information from a graph.

Use this activity to start students describing what is happening in the graph. Encourage their contribution by asking what happens to the time as the line goes to the right. What happens to the distance as the line goes upwards or downwards?

It will be very challenging for students who are not yet multiplicative to be able to relate the distance and the time to a speed. Support their understanding by restricting the time frame they are considering to one minute. How far did Kurt travel in the first minute? Support the language of speed by using the following sequencing: he travelled 120 m in one minute so he travelled 120 m per minute.

As a class, select one or two students to model out the walk. Have them talk through what they are doing. They need not walk for 10 minutes in total but can say 'I walk for 240 metres from home and it takes me 2 minutes, then I stand still for another 2 minutes, etc.'.

Differentiated pathways

Below Level	At Level	Above Level
1–6, 11, 16, 17	1–12, 15–18	1–10, 13–15, 17, 19, 20
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q14 and Q15

Focus: To demonstrate an understanding of a relationship between distance and time to indicate speed

- Direct students to complete **SS 4G-1 Calculating speed** (see Resources) if they had difficulty with these questions or require more practice at this skill. Students are introduced to the distance formula. They are guided through the solution of a problem that requires substitution of values into the formula. This task involves students using a formula to calculate speed from the distance travelled in the time taken. Students who are not multiplicative thinkers will be unable to successfully complete this task.
- You may need to undertake some explicit teaching so students understand that distance travelled over a specific time can be translated into a unit of speed, written as $\frac{\text{distance}}{\text{time}}$.
- To give this a real-life context, record the time for each student to run 100 metres. Demonstrate how to convert their times into m/s. Students could use a number line to compare their times to those of other students and champion athletes. Look up the current world record and compare it to the times collected.

At Level

At Level
1–12, 15–18

- Demonstrate **4G eTutor** or direct students to do this independently.
- In Q1–7, students examine the four sections of Kurt's travel graph from *4G Start thinking!*. They appreciate that graphs of this type show upward-trending line sections, downward-trending line sections and horizontal sections and learn that the steeper a line section, the faster the travel
- It is important that students can identify the variable that is represented on each axis and recognise that there is a relationship between the two variables. Explain the graph to students in detailed terms: in Q8 for example, after 1 hour Sarah had travelled 2 km. Ask students to explain what happened during the second hour. What does it mean if she has travelled no distance? Ask students to work out how long it took to walk the remaining 2 km. What is the difference in time taken for the first 2 km and the last 2 km? By explicitly working through a graph with students, they will be more able to see that they are extracting information that is provided in a different format.
- Direct students to complete **SS 4G-1 Calculating speed** (see Resources) if they have difficulty completing Q9 and Q10.
- For Q15, students are asked to draw a graph to represent their own travel between home and school. The BLM **1-cm grid paper** (see Resources) can be provided to students for this task.

- To examine how a line graph changes over time, refer students to **WS 4G-2 Describing travel graphs** (see Resources). Students are guided through the analysis of a given travel graph and identify the key features. They are given a story to accompany the graph and match different segments of the graph to specific components of the story. Students then practise reading a travel graph and independently complete questions.

- To explore the term *gradient*, refer students to **WS 4G-3 Interpreting the slope on travel graphs** (see Resources). Students first recognise whether the line is rising or falling before they judge the magnitude of that change and compare it with other portions of the graph. Students then attempt to work out from the graph how far is travelled in each minute and articulate this as a speed. This can then be related to the formula speed = $\frac{\text{distance}}{\text{time}}$.

Below Level

Below Level
1–6, 11, 16, 17

- Demonstrate **4G eTutor** or direct students to do this independently.

- Focus student attention on what the line is doing during each period of time and use the language of increasing, decreasing or is unchanged to consolidate understanding of academic vocabulary.

- It might support understanding for some students if they create a graph of their own distance against time. For example, students are paired off and are given a stopwatch, chalk and a measuring tape. Ask them to walk from the door in a straight line and make a chalk mark every 10 seconds, encouraging them to go at different speeds or stopping occasionally. Do this for 1 minute and then measure the distance between each mark. This helps students to visualise what the line graphs are representing.

POTENTIAL DIFFICULTY
Students can find it challenging to visualise the physical situation that a line graph is representing.

- To examine how a line graph changes over time, refer students to **WS 4G-2 Describing travel graphs** (see Resources)**.** Students are guided through the analysis of a given travel graph and identify the key features. They are given a story to accompany the graph and match different segments of the graph to specific components of the story. Students then practise reading a travel graph and independently complete questions.

Above level

Above Level
11–10, 13–15, 17, 19, 20

- Demonstrate **4G eTutor** or direct students to do this independently.

- Introduce the idea of gradient with students who are able to understand the concept. Relate the gradient to the rate of change of the line; how much the line goes up for each box it goes along.
- Encourage Above Level students to formalise their understanding of the relationship between speed and distance and time $\left(\text{speed} = \dfrac{\text{distance travelled}}{\text{time taken}}\right)$
- The graph in Q13 does not contain units on the axes, so students focus on the slope of the graph to identify different sections.
- To explore the term *gradient*, refer students to **WS 4G-3 Interpreting the slope on travel graphs** (see Resources). Students first recognise whether the line is rising or falling before they judge the magnitude of that change and compare it with other portions of the graph. Students then attempt to work out from the graph how far is travelled in each minute and articulate this as a speed. This can then be related to the formula speed = $\dfrac{\text{distance}}{\text{time}}$.
- For more problem-solving tasks and investigations, refer students to **INV 4G-4 Is the drought over?** (see Resources). Students are provided with a graph representing water storages in Melbourne over year-long periods of the last decade. They answer questions related to the information in the graph. The opportunity to explore this graph and add in the data for additional years is provided.
- For further problem-solving and investigating, refer students to **INV 4G-5 The information super highway** (see Resources). Students are provided with information from 2001 to 2006 related to the number of Australians who use the Internet. They explore the relationship over time and answer a series of questions interpreting the data.

Extra activities

Whole class

One way to start this section would be to discuss Q1–7 as a class. Ask students to identify the variables and to identify whether distance is dependent on time or whether time is dependent on distance. Ask students to identify the unit of the variable on each axis.

It would be useful to collect some graphs from newspapers and magazines and discuss these with students. What story is each graph telling?

Answers

4G Interpreting graphs

4G Start thinking!

1 a time b distance from home

2 Kurt leaves home and travels 240 m in 2 minutes (120 m each minute). He stops at the shop for 2 minutes to buy a magazine. He then takes 4 minutes to walk 120 m to his friend's house (30 m each minute) but finds that he is not there. He then jogs 360 m in 2 minutes to return home (180 m each minute).

Exercise 4G Interpreting graphs

1 a i 0 m ii 120 m iii 240 m
 b i 120 m ii 120 m
 c Kurt travelled the same distance in each minute, so he was travelling at a constant speed.

2 a i 240 m ii 240 m iii 240 m
 b i 0 m ii 0 m
 c Kurt was stationary.

3 a i 240 m ii 270 m iii 300 m iv 330 m v 360 m
 b 30 m
 c Kurt travelled the same distance in each minute, so he was travelling at a constant speed.

4 a 360 m b 2 minutes

5 D; Kurt travels 180 m each minute compared to 120 m in section A, 0 m in section B and 30 m in section C.

6 Section B is horizontal; it has no slope. This means that Kurt is not moving.

7 a B: 0 m; D: 180 m b D
 c The steeper the slope, the higher the speed.

8 a i 0 km ii 2 km iii 2 km iv 3 km v 4 km
 b one hour into her journey
 c 1 hour d 2 km e A
 f Sarah's distance from home has not changed.
 g Sarah leaves home and travels 2 km in 1 hour. She stops at her friend's house for 1 hour. She then takes 2 hours to slowly walk 2 km to the music store.

9 a 2 km/h b 0 km/h c 1 km/h

10 a 120 m/min b 0 m/min c 180 m/min

11 a 2 km b 4 h
 c from home to school; steeper line
 d 4 km

12 a 3 km b $6\frac{1}{2}$ hours after leaving home
 c 1 km d 1 h e 4 pm
 f from dental clinic to home; 4 km/h
 g Nina leaves home and travels 3 km in 1 hour to reach school. She stays at school for $5\frac{1}{2}$ hours before taking 1 hour to walk 1 km to the dental clinic. After a 1-hour dental appointment, she takes $\frac{1}{2}$ hour to travel 2 km home.

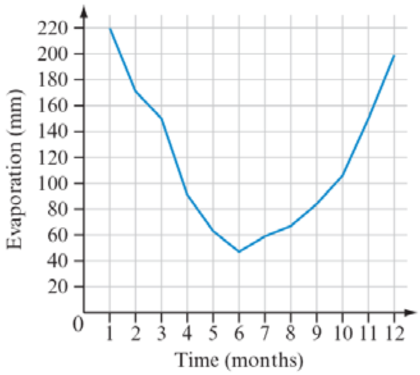
13 a false b true c false d true

16 a Tom b Sue

17 No; you can't be in different places at the same time.

18 a −10°C b −5°C
 c after 3 min d 35°C
 e from 3 to $3\frac{1}{2}$ min; from 5 to $5\frac{1}{2}$ min
 f from $3\frac{1}{2}$ to 5 min and from $5\frac{1}{2}$ to 6 min
 g 1 min
 h Temperature of the frozen soup after 3 minutes of heating rose by 15°C. The heating was paused for 30 seconds for the soup to be stirred. It was then heated for $1\frac{1}{2}$ minutes to increase the temperature by 30°C. Heating was again paused for 30 seconds for the soup to be stirred. After another one minute of heating, the temperature of the soup rose 10°C to reach 45°C.

20 a 220 mm b 150 mm
 d



 e January, 220 mm f June, around 47 mm
 g Between March and April; steepest line
 h Higher evaporation during hotter, dry months. Factors such as temperature, humidity and wind affect the amount of evaporation.

Reflect

Possible answer: The slope of a graph shows you how much one quantity is changing in relation to the other; for example, how much distance changes over time. The slope of a travel graph can also be explained in terms of speed.

Resources

SupportSheet

SS 4G-1 Calculating speed

Focus: To calculate the speed of objects travelling at a constant or steady pace

Students are introduced to the distance formula. They are guided through the solution of a problem that requires substitution of values into the formula. This task involves students using a formula to calculate speed from the distance travelled in the time taken. Students who are not multiplicative thinkers will be unable to successfully complete this task.

WorkSheets

WS 4G-2 Describing travel graphs

Focus: To describe some key features of a travel graph

Resources: ruler, **1-cm grid paper** (BLM) (optional)

Students are guided through the analysis of a given travel graph and identify the key features. They are given a story to accompany the graph and match different segments of the graph to specific components of the story. Students then practise reading a travel graph and independently complete questions.

WS 4G-3 Interpreting the slope on travel graphs

Focus: To interpret the slope on travel graphs and relate it to speed

Resources: coloured pencils or highlighters

Students are given a story to accompany a given graph and match different segments of the graph to specific components of the story. They are guided through questions related to the slope of the graph and use the formula speed $= \frac{\text{distance travelled}}{\text{time taken}}$ to calculate speed.

Investigations

INV 4G-4 Is the drought over?

Focus: To interpret data presented in a graph for water storages in Melbourne over the past decade

Resources: pencils, graph paper (optional), ruler, Internet access (optional)

Students are provided with a graph representing water storages in Melbourne over year-long periods of the last decade. They answer questions related to the information in the graph. The opportunity to explore this graph and add in the data for additional years is provided.

INV 4G-5 The information super highway

Focus: To interpret data presented in a graph for Internet usage in Australia

Resources: pencils, ruler

Students are provided with information from 2001 to 2006 related to the number of Australians who use the Internet. They explore the relationship over time and answer a series of questions interpreting the data.

BLM

1-cm grid paper

Interactives

4G eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 226–9

Additional teaching strategies

Multiple-choice

1 Answer: C.
A: incorrectly chose –5, which is smaller than –3 and so does not lie between –3 and 3.
B: incorrectly chose –1.5, which lies between –3 and 3 but is not an integer.
D: incorrectly chose 4, which is larger than 3 and so does not lie between –3 and 3.

2 Answer: A. (+5) + (–8) = –3.
B: (+5) + (–2) = +3
C: (+5) + (+2) = +7
D: (+5) + (+8) = +13

3 Answer: D. Start at –4 and move 6 units to the right to +2; that is, (–4) – (–6) = (+2).
A: correctly started at –4 but calculation incorrectly indicates to move 6 units left to reach +2.
B: correctly started at –4 but calculation indicates to move 2 units right to reach –2. Should end at +2.
C: correctly started at –4 but calculation indicates to move 2 units left to reach –6. Should end at +2.

4 Answer: D. (–4) + (–5) = –9; (–4) – (–5) = +1
A: (–4) + (+5) = +1; (–4) – (+5) = –9; incorrectly chose option that gives a sum of 1 and a difference of –9.
B: 4 + 5 = 9; 4 – 5 = –1; incorrectly chose option that gives a sum of 9 and a difference of –1.
C: 4 + (–5) = –1; 4 – (–5) = 9; incorrectly chose option that gives a sum of –1 and a difference of 9.

5 Answer: B. –12 + 24 = 12 (positive result)
A: –12 – 24 = –36 (negative result).
C: 12 – 24 = –12 (negative result).
D: –12 + 12 = 0 (non-positive result).

6 Answer: C. Origin has coordinates (0, 0).

7 Answer: B. Points in quadrant 2 have negative *x*-coordinates and positive *y*-coordinates.

8 Answer: A. A horizontal line indicates no change in distance over the period of time indicated.

B: If Justin did change his distance over that period of time, the line would trend upwards or downwards to show that change.
C: An increase in speed would be indicated by a rising line.
D: A decrease in speed would be indicated by a falling line.

Short answer

1 **a** –3 > +3 False, +3 is greater than –3.

b –12 < –5 True, –12 is less than –5.

c 0 > –7 True, 0 is greater than –7.

d 1 < – 1 False, +1 is greater than –1.

2 Ascending order is smallest to largest.

a –10, –5, 0, 2, 12, 25

b –12, –6, –3, 0, 6, 9

3 **a** –86

b +615

c +8840

d –35

4 **a** (–6) + (–3) = –9

b (+7) + (+8) = +15

c (+2) + (–4) = –2

d (–10) + (+9) = –1

5 **a** (+1) – (+9) = –8

b (–4) – (–5) = +1

c (–8) – (+6) = –14

d (+11) – (+3) = +8

6 **a** –5 + 7 = 2

b –1 – 6 = –7

c 9 – 12 = –3

d –8 + 3 = –5

e –15 + 15 = 0

f –20 – 20 = –40

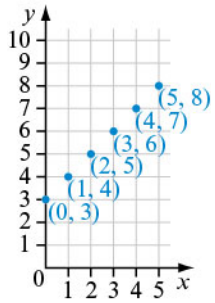
7 49 – (– 29)
= 49 + 29
= 78°C

8 23 – (– 15)
= 23 + 15
= 38 m

9 **a**

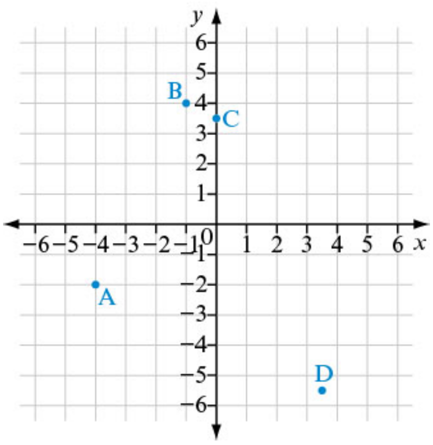
x-coordinate	0	1	2	3	4	5
y-coordinate	3	4	5	6	7	8
coordinates	(0, 3)	(1, 4)	(2, 5)	(3, 6)	(4, 7)	(5, 8)

b



10 A(3, 1), B(–3, 3), C(4, –2), D(1, 0), E(–2, –3), F(0, –2)

11



12 **a** Ben walks 4 km from home to school.

b Ben walks the fastest in Section A

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{4\text{ km}}{1\text{ h}} = 4\text{ km/h}$$

c Section A – Ben walks from home to school.

Section B – Ben is at school.

Section C – Ben walks from school to the shop.

Section D – Ben stays at the shop with friends.

Section E – Ben walks home from the shop.

NAPLAN-style practice

Multiple-choice options have been listed as A, B, C and D for ease of reference.

1 –5; as each interval represents one unit and the arrow is pointing to a position five units to the left of 0.
Refer to *4A Understanding negative numbers*.

2 Answer: C. 2 is the largest number.
Refer to *4A Understanding negative numbers*.

3 Answer: B. –20 is the smallest number.
Refer to *4A Understanding negative numbers*.

4 (–3) + (+7) = +4 Temperature at midday is 4°C.
Refer to *4B Adding integers.*

5 Answer: A. (–5) + (+6) = +1. Miners leave the lift at Level 1 above the surface.
B: incorrectly calculated (–5) + (+6) as –1 and chose Level 1 below the surface.
C: incorrectly calculated (–5) + (+6) as +11 and chose Level 11 above the surface.
D: incorrectly calculated (–5) + (+6) as –11 and chose Level 11 below the surface.
Refer to *4B Adding integers.*

6 (–5) + (+2) = –3
Lift doors open at Level 3 below the surface.
Refer to *4B Adding integers.*

7 (–5) + (–6) = –11
Refer to *4B Adding integers.*

8 Answer: B. (–4) – (–3) = –1
A: may have moved 3 units left of –4 instead of 3 units right on a horizontal number line to obtain –7.
C: may have moved 3 units left of +4 instead of 3 units right of –4 on a horizontal number line to obtain 1.
D: may have moved 3 units right of +4 instead of –4 on a horizontal number line to obtain 7.
Refer to *4C Subtracting integers*.

9 4 – 10 = –6
New temperature is –6°C.
Refer to *4C Subtracting integers* and *4D Simplifying addition and subtraction of integers*.

10 (+2) – (+16) = –14
Seabed at –14 so water is 14 m deep.
Refer to *4C Subtracting integers.*

11 –4 – (–16) = –4 + 16 = 12
Refer to *4D Simplifying addition and subtraction of integers*.

12 (+86) + (+25) + (–15) + (–33) + (–243) + (+200)
= 86 + 25 – 15 – 33 – 243 + 200
= 20
Amount in Peter's account is $20.
Refer to *4D Simplifying addition and subtraction of integers.*

13 6 – 8 = –2
Tess travelled 8 floors.
Refer to *4D Simplifying addition and subtraction of integers.*

14 Answer: D. Point A at (4, 3).
A: chose coordinates of point B.
B: swapped the order of the *x*- and *y*-coordinates.
C: looked at point C and swapped the order of the coordinates.
Refer to *4E Introducing the Cartesian plane.*

15 Answer: B.
Refer to *4E Introducing the Cartesian plane.*

16 First coordinate in ordered pair is the *x*-coordinate.
The *x*-coordinate is 2.
Refer to *4E Introducing the Cartesian plane.*

17 Answer: C. Section D represents their time at the cinema.

A: Section B represents the time when Emma is at her friend's house.
B: Section C represents the time when they walk to the cinema.
D: Section E represents the time when they walk to a cafe.
Refer to *4G Interpreting graphs.*

18 Emma's home is closer to her friend's home (3 km) than to the cafe (4 km), so the answer is false.
Refer to *4G Interpreting graphs.*

19 Section G has the steepest slope and hence represents the fastest speed (4 km/h).
Refer to *4G Interpreting graphs.*

20 Point C has coordinates (6, –3).
Refer to *4F Negative numbers and the Cartesian plane.*

21 Answer: D. Point H has coordinates (–3, 6).
A: Point A has coordinates (–2, 4).
B: Point C has coordinates (6, –3); *x*- and *y*-coordinates swapped.
C: Point F has coordinates (–4, –4).
Refer to *4F Negative numbers and the Cartesian plane.*

22 Points in first quadrant have positive coordinates. Points B(2, 5) and E(1, 3) are in the first quadrant.
Refer to *4F Negative numbers and the Cartesian plane.*

23 Point I has coordinates (–2, –3). Point A has coordinates (–2, 4). Both have an *x*-coordinate of –2.
Refer to *4F Negative numbers and the Cartesian plane.*

Analysis

a **i** Seagull +10 or 10

ii Fish –2

iii Swimmer 0

iv Seabed –8

b **i** Distance between the seagull and the fish = 10 – (–2) = 10 + 2 = 12 m

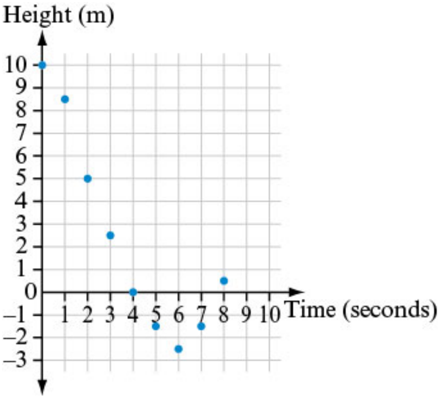
ii Distance between the seabed and the fish = –2 – (–8) = –2 + 8 = 6 m

c Parasailor is at half of the height of the seagull so height is +5 m.

d The sting ray is at –6.

e (0, 10), (1, 8.5), (2, 5), (3, 2.5), (4, 0), (5, –1.5), (6, –2.5), (7, –1.5), (8, 0.5)

f



g The seagull swoops down towards the water and hits the surface at 4 seconds. It spends 3 seconds under the water, diving to a depth of 2.5 m, to catch the fish and resurfaces with the fish just before the 8-second mark.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 4 Chapter test A

Chapter 4 Chapter test B

Test answers

Chapter 4 Chapter test answers

Connect

Teaching support for pages 230–1

Teaching strategies

Temperatures around the world

Focus: To use integers and graphing skills in a real-life context

- Students explore temperatures around the world as they plan a holiday where cost is not a consideration.
- The task requirements are expressed in everyday language. Students compare temperatures at possible holiday locations and recognise that some of the temperatures are expressed in degrees Celsius and others are in degrees Fahrenheit.
- You may like to have students discuss the task requirements in small groups to:
  - identify which cities have temperatures shown in degrees Fahrenheit
  - decide whether to convert all temperatures to degrees Celsius or to degrees Fahrenheit for easier comparison
  - suggest factors that would influence their decision on a holiday destination.
- At this point students may need to work individually to:
  - decide which holiday destination suits them the best
  - decide which holiday destinations may suit them better at different times of the year.
- Direct students to complete the **Connect worksheet** (see Resources). This provides scaffolding to the task to guide students through the problem-solving process. They can use this as a foundation to presenting their findings in a report.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided to the **Connect worksheet**.
- An assessment rubric is available (see Resources).
- Two additional Connect investigations are provided: **CI 4-1 Atoms and ions** and **CI 4-2 Analysing experimental data** (see Resources).

Additional Connect investigations

CI 4-1 Atoms and ions

Focus: To examine the structure of atoms and ions and relate this to integers

Students consider the charges within atoms and then the overall charge on an ion and relate this to integers. They add integers to calculate the charge on molecules formed by combining different ions together.

Students consider:

- the particles (electrons, protons and neutrons) that make up an atom
- the charge (negative, positive or neutral) on each particle in an atom
- how an atom becomes an ion by losing or gaining electrons
- the overall charge on a molecule when two or more ions combine.

As an extension, students use their knowledge of integers to predict the structure of a water molecule.

An assessment rubric is available (see Resources**).**

CI 4-2 Analysing experimental data

Focus: To analyse experimental data using knowledge of integers and the Cartesian plane

Resources: ruler, graph or grid paper

Students analyse data recorded by two groups of students during a Science experiment. Each group heats a substance and records the temperature every two minutes.

Students consider:

- the difference between the two sets of data over the 30-minute heating period
- how to represent the data on a Cartesian plane
- whether each temperature-time relationship is linear
- whether the two compounds being heated are the same or different.

As an extension, students try to identify the two compounds using the information provided and by completing some research on the Internet.

An assessment rubric is available (see Resources**).**

Resources

Connect worksheet

CW 4 Temperatures around the world

Additional Connect investigations

CI 4-1 Atoms and ions

CI 4-2 Analysing experimental data

Assessment rubrics

Temperatures around the world

Atoms and ions

Analysing experimental data

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