13

Mathematics and health

This chapter deals with body measurements, medication calculations and life expectancy.

The main mathematical ideas investigated are:

- using scatterplots
- calculating correlation coefficients
- working with least-squares line of best fit
- converting units including rates
- calculating medical dosages
- interpreting life expectancy data
- calculating life expectancy.

FOCUS STUDY

Syllabus references: FSHe1, FSHe2, FSHe3

Outcomes: MG2H–1, MG2H-2, MG2H-3, MG2H-5, MG2H-7, MG2H-9, MG2H-10

13A Scatter diagrams

The aim of many statistical investigations is to determine whether there is a relationship between the two variables being investigated. For instance, medical researchers might be interested in the relationship, if any, between the amount of a drug administered and the number of patients cured. A business enterprise might be interested in the relationship, if any, between the amount of money spent on advertising and the change in sales.

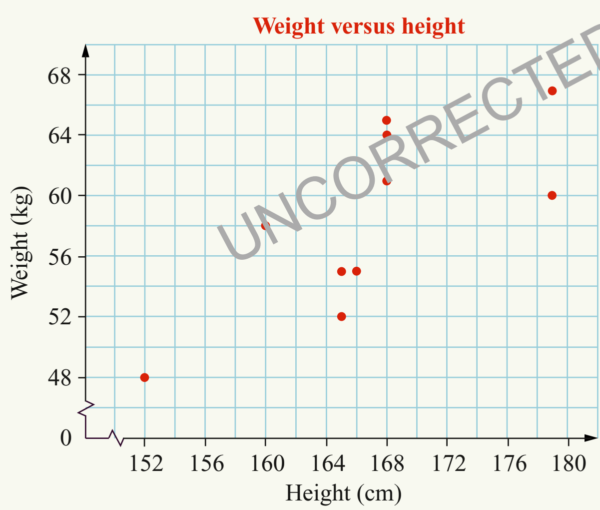
In this section we will investigate ways of illustrating data so that a relationship, if it does exist, can be seen. A simple method of illustrating numerical data that relates two variables is to plot it as ordered pairs on a number plane. The resulting diagram is known as a scatterplot or scattergram.

WORKED EXAMPLE 1

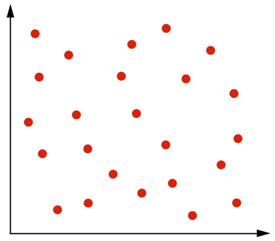
The heights and weights of 10 students were measured and the results shown in the table below.

Student	1	2	3	4	5	6	7	8	9	10
Height	179	165	160	179	152	168	168	165	168	166
Weight	60	55	58	67	48	64	61	52	65	55

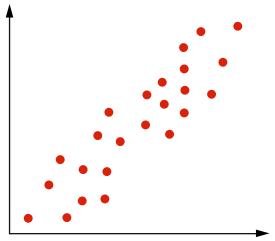
Illustrate this data on a scatterplot and determine whether there is a possible relationship between the two variables.

Solve	Think	Apply
 From the distribution of points plotted, there appears to be a trend that as height increases so does weight. This might indicate a relationship between these two variables, but because of the scatter of the points, there does not appear to be a strong link. There would not appear to be a mathematical relationship that would allow the weight of a student to be predicted from his or her height.	The data is plotted as ordered pairs with height on the horizontal axis and weight on the vertical axis.	The chart option on a spreadsheet makes drawing these graphs very easy. Using a spreadsheet: *Step 1:* Put the data into a table. *Step 2:* Highlight the table. *Step 3:* From the Insert menu select X-Y Scatter Chart type.

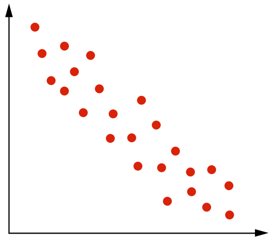
In general, if the points are scattered at random over the grid, as in Graph A below, the variables are not mathematically related. If the points are scattered along a straight line, as in Graph B and Graph C, there may be a mathematical relationship between the two variables. The strength of this relationship, or how closely linked the variables are, is called **correlation**. Correlation will be investigated in more detail later.



Graph A



Graph B

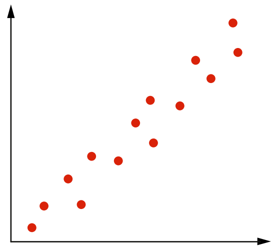


Graph C

EXERCISE 13A

1 State whether or not there appears to be a linear relationship between the variables plotted on these scatterplots.

a



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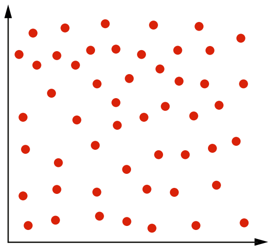
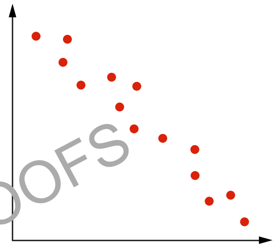


c



2 **i** Draw scatterplots for the data in the tables.

ii Comment on any possible linear relationship between the variables.

a

x	40	50	60	70	80	90
y	42	47	51	59	62	72

b

x	10	15	20	25	30	35	40
y	12	17	19	24	29	31	37

c

x	30	40	45	50	60	70	80	90
y	45	35	85	50	50	40	90	60

13B Line of best fit

If a pair of variables appears to be related, as indicated by a linear pattern of dots on a scatterplot, then we can draw a straight line that fits the points plotted and use this line to predict the value of one variable given the value of the other. This line is known as the:

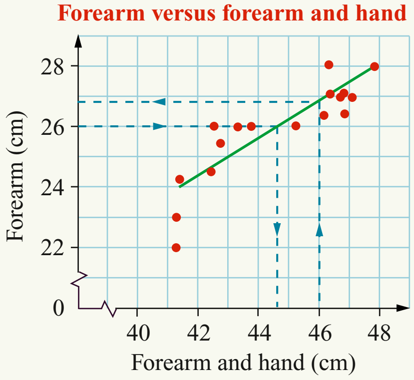
- 'line of best fit' or
- 'line of good fit' or
- 'regression line' or
- 'trendline' or
- 'least-squares line of best fit'.

WORKED EXAMPLE 1

This scatterplot shows the forearm and hand against forearm only measurements for a group of students. Use line of best fit to predict:

a the forearm measurement for a student with a forearm and hand measurement of 46 cm

b the forearm and hand measurement for a student with a forearm measurement of 26 cm.



	Solve	Think	Apply
a	The forearm measurement is about 26.8 cm.	Draw a vertical line from 46 on the forearm and hand axis to meet the line. From this point of intersection, draw a horizontal line to meet the forearm axis. Read off the approximate forearm measurement.	The line of best fit will approximate the values. It is not a good indicator for values outside the range of the plotted points. The greater the number of points and the closeness of the points to the line, the better the line is as a predictor.
b	The forearm and hand measurement is about 44.6 cm.	Start at 26 on the forearm axis and reverse the process in part a.	

EXERCISE 13B

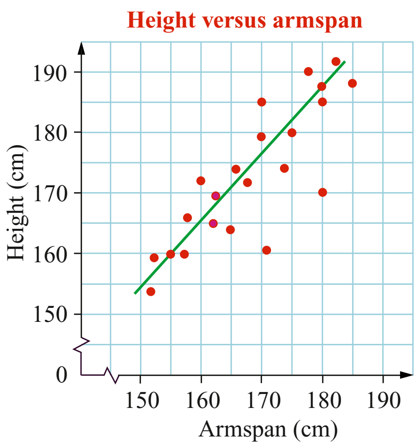
1 This scatterplot shows the heights and armspans of a group of students. A line of best fit has been drawn for these points. Use the line of best fit to predict:

a the heights of students with these armspans

i 160 cm **ii** 175 cm **iii** 180 cm

b the armspans of students with these heights

i 160 cm **ii** 175 cm **iii** 185 cm



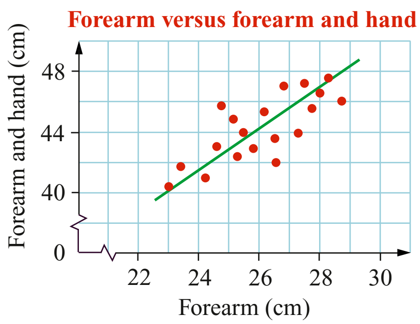
2 This scatterplot shows the forearm and forearm and hand measurements of a group of students. A line of best fit has been drawn for these points. Use the line of best fit to predict:

a the forearm and hand measurements of a student with a forearm measurement of

i 24 cm **ii** 28 cm **iii** 27 cm

b the forearm measurements of a student with a forearm and hand measurement of

i 42 cm **ii** 44 cm **iii** 46 cm



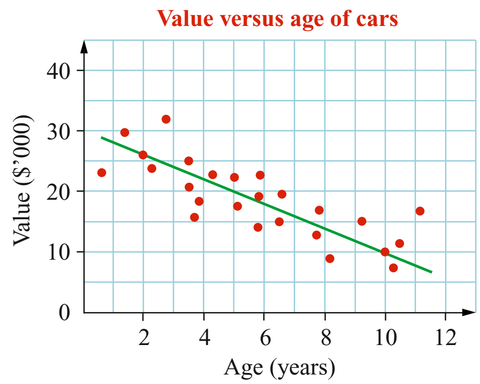
3 This scatterplot shows the age and value for a sample of cars of a particular model. A line of best fit has been drawn for these points. Use the line to predict:

a the value of a car of this model of age

i 2 years **ii** 5 years **iii** 10 years

b the age of a car of this model with a value of

i \$28 000 **ii** \$18 000 **iii** \$12 000



WORKED EXAMPLE 2

The equation of the line of best fit in Worked example 1 is $F = 0.6 \times A - 0.7$, where F represents the forearm length and A represents the forearm and hand length. Use the equation to predict:

a the forearm length of a student with a forearm and hand length of 45 cm

b the forearm and hand length of a student with a forearm length of 25 cm.

	Solve	Think	Apply
a	$F = 0.6 \times 45 - 0.7$ $= 26.3$ cm	Substitute $A = 45$ into the equation and calculate F.	Care must be taken if using the equation without viewing the data points.
b	$25 = 0.6 \times A - 0.7$ $25.7 = 0.6A$ $A = \frac{25.7}{0.6}$ $= 42.8$ cm	Substitute $F = 25$ then solve the equation for A.	

4 The equation of the line of best fit connecting height (H) and armspan (A), plotted on a scatterplot, is $H = 1.2A - 36$. Use the equation to predict:

a the height of a student with an armspan length of

i 160 cm **ii** 170 cm **iii** 178 cm

b the armspan length of a student with a height of

i 160 cm **ii** 175 cm **iii** 183 cm

5 The equation of the line of best fit connecting hip measurement (H), in cm, and waist measurement (W), in cm, is $W = 0.7H - 2.1$. Use the equation to predict:

a the waist of a person with a hip measurement of

i 85 cm **ii** 96 cm **iii** 100 cm

b the hip measurement of a person with a waist of

i 60 cm **ii** 65 cm **iii** 71 cm

6 The equation of the line of best fit connecting grape yield (G), in tonnes, of a vineyard and the number of frosts (n) during the growing season is $G = -0.14 \times n + 5.6$, when plotted on a scatterplot. Use the equation to predict:

a the yield when there are

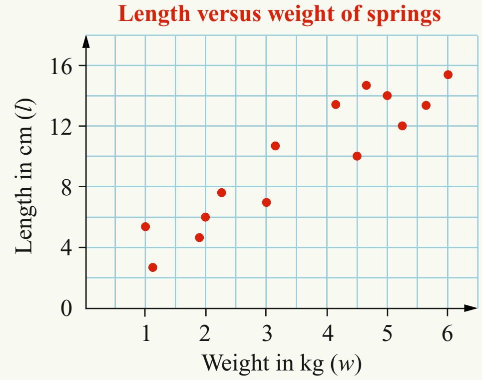
i 5 frosts **ii** 12 frosts **iii** 20 frosts

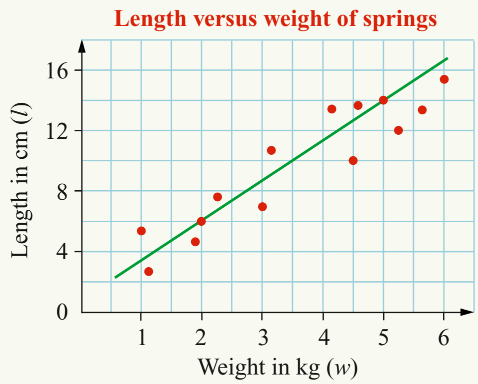
b the number of frosts given that the yield was

i 4.2 t **ii** 3.5 t **iii** 2.1 t

WORKED EXAMPLE 3

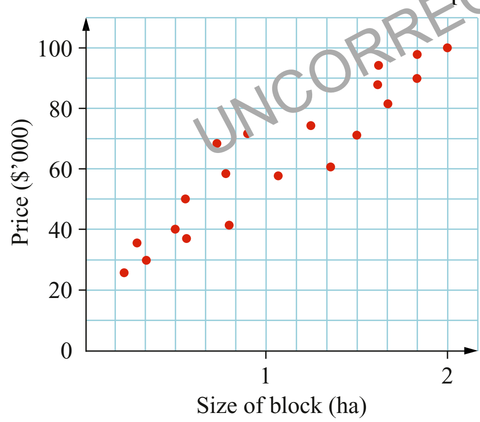
Draw a line of best fit on the scatterplot.



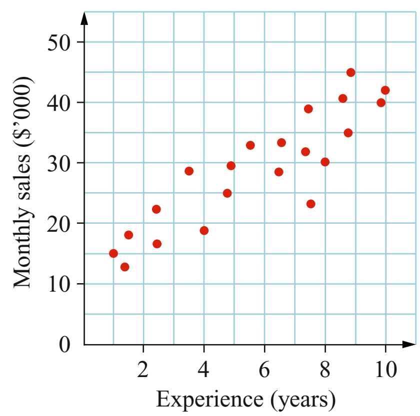
Solve	Think	Apply
	The line must have about the same number of dots above and below it.	The line need not pass through any of the points but must balance the points above and below.

7 Draw a line of best fit on each of the scatterplots below.

a



b



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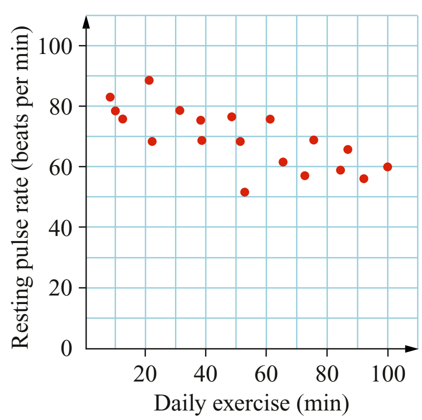

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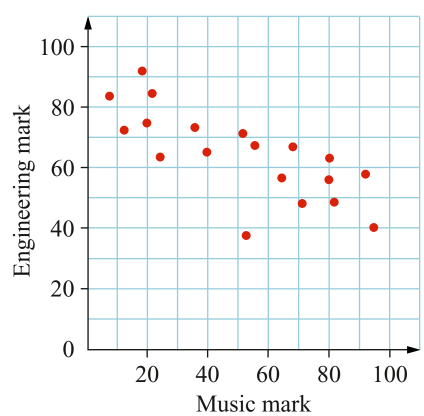
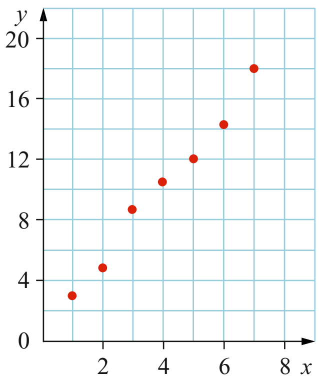


8 Draw the line of best fit.

a



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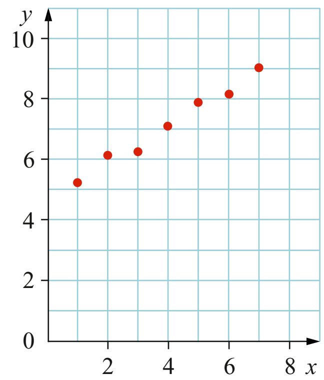

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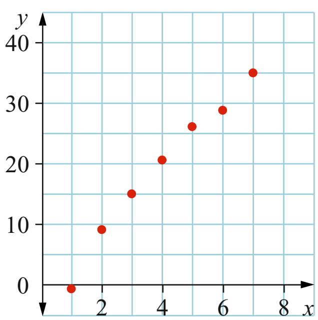

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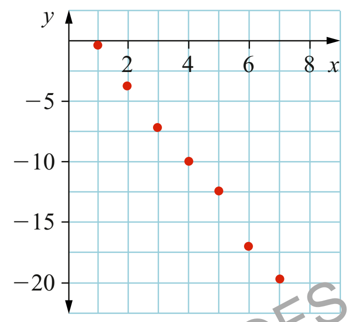

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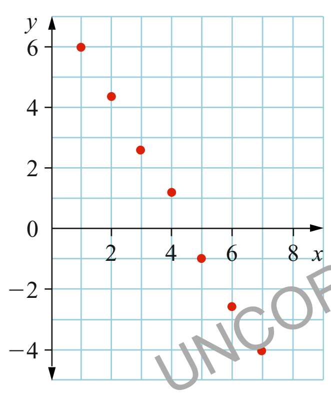
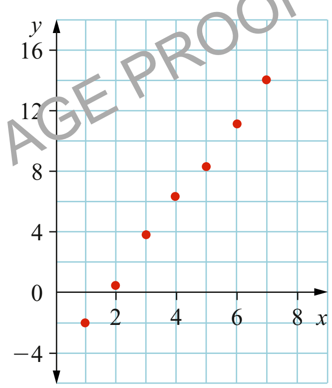


f



9 Draw a scatterplot and line of best fit for the following data.

a

x	100	120	125	140	170	180	190	210	220	240
y	90	85	100	90	100	115	105	125	110	120

b

x	10	14	20	22	28	35	38	43	47
y	9	15	16	13	24	20	29	22	27

c

x	8	14	17	17	22	27	30	33
y	11	14	20	16	22	29	28	35

d

x	5	10	14	15	24	27	32	33
y	10	11	12	14	15	17	20	20

e

x	86	95	100	90	96	105	94	98	110	100	93
y	120	74	20	104	46	50	80	96	10	25	100

FOCUS STUDY

13C Correlation

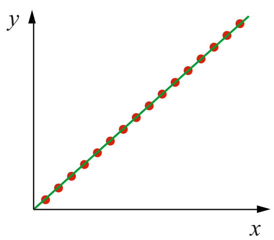
In Section 13B we looked for the relationship between two variables by plotting the data as ordered pairs and drawing the **regression line** (straight line of best fit). We then used the line to predict values of one variable given values of the other. This process does not take into account how closely the points fit the straight line.

In some cases the points fit almost exactly on a line, and hence the predictions based on the algebraic relationship found between the two variables (the equation of the line) are quite accurate. In other cases the points are considerably spread about the line, and hence any predictions based on its equation are less valid.

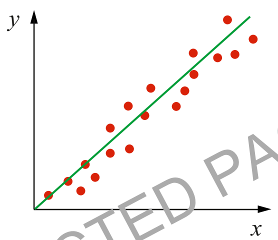
> The study of how closely two variables are related is called **correlation**. The numerical measure of this property is called the **correlation coefficient** and is usually denoted by r.

The methods of determining the numerical value of r are beyond this course, but it can be shown that r takes values from 1 through 0 to -1. A spreadsheet can be used to calculate the value of r.

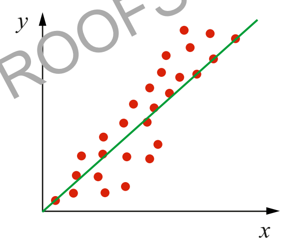
Let x and y be two variables. If large values of x are associated with large values of y, and small values of x are associated with small values of y, then we say that there is a positive correlation between the variables x and y. This is illustrated by an **upwards trend** (as x increases y increases) on a scatterplot as shown below.



Graph A: $r = +1$
Perfect positive correlation



Graph B: $r = +0.8$
High positive correlation



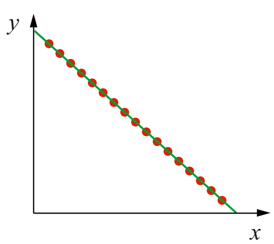
Graph C: $r \approx +0.3$
Low positive correlation

In Graph A there is clearly an upwards trend and all the points lie exactly on a straight line. This is called a **perfect positive correlation** and for this case the correlation coefficient $r = +1$. The two variables are directly related: as one variable increases there is a proportional increase in the other.

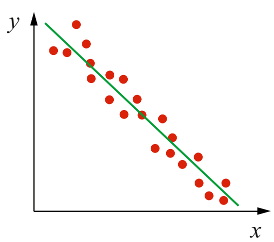
Graph B shows an example of high positive correlation; there is an obvious upwards trend and the points are closely spread about the line of best fit. The variables are **closely related**.

In Graph C there is an upwards trend, but the points are widely spread about the line of best fit. This is an example of low positive correlation. The variables are **related but not closely**.

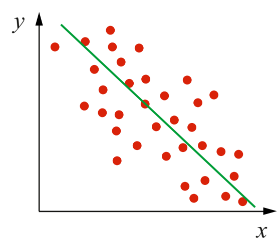
If large values of the variable x are associated with small values of the variable y, and small values of x are associated with large values of y, we say that there is a negative correlation between them. This is illustrated by a **downwards trend** (as x increases y decreases) on a scatterplot.



Graph D: $r = -1$
Perfect negative correlation



Graph E: $r = -0.8$
High negative correlation



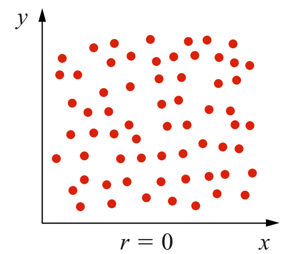
Graph F: $r \approx -0.3$
Low negative correlation

For **perfect negative correlation**, as in Graph D, the coefficient $r = -1$. For every increase in one variable there is a proportional decrease in the other. These variables are said to be inversely proportional to each other.

In Graph E there is clearly a downwards trend and the points are closely spread about the line of best fit. This is an example of **high negative correlation** and the variables are closely related (inversely).

Graph F is an example of **low negative correlation**; there is a weak (inverse) relationship between the variables.

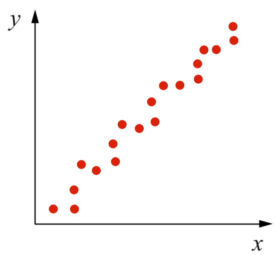
If there is no upwards or downwards trend, the correlation coefficient $r = 0$ and the variables are not related. This is shown in the graph on the right.

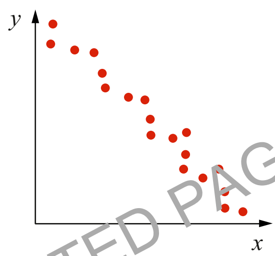


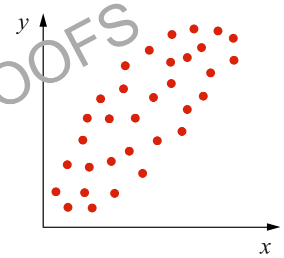
It should be clear that the magnitude of the correlation coefficient determines the accuracy of the predictions made from the equation of the regression line; that is, the closer r is to $+1$ or -1 the closer is the relationship between the variables and the more accurate are the predictions from the equation of the line of best fit. The closer r is to 0 the weaker the relationship between the variables and the less accurate the predictions made.

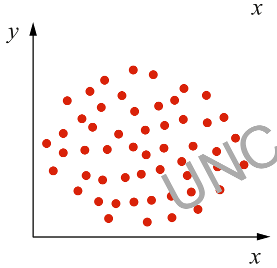
EXERCISE 13C

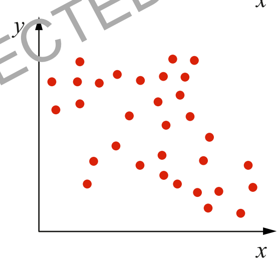
1 For each of the scatterplots drawn below state whether the correlation coefficient is positive, negative or zero. Give reasons.

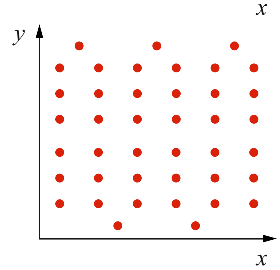
a 

b 

c 

d 

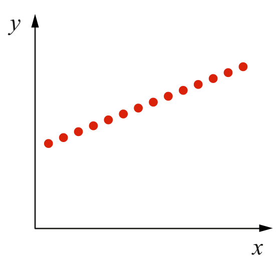
e 

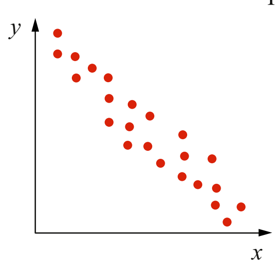
f 

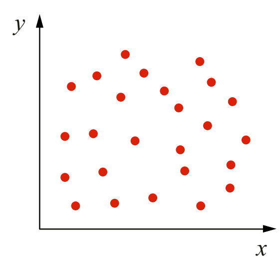
2 Consider the pairs of variables graphed below.

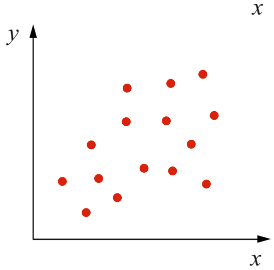
i State whether they have perfect, high, low or zero correlation coefficients.

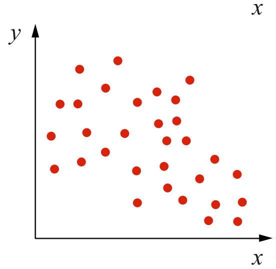
ii How accurate would be the predictions made from the equation of the line of best fit?

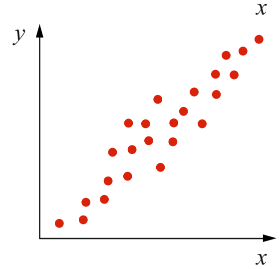
a 

b 

c 

d 

e 

f 

3 Draw a scatterplot for two variables that have the following correlations.

a high positive
b low negative
c perfect negative
d zero
e perfect positive
f high negative

WORKED EXAMPLE 1

Discuss the expected strength of the relationship (correlation) between these variables.

a speed and distance travelled
b speed and time taken
c age and weight of a baby, up to 12 months of age
d height and weight of 18-year-old girls
e height of 18-year-old girls and mark in Mathematics in the HSC exam

	Solve/Think	Apply
a	As speed increases there is a proportional increase in the distance travelled. This is an example of perfect positive correlation.	Sometimes both quantities increase or decrease but are unrelated; that is, there is zero correlation.
b	As speed increases there is a proportional decrease in the time taken. This is an example of perfect negative correlation.	
c	As a baby's age increases so does its weight. However, this will happen at different rates for different babies, hence this is an example of high positive correlation.	
d	In general, taller girls weigh more than shorter girls; that is, larger heights are associated with larger weights and smaller heights are associated with smaller weights, but there are many exceptions. This is an example of low positive correlation.	
e	There is no reason to suspect that there is any relationship between these two variables; that is, height will have no bearing on performance in the HSC or vice versa. This is an example of zero correlation.	

4 Discuss the expected strength of the relationship between the following variables.

a the distance travelled and the cost for a taxi journey
b the volume of water remaining in a tank and the time the tap is on
c the number of police cars and the number of accidents on a highway
d the height and shoe size of male adults
e the age of cars and their price
f the number of sunny days and the sales of umbrellas for a month
g the speed of a car and the stopping distance
h family income and the number of family pets
i lengths of left arm and right arm of people
j eyesight and age
k hours spent studying and examination marks
l smoking and lung cancer

Note: A high degree of correlation between two variables does not necessarily imply that one causes the other.

WORKED EXAMPLE 2

Comment on the following findings.

a The heights and reading speeds of children were measured and a high positive correlation was found.

b The number of televisions sold in Newcastle and the number of stray dogs in Wollongong were recorded over several years and a high positive correlation was found between these variables.

	Solve/Think	Apply
a	Increases in height were associated with increases in reading speed. However, height does not affect reading speed and reading speed does not affect height. The high correlation may be attributed to the fact that both variables are closely linked to a third variable, age. That is, as age increases so do height and reading speed.	These are examples of what is known as spurious correlation. The high correlation occurs because of the existence of a third related variable or because both variables happen by chance to be increasing or decreasing at the same time. When variables are related such that one variable does cause an effect on the other (i.e. if one is changed the other will change), we say that a causal relationship exists. This is referred to as **causality**.
b	Obviously an increase in the number of TVs sold in Newcastle does not cause an increase in the number of stray dogs in Wollongong or vice versa. Both variables must simply happened to be increasing over this period.	

5 The following pairs of variables were measured and a high positive correlation between them was found. Discuss whether a cause and effect relationship exists or whether it is a case of spurious correlation:

a the length of a person's left arm and right foot
b company expenditure on advertising and sales
c daily temperature and ice-cream consumption
d the damage caused by a fire and the number of firemen who attend the fire
e the number of people unemployed and the price of eggs
f the height of parents and the height of adult offspring
g the number of hotels and the number of churches in rural towns

13D Least-squares line of best fit

The simplest method of finding the equation of the **least-squares line of best fit** is to use a spreadsheet.

To find the least-squares line of best fit equation from a table of values comparing variables x and y, we need to calculate r, the mean and standard deviation of the x scores, and the mean and standard deviation of the y scores.

$$\text{Gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

$$y\text{-intercept} = \text{mean of } y \text{ scores} - (\text{gradient} \times \text{mean of } x \text{ scores})$$

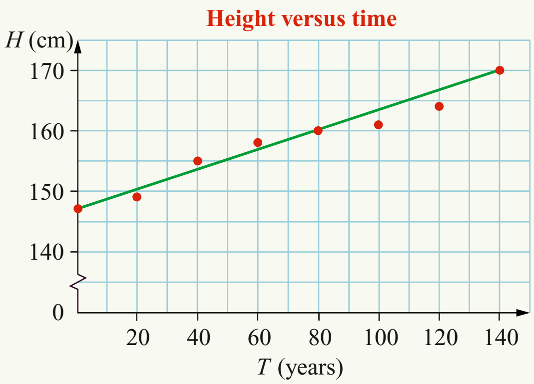
WORKED EXAMPLE 1

The data in the table gives the average height, H, in centimetres, of 17 year-old girls for the years 1850 to 1990. The time, T, is measured in years since 1850.

Year	1850	1870	1890	1910	1930	1950	1970	1990
Time (years)	0	20	40	60	80	100	120	140
Height (cm)	147	149	155	158	160	161	164	170

a Draw a scatterplot comparing T and H and add a line of best fit.

b Given that $r = 0.983$, calculate the equation of the least-squares line of best fit.

	Solve	Think	Apply
a		Plot each point on the grid with time (T) on the horizontal axis and height (H) on the vertical axis. Draw a line of best fit.	Draw a scatterplot, find the gradient and calculate the value the equation of the line of best fit using the given value of r.
b	Mean height = 158 Standard deviation height = 7.1 Mean time = 70 Standard deviation time = 45.8 Gradient $= 0.983 \times \frac{7.1}{45.8}$ $= 0.152$ y-intercept $= 158 - (0.152 \times 70)$ $= 147.36$ The equation of the line of best fit is $H = 0.152T + 147.36$.	Use a scientific calculator to find the means and standard deviations. Gradient $= r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$ y-intercept = mean of y scores − (gradient × mean of x scores). The equation found may not be the line of best fit by eye, but it is the equation of the least-squares line of best fit.	There are many calculations required to the find the equation of the line. It is much more practical to use a spreadsheet.

EXERCISE 13D

1 The data below gives the average height (H) in centimetres, of 17-year-old boys for the years 1850 to 1990. The time (T) is measured in years since 1850.

Year	1850	1870	1890	1910	1930	1950	1970	1990
Time (T)	0	20	40	60	80	100	120	140
Height (H)	153	155	159	161	164	168	171	175

a Illustrate the data (T vs H) on a scatterplot.

b Draw a line of best fit.

c Given that $r = 0.997$, calculate the equation of the least-squares line of best fit.

d Substitute the value of $T = 200$ for the year 2050 and obtain a value for height. Comment on this value.

2 The results of a group of students on Mathematics and Science tests are compared.

Student	1	2	3	4	5	6	7	8	9	10
Maths test (*M*)	64	67	69	70	73	74	77	82	84	85
Science test (*S*)	68	73	68	75	78	73	77	84	86	89

a Illustrate the given data on a scatterplot.
b Draw a line of best fit.
c Given that $r = 0.94$, calculate the equation of the least-squares line of best fit.
d Use the equation to predict the (average) score in Science of students who score 80 in Mathematics.
e Use the line of best fit equation to predict the (average) score in Mathematics of students who score 70 in Science.

3 The results of a group of students in History and Geography tests are compared.

Student	1	2	3	4	5	6	7	8
History test (*H*)	84	65	63	74	68	79	70	61
Geography test (*G*)	52	72	75	64	70	54	65	76

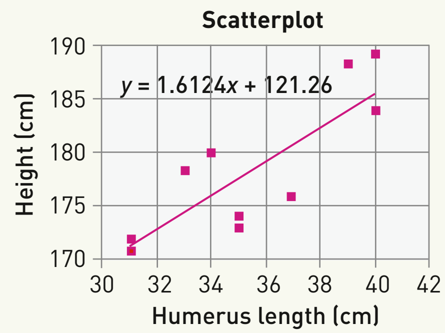
a Illustrate the given data on a scatterplot.
b Draw a line of best fit.
c Given $r = 0.986$, calculate the equation of the least-squares line of best fit.
d Use the line of best fit equation to predict the Geography score of students who score 75 in History.
e Use the line of best fit equation to predict the History score of students who score 75 in Geography.

WORKED EXAMPLE 2

For a group of girls, the humerus length (elbow to shoulder) was measured and compared with height. The results are listed in this table.

Humerus length (cm)	37	35	40	31	35	33	31	40	34	39
Height (cm)	176	174	184	172	173	178	171	189	180	188

a Enter this data into a spreadsheet.
b Calculate *r*.
c Draw a scatterplot.
d Add the trendline (least-squares line of best fit) and show the equation.

	Solve	**Think**	**Apply**
a	Enter the data into a spreadsheet.	Put the data into two columns.	Put heading into first row. Use built-in formula.
b	$r = 0.836\ 269$	Use =CORREL(A2:A11, B2:B11).	
c **d**		From the Insert menu select Scatterplot then first scatterplot type. From the Chart tools select Linear Trendline. Right click on the line and select Format Trendline from the drop-down menu. Check the Display equation on chart box and close.	

FOCUS STUDY

4 The data below gives the average height, in centimetres, of 17-year-old girls for the years 1860 to 2000. The time is measured in years since 1860.

Year	1860	1880	1900	1920	1940	1960	1980	2000
Time (years)	0	20	40	60	80	100	120	140
Height (cm)	147	149	155	158	160	161	164	170

a Enter this data into a spreadsheet. Calculate r.

b Draw a scatterplot.

c Add the least-squares line of best fit and show the equation.

5 The population of a town over a period of 10 years is shown in the table. The time is measured in years from the start of 1990; that is, $T = 1$ is the start of 1991, $T = 2$ is the start of 1992, etc.

Time (years)	1	2	3	4	5	6	7	8	9	10
Population	3400	4100	4500	4900	5600	6100	6500	6900	7400	8000

a Use a spreadsheet to illustrate the data on a scatterplot.

b Draw the trendline and show the equation of this line.

c Use this equation to predict the population:

i after 4.5 years **ii** after 7.5 years **iii** after 12 years **iv** at the start of 2007.

d Which of the answers in part **c** are the least reliable? Give reasons for your answer.

e Use this equation to estimate when the population:

i was 5000 **ii** will reach 10 000.

6 **a** The table below shows the production costs of DVDs. Use a spreadsheet to illustrate this data on a scatterplot.

Number of DVDs produced ('000s)	5	10	20	40	80	100
Cost of production \$/DVD	9.80	9.60	8.70	7.30	5.80	4.90

b Draw the trendline.

c Find the algebraic relationship connecting the number of DVDs produced and the cost per DVD; that is, find the equation of the line.

d Use this equation to estimate the cost per DVD of producing:

i 15 000 DVDs **ii** 50 000 DVDs **iii** 3000 DVDs **iv** 120 000 DVDs.

e Which of the results in part **d** are the least reliable? Give reasons for your answer.

f Use the equation to find the cost per DVD of producing 200 000 DVDs. Comment on this answer.

INVESTIGATION 13.1

13E Regression line by calculator: extension

For most scientific calculators, the regression line is usually expressed in the form $y = A + Bx$.

Follow the steps below to find the equation of the regression line for this table of data.

Income ('000s)	10	12	16	22	26	29	33	37	42	49
Expenditure ('000s)	2.5	2.8	3	3.3	3.6	3.8	3.9	4.1	4.5	4.9

First put the calculator in REG mode (regression mode).

On a CASIO fx-82TL press MODE 3 1 for linear regression.

To enter the data from the table, press:

10 , 2.5 DT , 12 , 2.8 DT , 16 , 3 DT , 22 , 3.3 DT , …, 49 , 4.9 DT

To obtain the linear coefficients A and B, press SHIFT A = 2.15 (2 decimal places)

SHIFT B = 0.05 (2 decimal places)

The equation of the regression line for this data is $y = 2.15 + 0.05x$, where x represents income and y represents expenditure.

Note: The calculator finds the equation of the line of best fit, called the least-squares regression line.

EXERCISE 13E

1 Using your calculator, find the equation of the least-squares regression line for the data below.

a

x	100	120	125	140	170	180	190	210	220	240
y	90	85	100	90	100	115	105	125	110	120

b

x	10	14	20	22	28	35	38	43	47
y	9	15	16	13	24	20	29	22	27

c

x	8	14	17	17	22	27	30	33
y	11	14	20	16	22	29	28	35

d

x	5	10	14	15	24	27	32	33
y	10	11	12	14	15	17	20	20

e

x	86	95	100	90	96	105	94	98	110	100	93
y	120	74	20	104	46	50	80	96	10	25	100

13F Measurement calculations

The most common forms of a drug are tablets and liquids. As the amount of active drug taken (the dosage) is usually small, it is measured in milligrams mg. (1000 mg is equal to 1 g.)

WORKED EXAMPLE 1

Convert the following to milligrams.

a 2 g **b** 0.6 g **c** 0.35 g

	Solve	Think	Apply
a	$2\text{ g} = 2 \times 1000\text{ mg}$ $= 2000\text{ mg}$	Multiply each measurement by 1000, as $1\text{ g} = 1000\text{ mg}$.	Grams are larger than milligrams and hence the measurement in grams must be multiplied by the conversion factor of 1000.
b	$0.6\text{ g} = 0.6 \times 1000\text{ mg}$ $= 600\text{ mg}$		
c	$0.35\text{ g} = 0.35 \times 1000\text{ mg}$ $= 350\text{ mg}$		

EXERCISE 13F

1 Convert the following to milligrams.

a 3 g **b** 5 g **c** 7 g **d** 9 g
e 2.5 g **f** 2.2 g **g** 1.3 g **h** 3.4 g
i 0.4 g **j** 0.3 g **k** 0.15 g **l** 0.22 g
m 0.05 g **n** 0.037 g **o** 0.002 g **p** 0.003 g

WORKED EXAMPLE 2

Convert the following to grams.

a 3000 mg **b** 200 mg **c** 43 mg

	Solve	Think	Apply
a	$3000\text{ mg} = \frac{3000}{1000}\text{ g}$ $= 3\text{ g}$	Divide each measurement in mg by 1000, as $1\text{ g} = 1000\text{ mg}$.	Milligrams are smaller than grams and hence the measurement in mg must be divided by the conversion factor of 1000.
b	$200\text{ mg} = \frac{200}{1000}\text{ g}$ $= 0.2\text{ g}$		
c	$43\text{ mg} = \frac{43}{1000}\text{ g}$ $= 0.043\text{ g}$		

2 Convert the following to grams.

a 3000 mg	**b** 7000 mg	**c** 4000 mg	**d** 8000 mg
e 2500 mg	**f** 4200 mg	**g** 7500 mg	**h** 6200 mg
i 400 mg	**j** 350 mg	**k** 270 mg	**l** 120 mg
m 60 mg	**n** 38 mg	**o** 4 mg	**p** 2.5 mg

WORKED EXAMPLE 3

A patient is prescribed 400 mg of a painkiller. The medication available contains 80 mg in 10 mL. How much medication should be given to the patient.

Solve	Think	Apply
$400 \div 80 = 5$ $\text{Amount} = 5 \times 10\text{ mL}$ $= 50\text{ mL}$ Or volume required $= \dfrac{\text{strength required}}{\text{stock strength}} \times \text{volume of stock}$ $= \dfrac{400}{80} \times 10$ $= 50\text{ mL}$	Calculate how many lots of 10 mL is required by dividing the amount needed by the amount supplied. Multiply this answer by 10 mL to obtain the amount. In this formula: Strength required = 400 mg Stock strength = 80 mg Volume of stock = 10 mL	As the amount of medication required is greater than the amount in the medication available, more of the medication will be given. Deciding whether more or less than 10 mL is to be given is the key to answering the question.

3 A patient is prescribed 600 mg of a painkiller. Calculate how much must be given if the medication is available in these concentrations.

a 20 mg in 5 mL	**b** 30 mg in 10 mL	**c** 50 mg in 1 mL
d 120 mg in 5 mL	**e** 100 mg in 20 mL	**f** 60 mg in 5 mL
g 5 mg in 1 mL	**h** 50 mg in 5 mL	**i** 75 mg in 5 mL

4 A patient is prescribed 800 mg of an anti-nausea drug. Calculate how much must be given if the medication is available in the following concentrations.

a 100 mg in 5 mL	**b** 10 mg in 1 mL	**c** 50 mg in 5 mL
d 160 mg in 10 mL	**e** 200 mg in 20 mL	**f** 80 mg in 5 mL
g 20 mg in 5 mL	**h** 40 mg in 5 mL	**i** 80 mg in 10 mL

13G Medication calculations

This section examines dosages of various medications. Some terms are defined here.

- The dose is the amount of drug taken at any one time.
- The dosage regimen is the frequency at which the drug doses are given.
- The total daily dose is calculated from the dose and the number of times the dose is taken.
- The dosage form is the physical form of a dose of the drug. Common dosage forms include tablets, capsules, creams, ointments, aerosols and patches.
- The optimal dosage is the dosage that gives the desired effect with minimal side effects.

EXERCISE 13G

1 The dosage for a painkiller is as follows.

Age: 7–12 years: $\frac{1}{2}$–1 tablet every 4–6 hours (maximum 4 tablets in 24 hours)

Age: 12–adult: 1–2 tablets every 4–6 hours (maximum 8 tablets in 24 hours)

a How many tablets can an adults take in one dose?

b An adult plans to take two tablets every 4 hours for 24 hours.

i How may tablets would they take over 24 hours?

ii Why shouldn't they do this?

iii How many doses of two tablets can be taken over 24 hours?

c A child takes a $\frac{1}{2}$ tablet every 4 hours for 24 hours. Have they exceeded the maximum dosage? Explain.

2 The dosage for a very strong painkiller is given. Adults and children from 12 years: two caplets, then 1–2 caplets every 4–6 hours as necessary. (Maximum 6 caplets in 24 hours.)

a An adult takes two caplets now and then two more after 4 hours. How many more caplets can they take in that 24-hour period?

b Is two caplets initially, two more after 4 hours and two more after 6 hours then no more an acceptable dosage? Explain your answer.

WORKED EXAMPLE 1

The adult dose of a medication is 40 mL. Use Fried's formula for children 1 to 2 years old to calculate the dosage for a 20-month-old child.

$$\text{Dosage for children 1 to 2 years} = \frac{\text{age (in months)} \times \text{adult dosage}}{150}$$

Solve	Think	Apply
$\text{Dose} = \frac{20 \times 40}{150}$ $= 5.3 \text{ mL}$	Age = 20 months Adult dose = 40 mL	Ensure that the units are correct for the formula; that is, age in months.

3 Use Fried's formula to calculate the child's dosage.

a adult dose of 50 mL, child's age 15 months

b adult dose of 40 mL, child's age 21 months

c adult dose of 30 mL, child's age 18 months

d adult dose of 50 mL, child's age 13 months

e adult dose of 100 mL, child's age 17 months

f adult dose of 80 mL, child's age 23 months

4 **a** A child aged 17 months is given a dosage of 6 mL. Calculate the adult dosage.

b A child aged 11 months is given a dosage of 7 mL. Calculate the adult dosage.

WORKED EXAMPLE 2

Use Young's formula to calculate the dosage for a $5\frac{1}{2}$-year-old child if the adult dose is 60 mL.

$$\text{Dosage for children 1 to 12 years} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}$$

Solve	Think	Apply
$\text{Dose} = \frac{5.5 \times 60}{5.5 + 12}$ $= 19 \text{ mL}$	Age in years = 5.5 Adult dose = 60 mL	Check the units before substituting. Some formulas use age in years, others months.

5 Use Young's formula to calculate the child's dosage.

a adult dose of 50 mL, child's age 6 years
b adult dose of 40 mL, child's age 8 years
c adult dose of 80 mL, child's age 4.5 years
d adult dose of 20 mL, child's age 7.5 years
e adult dose of 100 mL, child's age 6.2 years
f adult dose of 75 mL, child's age 8.4 years

6 a Calculate the adult dose if the dosage for an 8-year-old child was 10 mL.
b Calculate the adult dose if the dosage for a 6-year-old child was 5 mL.

WORKED EXAMPLE 3

Use Clark's formula to calculate the dosage for a child weighing 19 kg. The adult dose is 50 mL.

$$\text{Dosage} = \frac{\text{child's weight in kg} \times \text{adult dose}}{70}$$

Solve	Think	Apply
$\text{Dose} = \frac{19 \times 50}{70}$ $= 13.6\text{ mL}$	Weight in kg and dose in mL.	Check units before substituting. The formula uses 70 kg as the average adult weight.

7 Use Clark's formula to calculate the child's dosage.

a adult dosage of 18 mL, child's weight 40 kg
b adult dosage of 27 mL, child's weight 60 kg
c adult dosage of 51 mL, child's weight 80 kg
d adult dosage of 39 mL, child's weight 75 kg
e adult dosage of 32 mL, child's weight 100 kg
f adult dosage of 40 mL, child's weight 35 kg

8 a Calculate the adult dose if a 35 kg child has a dose of 18 mL.
b Calculate the adult dose if a 25 kg child has a dose of 17 mL.
c Calculate the weight of a child receiving a dose of 40 mL given that the adult dose is 140 mL.

WORKED EXAMPLE 4

A patient is to receive 1.6 L of fluid over 10 h. What is the flow rate in mL/h?

Solve	Think	Apply
$\text{Flow rate} = \frac{\text{volume (mL)}}{\text{time (h)}}$ $= \frac{1600\text{ mL}}{10\text{ h}}$ $= 160\text{ mL/h}$	Convert 1.6 L to mL by multiplying by 1000.	Ensure that units are converted before dividing to find the rate.

9 Calculate the flow rate in mL/h for these volumes of fluid and times.

a volume of 1.4 mL over 8 h
b volume of 1.7 mL over 5 h
c volume of 0.8 mL over 6 h
d volume of 0.6 mL over 5 h
e volume of 0.085 mL over 3 h
f volume of 4.26 mL over 12 h

10 **a** The flow rate is 150 mL/h for 6 h. How much fluid is delivered?
b The flow rate is 200 mL/h for 7 h. How much fluid is delivered?
c The flow rate is 180 mL/h and 600 mL is delivered. For how long was the fluid delivered?

WORKED EXAMPLE 5

A patient is to receive 1.2 L of fluid over 4 h through an IV drip. There are 15 drops/mL. How many drops per minute are required?

Solve	Think	Apply
Flow rate $= \frac{1200 \text{ mL}}{240 \text{ min}} = 5$ mL/min Drops $= 5 \times 15$ drops/min $= 75$ drops/min	Convert 1.2 L to mL by multiplying by 1000. Convert 4 h to minutes by multiplying by 60.	The unit for drops/min is actually gtts/min. Large drops per mL is called a macrodrip and is used if the rate is greater than 100 mL/h.

11 Calculate the number of drops per minute required at a rate of 15 drops/mL in these situations.
a volume of 1.4 L over 7 h
b volume of 1.5 L over 6 h
c volume of 800 mL over 4 h
d volume of 600 mL over 2 h
e volume of 750 mL over 5 h
f volume of 900 mL over 6 h

12 What is the drip rate per minute for:
a 1.3 L of fluid over 6 h with a drip size giving 12 drops/mL?
b 850 mL of fluid over 5 h with a drip size giving 8 drops/mL?

WORKED EXAMPLE 6

An IV drip is delivering 30 drops/min. There are 20 drops/mL and 900 mL of liquid to be delivered. How long will the drip take?

Solve	Think	Apply
Number of drops $= 900 \times 20 = 18\,000$ Time $= \frac{18\,000}{30}$ min $= 600$ min $= 10$ h	First calculate the number of drops needed. Calculate time using number of drops/min. Divide minutes by 60 to convert to hours.	Be aware of the units and convert where necessary. Drop size can be varied as well as drop rate.

13 Calculate the time it will take to deliver IV liquid at these rates.
a 800 mL delivered at 20 drops/min and there are 15 drops/mL
b 600 mL delivered at 15 drops/min and there are 10 drops/mL
c 500 mL delivered at 10 drops/min and there are 12 drops/mL
d 1.2 L delivered at 25 drops/min and there are 20 drops/mL
e 1.5 L delivered at 15 drops/min and there are 12 drops/mL
f 1.8 L delivered at 20 drops/min and there are 15 drops/mL

13H Life expectancy

Life expectancy is an indicator of how long a person can expect to live, on average, given prevailing mortality rates. Technically, it is the average number of years of life remaining to a person at a specified age, assuming current age-specific mortality rates continue during the person's lifetime.

Life expectancy is a common measure of population health in general, and is often used as a summary measure when comparing different populations (such as for international comparisons). For example, high life expectancy indicates low infant and child mortality, an ageing population, and a high quality of healthcare delivery. Life expectancy is also used in public policy planning, especially as an indicator of future population ageing in developed nations.

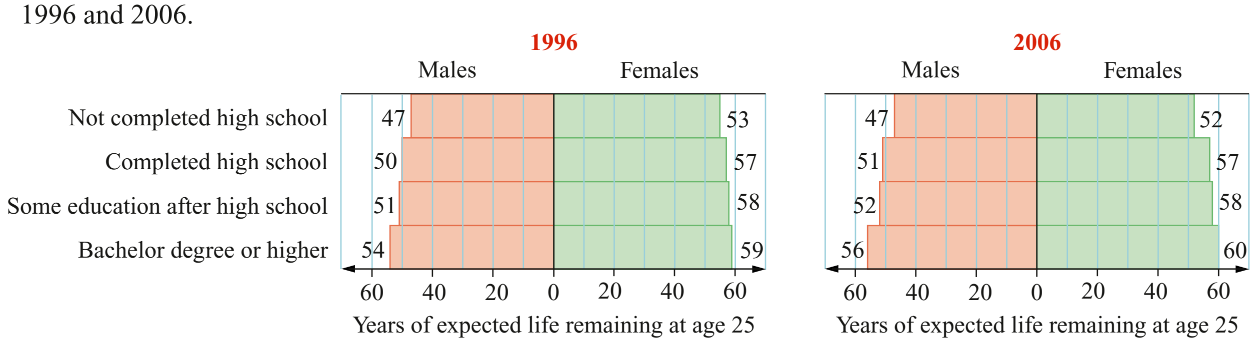
The expected length of a life is inversely related to the mortality rates at that time. In Australia, life expectancy has increased significantly over the past century, reflecting the considerable falls in mortality rates, initially from infectious diseases and, in later years, from cardiovascular disease.

Based on the latest mortality rates, a boy born in 2006 would be expected to live to 78.7 years on average, while a girl would be expected to live to 83.5 years. However, a man and woman aged 25 in 2006 would be expected to live to ages 79.7 and 84.2 years respectively. This shows that once people survive through childhood, the chance of dying as a young adult is very low and hence life expectancy increases.

Life expectancy is calculated using a mathematical tool called a 'life table'. These are constructed by taking death rates from the population in question (such as Australian males in 2006) and applying them to a hypothetical cohort of persons. The life table is then able to provide probabilities concerning the likelihood of someone in this hypothetical population dying before or surviving to their next birthday. Life expectancy can be provided for any age in the life table, by summing the number of person years (the total number of years lived by all persons in the life table) and dividing this by the number of persons still alive in the life table.
Source: *www.aihw.gov.au*

EXERCISE 13H

1 The graph below shows life expectancy at age 25 years for a developed country, by sex and education level in 1996 and 2006.



a What was the difference in life expectancy in 1996 between:
  i the most-educated and the least-educated males
  ii the most-educated and the least-educated females.

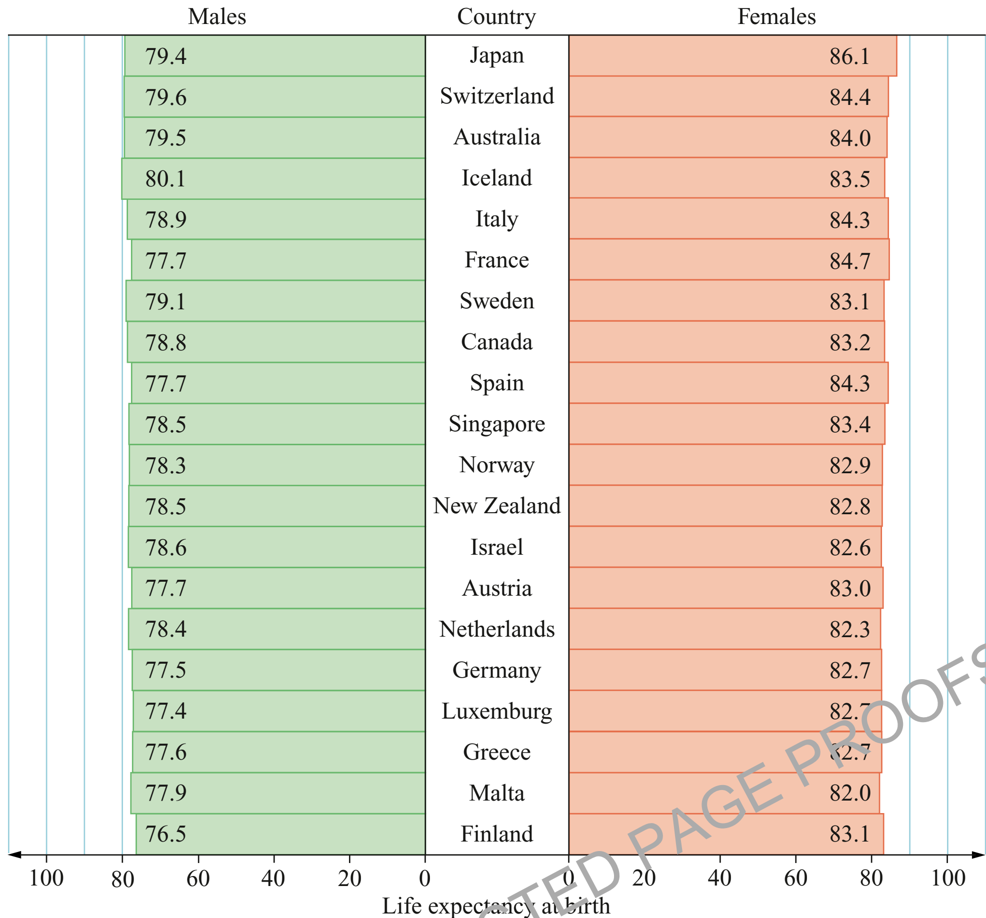
b Describe the change in life expectancy for males from 1996 to 2006.

c For women there were only two categories that changed from 1996 to 2006. Which were they and what was the impact of the changes?

d Compare the life expectancies of males and females for each category of education for the year 2006.

2 The graph shows life expectancy for the top 15 WHO countries by sex in 2010.

Life expectancy by country and sex, 2010



a Which country has the greatest life expectancy for:

i males? **ii** females?

b What is the difference, in years, between life expectancy in Australia and the top life expectancy countries for males and females?

c What is the difference in life expectancy, in years, between the highest and lowest ranked countries in this table for males and females?

d The country with the lowest life expectancy is Swaziland, with 40 years for males and 39 years for females. Calculate the difference in the number of years of life expectancy between Australia and Swaziland.

3 The graph shows survivors by age and country income group for a particular year.

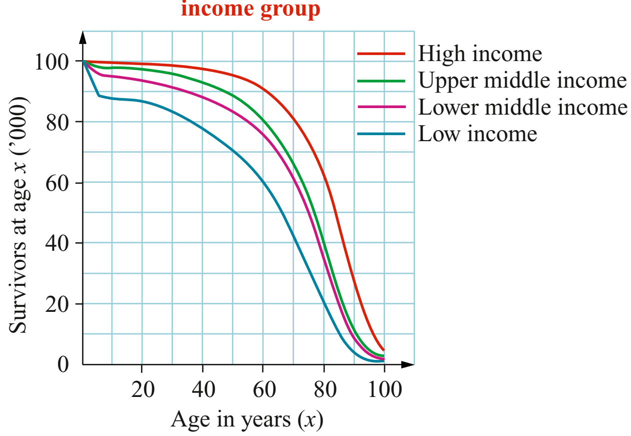
a Comment on the statement that: 'Wealthier countries have a longer life expectancy', using information from the graph.

b Where is the largest drop in survivors?

c One of the strongest factors in life expectancy is infant mortality rate. Which income level has the largest infant mortality rate?

d What could be done to increase life expectancy in low income countries?

Survivors by age and country income group

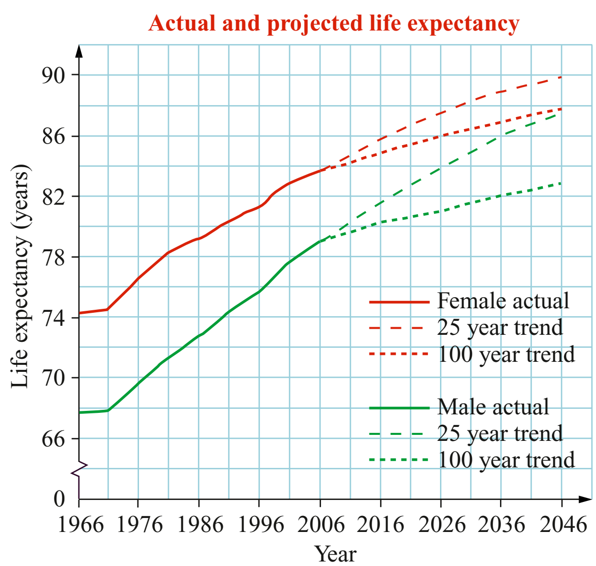


4 This table shows the life expectancy (years to live) for males and females aged from 50 to 89 for a particular year.

Age (years)	Male	Female	Age (years)	Male	Female
50	31.43	35.17	70	14.76	17.42
51	30.53	34.24	71	14.04	16.61
52	29.63	33.31	72	13.33	15.82
53	28.73	32.38	73	12.64	15.03
54	27.84	31.45	74	11.96	14.27
55	26.95	30.53	75	11.31	13.51
56	26.08	29.61	76	10.68	12.78
57	25.20	28.70	77	10.07	12.05
58	24.34	27.79	78	9.48	11.35
59	23.48	26.89	79	8.92	10.67
60	22.63	26.00	80	8.38	10.01
61	21.79	25.11	81	7.86	9.37
62	20.96	24.23	82	7.36	8.75
63	20.14	23.35	83	6.89	8.17
64	19.34	22.48	84	6.45	7.61
65	18.54	21.62	85	6.03	7.08
66	17.76	20.76	86	5.64	6.58
67	16.99	19.92	87	5.27	6.11
68	16.24	19.08	88	4.94	5.68
69	15.49	18.24	89	4.63	5.28

a Calculate the age expected for a male who is currently aged:
i 50 years **ii** 55 years **iii** 60 years **iv** 85 years
What do you notice about the answers?
b Repeat part **a** for females.
c Use data from the table to support the statement that:
'The longer you have lived, the older you will be when you die.'

5 The line graph shows actual and projected life expectancy at birth from 1966 to 2046.
a What is the life expectancy in 2026 according to the 100 year trend for:
i males? **ii** females?
b For the year 2036, what is the difference between the 25 year trend and 100 year trend for:
i males? **ii** females?
c Since 1980 enormous advances have been made in the successful treatment of cardiovascular disease. How is this reflected in the graph?



13I Interpreting life expectancy data

What affects life expectancy?

There are both positive and negative influences on mortality and life expectancy, such as improving socioeconomic conditions (positive) or increases in a certain cause of death (negative). In many developing countries, increasing child survival and infectious disease control is leading to increasing life expectancies. However, in some sub-Saharan countries, severely affected by the HIV/AIDS pandemic, life expectancy has decreased in the past two decades due to increases in premature death (*www.unaids.org*).

In most developed countries, life expectancy has been increasing steadily since the middle of the 20th century, due mainly to the near-eradication of infectious disease and high standards of living (which includes diet, sanitation and healthcare). However, even in developed countries, these positive influences on life expectancy may change when looking at population sub-groups. For example, life expectancy among African-Americans decreased throughout the late 1980s, due in part to increasing rates of HIV infection and homicide, which offset other positive influences.
Source: Kochanek, K. D. et al. 1994. *American Journal of Public Health* 84(6): 938–44

Public health campaigns and cultural change may also have a measurable influence on life expectancy. In Australia, the rise in cigarette smoking in the middle of the 20th century resulted in large increases in mortality from lung cancer, cardiovascular disease, respiratory and other conditions. These increases in mortality had a retarding effect on life expectancy, especially in the 1960s. Public health campaigns and changes in public health regulation began to reduce smoking rates. The effect of legislation, rises in tobacco taxes and other health promotion activities are starting to become evident in the mortality rates and other measures. A sharp decline in the proportion of males who are smoking has been followed by a decline in the incidence of male lung cancer. A rise in smoking prevalence among females in the latter part of the 20th century has been followed by a rise in the incidence of female lung cancer (although female smoking rates are also now in decline).
Source: *www.aihw.gov.au*

Increasing rates of chronic disease may now have a growing negative influence on life expectancy in both developed and developing countries. This is also the case for chronic disease risk factors, such as obesity and overweight. Indeed, recent research in the United States suggests that high obesity levels may lead to decreasing life expectancy in that country during the 21st century.
Source: Olshansky, S. et al. 2005. *Obstetrics and Gynecological Survey* 60: 450–52

EXERCISE 13I

1 The table shows the life expectancy, years to live, at ages 0, 30 and 65 years, for males in a developed country.

Year	Age 0 years	Age 30 years	Age 65 years
1880	47.2	33.64	11.06
1890	51.08	35.11	11.25
1900	55.2	36.52	11.31
1910	59.15	38.44	12.01
1920	63.48	39.9	12.4
1930	66.07	40.4	12.25
1940	67.14	40.9	12.33
1950	67.92	41.12	12.47
1960	67.63	40.72	12.16
1970	68.1	41.1	12.37
1980	71.23	43.51	13.8
1990	74.32	46.07	15.41
2000	77.64	49.07	17.7
2010	79.02	50.2	18.54

a Give the expected final age for these cases.

i Year 1890, aged 30 years

ii Year 1940, aged 0 years

iii Year 2000, aged 65 years

b **i** Plot this data on a scatterplot. (A spreadsheet would be useful.)

ii Draw a line of best fit or have the spreadsheet display the least-squares line of best fit.

iii Use your line of best fit to estimate the years to live for the 3 ages in 2050. Comment on your answers.

iv There is a flattening of the increase in life expectancy from 1920–1950 followed by a much larger increase to 2000. Give an explanation for this.

v Use a spreadsheet to calculate the correlation coefficient for each set of data.

2 The table shows the life expectancy from age 0 years in five countries from 1980 to 2010.

	Life expectancy			
Country	**1980**	**1990**	**2000**	**2010**
Australia	71.0	73.9	76.6	79.4
Estonia	64.2	64.5	65.1	69.8
France	70.2	72.8	75.2	77.8
Korea	61.8	72.3	73.5	76.8
USA	70.0	71.8	74.1	76.2

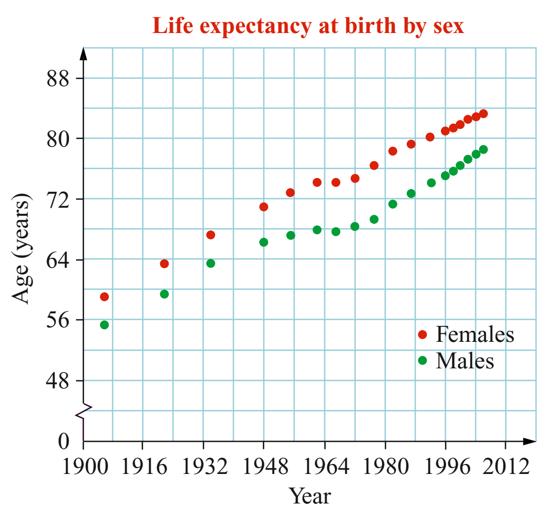
a Use a spreadsheet, or otherwise, to draw a scatterplot representing each country.

b Calculate the correlation coefficient, r, for each country.

c List the countries in order of their correlation with a least-squares regression line.

d Predict the life expectancy in Australia in 2050.

3 The graph shows the expected length of life in Australia at birth, by sex, from 1900 to 2006.

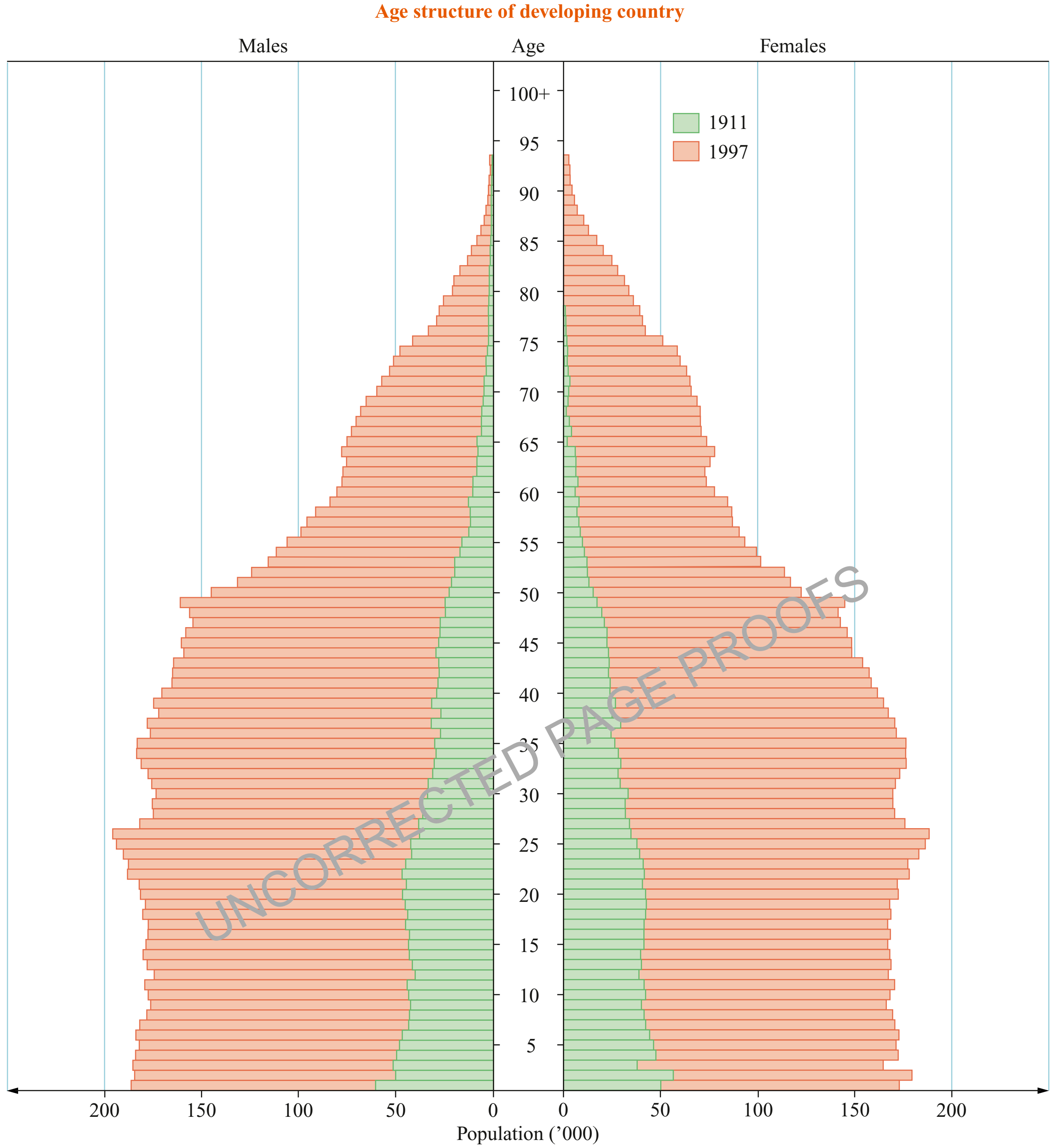


a Male life expectancy decreased due to an increase in the death rate due to circulatory disease. When was this?

b Life expectancy at birth increased significantly early in the 20th century. Calculate the percentage increase between 1900 and 1935 (first and third data points) for males and females.

c Advances in the treatment of circulatory disease ended the plateau through the 1960s and led to life expectancy again increasing. Extend a line of best fit for the final six points and estimate the year male and female life expectancies will be equal. Do you think this will happen? Explain your answer.

4 The population pyramid shows a profile of Australia's population in 1911 and 1997.



a Which age group had the highest number of females in:

i 1911? **ii** 1997?

b Which age group had the greatest number of males in:

i 1911? **ii** 1997?

c At what age did the population start to steadily decrease in:

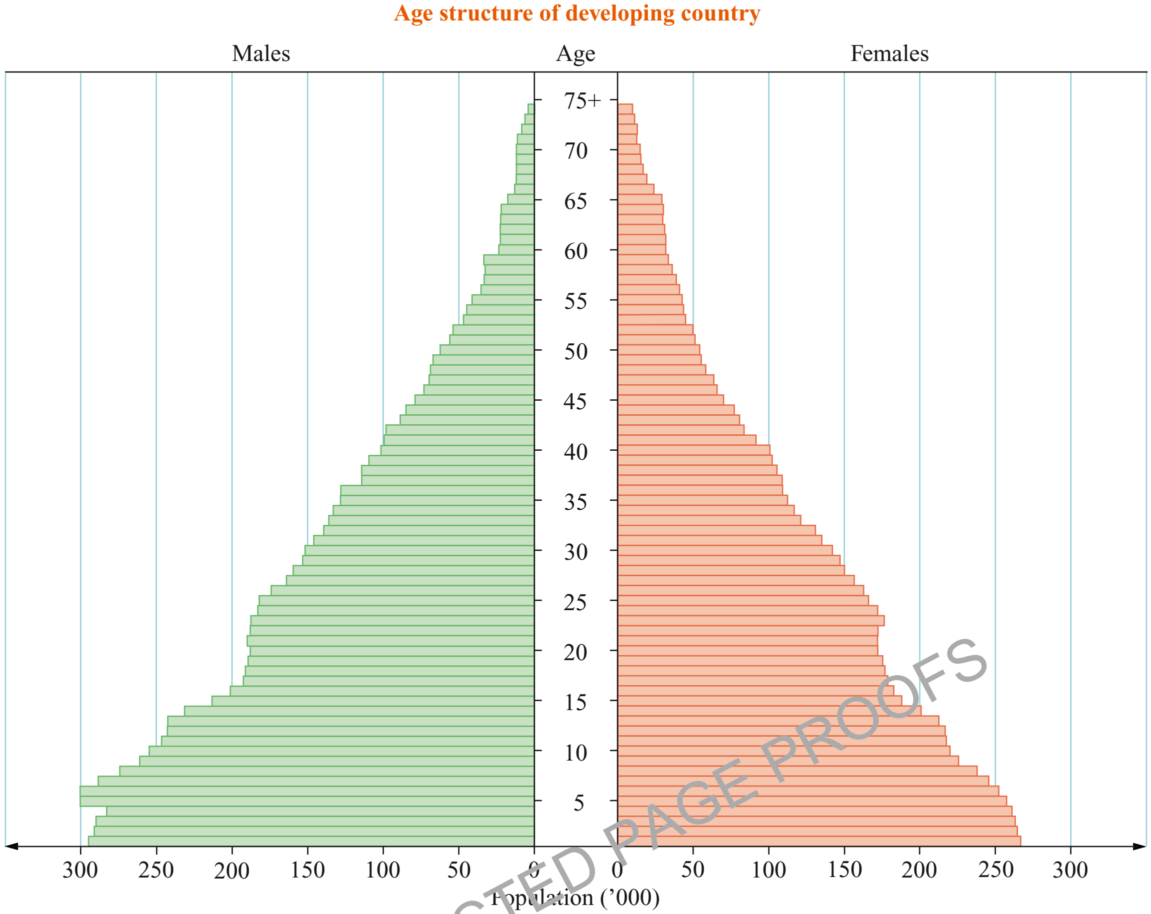
i 1911? **ii** 1997?

d In 1997 the number of males in the 0–20 age group decreases slightly while the number of females remained basically steady. Give a possible explanation for this.

e Comment on the life expectancy for both males and females in:

i 1911 **ii** 1997.

5 The following population pyramid shows the age structure for a developing country. Compare this with the population pyramid in question **4** for Australia, to answer the questions below.



a Which pyramid is wider at the base? What does this indicate about the birth rates in these two countries?
b Which pyramid narrows immediately from the base? Comment on the infant mortality rate in these countries.
c Comment on the mortality rates in all age groups.
d Which country has the higher life expectancy?

INVESTIGATION 13.2

INVESTIGATION **13.1**

Correlating data

1 **a** Measure the height and handspan of the students in your class (separate results for males and females) and record the information in a table (or directly into a spreadsheet).
b Plot the data as a scatterplot.
c Discuss the correlation between these two variables. (Is it positive, zero, negative, high, low?) Is one variable a good predictor of the other? Calculate the value of r.
d Draw the least-squares line of best fit line for the data.
e Find the equation of this line of best fit.
f Measure the heights of some students from another class and predict the handspans of these students, using the equation.
g Find the handspans of these students by actual measurement and compare them with your predictions.
h Discuss the results in relation to your answer to part **c**.

2 Repeat the procedures in question **1** for these measurements.
a height and shoe size
b head circumference and height
c length of femur (thigh bone) and height
d length from hip to the ground and height

For the following questions use a spreadsheet and the basic procedure outlined below.
Step 1: Put the data in a table.
Step 2: Illustrate the data as a scatterplot.
Step 3: Discuss the correlation between the two variables. (Is it positive, zero, negative, high, low?). Is one variable a good predictor of the other? Calculate the value of r.
Step 4: Draw the least-squares line of best fit for the data.
Step 5: Find the equation of this line of best fit.

3 Collect data about the world records for a particular sporting event over past years (for example, the 200 m sprint) and investigate the relationship, if any, between the world record time and the year. Make some predictions using your equation and discuss their reliability.

4 Collect the results for the students in your class in the last Mathematics and English tests. Investigate whether the mark in one subject can be used to predict the mark in the other.

5 Investigate the correlation between the scaled school assessment mark and the scaled HSC examination mark in Mathematics General in last year's HSC. (Names of students are not required.)

INVESTIGATION **13.2**

Life expectancy calculations

1 Use an online life expectancy calculator to make an assessment of how variables such as smoking or low income affect life expectancy.

2 Research the way in which life expectancy data is calculated, particularly with respect to infant mortality, calculation of death rates, healthcare and medical advancements.

3 Research John Graunt (1620–1674) and his influence on the calculation of life expectancy.

REVIEW 13 MATHEMATICS AND HEALTH

Language and terminology

Here is a list of terms used in this chapter. Explain each term in a sentence.

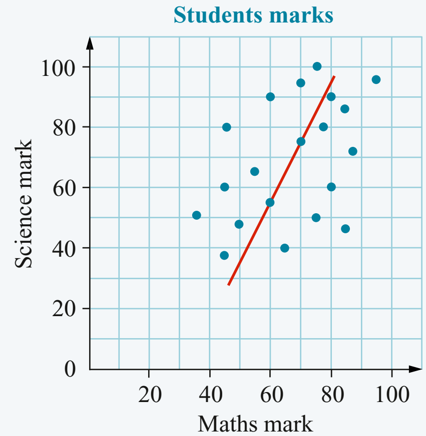
causality, concentration, correlation, correlation coefficient, dosage, dosage strength, drip rate, extrapolation, interpolation, least-squares line of best fit, life expectancy, line of fit, linear, medication, ordered pair, regression line, scatterplot, standard deviation, trendline, y-intercept

Having completed this chapter you should be able to:

- plot data on a scatterplot
- use a spreadsheet to calculate the correlation factor r
- add a least-squares line of best fit and find the equation
- convert units and rates
- calculate medical dosages
- interpret life expectancy data
- perform calculations related to life expectancy.

13 REVIEW TEST

This graph is a scatterplot, with a line of best fit, of the Mathematics and Science marks for a group of students. Use the graph to answer questions **1** and **2**.



1 The Science mark for a student who scores 70 in Mathematics is:

A 60 **B** 65
C 70 **D** 75

2 The Mathematics mark of a student who scores 55 in Science is:

A 50 **B** 55
C 60 **D** 65

Use the equation $H = 0.3 \times T + 4.5$ to answer questions **3** and **4**.

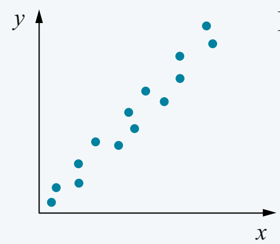
3 If $T = 12$, then $H =$

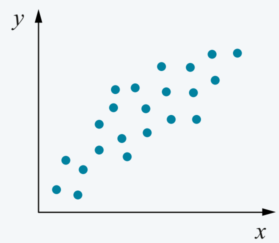
A 16.5 **B** 7.1 **C** 8.1 **D** 4.95

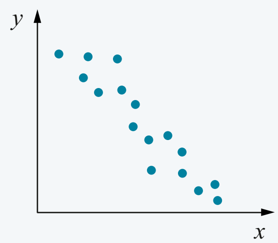
4 If $H = 6.9$ then $T =$

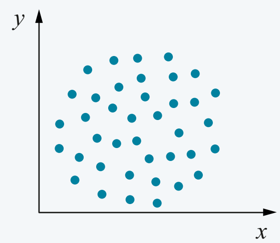
A 18.5 **B** 2.1 **C** 38 **D** 8

5 Which of the following scatterplots shows a high negative correlation?

A


B


C


D


FOCUS STUDY

6 There is a high degree of correlation between the lengths of the left and right feet of individuals. This is an example of:

A causality **B** spurious correlation **C** interpolation **D** extrapolation

7 The table shows the distance travelled in 1000 km versus the servicing costs in $1000 for a motor vehicle.

Distance ('000 km)	50	100	180	200	230	270	330	350	400
Cost ($'000)	3.2	4.1	4.4	6	7.3	8.5	9.1	9.8	13.5

Given that $r = 0.957$, the equation of the least-squares line of best fit is:

A $y = 33.6x - 11.2$ **B** $y = 0.027x + 0.95$
C $y = 0.45x + 231$ **D** $y = 0.03x + 234$

8 Convert 0.08 g to mg.

A 8000 mg **B** 800 mg **C** 80 mg **D** 8 mg

9 A patient is prescribed 500 mg of a drug that is available as 40 mg in 5 mL. What is the amount of medication that should be given to the patient?

A 2.5 mL **B** 40 mL **C** 12.5 mL **D** 62.5 mL

10 The adult dose of a medication is 50 mL. Using Fried's formula below, what is the dosage for a 1-year-old child?

$$\text{Child dose} = \frac{\text{age (in months)} \times \text{adult dose}}{150}$$

A 4 mL **B** 0.3 mL **C** 3 mL **D** 0.4 mL

11 A patient is to receive 1.2 L of fluid over 8 h. What is the flow rate in mL/h?

A 9.6 mL/h **B** 15 mL/h **C** 150 mL/h **D** 6.6 mL/h

12 A patient is to receive 800 mL of liquid through an IV drip delivering 25 drops/min. If there are 16 drop/mL, how long will it take?

A 0.52 h **B** 8h 32 min **C** 12 h **D** 20 h 50 min

13 Using the life expectancy table in Exercise 13H question **4,** what is the difference in life expectancy between a 58-year-old male and a 58-year-old female?

A 3.45 years **B** 82.34 years **C** 85.79 years **D** 5.87 years

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–4	5, 6	7	8, 9	10–12	13
Section	A, B	C	D	F	G	H, I

13A REVIEW SET

1 If $L = 4.2w + 8.5$, determine these values.

a L when $w = 6.5$ **b** w when $L = 50.5$

2 **a** Draw a scatterplot for the following table of data.

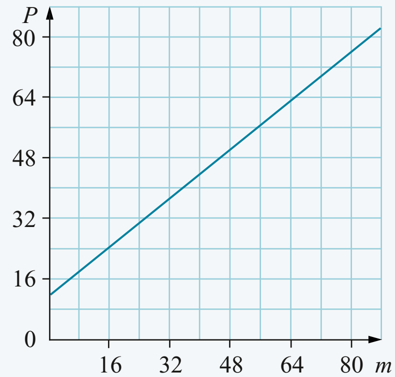
x	10	20	30	40	50	60	70	80
y	2.2	1.9	1.8	1.8	1.4	1.3	0.9	0.8

b Is the correlation between x and y:

i perfect, high or low? **ii** positive, negative or zero?

3 Use the graph on the right to find:

a P when $m = 80$ **b** m when $P = 50$.



4 Use Fried's formula to calculate the dosage for a 9-month-old child if the adult dose is 60 mL.

5 Use Young's formula to calculate the dosage for a $6\frac{1}{2}$-year-old child if the adult dose is 45 mL.

6 A patient is to receive 1.5 L of fluid over 6 h through an IV drip. If there are 12 drops/mL, how many drops per minute are required?

7 This table shows the life expectancy (number of years to live for a person at the current age) for males and females aged 39 to 55.

Current age (years)	39	40	41	42	43	44	45	46	47
Female	45.66	44.70	43.73	42.77	41.81	40.85	39.90	38.95	38.00
Male	41.66	40.71	39.77	38.83	37.89	36.96	36.03	35.10	34.18

Current age (years)	48	49	50	51	52	53	54	55
Female	37.05	36.11	35.17	34.24	33.31	32.38	31.45	30.53
Male	33.26	32.34	31.43	30.53	29.63	28.73	27.84	26.95

Use the data in the table to calculate the life expectancy of:

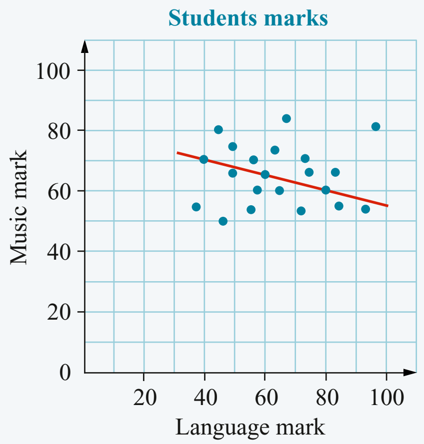
a a 46-year-old female **b** a 50-year-old male.

13B REVIEW SET

1 The graph shows a scatterplot with a line of best fit for the Language and Music test marks of a group of students. Use the line of fit to predict:

a the Music mark of a student who scores 60 in the Language test

b the Language mark of a student who scores 60 in the Music test.



2 If $w = 1.6y - 0.3$, find:

a w when $y = 12$ **b** y when $w = 14.1$

3 Sketch a scatterplot that shows:

a high positive correlation **b** low negative correlation.

4 Use Young's formula to calculate the dosage for a $7\frac{1}{2}$-year-old child if the adult dose is 25 mL.

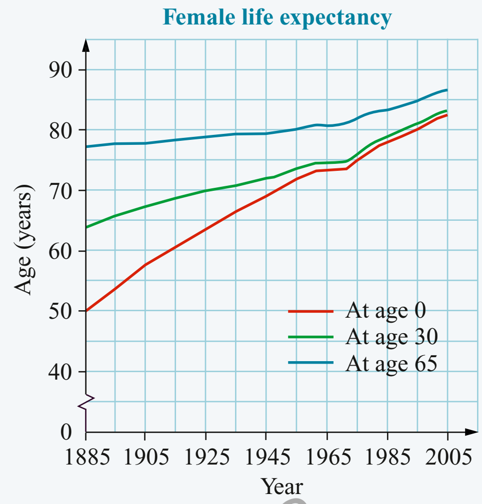
5 Use Clark's formula to calculate the dosage for a child weighing 15 kg. The adult dose is 30 mL.

6 A patient is to receive 600 mL of saline. An IV drip delivers 30 drops/min and there are 12 drops/mL. How long will it take?

7 The diagram shows the life expectancy of females from 1885 to 2005.

a What was the life expectancy at age 0 years in 1945?

b How much greater would the life expectancy of a 30-year-old be than that of a 0-year-old in 1945?



13C REVIEW SET

1 If $p = -1.2q - 1.5$, find:

a p when $q = 12$

b q when $p = -11.1$

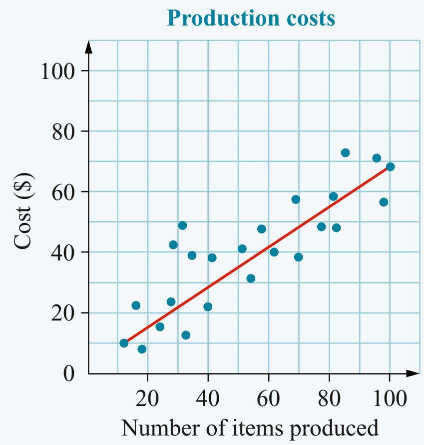
2 Draw a scatterplot for the data in this table.

x	5	10	15	20	25	30	35	40	45	50	55
y	0	8	21	28	43	51	60	69	82	94	102

3 Use this line of best fit to predict:

a the cost when the number of items produced is 80

b the number of items produced when the cost is \$35.



4 The following pairs of variables were measured and a high correlation found. State whether it is a cause and effect relationship or a case of spurious correlation.

a the number of umbrellas sold and the number of swimming costumes sold

b the number of storks nesting in chimneys and the birth rate

5 Use Clark's formula to calculate the dosage for a child weighing 38 kg, given the adult dose is 35 mL.

6 What is the drip rate per minute for 1.25 L of fluid over 8 h, with a drip size giving 14 drops/mL.

7 The table shows average life expectancy from age 0 to 20.

Current age	0	1	2	3	4	5	6	7	8	9	10
Female	83.67	83.04	82.07	81.08	80.10	79.11	78.11	77.12	76.13	75.14	74.14
Male	79.02	78.44	77.47	76.49	75.50	74.51	73.52	72.53	71.54	70.55	69.55

Current age	11	12	13	14	15	16	17	18	19	20
Female	73.15	72.15	71.16	70.16	69.17	68.19	67.20	66.22	65.24	64.25
Male	68.56	67.57	66.58	65.58	64.59	63.61	62.63	61.66	60.71	59.75

Calculate the life expectancy of:

a a 10-year-old male

b a 15-year-old female.

13D REVIEW SET

1 A transport company keeps a record of the annual maintenance costs of its fleet of semitrailers. The distance travelled, in thousands of kilometres, and cost, in thousands of dollars, is shown in the table below.

Distance travelled ('000 km)	50	100	180	200	230	270	330	350	400
Cost ('$000)	2.3	2.7	3.3	3.5	3.7	4.1	4.5	4.7	5.1

a Plot the data on a scatterplot.

b Comment on the correlation between the variables (high, low, positive, negative).

c Draw a line of best fit.

d Predict the annual cost for a semitrailer that has travelled 300 000 km.

e Using your answer in part **b**, comment on the accuracy of this prediction.

2 For the data in question 1 calculate the equation of the least-squares line of best fit, given that $r = 0.9995$.

3 Use Young's formula to calculate the dosage for a 10-year-old child. The adult dose is 20 mL.

4 A patient is to receive 1.1 L of fluid over 9 h.

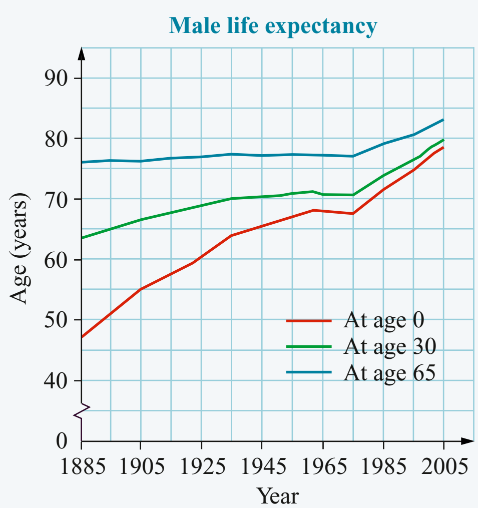
a What is the flow rate in mL/h?

b Find the drip rate in drops/min if there are 18 drops/mL.

5 The diagram shows the life expectancy of males from 1885 to 2005.

a When was there a big increase in life expectancy across all ages?

b Suggest a reason why the gap between 0 years and 30 years closed between 1885 and 1935?



13 EXAMINATION QUESTION (15 MARKS)

a **i** Draw a scatterplot for the data in this table. (2 marks)

T	0	20	40	60	80	100	120	140
H	38	35	43	54	55	68	72	73

ii Draw a line of best fit and estimate the value of T when $H = 50$. (1 mark)

iii The value of the correlation coefficient is 0.97. What is the meaning of a correlation coefficient of 0.97? (1 mark)

iv Calculate the gradient and y-intercept of the least-squares line of best fit. (2 marks)

b A patient is to receive 1.2 g of medication.

i Convert 1.2 g to mg. (1 mark)

ii The medication is available in tablets containing 300 mg. How many tablets should the patient take? (1 mark)

c Use Young's formula to calculate the dosage for a $4\frac{1}{2}$-year-old child, given the adult dose is 100 mL.

$$\text{Dosage for children 1 to 12 years} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}$$ (1 mark)

d A patient is to receive 1.8 L of fluid over 12 h.

i What is the required flow rate in mL/h? (2 marks)

ii If an IV drip is used with a drip size of 12 drops/mL, what drop rate is required? (2 marks)

e The life expectancy table for females and males aged 21 to 38 is shown.

i What is the life expectancy of a 26-year-old female? (1 mark)

ii What is the difference between the life expectancies of 30-year-old males and females? (1 mark)

Current age	Female	Male
21	63.27	58.80
22	62.29	57.84
23	61.31	56.88
24	60.32	55.93
25	59.34	54.97
26	58.36	54.02
27	57.38	53.06
28	56.40	52.11
29	55.42	51.16
30	54.44	50.20
31	53.46	49.25
32	52.48	48.30
33	51.50	47.35
34	50.52	46.40
35	49.55	45.45
36	48.58	44.50
37	47.60	43.55
38	46.63	42.60