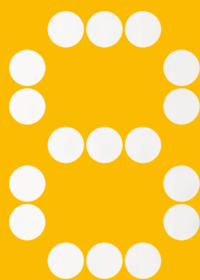


# MY MATHS



VICTORIAN CURRICULUM

AUTHORS: Jennifer Nolan / Melanie Koetsveld / Sonja Stambulic / Robert Bell

SAMPLE

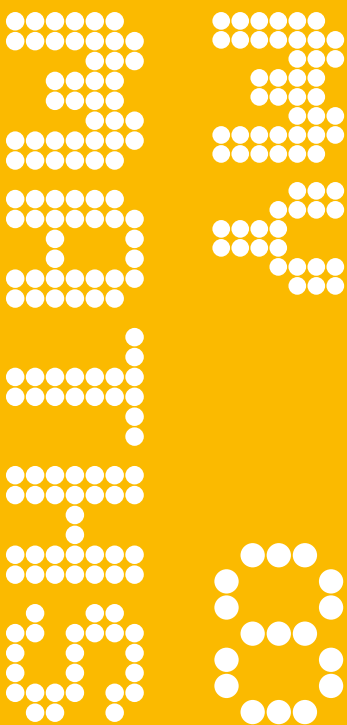
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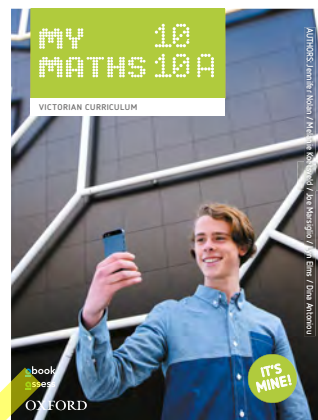
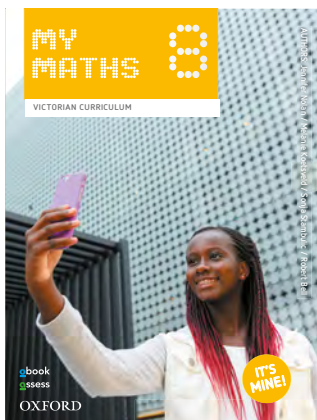
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# OXFORD MYMATHS VICTORIAN CURRICULUM



*Oxford MyMaths Victorian Curriculum* has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

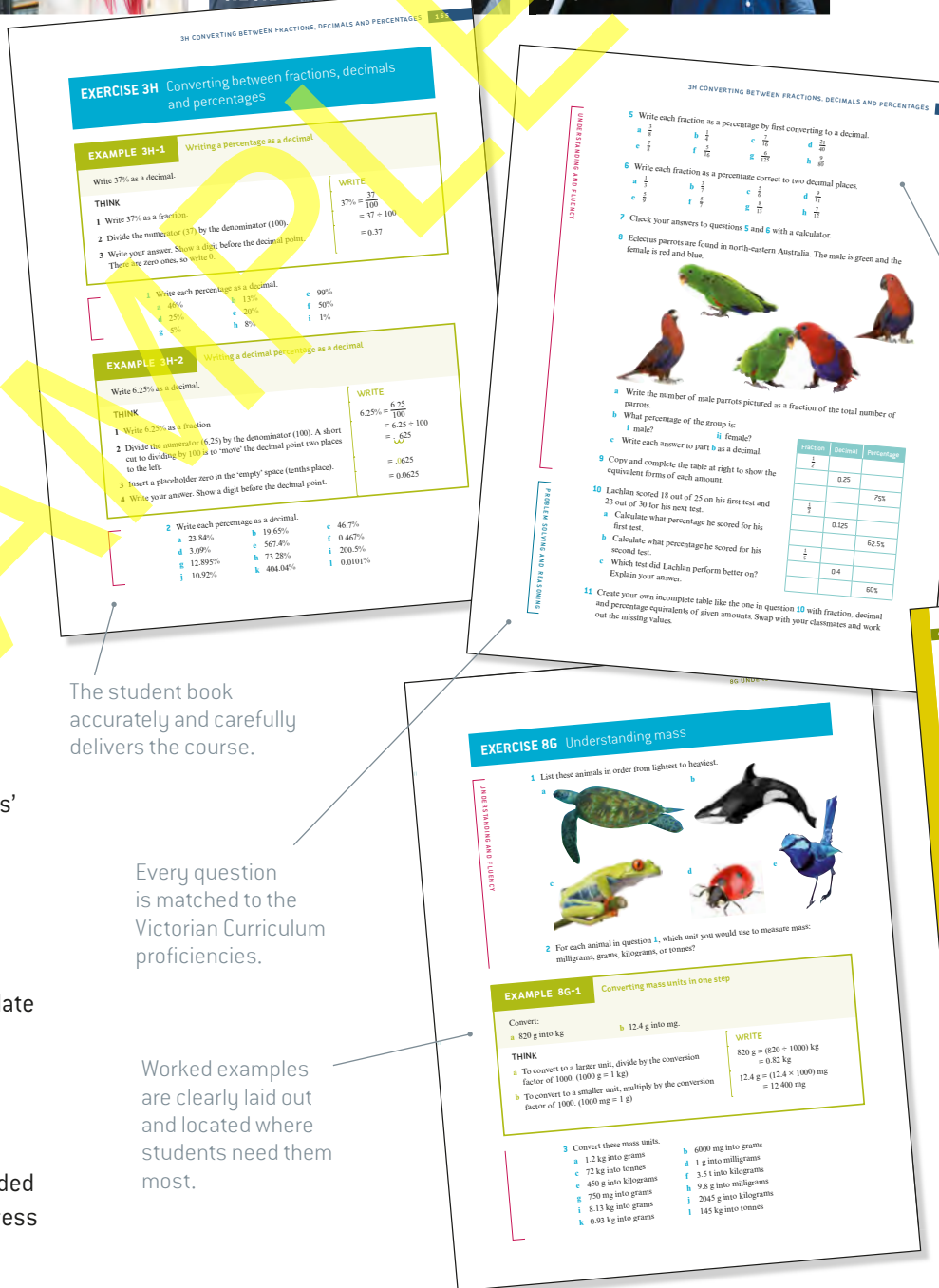
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- ▶ Integrated worked examples – right where your students need them
- ▶ Learning organised around the ‘big ideas’ of mathematics
- ▶ Discovery, practice, thinking and problem-solving activities to promote deep understanding
- ▶ A wealth of revision material to consolidate and prove learning
- ▶ Rich tasks to apply understanding
- ▶ Highly accessible and easy to navigate
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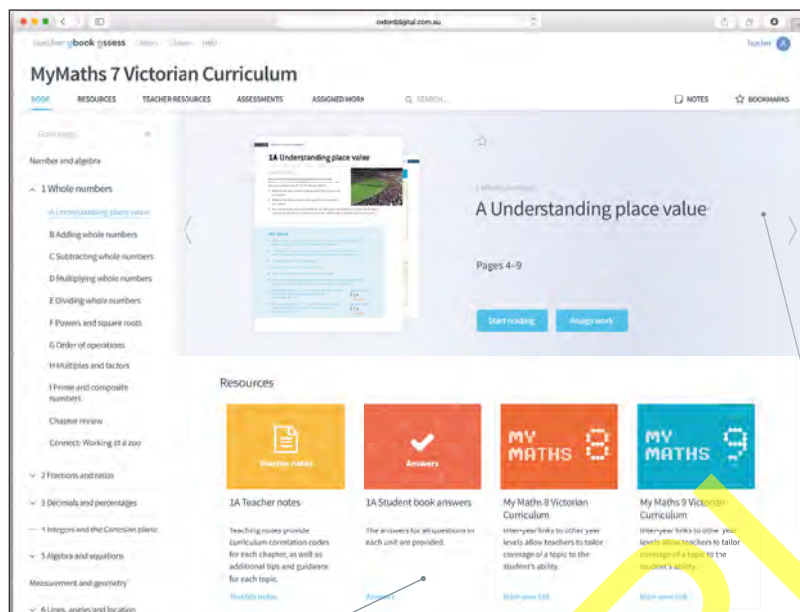
The student book accurately and carefully delivers the course.

Every question is matched to the Victorian Curriculum proficiencies.

Worked examples are clearly laid out and located where students need them most.



# YOUR 7-10 DIFFERENTIATION SOLUTION



## TEACHER QBOOK ASSESS

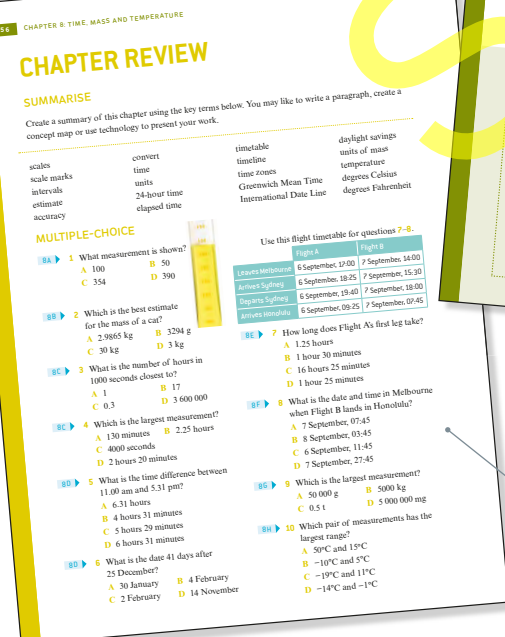
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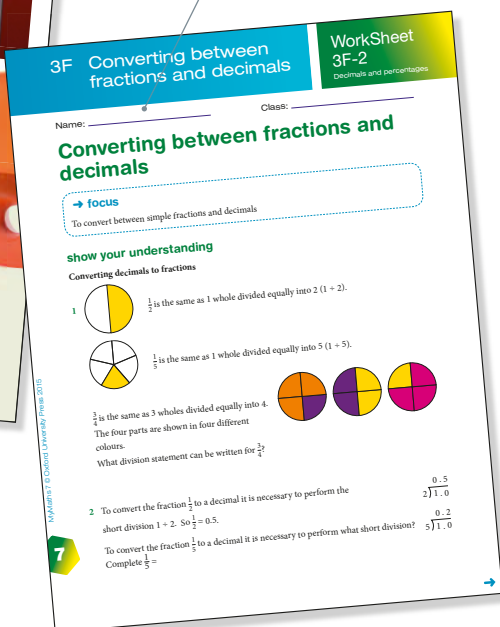
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# 1 NUMBER SKILLS

**1A** Estimating and rounding

**1B** Order of operations

**1C** Understanding fractions

**1D** Operations with fractions

**1E** Understanding decimals

**1F** Operations with decimals

**1G** Terminating, non-terminating and  
recurring decimals

**1H** Powers and roots

**1I** Index laws

## ESSENTIAL QUESTION

How does performing an operation on a number affect its size?



1A ➤ 1 a What is  $40 \times 1000$ ?

- A 40                      B 400  
C 4000                    D 40 000

b What is  $700\,000 \div 100$ ?

- A 70                      B 700  
C 7000                   D 70 000

1A ➤ 2 Calculate:

- a  $26 \times 100$               b  $51\,000 \div 10$   
c  $100\,000 \div 10\,000$

1A ➤ 3 a What is  $3000 \times 600$ ?

- A 18 000                B 180 000  
C 900 000              D 1 800 000

b What is  $8000 \div 40$ ?

- A 200                    B 760  
C 2000                   D 320 000

1B ➤ 4 a What is  $5^2$ ?

- A 2                      B 5                      C 10                    D 25

b What is  $9^2$ ?

1C ➤ 5 a What is the highest common factor of 45 and 72?

- A 3                      B 9                      C 18                    D 360

b What is the highest common factor of 24 and 30?

1C ➤ 6 a The fraction  $\frac{9}{4}$  is equivalent to which mixed number?

- A  $\frac{1}{4}$                     B  $2\frac{1}{4}$                     C  $2\frac{4}{9}$                     D  $2\frac{1}{2}$

b What is  $5\frac{3}{7}$  as an improper fraction?

1D ➤ 7 a What is the lowest common multiple of 6 and 8?

- A 6                      B 8                      C 24                    D 48

b What is the lowest common multiple of 4 and 10?

1D ➤ 8 a What is the lowest common denominator for  $\frac{2}{3} - \frac{1}{5}$ ?

- A 3                      B 5                      C 15                    D 35

b What is the lowest common denominator for  $\frac{1}{4} + \frac{5}{6}$ ?

1D ➤ 9 a What is  $\frac{1}{7} + \frac{2}{3}$ ?

- A  $\frac{17}{21}$                     B  $\frac{3}{10}$                     C  $\frac{2}{21}$                     D  $\frac{3}{21}$

b What is  $\frac{3}{5} - \frac{1}{4}$ ?

1E ➤ 10 a What is the value of 8 in 21.218 42?

- A 8 tenths                B 8 hundredths  
C 8 thousandths  
D 8 ten-thousandths

b What is the value of 3 in 510.347?

- A 3 hundreds            B 3 hundredths  
C 3 tens                   D 3 tenths

1E ➤ 11 a Which number is larger: 0.071 or 0.07?

b Which number is smaller: 4.3 or 4.03?

c What is the largest number in this list?  
0.105, 0.152, 0.251, 0.125, 0.05, 0.25

1F ➤ 12 a What is  $23.56 + 32.47 + 12.98$ ?

- A 56.03                    B 67.201  
C 67.81                    D 69.01

b What is  $157.62 - 78.57$ ?

- A 79.05                    B 79.15  
C 121.15                   D 236.19

1F ➤ 13 a What is  $3.546 \times 4$ ?

b What is  $15.42 \div 6$ ?

- A 9.42                    B 2.57                    C 2.20                    D 2.07

1H ➤ 14 a  $5^3$  is a number written in index form.

i What is the base?

ii What is the power?

b What is the expanded form of  $5^3$ ?

- A  $3 \times 3 \times 3 \times 3 \times 3$   
B  $5 + 5 + 5$             C  $5 \times 5 \times 5$   
D  $5 \times 3$



# 1A Estimating and rounding

## Start thinking!

The cost of running a school camp for one year is around \$350 000. It is expected 826 students will attend the camp in one year. Does \$385 seem a reasonable amount to charge each student to cover the cost?

You don't have a calculator and you really only need a rough estimate.

1 Is 826 closer to 800 or 900? How can you tell?

2 Is 385 closer to 300 or 400? How can you tell?

When you write a number as an approximate value with a first digit of 1, 2, 3 ... or 9 followed by zeros, it is called **rounding** to the **leading digit**.

3 Round 826 to its leading digit. (Hint: use your answer to question 1.)

4 Round 385 to its leading digit. (Hint: use your answer to question 2.)

5 What is important about the second digit in each number? How does it help you decide what the first digit of the rounded number should be?

6 Is the third digit important in deciding what the first digit should be in the rounded number? Explain.

7 Multiply the rounded numbers from questions 3 and 4 to find an estimate for  $826 \times 385$ .

Write your answer as  $826 \times 385 \approx \underline{\hspace{2cm}}$ . (The symbol  $\approx$  means 'approximately equals'.)

8 Is \$385 a reasonable amount to cover costs?



## KEY IDEAS

- ▶ To write an approximate value for a number, round the number to its first or leading digit. To start, look at the second digit in the number.
- ▶ If it is 0, 1, 2, 3 or 4, **keep** the first digit the same and replace the digits that follow with zero. For example,  $634 \approx 600$ . This is called rounding down.
- ▶ If the second digit is 5, 6, 7, 8 or 9, increase the first digit by one and replace the digits that follow with zero. For example,  $684 \approx 700$ . This is called rounding up.
- ▶ To estimate the result of a calculation, round each number to its leading digit before you perform any operations.
- ▶ You can also round to the nearest 10, 100, 1000, etc., using a similar method. For example:
  - ▷ rounded to the nearest 10,  $153 \approx 150$
  - ▷ rounded to the nearest 100,  $349 \approx 300$
  - ▷ rounded to the nearest 1000,  $4501 \approx 5000$

leading digit

6 3 4

second digit

leading digit

6 8 4

second digit

## EXERCISE 1A Estimating and rounding

1 Decide whether each number is closer to 400 or 500.

- a 438      b 477      c 462  
d 455      e 433      f 449

2 Decide whether each number is closer to 6000 or 7000.

- a 6789      b 6306      c 6010  
d 6505      e 6880      f 6448

### EXAMPLE 1A-1

#### Rounding a number to its leading digit

Round each number to its leading digit.

- a 26      b 739

#### THINK

- a Since the second digit is 6 (5 or more), 26 is closer to 30 than 20.  
Increase the leading digit by one and replace the other digit with zero.
- b Since the second digit is 3 (4 or less), 739 is closer to 700 than 800.  
Keep the leading digit the same and replace the other digits with zero.

#### WRITE

- a  $26 \approx 30$   
b  $739 \approx 700$

3 Round each number to its leading digit.

- a 77      b 42      c 81      d 347      e 160      f 555  
g 909      h 2489      i 6902      j 22 117      k 982      l 10 999

4 Round each number to its leading digit.

- a 94      b 99      c 952  
d 92 949      e 9008      f 960

5 Give three examples of numbers that round to these approximations if you round to the leading digit.

- a 40      b 700      c 3000  
d 50 000      e 800 000      f 2 000 000

6 Calculate:

- a  $900 + 700$       b  $300 - 10$       c  $30 \times 20$   
d  $80 \div 4$       e  $500 \times 40$       f  $800 + 3000$   
g  $200 \div 8$       h  $6000 - 500$       i  $40\,000 \div 20$   
j  $7000 \times 300$       k  $20\,000 - 9000$       l  $5000 + 10\,000$

**EXAMPLE 1A-2****Estimating the result of a calculation**

Estimate the result of each calculation by first rounding each number to its leading digit.

**a**  $1307 + 4875$

**b**  $576 \times 42$

**c**  $85\,229 \div 5$

**THINK**

- a**
- 1 Round each number to its leading digit.
  - 2 Perform the addition.
  - 3 Write your answer.
- b**
- 1 Round each number to its leading digit.
  - 2 It may be easier to rewrite the calculation.
  - 3 Perform the multiplication.
  - 4 Write your answer.
- c**
- 1 Round the first number. Single-digit numbers are not changed.
  - 2 Perform the division.
  - 3 Write your answer.

**WRITE**

- a**  $1307 + 4875$   
 $\approx 1000 + 5000$   
 $= 6000$   
 $1307 + 4875 \approx 6000$
- b**  $576 \times 42$   
 $\approx 600 \times 40$   
 $= 6 \times 100 \times 4 \times 10$   
 $= 24 \times 1000$   
 $= 24\,000$   
 $576 \times 42 \approx 24\,000$
- c**  $85\,229 \div 5$   
 $\approx 90\,000 \div 5$   
 $= 18\,000$   
 $85\,229 \div 5 \approx 18\,000$

- 7** Estimate the result of each calculation by first rounding each number to its leading digit.

**a**  $468 + 731$

**b**  $92 - 38$

**c**  $27 \times 49$

**d**  $83 \div 2$

**e**  $582 \times 17$

**f**  $245 + 6379$

**g**  $4512 \div 43$

**h**  $137 - 51$

**i**  $3694 \div 442$

**j**  $187 \times 9364$

**k**  $7085 - 750$

**l**  $964 + 5803$

**m**  $8277 \times 65\,234$

**n**  $13\,761 + 8036$

**o**  $94\,113 \div 587$

**p**  $24\,905 - 780$

- 8** Tickets to a concert are \$96. Estimate the cost of 32 tickets.
- 9** Tina thinks 9648 rounded to its leading digit is 10 000. Ruby isn't sure, as she thinks the rounded number should have the same number of digits as the original number. Who is correct? Explain.



**EXAMPLE 1A-3****Rounding a number to the nearest ten or hundred**

Round 2638 to the nearest:

- a** ten                      **b** hundred.

**THINK**

- a** Since the digit to the right of the tens digit is 8 (5 or more), 2638 is closer to 2640 than to 2630.
- b** Since the digit to the right of the hundreds digit is 3 (4 or less), 2638 is closer to 2600 than to 2700.

**WRITE**

- a**  $2638 \approx 2640$
- b**  $2638 \approx 2600$

- 10 a** Round each number to the nearest ten.

**i** 482      **ii** 6377      **iii** 56 026

**iv** 738 494      **v** 8075      **vi** 904 507

- b** Round each number in part **a** to the nearest hundred.

- 11 a** Round each number to the nearest thousand.

**i** 36 428      **ii** 7510      **iii** 183 915

**iv** 50 703      **v** 6052      **vi** 825

- b** Round each number in part **a** to the nearest hundred.

- 12** An office building has 35 782 panes of glass. Write an approximation to this number if you round to:

- a** the leading digit  
**b** the nearest hundred  
**c** the nearest thousand  
**d** the nearest ten  
**e** the nearest ten thousand.

- 13** Russia is the largest country in the world, with an area of 17 075 242 km<sup>2</sup>. Round this value to:
- a** its leading digit  
**b** the nearest hundred  
**c** the nearest thousand  
**d** the nearest ten thousand  
**e** the nearest hundred thousand  
**f** the nearest million.



- 14 Daniel is saving for a quad bike. He has saved \$60 per month for the past 28 months.

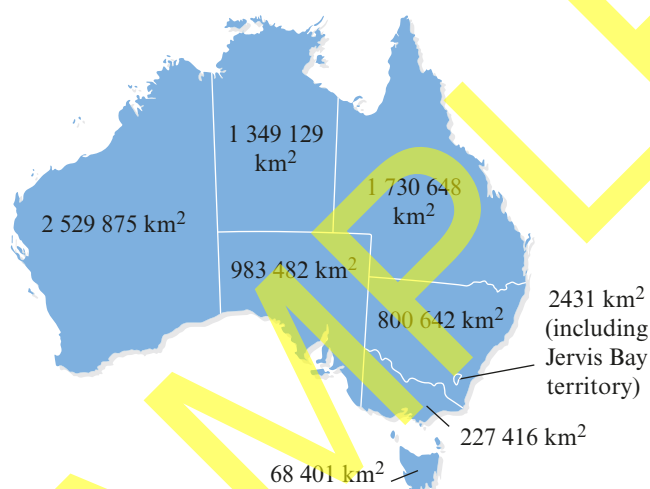


- a Write an approximation for the number of months by rounding the value to its leading digit.
  - b Using your answer to part a, estimate the amount of money Daniel has saved.
  - c Does he have enough money to buy the quad bike?
  - d If he doesn't have enough, estimate how many more months he needs to save.
- 15 Paving tiles cost \$7 each. Antonia needs 385 tiles for the back patio.
- a Write an approximation of the number of tiles she needs by rounding to the leading digit.
  - b Estimate the cost of the tiles, using your answer to part a.
  - c Compare your estimate to the exact cost of the tiles.
- 16 The school fundraising committee wants to raise money by buying boxes of sunscreen from a wholesaler to sell for profit. Each box has a wholesale price of \$47.
- a Estimate the number of boxes the committee could buy with \$2000.
  - b If the committee used this estimate, without performing the exact calculation decide whether there would be money left over or money owed to the wholesaler. Explain your answer.
- 17 Australia has an area of 7 692 024 km<sup>2</sup> and Indonesia has an area of 1 919 931 km<sup>2</sup>.
- a Round each value to its leading digit.
  - b Use the approximations to estimate:
    - i the difference in area between the two countries
    - ii the area of Australia compared to the area of Indonesia.
  - c Check how close your estimations are to the exact answers for part b.

- 18** People attending a State of Origin rugby league match entered the stadium through one of five gates. The number of people passing through each gate is shown.

Gate	Number of people
A	9 361
B	10 758
C	12 196
D	8 844
E	11 037

- Estimate the number of people attending the match by rounding the numbers at each gate to the leading digit.
  - Calculate the exact number of people attending the match.
  - Which answer would a sports commentator be more likely to use when reporting the match?
- 19** Australia is the sixth largest country in the world. The area of the states and territories (including islands) of Australia is shown on the map.



- Round each area to its leading digit.
  - Estimate the area of Australia by adding the approximate values from part **a**.
  - The area of NSW, rounded to the nearest thousand, is 801 000 km<sup>2</sup>. Round each of the other areas to the nearest thousand.
  - Estimate the area of Australia by adding the approximate values from part **c**.
  - Compare the estimates you obtained in parts **b** and **d** with the accepted value for the area of Australia of 7 692 024 km<sup>2</sup>.
- 20** Approximately 600 000 bacteria fit on the head of a pin of diameter 2 mm. Write five different numbers that would round to 600 000.
- 21** Write three examples of numbers that would:
- round to 5700 when rounded to the nearest hundred
  - round to 836 000 when rounded to the nearest thousand.

**Reflect**

Why is rounding useful in estimating the result of a calculation?



# 1B Order of operations

## Start thinking!

Students in a Year 8 class were asked to calculate  $130 + 70 \times 5 - 10^2$  without using a calculator.

1 How many operations need to be performed? List them.

Hanna, Josh and Imogen produced the workings shown at right.

2 Explain how each student produced a different answer.

3 Who do you think is correct?

There is a correct order to follow when you perform a number of operations in one problem. This is the **order of operations**.

Order	Operations
First	<b>B</b> rackets (operations inside grouping symbols)
Second	<b>I</b> ndices (powers and roots)
Third	<b>D</b> ivision and <b>M</b> ultiplication
Fourth	<b>A</b> ddition and <b>S</b> ubtraction

Some people use **BODMAS**, which has 'O' for 'power of' in place of 'I' for 'indices'.

4 To remember this, think **BIDMAS**.

Can you see why?

5 Follow the order of operations to calculate  $130 + 70 \times 5 - 10^2$ .

Which student produced the correct answer?

Hanna

$$\begin{aligned}
 &130 + 70 \times 5 - 10^2 \\
 &= 200 \times 5 - 10^2 \\
 &= 200 \times 5 - 100 \\
 &= 1000 - 100 \\
 &= 900
 \end{aligned}$$

Josh

$$\begin{aligned}
 &130 + 70 \times 5 - 10^2 \\
 &= 200 \times 5 - 10^2 \\
 &= 1000 - 10^2 \\
 &= 990^2 \\
 &= 980\,100
 \end{aligned}$$

Imogen

$$\begin{aligned}
 &130 + 70 \times 5 - 10^2 \\
 &= 130 + 70 \times 5 - 100 \\
 &= 130 + 350 - 100 \\
 &= 480 - 100 \\
 &= 380
 \end{aligned}$$

## KEY IDEAS

- ▶ The order of operations is:
  - first: **B**rackets (operations inside grouping symbols)
  - second: **I**ndices (powers and roots)
  - third: **D**ivision and **M**ultiplication (work from left to right)
  - fourth: **A**ddition and **S**ubtraction (work from left to right)
- ▶ Operations of the same ranking are performed in order from left to right.
- ▶ Where there is more than one set of grouping symbols, calculate the operations in the innermost brackets first.

Remember  
**BIDMAS**  
(or **BODMAS**).

## EXERCISE 1B Order of operations

### EXAMPLE 1B-1

#### Listing the order of operations

List in order the operations to be performed for  $7 + 3^2 \times (6 - 2)$ .

#### THINK

$6 - 2$  is performed first (brackets), then  $3^2$  (indices), then the multiplication and then the addition.

**NOTE** You may find it easier to think of BIDMAS.

#### WRITE

Order is: subtraction within brackets, squaring, multiplication, addition

1 For each calculation, list the order of operations.

a  $28 - 3 \times 7 + 11$

b  $50 + (34 - 19) \div 5$

c  $12 + 3 \times 4^2 - 41$

d  $20 - 2 \times 6 + 3^2$

e  $8 + [9 - (3 + 2)]$

f  $24 \div 6 + (5 + 1)^2$

### EXAMPLE 1B-2

#### Using order of operations

Calculate  $5^2 \times 3 - 36 \div 4$ .

#### THINK

- 1 Perform squaring first (indices).
- 2  $\times$  and  $\div$  come before  $-$  so, working from left to right, perform the multiplication first then the division.
- 3 Perform the subtraction.

#### WRITE

$$\begin{aligned}
 &5^2 \times 3 - 36 \div 4 \\
 &= 25 \times 3 - 36 \div 4 \\
 &= 75 - 36 \div 4 \\
 &= 75 - 9 \\
 &= 66
 \end{aligned}$$

2 Calculate:

a  $16 + 28 \div 4$

b  $60 - 2 \times 10$

c  $27 \div 3 + 6 \times 11$

d  $22 + 30 \times 2 \div 15$

e  $10 \times 8 - 7 \times 9$

f  $6^2 - 8 \times 3 + 35$

g  $5 \times 2^2 - 3^2 \times 2 + 8$

h  $7 + 7 \times 6 - 7^2$

i  $50 - 3 \times 12 + 4^2 \div 4$

j  $5^2 + 8 \times 3 - 2^2$

**EXAMPLE 1B-3****Using order of operations (with brackets)**

Calculate:

**a**  $9 \times (15 - 11) + 24 \div 3$       **b**  $42 + [18 \div (12 - 3)]^2 - 17$ .

**THINK**

- a** 1 Perform operations inside brackets first, so calculate  $15 - 11$ .
- 2 Perform  $\times$  and  $\div$  before  $+$ . Working from left to right, calculate  $9 \times 4$ .
- 3 Perform  $\div$  before  $+$  so calculate  $24 \div 3$ .
- 4 Perform the addition.
- b** 1 There are two sets of grouping symbols. Perform the operation in the innermost brackets first, so calculate  $12 - 3$ .
- 2 Perform the operation in the remaining set of brackets.
- 3 Perform the operation of squaring as 'indices' comes before  $+$  and  $-$ .
- 4 Working from left to right, perform the addition then the subtraction.

**WRITE**

**a**  $9 \times (15 - 11) + 24 \div 3$   
 $= 9 \times 4 + 24 \div 3$   
 $= 36 + 24 \div 3$   
 $= 36 + 8$   
 $= 44$

**b**  $42 + [18 \div (12 - 3)]^2 - 17$   
 $= 42 + [18 \div 9]^2 - 17$   
 $= 42 + 2^2 - 17$   
 $= 42 + 4 - 17$   
 $= 46 - 17$   
 $= 29$

**3** Calculate:

**a**  $72 \div (7 + 2) - 2 \times 3$

**c**  $9^2 + 7 \times (34 - 28)$

**e**  $60 \div [6 + (3 \times 5 - 1)] + 48$

**b**  $(27 - 14) \times (19 + 11)$

**d**  $10 \times 8 - 4 \times (3^2 + 2)$

**f**  $12 + [22 \div (37 - 26)]^2 - 9$

**4** Perform the calculations in question 1 using the order of operation rules.

**5** Repeat question 4 using a calculator. Enter each number, each operation and any grouping symbols from left to right. Does your calculator perform the operations in the correct order?

**6** The **average** of a set of scores is the total sum of the scores divided by the number of scores. For example, the average of 25, 30 and 23 is  $(25 + 30 + 23) \div 3$  or  $\frac{25 + 30 + 23}{3}$ .

**a** Use the order of operations to find the average of 25, 30 and 23.**b** Find the average of each set of numbers.

**i** 18, 21, 19, 25, 24, 25

**ii** 344, 363, 351, 346

**iii** 71, 69, 75, 78, 67

**iv** 2, 4, 7, 0, 1, 5, 3, 2

**v** 3, 2, 1, 1, 2, 1, 4

**vi** 28, 35, 49, 36, 22

**vii** 1052, 2711, 1949

**viii** 17, 15, 0, 9, 12, 13

**ix** 31, 35, 37, 36, 34, 31



- 7 a** The **commutative law** relates to the order of operations. Calculate:
- i**  $3 + 6$                       **ii**  $6 + 3$ .
- b** Does the order matter when you add two numbers? Try two more examples.  
Your answer to part **b** confirms that addition obeys the commutative law.
- c** Does subtraction obey the commutative law? Try these calculations.
- i**  $6 - 3$                       **ii**  $3 - 6$
- d** Does multiplication obey the commutative law? Try these.
- i**  $6 \times 3$                       **ii**  $3 \times 6$
- e** Does division obey the commutative law? Try these.
- i**  $6 \div 3$                       **ii**  $3 \div 6$
- 8 a** The **associative law** also relates to the order of operations. Calculate:
- i**  $(5 + 6) + 7$               **ii**  $5 + (6 + 7)$               **iii**  $(5 + 7) + 6$
- b** Does it matter in which order you add three numbers? Try two more examples.  
Your answer to part **b** confirms that addition obeys the associative law.
- c** Do subtraction, multiplication and division obey the associative law? Try some examples to help you decide.
- 9** The commutative and associative laws can make mental calculations easier.
- a** To calculate  $54 + 118 + 16$ , you can add any two of the numbers together first.
- i** Which two numbers are easiest to add together first? Explain.
- ii** Perform the calculation.
- b** To calculate  $87 \times 25 \times 4$ , you can multiply any two of the numbers together first.
- i** Which two numbers are easiest to multiply together first? Explain.
- ii** Perform the calculation.
- c** Explain how these laws can make some calculations easier to perform.

**EXAMPLE 1B-4****Using mental strategies**

Use mental strategies to calculate:

**a**  $37 + 14 + 6$

**b**  $20 \times 63 \times 5$ .

**THINK**

- a** Decide which two numbers to add together first.  
14 and 6 are best as the result of 20 is easy to add to 37.
- b** Decide which two numbers to multiply together first.  
20 and 5 are best as the result of 100 is easy to multiply by 63.

**WRITE**

**a**  $37 + 14 + 6$   
 $= 37 + (14 + 6)$   
 $= 37 + 20$   
 $= 57$

**b**  $20 \times 63 \times 5$   
 $= (20 \times 5) \times 63$   
 $= 100 \times 63$   
 $= 6300$

**10** Use mental strategies to calculate:

**a**  $17 + 28 + 13$

**b**  $219 + 54 + 16$

**c**  $39 + 74 + 21$

**d**  $166 + 88 + 12$

**e**  $4 \times 56 \times 25$

**f**  $921 \times 5 \times 20$

**g**  $35 \times 4 \times 5$

**h**  $29 \times 250 \times 4$

**i**  $200 \times 186 \times 5$

**11 a** Calculate these, then compare your answers.

**i**  $4 \times (5 + 6)$

**ii**  $4 \times 5 + 4 \times 6$

**b** Calculate these, then compare your answers.

**i**  $6 \times (10 - 2)$

**ii**  $6 \times 10 - 6 \times 2$

**c** Parts **a** and **b** are examples of the **distributive law** at work. Describe this law.

**d** Show how the distributive law works by providing two more examples.

**12** The distributive law can be used to make some multiplication calculations easier to perform.

**a** The calculation  $16 \times 24$  can be written as  $16 \times (20 + 4)$  or  $16 \times 20 + 16 \times 4$ .

**i** Calculate  $16 \times 20$ .

**ii** Calculate  $16 \times 4$ .

**iii** Add your results and write the answer to  $16 \times 24$ .

**b** The calculation  $45 \times 998$  can be written as  $45 \times (1000 - 2)$  or  $45 \times 1000 - 45 \times 2$ .

**i** Calculate  $45 \times 1000$ .

**ii** Calculate  $45 \times 2$ .

**iii** Subtract your results and write the answer to  $45 \times 998$ .

**c** Explain how the distributive law can be used to make some multiplication calculations easier to perform.

### EXAMPLE 1B-5

#### Using the distributive law

Use the distributive law to calculate:

**a**  $25 \times 41$

**b**  $32 \times 97$ .

#### THINK

**a 1** Write 41 as the sum of 40 and 1 as these numbers will be easier to multiply.

**2** Use the distributive law to write the calculation as the sum of two products.

**3** Calculate the result.

**NOTE**  $25 \times 40$  could be worked out as  $25 \times 4 \times 10$ .

**b 1** Write 97 as the difference of 100 and 3 as these numbers will be easier to multiply.

**2** Use the distributive law to write the calculation as the difference of two products.

**3** Calculate the result.

#### WRITE

$$\begin{aligned} \mathbf{a} \quad 25 \times 41 &= 25 \times (40 + 1) \\ &= 25 \times 40 + 25 \times 1 \\ &= 1000 + 25 \\ &= 1025 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad 32 \times 97 &= 32 \times (100 - 3) \\ &= 32 \times 100 - 32 \times 3 \\ &= 3200 - 96 \\ &= 3104 \end{aligned}$$

- 13** Use the distributive law to calculate:

**a**  $21 \times 42$       **b**  $54 \times 102$       **c**  $32 \times 203$       **d**  $14 \times 305$   
**e**  $17 \times 98$       **f**  $83 \times 99$       **g**  $24 \times 95$       **h**  $41 \times 19$   
**i**  $62 \times 101$       **j**  $29 \times 1001$       **k**  $11 \times 97$       **l**  $15 \times 999$

- 14** Scoring in basketball depends on where a player shoots from. A successful shot from on or inside the three-point line scores two points; from outside the three-point line, it scores three points. Each successful free shot from the foul line scores one point.

The number of successful shots for two teams during a match is shown in the table.

Number of successful shots	Titans	Giants
inside the three-point line	32	35
outside the three-point line	7	5
from a free throw	4	2

- a** Copy and complete these statements to show the numbers and operations needed to calculate the total points scored by each team.  
 points scored by Titans =  $\_\_\_ \times 2 + \_\_\_ \times 3 + \_\_\_ \times 1$   
 points scored by Giants =  $\_\_\_ \times \_\_\_ + \_\_\_ \times \_\_\_ + \_\_\_ \times \_\_\_$
- b** Calculate the number of points each team scored.
- c** Which team won and by how much?

- 15** You have \$195 to buy three shirts priced at \$58 each.

- a** Write a calculation to show how to work out how much money you will have left over.
- b** Estimate how much money is left over by rounding each number to its leading digit then performing the calculation.
- c** Perform the calculation in part **a** and compare your estimate to the exact amount of money you have left over.

- 16** Using the images on the right, calculate the cost of buying lunch for the class: 14 salad rolls, 11 samosa packs, 9 flavoured milks, 16 orange juices and 25 apples. Show the numbers and operations you used in the calculation.

- 17** Using 4 as many times as you like and with any of the operations (+, −, ×, ÷), write a calculation to obtain every number from 1 to 10. For example,  $4 \div 4 = 1$ ,  $(4 + 4) \div 4 = 2$  and so on. You can also square numbers and use brackets. Remember to follow the correct order of operations.

- 18** Repeat question **17** using:

**a** 3      **b** 7.



### Reflect

How can you remember the order of operations?

# 1C Understanding fractions

## Start thinking!

Paolo has to share this cake with his two sisters.

- 1 Draw a rectangle to represent the cake and divide it into three equal sections. Shade a section to show Paolo's share.
- 2 What fraction of the cake is Paolo's share?
- 3 Visitors have arrived, so now the whole cake has to be divided into 15 slices. Draw a diagram to show this.
- 4 Paolo still wants to eat his original share. How many slices is this now? Shade Paolo's share on your second rectangle.
- 5 Write a fraction for Paolo's share of the cake after it has been cut into 15 slices. (Hint: the denominator will be 15.)
- 6 Do the fractions for questions 2 and 5 represent the same amount of cake? Explain.
- 7 These fractions are called **equivalent fractions**. Write an equivalent fraction for Paolo's share, if the cake is divided into:
  - a 12 slices
  - b 18 slices.
- 8 The simplest fraction for Paolo's share was  $\frac{1}{3}$ . Look at the **numerator** and **denominator** of the equivalent fraction in question 7a and compare this to the numerator and denominator of  $\frac{1}{3}$ . What relationship can you see?
- 9 Repeat question 8 for the equivalent fraction in question 7b.



## KEY IDEAS

- ▶ Equivalent fractions have the same numerical value.
- ▶ An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.
- ▶ Dividing the numerator and the denominator of a fraction by the **highest common factor (HCF)** produces a fraction in its simplest form. This is called simplifying or cancelling.
- ▶ Mixed numbers can be simplified by keeping the whole number the same and simplifying the fraction component.

$$\frac{2}{3} = \frac{10}{15}$$

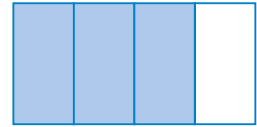
$\times 5$  (above the arrow from 2 to 10)  
 $\times 5$  (below the arrow from 3 to 15)

$$\frac{24}{54} = \frac{4}{9}$$

$\div 6$  (above the arrow from 24 to 4)  
 $\div 6$  (below the arrow from 54 to 9)

## EXERCISE 1C Understanding fractions

- 1 a What fraction of the rectangle has been shaded?
- b Draw a rectangle that is divided into 12 equal sections. Shade the rectangle so that it represents an equivalent fraction to that in part a.
- c Write the equivalent fraction.



### EXAMPLE 1C-1

#### Writing equivalent fractions

Fill the gap to produce equivalent fractions.

$$\frac{3}{7} = \frac{18}{\quad}$$

#### THINK

- 1 Look for a relationship between the two numerators.  $3 \times 6 = 18$ .
- 2 Use the same relationship (multiply by 6) to obtain the denominator of the second fraction.

$$\begin{array}{c} \times 6 \\ \frac{3}{7} = \frac{18}{\quad} \\ \times 6 \\ \frac{3}{7} = \frac{18}{42} \\ \times 6 \end{array}$$

#### WRITE

$$\frac{3}{7} = \frac{18}{\quad}$$

$$\frac{3}{7} = \frac{18}{42}$$

- 2 Fill the gaps to produce equivalent fractions.

a  $\frac{3}{10} = \frac{6}{\quad}$

b  $\frac{2}{7} = \frac{\quad}{35}$

c  $\frac{18}{33} = \frac{\quad}{11}$

d  $\frac{16}{48} = \frac{2}{\quad}$

e  $\frac{4}{5} = \frac{\quad}{30}$

f  $\frac{8}{36} = \frac{2}{\quad}$

g  $1\frac{3}{8} = \frac{\quad}{40}$

h  $3\frac{3}{4} = \frac{75}{\quad}$

i  $\frac{1}{4} = \frac{2}{\quad} = \frac{8}{20} = \frac{20}{100}$

j  $\frac{3}{7} = \frac{15}{49} = \frac{\quad}{70} = \frac{300}{210}$

k  $1\frac{1}{5} = \frac{60}{10} = \frac{\quad}{250} = 1\frac{72}{45}$

l  $2\frac{1}{3} = \frac{14}{15} = \frac{49}{\quad} = 2\frac{3}{9} = \frac{\quad}{300}$

- 3 From this list, select which fractions are equivalent to:

a  $\frac{2}{5}$

b  $\frac{4}{3}$

$\frac{6}{10}, \frac{20}{15}, \frac{8}{20}, \frac{7}{10}, \frac{28}{21}, \frac{22}{55}, \frac{4}{10}, \frac{16}{9}, 1\frac{4}{12}, \frac{10}{25}, \frac{18}{45}, 2\frac{6}{15}$



**EXAMPLE 1C-2****Writing fractions in simplest form**

Write each fraction in simplest form.

**a**  $\frac{24}{40}$

**b**  $6\frac{25}{35}$

**THINK**

- a** 1 Find the highest common factor (HCF) of the numerator and the denominator. HCF = 8.
- 2 Divide both the numerator (24) and the denominator (40) by the HCF (8).
- 3 Write the answer.
- b** 1 Find the HCF of the numerator and the denominator. HCF = 5.
- 2 Leave the whole number as it is and divide both the numerator (25) and the denominator (35) by the HCF (5).
- 3 Write the answer.

**WRITE**

**a**  $\frac{24}{40}$

$$= \frac{24^3}{40^5}$$

$$= \frac{3}{5}$$

**b**  $6\frac{25}{35}$

$$= 6\frac{25^5}{35^7}$$

$$= 6\frac{5}{7}$$

**4** Write each fraction in simplest form.

**a**  $\frac{8}{10}$

**b**  $\frac{14}{21}$

**c**  $\frac{42}{54}$

**d**  $\frac{45}{72}$

**e**  $\frac{40}{12}$

**f**  $\frac{55}{30}$

**g**  $3\frac{24}{32}$

**h**  $5\frac{35}{63}$

**i**  $4\frac{9}{33}$

**j**  $1\frac{24}{42}$

**k**  $2\frac{15}{40}$

**l**  $7\frac{45}{54}$

**5 a** Write each fraction in simplest form.

**i**  $\frac{6}{15}$

**ii**  $\frac{33}{77}$

**iii**  $\frac{18}{45}$

**iv**  $\frac{30}{18}$

**v**  $\frac{32}{36}$

**vi**  $\frac{18}{42}$

**b** In part **a**, which are equivalent fractions?

**6** Why is a fraction that has been simplified also an equivalent fraction?

**7** Create three fractions that are equivalent to each fraction.

**a**  $\frac{3}{5}$

**b**  $\frac{7}{4}$

**c**  $\frac{5}{6}$

**d**  $\frac{8}{12}$

**8** Create three mixed numbers that are equivalent to each fraction.

**a**  $1\frac{2}{5}$

**b**  $3\frac{18}{24}$

**c**  $\frac{10}{7}$

**d**  $\frac{21}{9}$

9 Write five fractions that simplify to  $\frac{2}{3}$ .

10 Decide which fraction is larger in each pair.

a  $\frac{5}{9}, \frac{7}{9}$       b  $\frac{11}{4}, \frac{9}{4}$       c  $2\frac{3}{5}, \frac{12}{5}$

d  $\frac{43}{6}, 7\frac{5}{6}$       e  $3\frac{7}{11}, \frac{37}{11}$       f  $\frac{29}{2}, 8\frac{1}{2}$

g  $4\frac{2}{3}, \frac{16}{3}$       h  $\frac{55}{9}, 6\frac{2}{9}$       i  $3\frac{5}{7}, \frac{25}{7}$

11 Decide which fraction is larger in each pair.

(Hint: write equivalent fractions with the same denominator.)

a  $\frac{3}{8}, \frac{1}{2}$       b  $\frac{4}{7}, \frac{5}{14}$       c  $\frac{9}{10}, \frac{4}{5}$

d  $\frac{5}{12}, \frac{2}{3}$       e  $1\frac{5}{6}, 1\frac{19}{24}$       f  $\frac{17}{15}, \frac{7}{3}$

g  $\frac{2}{3}, \frac{3}{4}$       h  $\frac{5}{6}, \frac{4}{5}$       i  $\frac{11}{8}, \frac{5}{3}$

j  $2\frac{3}{4}, \frac{17}{6}$       k  $\frac{29}{3}, 9\frac{1}{2}$       l  $3\frac{5}{12}, \frac{25}{8}$

12 Write each list of fractions in **ascending order**.

a  $\frac{2}{5}, \frac{7}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$       b  $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$

c  $1\frac{1}{2}, \frac{2}{5}, 1\frac{9}{10}, \frac{9}{2}, \frac{3}{10}, \frac{9}{5}$       d  $\frac{2}{3}, 1\frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{4}{3}, 1\frac{1}{4}$

13 Write each list of fractions in **descending order**.

a  $\frac{13}{11}, \frac{5}{11}, \frac{19}{11}, \frac{9}{11}, \frac{3}{11}, 1\frac{5}{11}$       b  $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$

c  $1\frac{1}{2}, \frac{2}{5}, 1\frac{9}{10}, \frac{9}{2}, \frac{3}{10}, \frac{9}{5}$       d  $\frac{2}{3}, 1\frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{4}{3}, 1\frac{1}{4}$

### EXAMPLE 1C-3

#### Writing an amount as a fraction of a total amount

Write each amount as a fraction of the total amount in simplest form.

a 12 cm out of 28 cm

b 20 seconds out of 2 minutes

#### THINK

a 1 As each amount is in the same unit, write 12 out of 28 as a fraction.

2 Find the HCF (4) and simplify the fraction.

b 1 The amounts are not in the same unit so convert 2 minutes to seconds. It is usually easier to convert to the smaller unit.

2 Write 20 out of 120 as a fraction.

3 Find the HCF (20) and simplify the fraction.

#### WRITE

a  $\frac{12}{28}$

$= \frac{3}{7}$

b 2 minutes = 120 seconds

$\frac{20}{120}$

$= \frac{1}{6}$

**14** Write each amount as a fraction of the total amount in simplest form.

- |                                     |                                       |
|-------------------------------------|---------------------------------------|
| <b>a</b> 15 kg out of 25 kg         | <b>b</b> 80 m out of 360 m            |
| <b>c</b> \$35 out of \$120          | <b>d</b> 18 seconds out of 60 seconds |
| <b>e</b> 50c out of \$3             | <b>f</b> 14 days out of 5 weeks       |
| <b>g</b> 20 mm out of 7 cm          | <b>h</b> 300 g out of 2 kg            |
| <b>i</b> 28 seconds out of 1 minute | <b>j</b> 75c out of \$4.25            |
| <b>k</b> 20 cm out of 6 m           | <b>l</b> 16 minutes out of 2 hours    |

**15** Write three fractions that are:

- |  |  |
|--|--|
| <b>a</b> smaller than $\frac{3}{5}$                | <b>b</b> larger than $\frac{4}{7}$                 |
| <b>c</b> smaller than $2\frac{1}{2}$               | <b>d</b> larger than $3\frac{2}{3}$                |
| <b>e</b> between $\frac{1}{12}$ and $\frac{5}{6}$  | <b>f</b> between $\frac{2}{3}$ and $\frac{6}{5}$   |
| <b>g</b> between $1\frac{1}{4}$ and $2\frac{3}{8}$ | <b>h</b> between $4\frac{1}{2}$ and $4\frac{1}{7}$ |

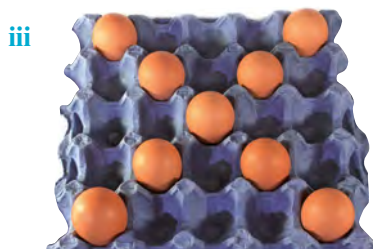
**16** Find the fraction that is halfway between each pair of fractions.

- |   |  |  |  |
|---|--|--|--|
| <b>a</b> $\frac{3}{7}$ and $\frac{5}{7}$  | <b>b</b> $\frac{5}{11}$ and $\frac{9}{11}$ | <b>c</b> $\frac{3}{8}$ and $\frac{5}{8}$   | <b>d</b> $\frac{1}{3}$ and $\frac{5}{9}$   |
| <b>e</b> $\frac{4}{15}$ and $\frac{2}{5}$ | <b>f</b> $\frac{1}{2}$ and $\frac{3}{4}$   | <b>g</b> $1\frac{1}{4}$ and $1\frac{1}{3}$ | <b>h</b> $2\frac{2}{3}$ and $3\frac{1}{5}$ |

**17** Match one of the fractions  $\frac{3}{4}$ ,  $\frac{2}{3}$ ,  $\frac{3}{5}$  or  $\frac{5}{8}$  to the amount shaded in each diagram.



**18 a** Write a fraction to represent the amount eaten from each item.



- b** For each fraction in part **a**, write three equivalent fractions.  
**c** For each item, write a fraction for the amount not eaten.  
**d** For each fraction in part **c**, write three equivalent fractions.

- 19 If you have read 56 out of 224 pages of a novel, what fraction have you read?
- 20 Brodie scored 25 out of 30 on the first Maths test and 21 out of 28 in the second. On which test did Brodie perform better?
- 21 *Guinness World Records* states that Stephen Taylor from the UK has the longest tongue of any person. His tongue measures about 10 cm from the tip to the middle of his closed top lip. Typically, the human tongue is about 4 cm long.
- Write a fraction to compare the typical tongue length to that of Stephen Taylor.
  - Measure or estimate the length of your tongue to the nearest centimetre.
  - Write a fraction, in simplest form, to compare the length of your tongue to the world record.

Gene Simmons (from Kiss) is also known for the length of his tongue, which measures about 8 cm.

- Write a fraction to compare the typical length of a human tongue to that of Gene Simmons.
- Write three different fractions using the information in this question and explain what each fraction represents.



- 22 Tony and Lisbeth obtained the same score for hitting a target with a ball. Tony hit the target four times in five shots. Lisbeth hit the target 12 times. Use equivalent fractions to work out how many shots Lisbeth had.

- 23 The number of faults out of the total number of serves was recorded for each player in a tennis match.

	Serena	Maria
total number of serves	119	102
number of serving faults	14	10

- Who had the better success rate in serving?
  - If Serena served at the same fault rate in her next match, how many faults did she make out of 85 serves?
  - If Maria served at the same fault rate in her next match and made 15 faults, what was her total number of serves?
- 24 What could the denominators of two different fractions be, if a common denominator is 36?
- 25 What could the denominators of three different fractions be, if a common denominator is 40?

### Reflect

How can you compare the size of fractions?

# 1D Operations with fractions

## Start thinking!

True or false: to multiply or divide two fractions, the denominators must be the same.

To answer this, first consider all four of the operations (addition, subtraction, multiplication and division) with fractions.

You may like to work with a partner.

- ☐ yes  
☐ no  
☒ maybe



- Copy and complete these four calculations involving  $\frac{3}{4}$  and  $\frac{2}{3}$ .
- Describe the method you used to add the two fractions. Do you need to write each fraction with the same denominator before adding? Explain.
- Repeat question 2 for:
  - subtraction
  - multiplication
  - division.
- Decide whether the statement at the start of this task is true or false. Provide a reason for your answer.

Addition	Subtraction	Multiplication	Division
$\frac{3}{4} + \frac{2}{3}$	$\frac{3}{4} - \frac{2}{3}$	$\frac{3}{4} \times \frac{2}{3}$	$\frac{3}{4} \div \frac{2}{3}$
$= \frac{12}{12} + \frac{8}{12}$	$= \frac{12}{12} - \frac{8}{12}$	$= \frac{3}{2} \times \frac{1}{1}$	$= \frac{3}{4} \times \frac{2}{2}$
$= \frac{20}{12}$	$= \frac{4}{12}$	$= \frac{3}{2}$	$= \frac{3}{8}$
$= 1\frac{5}{3}$			$= 1\frac{3}{8}$

## KEY IDEAS

- ▶ To add (or subtract) fractions that have different denominators:
  - find equivalent fractions with the **lowest common denominator (LCD)**
  - add (or subtract) the numerators.
- ▶ To multiply fractions:
  - look for any factors common to the numerator and denominator. Cancel a number in the numerator and a number in the denominator by dividing both numbers by the highest common factor (HCF).
  - multiply the numerators together and the denominators together.
- ▶ To divide fractions:
  - change from a division to a multiplication problem by replacing the division sign with a multiplication sign and turning the fraction that follows upside down
  - continue as for a multiplication problem.
- ▶ Convert mixed numbers into improper fractions before performing any calculation.
- ▶ If a result is an improper fraction, convert it to a mixed number.



## EXERCISE 1D Operations with fractions

### EXAMPLE 1D-1

#### Adding and subtracting fractions

Calculate:

**a**  $1\frac{3}{8} + \frac{5}{6}$

**b**  $2\frac{1}{4} - 1\frac{5}{12}$

#### THINK

- a** 1 Convert the mixed number to an improper fraction.
- 2 Check if the denominators are the same (no). Identify the LCD (24) and write equivalent fractions with a denominator of 24.
- 3 Add the numerators together. The denominator stays the same.
- 4 Convert the improper fraction to a mixed number.
- b** 1 Convert each mixed number to an improper fraction.
- 2 Check if the denominators are the same (no). Identify the LCD (12) and write equivalent fractions with a denominator of 12.
- 3 Subtract the numerators. The denominator stays the same.
- 4 Simplify the fraction. HCF is 2.

#### WRITE

**a**  $1\frac{3}{8} + \frac{5}{6}$   
 $= \frac{11}{8} + \frac{5}{6}$   
 $= \frac{33}{24} + \frac{20}{24}$

$$= \frac{53}{24}$$

$$= 2\frac{5}{24}$$

**b**  $2\frac{1}{4} - 1\frac{5}{12}$   
 $= \frac{9}{4} - \frac{17}{12}$   
 $= \frac{27}{12} - \frac{17}{12}$

$$= \frac{10}{12}$$

$$= \frac{5}{6}$$

1 Calculate:

**a**  $\frac{5}{17} + \frac{8}{17}$

**b**  $\frac{3}{4} - \frac{2}{5}$

**c**  $\frac{9}{10} + \frac{2}{3}$

**d**  $\frac{3}{8} - \frac{1}{12}$

**e**  $\frac{11}{15} - \frac{7}{10}$

**f**  $\frac{5}{6} + \frac{3}{4}$

**g**  $1\frac{1}{4} - \frac{7}{8}$

**h**  $2\frac{3}{5} + \frac{13}{20}$

**i**  $2\frac{2}{3} + 1\frac{4}{5}$

**j**  $3\frac{7}{11} - 1\frac{1}{2}$

**k**  $4\frac{3}{4} + 2\frac{2}{3}$

**l**  $5\frac{5}{6} - 3\frac{2}{15}$

2 Nine numbers are arranged in a square.

**a** Add the three numbers in:

- i** the first row    **ii** the second row    **iii** the third row.

**b** Add the three numbers in:

- i** the first column  
**ii** the second column  
**iii** the third column.

$\frac{4}{5}$	$\frac{1}{10}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{1}{2}$	$\frac{7}{10}$
$\frac{2}{5}$	$\frac{9}{10}$	$\frac{1}{5}$

- c** Add the three numbers in each diagonal.
- d** What do you notice about your answers to parts **a**, **b** and **c**?
- e** Your answer to part **d** is the reason why the nine numbers form a magic square. The sum in each row, column and diagonal is called the magic sum. What is the magic sum for this set of numbers?

- 3** Copy and complete each magic square. (Hint: first work out the magic sum.)

**a**

	$\frac{1}{6}$	$\frac{1}{4}$
	$\frac{1}{3}$	
	$\frac{1}{2}$	

**b**

		$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$

**c**

		$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	
$\frac{1}{3}$		

**EXAMPLE 1D-2****Multiplying and dividing fractions**

Calculate:

**a**  $\frac{8}{9} \times \frac{5}{6}$

**b**  $1\frac{7}{8} \div 2\frac{1}{4}$

**THINK**

- a 1** Look for any common factors between numerators and denominators. The numbers 8 and 6 have HCF 2 so cancel by dividing both numbers by 2 (shown in green).
- 2** Write the result obtained after cancelling.
- 3** Multiply the numerators together ( $4 \times 5 = 20$ ) and multiply the denominators together ( $9 \times 3 = 27$ ).
- b 1** Convert each mixed number to an improper fraction.
- 2** Change to a multiplication problem. Replace  $\div$  with  $\times$  and turn the fraction that follows upside down.
- 3** Look for any common factors between numerators and denominators. The numbers 15 and 9 have HCF 3 so cancel by dividing both numbers by 3 (shown in green). The numbers 8 and 4 have HCF 4, so cancel by dividing both numbers by 4 (shown in blue).
- 4** Write the result obtained after cancelling.
- 5** Multiply the numerators together ( $5 \times 1 = 5$ ) and multiply the denominators together ( $2 \times 3 = 6$ ).

**WRITE**

**a**  $\frac{8}{9} \times \frac{5}{6}$   
 $= \frac{4}{9} \times \frac{5}{3}$

$= \frac{4}{9} \times \frac{5}{3}$   
 $= \frac{20}{27}$

**b**  $1\frac{7}{8} \div 2\frac{1}{4}$   
 $= \frac{15}{8} \div \frac{9}{4}$   
 $= \frac{15}{8} \times \frac{4}{9}$

$= \frac{5}{2} \times \frac{1}{3}$

$= \frac{5}{6}$

4 Calculate:

a  $\frac{3}{8} \times \frac{2}{5}$

b  $\frac{9}{10} \times \frac{4}{3}$

c  $\frac{5}{7} \div \frac{3}{2}$

d  $\frac{3}{14} \div \frac{12}{35}$

e  $\frac{18}{25} \times \frac{35}{36}$

f  $\frac{9}{28} \div \frac{6}{7}$

g  $1\frac{7}{8} \times \frac{3}{5}$

h  $2\frac{3}{4} \div \frac{5}{12}$

i  $3\frac{1}{5} \times 1\frac{1}{4}$

j  $1\frac{5}{8} \div 2\frac{8}{9}$

k  $9\frac{1}{6} \times 1\frac{1}{11}$

l  $2\frac{1}{3} \div 4\frac{2}{3}$

5 Calculate:

a  $\frac{5}{6} - \frac{4}{9}$

b  $\frac{3}{8} + \frac{2}{5}$

c  $\frac{32}{49} \times \frac{21}{40}$

d  $\frac{15}{22} \div \frac{5}{33}$

e  $\frac{5}{18} \times 2\frac{7}{10}$

f  $3\frac{1}{6} + \frac{7}{8}$

g  $\frac{50}{63} \div 3\frac{4}{7}$

h  $4\frac{3}{5} - \frac{1}{2}$

i  $2\frac{4}{7} + 3\frac{1}{4}$

j  $5\frac{1}{3} \times 2\frac{2}{5}$

k  $3\frac{2}{11} - 1\frac{2}{3}$

l  $2\frac{1}{3} \div 1\frac{5}{9}$

6 Calculate:

a  $(\frac{6}{11} + \frac{2}{3}) \div \frac{5}{6}$

b  $3\frac{1}{4} - 2\frac{1}{2} \times \frac{7}{10}$

c  $\frac{8}{9} \times (\frac{4}{5} - \frac{3}{4}) \div 2\frac{2}{3}$

d  $2\frac{3}{5} \times 3\frac{1}{3} + \frac{2}{11} \times 1\frac{3}{8}$

Remember to think carefully about which operations are performed first.

**EXAMPLE 1D-3****Calculating a fraction of an amount**

Calculate each amount in the units shown in brackets.

a  $\frac{2}{3}$  of 27 m (m)

b  $\frac{1}{2}$  of 3 days (hours)

**THINK**

- a** 1 Replace 'of' with '×' and write the whole number as a fraction with a denominator of 1.
- 2 Cancel factors in the numerator and the denominator. HCF of 27 and 3 is 3.
- 3 Write the result after cancelling.
- 4 Complete the multiplication.
- 5 Write your answer.
- b** 1 Convert days to hours. ( $3 \times 24 = 72$ .)
- 2 Replace 'of' with '×' and write the whole number as a fraction with a denominator of 1.
- 3 Cancel any factors in the numerator and the denominator. HCF of 72 and 2 is 2.
- 4 Complete the multiplication.
- 5 Write your answer.

**WRITE**

a  $\frac{2}{3}$  of 27 m

$= \frac{2}{3} \times \frac{27}{1}$

$= \frac{2}{1} \times \frac{27}{3}$

$= \frac{2}{1} \times \frac{9}{1}$

$= \frac{18}{1}$

$= 18$

$\frac{2}{3}$  of 27 m = 18 m

b  $\frac{1}{2}$  of 3 days

$= \frac{1}{2}$  of 72 hours

$= \frac{1}{2} \times \frac{72}{1}$

$= \frac{1}{2} \times \frac{72}{1}$

$= \frac{36}{1}$

$= 36$

$\frac{1}{2}$  of 3 days = 36 hours

7 Calculate each amount in the units shown in brackets.

- |                                     |  |                                       |
|-------------------------------------|--|---------------------------------------|
| a $\frac{1}{2}$ of 14 hours (hours) | b $\frac{3}{4}$ of \$48 (dollars)      | c $\frac{2}{5}$ of 35 kg (kg)         |
| d $\frac{4}{9}$ of 72 cm (cm)       | e $\frac{2}{7}$ of 3 weeks (days)      | f $\frac{3}{5}$ of 2 cm (mm)          |
| g $\frac{3}{5}$ of 1 km (m)         | h $\frac{5}{6}$ of 4 minutes (seconds) | i $\frac{2}{3}$ of 6 L (mL)           |
| j $\frac{3}{8}$ of \$2 (cents)      | k $\frac{7}{10}$ of 5 kg (g)           | l $\frac{1}{12}$ of 8 hours (minutes) |

8 Craig wants to combine two open cartons of milk into one 2-L container. If one carton has  $\frac{1}{3}$  L and the other has  $1\frac{3}{4}$  L, will the 2-L container be large enough? Explain.

9 On Monday evening, Eleanor spent  $1\frac{1}{4}$  hours on the computer researching an assignment for school, Adam spent  $2\frac{1}{2}$  hours on the computer catching up with friends on Facebook, and their mother spent  $\frac{5}{6}$  of an hour typing up a report for work. How much time was spent on the computer Monday evening?

10 One-fifth of the student population at a local high school ride to school, one-tenth walk to school and two-thirds catch public transport. The remainder is driven to school.

What fraction of students:

- ride or walk to school?
- catch public transport, ride or walk to school?
- are driven to school?

11 The seating capacity of an interstate train carriage is 88 people. If the carriage is three-quarters full, how many people are seated in the train carriage?

12 John had a pizza home delivered and ate half of it before going to soccer training. Jen arrived home and ate half of what was left. Oscar then ate one-third of the remaining pizza. What fraction of the original pizza is now left over?

13 To bake 12 chocolate cupcakes, you need these ingredients.



#### Chocolate cupcakes

(makes 12)

100 g butter	1 teaspoon baking powder
$\frac{1}{2}$ cup sugar	$\frac{1}{2}$ cup milk
2 large eggs	$\frac{3}{4}$ cup cocoa powder
$1\frac{1}{2}$ cups flour	1 cup boiling water

#### Chocolate icing

$1\frac{1}{2}$ cups icing sugar	120 g chocolate
150 g butter	$\frac{1}{4}$ cup thickened cream

a Work out the quantities of each ingredient (excluding the icing) if you want to bake:

- 24 cupcakes
- 6 cupcakes
- 8 cupcakes.



- b** Finn bakes 12 cupcakes but before he can ice them, one-quarter are eaten. Adjust the ingredient list for the chocolate icing so it makes only enough for the remaining cupcakes.

- 14** Anna takes  $\frac{3}{4}$  hour to ride her bike from home to school.  
How many trips could she possibly make between home and school in  $2\frac{1}{4}$  hours?

- 15** Grapes are to be sold in  $2\frac{1}{2}$ -kg bags.  
**a** How many bags must Tricia pack if she has a 20-kg crate of grapes?  
**b** From another crate, Tricia packs 11 bags and has  $1\frac{1}{2}$  kg of grapes left over. How many kilograms of grapes were in the second crate?



- 16** Metal screws come in different sizes, with diameters often listed in inches.  
**a** Find the difference in size of two screws with diameters  $\frac{3}{50}$  inches and  $\frac{41}{250}$  inches.  
**b** Wes has drilled a hole in the wall of diameter  $\frac{3}{20}$  inches to hang a painting. Which of these two screws would be best for him to use?



- 17** Write three different addition problems that give an answer of  $\frac{11}{20}$ .  
**18** Write three different subtraction problems that give an answer of  $\frac{5}{6}$ .  
**19** Write three different multiplication problems that give an answer of  $\frac{3}{8}$ .  
**20** Write three different division problems that give an answer of  $\frac{7}{12}$ .  
**21** The answer to a fraction problem is  $2\frac{3}{4}$ .  
**a** Write a sample problem that uses each of these operations.  
**i** addition  
**ii** subtraction  
**iii** multiplication  
**iv** division  
**b** Write a sample problem that uses two or more of the operations.

### Reflect

What tips would you give a classmate to help them remember how to perform the four operations on fractions?



# 1E Understanding decimals

## Start thinking!

Results in a triple-jump competition are listed in metres as decimal numbers: 12.356, 11.8, 12.64, 11.704, 12.460 and 12.08.

- 1 The digit 3 is the first digit after the decimal point in 12.356. This is the first **decimal place**.
  - a What digit is in:
    - i the second decimal place?
    - ii the third decimal place?
  - b How many decimal places does 12.64 have?
  - c List the number of decimal places in each of the other measurements.
- 2 Which measurement is the largest? Which is the smallest? How can you tell? Does it depend on how many decimal places there are?
- 3 Look at the measurements containing zeros: 11.704, 12.460 and 12.08.
  - a What would happen if you wrote each number without its zero (for example, if you wrote 11.704 as 11.74 or 12.460 as 12.46)?
  - b Zeros between non-zero digits are called **placeholder zeros**. Why is it important that these zeros are never left out?
  - c Zeros at the end of a decimal are called **trailing zeros**. What difference does it make if these zeros are left out?
  - d Which measurements have:
    - i placeholder zeros?
    - ii trailing zeros?



## KEY IDEAS

- ▶ Zeros between non-zero digits in a decimal (called placeholder zeros) must never be left out, or the value of the number will be changed.
- ▶ Zeros at the end of a decimal (called trailing zeros) do not change the value of the number.
- ▶ The number of decimal places in a number is the number of digits after the decimal point.
- ▶ A decimal number can be rounded to a given number of decimal places by considering the next digit to the right.
  - ▷ If it is 0, 1, 2, 3 or 4, round down. For example, 1.263 rounded to two decimal places is 1.26.
  - ▷ If it is 5, 6, 7, 8 or 9, round up. For example, 1.268 rounded to two decimal places is 1.27.

## EXERCISE 1E Understanding decimals

- How many decimal places does each number have?
 

a 19.511	b 0.472 08	c 34.8002
d 6.492 177	e 0.005	f 90.7100
g 5893.2	h 802.713 204 15	i 7601.004
- Decide whether each decimal contains placeholder zeros (P), trailing zeros (T) or both (B).
 

a 4.705	b 29.300	c 605.9
d 1740.50	e 5.3720	f 88.302 000
- How many decimal places does each number in question 2 have?
- Use trailing zeros to write each number to five decimal places.  
For example,  $4.3 = 4.300\ 00$ .
 

a 5.46	b 76.3	c 0.306
d 29.7843	e 9.0	f 15

### EXAMPLE 1E-1

#### Ordering decimals

Write this list of numbers in ascending order: 5.74, 5.7, 5.47, 5.04, 5.4, 5.407.

#### THINK

- To make them easier to compare, write all the numbers with the same number of decimal places (three) by adding trailing zeros where necessary.
- Write the numbers in order from smallest to largest by comparing the place value of the digits, moving from left to right.

#### WRITE

5.740, 5.700, 5.470, 5.040, 5.400, 5.407

5.040, 5.400, 5.407, 5.470, 5.700, 5.740

In ascending order, the numbers are:

5.04, 5.4, 5.407, 5.47, 5.7, 5.74

- Write each list of numbers in ascending order.
 

a 0.214, 0.142, 0.2, 0.41, 0.104, 0.14
b 8.702, 9.87, 8.072, 9.782, 8.2, 8.97
- Write each list of numbers in descending order.
 

a 13.7, 13.07, 13.173, 13.31, 13.137, 13.713
b 0.658, 0.6058, 0.6508, 0.685, 0.6085, 0.65
- Explain why 9.46 is larger than 9.452 even though 9.452 has more digits.

**EXAMPLE 1E-2****Rounding decimals to two decimal places**

Round each number to two decimal places.

**a** 5.7381

**b** 12.054 76

**THINK**

- a** 1 Look at the digit in the third decimal place.  
Since it is 8, round up.
- 2 Write the number to two decimal places, with the digit in the second decimal place increased by one. The digits to the left of the second decimal place remain the same and the digits to the right are removed.
- b** 1 Look at the digit in the third decimal place.  
Since it is 4, round down.
- 2 Write the number to two decimal places, with the digit in the second decimal place remaining the same. The digits to the left of the second decimal place also remain the same and the digits to the right are removed.

**WRITE**

**a** 5.7381

$\approx 5.74$

**b** 12.054 76

$\approx 12.05$

**8** Round each number to two decimal places.

**a** 25.679

**b** 673.8234

**c** 0.784 231

**d** 101.0157

**e** 8148.255

**f** 54.2618

**g** 330.406

**h** 4.854 793

**i** 71.006 24

**9** Round each decimal to the number of decimal places shown in the brackets.

**a** 57.234 (2)

**b** 8.9137 (3)

**c** 92.087 12 (4)

**d** 0.756 825 (3)

**e** 135.152 79 (1)

**f** 65.071 226 33 (5)

**g** 7.510 429 (3)

**h** 208.009 862 (4)

**i** 1.727 727 727 (5)

**j** 845.845 845 (1)

**k** 0.123 456 78 (4)

**l** 0.123 456 78 (7)

**10** Wendy thinks 8.2735 rounded to three decimal places is 8.273.  
Rosa thinks it is 8.274. Who is correct? Explain.

**11** Jayden thinks that 24.7963 rounded to two decimal places is 24.80.  
Oscar thinks it is 24.71. Who is correct? Explain.

**12** Round each number to three decimal places.

**a** 6.8134

**b** 0.741 52

**c** 21.8306

**d** 4.0798

**e** 30.2995

**f** 15.9999

**EXAMPLE 1E-3****Using place value to write decimals as fractions**

Use your knowledge of place value to write each decimal as a fraction.

**a** 0.3**b** 0.09**THINK**

**a** Consider the place value of 3. The first decimal place shows tenths.

**b** Consider the place value of 9. The second decimal place shows hundredths.

**WRITE**

**a**  $0.3 = \frac{3}{10}$

**b**  $0.09 = \frac{9}{100}$

**13** Use your knowledge of place value to write each decimal as a fraction.

**a** 0.03**b** 0.9**c** 0.007**d** 0.0001**e** 0.7**f** 0.000 03**g** 0.000 009**h** 0.000 000 7

**14** Write each fraction as a decimal.

**a**  $\frac{7}{10}$

**b**  $\frac{29}{100}$

**c**  $\frac{451}{1000}$

**d**  $\frac{2047}{10\ 000}$

**e**  $\frac{4}{100}$

**f**  $\frac{9}{1000}$

**g**  $\frac{5}{10\ 000}$

**h**  $\frac{63}{1000}$

**i**  $\frac{19}{10\ 000}$

**j**  $\frac{368}{10\ 000}$

**k**  $\frac{20}{100}$

**l**  $\frac{70}{1000}$

**15** Copy and complete.

**a**  $0.27 = \frac{2}{10} + \frac{7}{100}$  or  $\frac{27}{100}$

**b**  $0.589 = \frac{5}{10} + \frac{8}{100} + \frac{9}{1000}$  or  $\frac{589}{1000}$

**c**  $0.343 = \frac{3}{10} + \frac{4}{100} + \frac{3}{1000}$  or  $\frac{343}{1000}$

**d**  $0.6251 = \frac{6}{10} + \frac{2}{100} + \frac{5}{1000} + \frac{1}{10\ 000}$  or  $\frac{6251}{10\ 000}$

**e**  $0.109 = \frac{1}{10} + \frac{9}{1000}$  or  $\frac{109}{1000}$

**f**  $0.0867 = \frac{8}{100} + \frac{6}{1000} + \frac{7}{10\ 000}$  or  $\frac{867}{10\ 000}$

**16** Copy this table and allow enough rows for nine decimal numbers.

Decimal number	Number of decimal places	Value of digit in:				Fractional form
		first decimal place	second decimal place	third decimal place	fourth decimal place	
0.473	3	$\frac{4}{10}$	$\frac{7}{100}$	$\frac{3}{1000}$		$\frac{473}{1000}$
0.8619	4	$\frac{8}{10}$	$\frac{6}{100}$	$\frac{1}{1000}$	$\frac{9}{10\ 000}$	$\frac{8619}{10\ 000}$

**a** Complete a row in the table for 0.357.

**b** How many decimal places does 0.357 have?

**c** How many zeros are in the denominator when 0.357 is written as a fraction?

**d** Repeat parts **a–c** for each of these numbers.

**i** 0.7253    **ii** 0.691    **iii** 0.49

**iv** 0.103    **v** 0.9    **vi** 0.007

**e** What is the relationship between the number of decimal places in a number and the number of zeros in the denominator of the matching fraction?

**EXAMPLE 1E-4****Writing decimals as fractions**

Write each number as a fraction in simplest form.

**a** 0.24

**b** 0.365

**THINK**

- a** 1 As 0.24 has two decimal places, the denominator is 100.  
 2 Simplify the fraction. HCF is 4.
- b** 1 As 0.365 has three decimal places, the denominator is 1000.  
 2 Simplify the fraction. HCF is 5.

**WRITE**

$$\begin{aligned}\mathbf{a} \quad 0.24 &= \frac{24}{100} \\ &= \frac{6}{25} \\ \mathbf{b} \quad 0.365 &= \frac{365}{1000} \\ &= \frac{73}{200}\end{aligned}$$

- 17** Write each number as a fraction in simplest form.

**a** 0.352

**b** 0.5724

**c** 0.803

**d** 0.04

**e** 0.095

**f** 0.0006

**g** 0.74

**h** 0.252

**i** 0.0015

**j** 0.0484

**k** 0.000 17

**l** 0.8002

- 18** Six students' heights were measured as 1.651 m, 1.63 m, 1.568 m, 1.615 m, 1.58 m and 1.657 m.

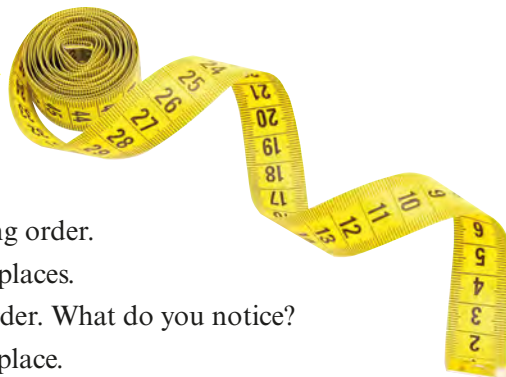
- a** Which is the smallest measurement?  
**b** Which is the largest measurement?  
**c** Write the list of measurements in ascending order.  
**d** Round each measurement to two decimal places.  
**e** Write your list from part **d** in ascending order. What do you notice?  
**f** Round each measurement to one decimal place.  
**g** Write your list from part **f** in ascending order. What do you notice?

- 19** The width of a human hair is measured to be 0.018 35 mm.  
 Write this value as an approximation to:

- a** two decimal places      **b** three decimal places      **c** four decimal places.

- 20** The diameter of an optical fibre is 0.2845 mm.

- a** Write this value as an approximation to:  
     **i** one decimal place  
     **ii** two decimal places  
     **iii** three decimal places.  
**b** Write the exact diameter as a fraction.





- 21** Another way to say that you have an approximate value after rounding is to say that a number is correct to a certain number of decimal places. For example, the length of my fingernail is 1.6 cm, correct to one decimal place.

- a** Find the length of one of your fingernails correct to one decimal place.
- b** List some reasons for not quoting the length to lots of decimal places.

- 22** The length of a mosquito is measured to be 0.68 cm. Write this value correct to one decimal place.



- 23** Write 2.734 829 954 correct to the following number of decimal places.

- |                |                |
|----------------|----------------|
| <b>a</b> three | <b>b</b> five  |
| <b>c</b> eight | <b>d</b> one   |
| <b>e</b> six   | <b>f</b> seven |

- 24** Round each amount to the nearest cent.

- |                   |                    |                    |
|-------------------|--------------------|--------------------|
| <b>a</b> \$3.437  | <b>b</b> \$28.084  | <b>c</b> \$117.206 |
| <b>d</b> \$59.115 | <b>e</b> \$62.7462 | <b>f</b> \$0.9191  |

- 25** Round each amount to the nearest five cents.

- |                  |                   |                  |
|------------------|-------------------|------------------|
| <b>a</b> \$42.37 | <b>b</b> \$207.62 | <b>c</b> \$7.24  |
| <b>d</b> \$15.03 | <b>e</b> \$60.71  | <b>f</b> \$99.99 |

- 26** Write three examples of numbers that, when rounded to two decimal places, equal 5.74.

- 27** Write three examples of numbers that, when rounded to one decimal place, equal 0.6.

- 28** Round 4708.612 95 to:

- a** the nearest thousand
- b** the nearest hundred
- c** the nearest ten
- d** the nearest one
- e** the nearest tenth
- f** the nearest hundredth
- g** the nearest thousandth
- h** the nearest ten thousandth.

### Reflect

Why are decimals often rounded to a certain number of decimal places?

# 1F Operations with decimals

## Start thinking!

Four friends share the cost of buying food for a holiday.  
Sasha writes a list.

- 1 What is the cost of six tins of soup?
- 2 Copy and complete the list by calculating each item's cost.
- 3 Sasha has \$100. Is this enough? Estimate the total cost by rounding the cost of each item to the nearest dollar.
- 4 Estimate how much money Sasha would expect to have left over.
- 5 Calculate the total cost.
- 6 How much does Sasha actually have left over?
- 7 The remaining money is shared between the four friends. How much does each person receive?
- 8 What is important to remember when:
  - a multiplying a decimal number by a whole number?
  - b adding or subtracting decimals?
  - c dividing a decimal number by a whole number?

Item	Unit cost	Amount	Cost of item
Tinned soup	\$3.95 each	6	
Packet of pasta	\$2.85 each	1	
Mince steak	\$13.65 per kg	0.8 kg	
Jar of pasta sauce	\$5.79 each	1	
Bread	\$3.45 each	1	
Sausages	\$8.98 per kg	0.5 kg	
Tomato sauce	\$4.70 each	1	
Oranges	\$0.85 each	4	
Apples	\$4.60 per kg	2 kg	
Box of cereal	\$5.99 each	1	
Milk	\$2.48 per L	2 L	
Tropical juice	\$6.75 each	1	

## KEY IDEAS

- ▶ To add or subtract decimals:
  - ▷ line up the decimal points, and digits with the same place value
  - ▷ include trailing zeros to fill blank spaces in columns to minimise mistakes
  - ▷ add or subtract digits in each column from right to left.
- ▶ To multiply two decimals:
  - ▷ multiply them as if they were whole numbers
  - ▷ the answer has the same number of decimal places as the total number of decimal places in the original problem.
- ▶ To divide a decimal by a whole number:
  - ▷ divide into the dividend from left to right
  - ▷ line up the decimal point and the digits with the same place value in the quotient with those in the dividend
  - ▷ add trailing zeros to the end of the decimal number until you can divide exactly, without a remainder.
- ▶ To divide a decimal by another decimal:
  - ▷ write an equivalent division problem with a divisor that is a whole number
  - ▷ continue as for dividing a decimal by a whole number.

**NOTE** Estimating an answer before performing the calculation helps you check your result.

$$\begin{array}{r}
 21.625 \leftarrow \text{quotient (the answer)} \\
 8 \overline{) 173.000} \\
 \text{divisor (the number we are dividing by)} \quad \text{dividend (the number being divided)}
 \end{array}$$

### Example

$$12.328 \div 0.04 = 1232.8 \div 4$$

## EXERCISE 1F Operations with decimals

UNDERSTANDING AND FLUENCY

- 1 Use vertical addition to calculate each result.
  - a  $23.82 + 17.13 + 124.06 + 82.95$
  - b  $5.613 + 9.27 + 11.3856 + 7.5$
  - c  $324.7 + 76.29 + 4.93 + 285.064$
  - d  $74.0106 + 2.93 + 28.738 + 0.9167$
- 2 Use vertical subtraction to calculate each result.
  - a  $158.29 - 75.46$
  - b  $2051.384 - 1265.73$
  - c  $0.35 - 0.2156$
  - d  $70.308 - 44.057$
  - e  $9.104 - 6.3728$
  - f  $265.7 - 88.504$
  - g  $5061.03 - 1266.735$
  - h  $73\,412 - 8743.56$
- 3 Use short multiplication to calculate each result.
  - a  $24.35 \times 8$
  - b  $305.6 \times 4$
  - c  $17\,352.7 \times 3$
  - d  $0.327\,51 \times 9$

### EXAMPLE 1F-1

#### Multiplying decimals using long multiplication

Use long multiplication to calculate  $24.756 \times 3.2$ .

#### THINK

- 1 Estimate the answer. Round each decimal to the nearest whole number.
- 2 Write the numbers under each other without the decimal points and use long multiplication.
- 3 Count the number of decimal places in the original calculation ( $3 + 1 = 4$ ). There will be the same number of decimal places in the answer (four).
- 4 Write the answer with the decimal point in the correct position. Check that it matches your estimate.

#### WRITE

Estimate is  $25 \times 3 = 75$ .

$$\begin{array}{r}
 \begin{array}{cccc}
 1 & 2 & 1 & 1 \\
 & 1 & 1 & 1 \\
 24 & 756 \\
 \times & 32 \\
 \hline
 49512 \\
 742680 \\
 \hline
 792192
 \end{array}
 \end{array}$$

$$24.756 \times 3.2 = 79.2192$$

- 4 Use long multiplication to calculate each result:
  - a  $8.27 \times 5.3$
  - b  $12.86 \times 2.4$
  - c  $235.2 \times 9.7$
  - d  $62.513 \times 1.8$
  - e  $9.2 \times 0.46$
  - f  $3.8927 \times 0.52$
  - g  $56.361 \times 0.14$
  - h  $157.93 \times 7.12$

**EXAMPLE 1F-2****Dividing a decimal by a whole number**

Use short division to calculate  $37.45 \div 4$ .

**THINK**

- 1 Estimate the answer. Round each number to its leading digit.
- 2 Divide into 37.45 from left to right. Use trailing zeros (if necessary) to obtain an exact decimal answer without a remainder. Remember to line up the decimal points and the digits that have the same place value.
- 3 Write your answer. Check that it matches your estimate.

**WRITE**

Estimate is  $40 \div 4 = 10$ .

$$\begin{array}{r} 9.3625 \\ 4 \overline{) 37.4510} \end{array}$$

$$37.45 \div 4 = 9.3625$$

- 5 Use short division to calculate each result.

a  $72.596 \div 4$

b  $6.724 \div 5$

c  $58.2743 \div 7$

d  $65.0955 \div 3$

e  $4.73 \div 8$

f  $0.627 \div 4$

g  $0.1536 \div 6$

h  $138.0714 \div 5$

**EXAMPLE 1F-3****Converting a fraction to a decimal using short division**

Write  $\frac{3}{4}$  as a decimal.

**THINK**

- 1 Write  $\frac{3}{4}$  as a division problem.
- 2 Use short division to divide 3 by 4. Include two trailing zeros to obtain an exact decimal answer without a remainder.
- 3 Write your answer.

**WRITE**

$$\frac{3}{4} = 3 \div 4$$

$$\begin{array}{r} 0.75 \\ 4 \overline{) 3.00} \end{array}$$

$$\frac{3}{4} = 0.75$$

- 6 Write each fraction as a decimal.

a  $\frac{4}{5}$

b  $\frac{3}{8}$

c  $\frac{1}{2}$

d  $\frac{5}{4}$

e  $\frac{1}{8}$

f  $\frac{2}{5}$

g  $\frac{7}{20}$

h  $\frac{9}{25}$

i  $\frac{1}{40}$

j  $\frac{3}{5}$

k  $\frac{11}{50}$

l  $\frac{5}{16}$

- 7 Dividing a decimal by another decimal is easier if you write an equivalent division problem where the divisor is a whole number.
- In  $3.568 \div 0.4$ , the divisor is 0.4. Should you multiply 0.4 by 10, 100 or 1000 to produce 4?
  - Copy and complete:  $3.568 \div 0.4 = \underline{\hspace{2cm}} \div 4$
  - Why is it important to multiply each decimal in the division problem by the same number?
  - Use your answer to part **b** to work out the division.

**EXAMPLE 1F-4****Dividing a decimal by another decimal**

- Write  $9.46 \div 0.5$  as an equivalent division problem where the divisor is a whole number.
- Calculate  $9.46 \div 0.5$ .

**THINK**

- The decimal 0.5 multiplied by 10 becomes a whole number. To keep the calculation equivalent to the original problem, also multiply the dividend by 10.
- 1 Divide 94.6 by 5. Include trailing zeros until you can divide exactly without a remainder.  
  
2 Write the answer.

**WRITE**

$$\begin{aligned} \text{a } 9.46 \div 0.5 \\ &= (9.46 \times 10) \div (0.5 \times 10) \\ &= 94.6 \div 5 \end{aligned}$$

$$\text{b } \begin{array}{r} 18.92 \\ 5 \overline{)94.60} \end{array}$$

$$9.46 \div 0.5 = 18.92$$

- 8 Write each calculation as an equivalent division problem where the divisor is a whole number.
- |                     |                       |                       |
|---------------------|-----------------------|-----------------------|
| a $7.24 \div 0.2$   | b $4.902 \div 0.6$    | c $19.764 \div 0.03$  |
| d $1.728 \div 0.08$ | e $1.4368 \div 0.004$ | f $7.491 \div 0.06$   |
| g $4.872 \div 1.5$  | h $2.808 \div 0.12$   | i $0.1638 \div 0.009$ |

- 9 Calculate the result of each division problem in question 8.

- 10 The circuit of a fun run has five different stages. The lengths of the stages are: 1.435 km, 2.790 km, 1.902 km, 2.285 km, 3.613 km. First estimate the distance around the full circuit by rounding each length to the nearest whole number and then calculate the exact distance.
- 11 Calculate the cost of each item. Write your answers correct to the nearest cent.
- four punnets of raspberries at \$7.35 per punnet
  - 3 kg of watermelon at \$1.95 per kg
  - 0.6 kg of snow peas at \$5.62 per kg
  - 2.5 kg of potatoes at \$3.95 per kg
  - 0.55 kg of bok choy at \$8.99 per kg
  - 1.74 kg of green beans at \$6.89 per kg





- 12 a** Estimate the total cost of the items listed in question **11** by first rounding the cost of each item to the nearest dollar.
- b** Find the total cost of these items.  
Round your answer to the nearest five cents.
- c** How much change would you receive if you paid with a \$100 note?
- 13** In each of these calculations, round your answer to the nearest cent.
- a** Calculate the cost of one item in each case.
- i** nine bread rolls cost \$5.40
  - ii** four cartons of milk cost \$11.56
  - iii** five DVDs cost \$94.75
- b** Calculate the cost of 1 kg in each case.
- i** 3 kg of tomatoes cost \$13.95
  - ii** 7 kg of sausages cost \$47.95
  - iii** 6 kg of dog food costs \$81.66
- c** Calculate the cost of 100 g in each case.
- i** 200 g of salami costs \$25.90
  - ii** 500 g of cheese costs \$7.95
  - iii** 300 g of cashews cost \$5.85.
- 14** A box of 10 coloured pencils costs \$7.50. These pencils can also be bought individually for 72 cents each.
- a** Find the cost per pencil in each case.
- b** Which option provides better value or is the best buy? Explain.
- 15** William bought a 400-g pack of mushrooms for \$3.69. Kate bought 300 g of loose mushrooms for \$2.98.
- a** Calculate the amount paid by each of them for 100 g of mushrooms.
- b** Who made the best buy? Explain.
- c** Why does a store offer both options?
- 16 a** How many icy-poles could you buy with \$24.50?
- b** How many icy-poles could you buy with \$10? What change would you receive?
- 17** A cup holds 0.25 L of liquid. How many cups could be filled from a 5-L container of water?



- 18 Which is the best buy: 0.8 kg of mince steak for \$11.20 from the supermarket, or 1.2 kg of mince steak for \$16.50 from the butcher?
- 19 Lee Redmond once held the record for the longest fingernails on both hands (see table), after not cutting them for 30 years.

	Fingernail lengths (cm)	
	right hand	left hand
thumb	76.4	80.0
index finger	72.3	76.4
middle finger	74.1	76.7
ring finger	73.6	76.2
little finger	71.6	73.6

- a What is the total length of the fingernails on:
- her right hand?
  - her left hand?
- b Which hand has the longer fingernails? Suggest a reason for this.
- c What is the total length of the fingernails on both hands?
- d What is the average length of her fingernails?
- e Measure the length of your fingernails in centimetres to one decimal place.
- f Find the average length of your fingernails.
- g How many times longer is Lee's average fingernail length than yours? First work out an estimate and then calculate a value correct to one decimal place.



20 Calculate:

- $(15.73 + 2.9) \times 1.5 - 26.07$
- $72.816 \div 8 + 4.9 \times 0.5$
- $6.9 + 0.528 \div 0.04 - 9.3$

**NOTE** Remember to think carefully about which operations are performed first.

21 The answer to a decimal problem is 4.72.

- a Write a sample problem that uses each of these operations.
- addition
  - subtraction
  - multiplication
  - division
- b Write three different sample problems that use two or more of the operations.

### Reflect

Why is it important to estimate your answer before performing calculations with decimals?

# 1G Terminating, non-terminating and recurring decimals

## Start thinking!

In some division problems, you can add trailing zeros to a decimal so that you obtain an exact answer with no remainder. Does this approach work every time?

- 1 Use short division to calculate:    **a**  $5 \div 8$             **b**  $5 \div 6$ .
- 2 What is different about the answers to parts **a** and **b** of question 1?
- 3 The result in question 1a is a **terminating decimal**.  
After how many decimal places does the resulting decimal 'terminate' or stop?
- 4 Is the result to question 1b a terminating decimal? Explain.
- 5 If a decimal does not terminate then it is called a **non-terminating decimal**.  
How many decimal places does a non-terminating decimal have?
- 6 Look at your answer to question 1b. Is there a pattern to the digits?
- 7 A special type of non-terminating decimal is a **recurring decimal**, where one or more digits form a repeating or recurring pattern. Explain why the answer to question 1b is a recurring decimal.

## KEY IDEAS

- ▶ A terminating decimal 'terminates' or stops after a number of decimal places. Some examples are: 24.3, 0.49, 5.768 234.
- ▶ A non-terminating decimal has an endless (infinite) number of decimal places. Some examples are: 17.326 35..., 0.567 743 ..., 0.788 888 ....
- ▶ The full stops written after the last decimal place indicate that an endless number of digits follow.
- ▶ A recurring decimal is a special type of non-terminating decimal. The digits after the decimal point form a recurring (repeating) pattern. Some examples are: 8.999 999 ..., 42.671 671 671..., 0.582 323 232....
- ▶ A recurring decimal can be written in an abbreviated form using dots or a horizontal line over repeating digits. Some examples are:  
 $8.9999... = 8.\dot{9}$ ,  $42.671\ 671... = 42.\overline{671}$ ,  $0.582\ 323\ 23... = 0.58\dot{2}3$ .
- ▶ **Rational numbers** are numbers that can be written exactly as fractions.
- ▶ **Irrational numbers** are numbers that cannot be written exactly as fractions.

## EXERCISE 1G Terminating, non-terminating and recurring decimals

UNDERSTANDING AND FLUENCY

- State whether each decimal is terminating (T) or non-terminating (N).
 

a 4.572 35...	b 56.803
c 21.577 77...	d 8.561 395 62
e 0.451 730 69...	f 17.565 656 5...
g 0.123 412 341 234...	h 0.787 878 78
i 30.082 174...	j 291.291 291 2
k 0.999...	l 5.043 127
- Which of the numbers in question 1 are recurring decimals?
- Explain the difference between terminating, non-terminating and recurring decimals.

### EXAMPLE 1G-1

#### Writing recurring decimals in abbreviated form

Write each recurring decimal in abbreviated form using dots or a horizontal line.

- a 7.682 222 222...  
b 24.503 150 315 031...

#### THINK

- a Look for a repeating pattern of digits. Since there is only one digit that repeats (2), show a dot above that digit.  
b Look for a repeating pattern of digits (5031). Show a horizontal line above those digits (or show a dot above the first and last digit in the pattern).

#### WRITE

- a  $7.682\ 222\ 222\dots$   
 $= 7.68\dot{2}$   
b  $24.503\ 150\ 315\ 031\dots$   
 $24.\overline{5031}$  (or  $24.\dot{5}03\dot{1}$ )

- Write each recurring decimal in abbreviated form using dots or a horizontal line.
 

a 0.999 999...	b 4.363 636 3...
c 87.255 555...	d 3.476 476 476...
e 0.738 222 222...	f 16.196 666 66...
g 483.888 88...	h 6.729 191 919...
i 0.183 673 673 67...	j 1.356 283 562 835...
k 28.707 070 7...	l 9.230 714 444 44...
- Write your answers to question 2 in abbreviated form.

**EXAMPLE 1G-2****Writing recurring decimals to 12 decimal places**

Write each recurring decimal to show 12 decimal places.

**a**  $0.\dot{4}$

**b**  $159.8\overline{372}$

**THINK**

- a** Identify the repeating digit (4). Write the decimal to 12 decimal places and indicate that more digits follow.
- b** Identify the repeating digits (372). Write the decimal to 12 decimal places and indicate that more digits follow.

**WRITE**

**a**  $0.\dot{4}$

$= 0.444\ 444\ 444\ 444\dots$

**b**  $159.8\overline{372}$

$= 159.837\ 237\ 237\ 237\dots$

**6** Write each recurring decimal to show 12 decimal places.

**a**  $0.\dot{1}$

**b**  $5.3\dot{7}$

**c**  $14.\overline{82}$

**d**  $21.5\dot{6}9$

**e**  $8.\overline{104}$

**f**  $0.\overline{6513}$

**g**  $232.68\dot{4}$

**h**  $44.761\overline{359}$

**i**  $94.57\overline{02}$

**j**  $1.\overline{73924}$

**k**  $11.812\dot{1}$

**l**  $6.\overline{904}$

**m**  $14.2\dot{4}1\dot{3}$

**n**  $0.5\overline{620}$

**o**  $139.\dot{2}74\dot{5}$

**7** Convert each fraction to a decimal by dividing the numerator by the denominator.

**a**  $\frac{7}{10}$

**b**  $\frac{1}{2}$

**c**  $\frac{1}{3}$

**d**  $\frac{5}{4}$

**e**  $\frac{2}{11}$

**f**  $\frac{5}{8}$

**g**  $\frac{8}{3}$

**h**  $\frac{3}{4}$

**i**  $\frac{9}{8}$

**j**  $\frac{7}{5}$

**k**  $\frac{5}{6}$

**l**  $\frac{1}{7}$

**8** Consider the fractions and your answers to question 7.

**a** Which fractions produce:

**i** terminating decimals?

**ii** non-terminating decimals?

**b** Which fractions produce recurring decimals?

**c** Do any of the fractions produce a non-terminating decimal that isn't a recurring decimal?

**9** Write each number as a fraction. (Hint: whole numbers can be written with a denominator of 1.)

**a** 0.5

**b** 4

**c** 6.2

**d** 0.37

**e** 18

**f** 2.509

- 10** Numbers that can be written exactly as fractions are rational numbers.
- a** Use your answers to question **9** to explain why whole numbers and terminating decimals are rational numbers.
  - b** Are recurring decimals rational numbers? Use your answers to questions **7** and **8** to help you decide. Explain your decision.
- 11** Numbers that cannot be written exactly as fractions are irrational numbers. Non-terminating decimals that are not recurring decimals fit into this category. They are decimals with an endless number of decimal places but no pattern of repeating digits. Write three examples of an irrational number.

**EXAMPLE 1G-3****Classifying numbers as rational or irrational**

Classify each number as rational or irrational.

**a**  $\frac{5}{6}$

**b** 8.724

**c** 4.333...

**d** 0.231 475 6...

**THINK**

- a** All fractions are rational.
- b** All terminating decimals are rational.
- c** All recurring decimals are rational.
- d** All non-terminating decimals that are not recurring decimals are irrational.

**WRITE**

- a**  $\frac{5}{6}$  is a rational number.
- b** 8.724 is a rational number.
- c** 4.333... is a rational number.
- d** 0.231 475 6... is an irrational number.

- 12** Classify each number as rational or irrational.

**a** 4.6

**b** 0.121 212 12...

**c** 0.5871

**d** 2.357 896 21...

**e** 35

**f** 734.946 518 24...

**g**  $0.872\overline{503}$

**h** 96.734 25

**i**  $\frac{3}{17}$

**j** 500 000

**k** 0.117 843 26...

**l**  $10\frac{3}{4}$

- 13 a** Convert each fraction to a decimal.

**i**  $\frac{1}{9}$

**ii**  $\frac{2}{9}$

**iii**  $\frac{3}{9}$

**iv**  $\frac{4}{9}$

- b** What type of decimal do you get?

- c** Use the pattern from part **a** to write these as decimals.

**i**  $\frac{5}{9}$

**ii**  $\frac{6}{9}$

**iii**  $\frac{7}{9}$

**iv**  $\frac{8}{9}$



- 14 a** Use a calculator to convert each fraction to a decimal.

Show at least eight decimal places.

**i**  $\frac{1}{9}$

**ii**  $\frac{1}{99}$

**iii**  $\frac{1}{999}$

**iv**  $\frac{1}{9999}$

- b** Write each recurring decimal in abbreviated form with dots or a horizontal line.

- c** Use the pattern from part **a** to write these as recurring decimals without using a calculator.

**i**  $\frac{1}{99\,999}$

**ii**  $\frac{1}{999\,999}$

**iii**  $\frac{1}{9\,999\,999}$

**iv**  $\frac{1}{99\,999\,999}$

- 15 a** Write  $\frac{1}{3}$  and  $\frac{2}{3}$  as decimals, showing at least the first five decimal places.

- b** Add the two decimals you found in part **a**.

- c** Calculate  $\frac{1}{3} + \frac{2}{3}$  and compare your answer to part **b**.

- 16** An irrational number that is commonly used is **pi** (pronounced 'pie'). It has the symbol  $\pi$ . This number has intrigued mathematicians for thousands of years and is related to measurements on a circle. (You will discover more about this in Chapter 8.)

Since it is a decimal with an endless number of decimal places and no obvious repeating pattern, an estimate or approximation of its value is used in calculations.

**NOTE** Mathematicians started using the Greek letter  $\pi$  as the symbol for pi from 1706.

- a** Find the symbol  $\pi$  on your calculator. Press the appropriate keys to obtain a decimal number for pi. Write the number shown on your calculator screen.

- b** Write an approximation for pi to the following number of decimal places.

**i** two

**ii** four

**iii** five

**iv** six

- 17** Another irrational number is  $\sqrt{2}$ .

- a** Write the number shown on your calculator screen for  $\sqrt{2}$ .

- b** Write an approximation for  $\sqrt{2}$  to the following number of decimal places.

**i** two

**ii** four

**iii** six

**iv** seven

- c** Write the number shown on your calculator screen for  $\sqrt{3}$ .

- d** Is  $\sqrt{3}$  an irrational number?

- e** Write an approximation for  $\sqrt{3}$  to the following number of decimal places.

**i** two

**ii** four

**iii** five

**iv** six



- 18** Before the use of calculators and computers, many mathematicians used a fraction as an estimate for the value of pi. Some examples are shown in the table that follows.

**a** Copy this table.

Place, person and era of origin	Fraction estimate	Decimal approximation [to six decimal places]
Ancient Egypt (1700 BCE)	$\frac{256}{81}$	
Ancient Greece Archimedes (287–212 BCE)	$\frac{223}{71}$	
	$\frac{22}{7}$	
China Zu Chongzhi (429–500 CE)	$\frac{355}{113}$	
India Bhaskara II (1114–1185 CE)	$\frac{3927}{1250}$	
Italy Leonardo of Pisa (1180–1250 CE)	$\frac{12\,960}{4126}$	
	$\frac{7200}{2291}$	



Archimedes thought that pi was between  $\frac{223}{71}$  and  $\frac{22}{7}$ .

**NOTE** Some mathematicians thought that the value of pi was between two fractions.

- b** Use a calculator to complete the third column of the table.
- c** Which value is closest to the approximation of pi you wrote for question 16b part iv?
- 19** Since the 1940s, computers have been used to calculate an approximation for pi to an increasing number of decimal places (from 2000 to over 1 700 000 000 000 decimal places). Here is an approximation for pi written to 120 decimal places.
- 3.141 592 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105 820 974 944 592 307 816 406 286 208 998 628 034 825 342 117 067 982 148 086 513 282 306 647

**a** Copy and complete this table to show how many times each digit appears in the approximation for pi shown.

Digit	0	1	2	3	4	5	6	7	8	9
Number of times it appears										

- b** Which digit appears:
- i** the most? **ii** the least?
- c** Does there appear to be any pattern in the digits?
- d** Comment on your answers.
- 20** Can you use a computer to write the *exact* value of pi as a fraction or a decimal? Explain.

### Reflect

How can you tell the difference between terminating, non-terminating and recurring decimals?

# 1H Powers and roots

## Start thinking!

- $3^5$  is written in **index form**.
  - What is the **power** or **index**?
  - What is the **base**?
  - How is this number read?
  - Write the number in **expanded form** as a repeated multiplication.
  - Perform the repeated multiplication to obtain the **basic numeral**.
- The number  $3^2$  can be read as '3 to the power of 2', 'the **square** of 3' or '3 squared'.
  - Use the diagram on the right to explain why the last two expressions are used.
  - What is the square of 3?
- What number, when squared, gives 16? Use the diagram on the right to help you find the answer.
- Another way of asking 'What number, when squared, gives 16?' is 'Find the **square root** of 16' or  $\sqrt{16}$ . Copy and complete:  $\sqrt{16} = \underline{\quad}$  as  $\underline{\quad}^2 = 16$ .
- Use your answer to question 2 to write the value of  $\sqrt{9}$ .
- Explain how finding the square of a number and finding the square root of a number are related.



## KEY IDEAS

- A repeated multiplication can be written in index form (or index notation), where the **base** is the number to be repeatedly multiplied and the **power** or **index** indicates the number of times the base is written.

- Numbers in index form can be written in expanded form.

The result of the repeated multiplication produces the **basic numeral**.

$$\begin{array}{ccccccc}
 3^5 & = & 3 \times 3 \times 3 \times 3 \times 3 & = & 243 \\
 \text{index form} & & \text{expanded form} & & \text{basic numeral}
 \end{array}$$

- The square of a number is the result of multiplying a number by itself. For example,  $3^2 = 3 \times 3 = 9$ .
- The square root of a number is the number that, when squared, results in the original number. For example,  $\sqrt{9} = 3$  (as  $3^2 = 9$ ).
- The **cube** of a number is the result of raising a number to the power of 3. For example,  $2^3 = 2 \times 2 \times 2 = 8$ .
- The **cube root** of a number is the number that, when cubed, results in the original number. For example,  $\sqrt[3]{8} = 2$  (as  $2^3 = 8$ ).

base  $3^5$  power or index

**NOTE** The plural of the word **index** is **indices**.

## EXERCISE 1H Powers and roots

1 Copy and complete this table.

	Index form	Base	Index or power	Expanded form	Basic numeral
a	$3^4$	3	4	$3 \times 3 \times 3 \times 3$	
b	$2^3$	2			8
c			2	$8 \times 8$	
d				$6 \times 6 \times 6$	

### EXAMPLE 1H-1

#### Calculating the value of a number in index form

Write  $5^3$  in expanded form and calculate its value.

#### THINK

Identify the base (5) and the index (3). This means that 5 is written 3 times. Multiply to calculate the basic numeral.

#### WRITE

$$\begin{aligned} 5^3 &= 5 \times 5 \times 5 \\ &= 25 \times 5 \\ &= 125 \end{aligned}$$

2 Write each number in expanded form and calculate its value.

- |          |         |         |         |
|----------|---------|---------|---------|
| a $2^5$  | b $4^3$ | c $1^9$ | d $3^3$ |
| e $10^7$ | f $2^8$ | g $3^6$ | h $5^4$ |
| i $7^3$  | j $6^4$ | k $8^5$ | l $9^4$ |

### EXAMPLE 1H-2

#### Finding the square

Find:

- a the square of 9      b  $6^2$ .

#### THINK

- a To find the square of a number, multiply the number by itself.
- b Change from index form to a repeated multiplication and calculate the result.

#### WRITE

- a  $9^2 = 9 \times 9$   
 $= 81$   
 The square of 9 is 81.
- b  $6^2 = 6 \times 6$   
 $= 36$

3 Find:

- |                   |                   |                   |
|-------------------|-------------------|-------------------|
| a the square of 5 | b the square of 2 | c the square of 1 |
| d 7 squared       | e 10 squared      | f 25 squared      |
| g $8^2$           | h $11^2$          | i $20^2$          |
| k $17^2$          | l $60^2$          | j $33^2$          |
|                   |                   | n $500^2$         |

### EXAMPLE 1H-3

#### Finding the square root

Find:

- a the square root of 49      b  $\sqrt{81}$ .

#### THINK

- a Think of a number that, when squared, gives 49.  
(The square of 7 is 49.)
- b Think of a number that, when squared, gives 81.  
( $9^2 = 81$ )

#### WRITE

- a The square root of 49 is 7.

b  $\sqrt{81} = 9$

4 Find these values.

- |                         |                          |
|-------------------------|--------------------------|
| a the square root of 16 | b the square root of 9   |
| c the square root of 25 | d the square root of 64  |
| e the square root of 1  | f the square root of 121 |
| g $\sqrt{4}$            | h $\sqrt{100}$           |
| i $\sqrt{36}$           | j $\sqrt{225}$           |
| k $\sqrt{144}$          | l $\sqrt{400}$           |
| m $\sqrt{169}$          | n $\sqrt{900}$           |

5 One student incorrectly wrote  $4^2 = 8$ . Why is  $4^2$  not the same as  $4 \times 2$ ?

6 Another student incorrectly wrote  $\sqrt{16} = 8$ . Why is  $\sqrt{16}$  not the same as  $16 \div 2$ ?

7 Write each in expanded form and calculate its value.

- a  $2^3 \times 3^4$       b  $7^2 \times 10^5$       c  $1^9 \times 5^3$       d  $4^3 \times 2^2$

8 Write each in expanded form and calculate its value.

- |                     |                      |                     |                      |
|---------------------|----------------------|---------------------|----------------------|
| a $(0.8)^2$         | b $(3.4)^2$          | c $(0.5)^3$         | d $(0.3)^4$          |
| e $(0.1)^5$         | f $(0.2)^7$          | g $(\frac{1}{4})^3$ | h $(\frac{7}{9})^2$  |
| i $(\frac{5}{6})^3$ | j $(\frac{3}{10})^4$ | k $(\frac{1}{2})^5$ | l $(3\frac{1}{3})^6$ |

9 Calculate:

- |  |                                      |
|--|--------------------------------------|
| a $2^3 \times 7 + 8^2 \div 4$                          | b $5.8 - (1.2)^2 + (0.1)^4$          |
| c $\frac{7}{10} + (\frac{2}{5})^3 \times 6\frac{1}{4}$ | d $\sqrt{81} - 5^2 \times 0.2 + 3^4$ |

**NOTE** Remember to use the correct order of operations.

**10** With larger numbers, a calculator is useful. Calculate:

- |   |  |                                       |
|---|--|---------------------------------------|
| <b>a</b> $57^2$                         | <b>b</b> $942^2$                       | <b>c</b> $101^3$                      |
| <b>d</b> $13^7$                         | <b>e</b> $85^4$                        | <b>f</b> $26^5$                       |
| <b>g</b> $(6.85)^3$                     | <b>h</b> $(1.9)^4$                     | <b>i</b> $(12.4)^5$                   |
| <b>j</b> $\left(\frac{18}{23}\right)^3$ | <b>k</b> $\left(\frac{7}{11}\right)^6$ | <b>l</b> $\left(\frac{3}{5}\right)^9$ |
| <b>m</b> $\sqrt{196}$                   | <b>n</b> $\sqrt{256}$                  | <b>o</b> $\sqrt{1089}$                |
| <b>p</b> $\sqrt{26\,244}$               | <b>q</b> $\sqrt{2.25}$                 | <b>r</b> $\sqrt{53.29}$               |
| <b>s</b> $\sqrt{67.24}$                 | <b>t</b> $\sqrt{475.24}$               | <b>u</b> $\sqrt{25.3009}$             |

**11 a** Calculate:

- i**  $\sqrt{64}$       **ii**  $\sqrt{5.76}$       **iii**  $\sqrt{3}$ .

**b** Explain how you can tell whether each result is rational or irrational.

**12** For each of the following:

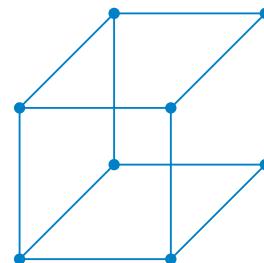
- i** find the value, using a calculator where necessary  
**ii** decide whether the result is rational or irrational.

- |   |  |                                       |
|---|--|---------------------------------------|
| <b>a</b> $\sqrt{144}$                   | <b>b</b> $\sqrt{20}$                   | <b>c</b> $\sqrt{54.76}$               |
| <b>d</b> $\sqrt{331\,776}$              | <b>e</b> $\sqrt{8.3}$                  | <b>f</b> $\sqrt{0.49}$                |
| <b>g</b> $\sqrt{\frac{1}{4}}$           | <b>h</b> $\frac{\sqrt{1}}{\sqrt{4}}$   | <b>i</b> $\frac{\sqrt{9}}{\sqrt{25}}$ |
| <b>j</b> $\frac{\sqrt{81}}{\sqrt{100}}$ | <b>k</b> $\frac{\sqrt{49}}{\sqrt{36}}$ | <b>l</b> $\frac{\sqrt{3}}{\sqrt{4}}$  |

**13** The number  $2^3$  can be read as '2 to the power of 3'.  
 It can also be read as 'the cube of 2' or '2 cubed'.

- a** Use this diagram to explain why the last two expressions can be used.  
**b** What is the cube of 2?  
**c** Find:

- i** 5 cubed      **ii** 3 cubed      **iii** the cube of 10      **iv** the cube of 7



**14 a** What number, when cubed, gives a result of 8?

**b** Another way of asking this is to find the cube root of 8 or  $\sqrt[3]{8}$ .

Copy and complete:  $\sqrt[3]{8} = \underline{\quad}$  as  $\underline{\quad}^3 = 8$ .

**c** Use your answers to question 13 to write the value of:

- |                               |                               |
|-------------------------------|-------------------------------|
| <b>i</b> the cube root of 125 | <b>ii</b> the cube root of 27 |
| <b>iii</b> $\sqrt[3]{1000}$   | <b>iv</b> $\sqrt[3]{343}$     |

**d** Explain how finding the cube of a number and finding the cube root of a number are related.

**15** Without using a calculator, find:

- |                           |                          |                           |
|---------------------------|--------------------------|---------------------------|
| <b>a</b> $\sqrt[3]{64}$   | <b>b</b> $\sqrt[3]{1}$   | <b>c</b> $\sqrt[3]{216}$  |
| <b>d</b> $\sqrt[3]{8000}$ | <b>e</b> $\sqrt[3]{729}$ | <b>f</b> $\sqrt[3]{1331}$ |



- 16 a** Find:
- i**  $1^2$       **ii**  $11^2$       **iii**  $111^2$       **iv**  $1111^2$
- b** Describe the pattern you can see.
- c** Without using a calculator, use the pattern to write the results for:
- i**  $11\ 111^2$       **ii**  $111\ 111^2$
- iii**  $1\ 111\ 111^2$       **iv**  $11\ 111\ 111^2$

- 17** For the six weeks of the school holidays, Sarah can earn \$30 per week doing extra jobs around home.

- a** How much could Sarah earn over the holidays?

Instead, Sarah suggests her parents pay her \$3 in the first week and then in each of the other weeks pay her three times as much as the previous week.

- b** How much would Sarah earn in each week with this new proposal?
- c** Compare the two possible amounts Sarah could earn over the school holidays. Which plan do you think Sarah's parents will agree to?
- d** Explain how you can use index notation to write the amount Sarah could earn in her eleventh week if her proposal was followed over a longer period of time.
- e** Use a calculator to work out the amount in part **d**.

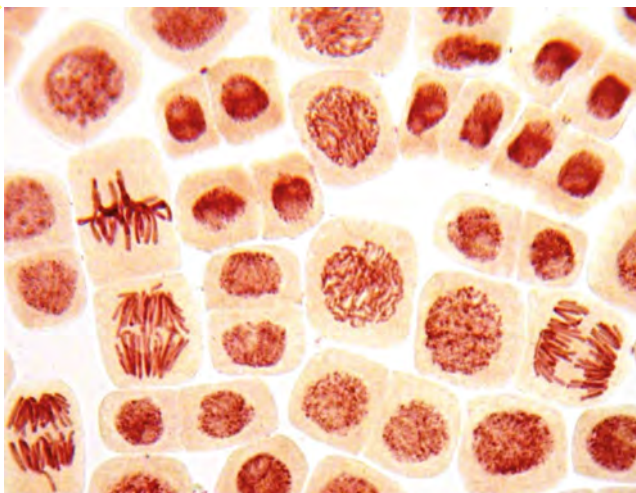


- 18** A colony containing 1000 cells doubles every hour.

- a** How large is the colony after:

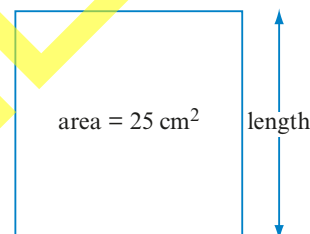
- i** one hour?
- ii** two hours?
- iii** three hours?

- b** Copy this table and complete the second row. (Hint: use your results from part **a** for the first few entries.)



Number of hours	1	2	3	4	5
Size of colony					
Number of times as large as original amount		4		16	

- c How many times larger is the new colony compared to the original colony after:  
 i one hour?      ii two hours?      iii three hours?
- d Complete the third row of the table. (Hint: use your answers from part c for the first few entries.)
- e Add a fourth row to your table and complete it by writing each number in the third row in index form with a base of 2.
- f Written in index form, how many times larger is the new colony than the original colony after:  
 i 6 hours?      ii 10 hours?      iii 24 hours?
- g Use a calculator to work out how many times larger the colony will be after 24 hours.
- h Use your answer to part g to write the number of cells in the colony after 24 hours.
- 19 a If a square has area  $25 \text{ cm}^2$ , explain how you can work out its length.
- b Calculate the length of a square with area:  
 i  $169 \text{ cm}^2$   
 ii  $200 \text{ m}^2$   
 iii  $268.96 \text{ km}^2$ .
- c Which of your answers is irrational?



- 20 In a similar way to finding the square root and cube root of a number, you can find the fourth root, the fifth root and so on.
- a If  $5^4 = 625$ , what is  $\sqrt[4]{625}$ ?
- b If  $7^5 = 16\,807$ , what is  $\sqrt[5]{16\,807}$ ?
- c Find each of these without using a calculator.
- |                   |                   |                         |
|-------------------|-------------------|-------------------------|
| i $\sqrt[4]{16}$  | ii $\sqrt[4]{81}$ | iii $\sqrt[4]{10\,000}$ |
| iv $\sqrt[5]{32}$ | v $\sqrt[5]{1}$   | vi $\sqrt[5]{243}$      |
- 21 a Use a calculator to find these values.
- |                         |                            |                         |
|-------------------------|----------------------------|-------------------------|
| i $\sqrt[3]{3375}$      | ii $\sqrt[3]{68.921}$      | iii $\sqrt[3]{99}$      |
| iv $\sqrt[4]{2401}$     | v $\sqrt[4]{73}$           | vi $\sqrt[4]{0.1}$      |
| vii $\sqrt[5]{32\,768}$ | viii $\sqrt[5]{0.000\,32}$ | ix $\sqrt[5]{200}$      |
| x $\sqrt[6]{729}$       | xi $\sqrt[6]{496}$         | xii $\sqrt[6]{15\,625}$ |
- b Classify each result you found as rational or irrational.
- 22 Calculate  $\sqrt{8} + \sqrt[3]{8}$ . Is your answer rational or irrational?

**Reflect**

How are powers and roots related?

# 11 Index laws

## Start thinking!

Index form	Basic numeral
$2^1$	2
$2^2$	4
$2^3$	8
$2^4$	
$2^5$	
$2^6$	
$2^7$	
$2^8$	
$2^9$	
$2^{10}$	

- Copy and complete this table.
- Use the table to answer these questions.
  - Write the values for:    **i**  $2^2$         **ii**  $2^3$         **iii**  $2^5$ .
  - Multiply your answers to part **a i** and **ii** to calculate  $2^2 \times 2^3$ .
  - How does this compare to your result for  $2^5$ ?
  - Write the values for:    **i**  $2^5$         **ii**  $2^4$         **iii**  $2^9$ .
  - Multiply your answers to part **d i** and **ii** to calculate  $2^5 \times 2^4$ .
  - How does this compare to your result for  $2^9$ ?
  - Describe the pattern or shortcut you can see. This is one of the **index laws**.
  - Provide three more examples to show this shortcut works for other values in the table.
- Use the table to answer these questions.
  - Write the values for:    **i**  $2^6$         **ii**  $2^4$         **iii**  $2^2$ .
  - Divide your answers to part **a i** and **ii** to calculate  $2^6 \div 2^4$ .
  - How does this compare to your result for  $2^2$ ?
  - Write the values for:    **i**  $2^{10}$         **ii**  $2^7$         **iii**  $2^3$ .
  - Divide your answers to part **d i** and **ii** to calculate  $2^{10} \div 2^7$ .
  - How does this compare to your result for  $2^3$ ?
  - Describe the pattern or shortcut you can see. This is another index law.
  - Provide three more examples to show this shortcut works for other values in the table.
- What must be the same for these two index laws to work?  
(Hint: do they work for calculations like  $2^3 \times 3^4$  and  $3^5 \div 2^3$ ?)

## KEY IDEAS

- ▶ Index laws make performing some calculations with numbers in index form easier.

- ▶ The index laws are shown in these examples:

$$2^5 \times 2^3 = 2^8$$

write the base and add the indices (powers).

$$2^5 \div 2^3 = 2^2$$

write the base and subtract the indices (powers).

$$(2^5)^3 = 2^{15}$$

write the base and multiply the indices

$$(2 \times 5)^3 = 2^3 \times 5^3$$

write each factor of the product to the same power

$$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$$

write the numerator and the denominator to the same power.

**NOTE** The first two index laws do not work for numbers in index form with different bases.

- ▶ A number in index form with a power of zero equals one. For example,  $2^0 = 1$ .
- ▶ To simplify a calculation involving indices, write the answer in index form.

## EXERCISE 11 Index laws

- 1 Copy and complete these sentences using words from this list:  
*multiply, divide, same, different, add, subtract, indices, powers.*
  - a When multiplying numbers in index form with the \_\_\_\_ base, write the base and \_\_\_\_ the indices (or \_\_\_\_).
  - b When dividing two numbers in index form with the \_\_\_\_ base, write the base and \_\_\_\_ the \_\_\_\_ (or powers).
- 2 Use an index law to copy and complete:
  - a  $2^3 \times 2^5 = 2^{3+5} = 2^-$
  - b  $2^4 \times 2^2 = 2^{-+2} = 2^6$
  - c  $2^3 \times 2^4 = 2^{-+4} = 2^-$
  - d  $2^5 \times 2^5 = 2^{-+-} = 2^-$
- 3 Use an index law to copy and complete:
  - a  $2^5 \div 2^2 = 2^{5-2} = 2^-$
  - b  $2^8 \div 2^3 = 2^{- - 3} = 2^5$
  - c  $2^9 \div 2^5 = 2^{9--} = 2^-$
  - d  $2^7 \div 2^4 = 2^{- --} = 2^-$
- 4 Use the table on the opposite page to write the result (basic numeral) of each calculation in:
  - i question 2
  - ii question 3.

### EXAMPLE 11-1

### Multiplying two numbers in index form

Use an index law to simplify

a  $7^5 \times 7^3$

b  $9^4 \times 9$

#### THINK

- a 1 Check the bases are the same. (yes)
- 2 Write the base and add the indices. ( $5 + 3 = 8$ .)
- b 1 Check the bases are the same. (yes)
- 2 Write the base and add the indices. ( $4 + 1 = 5$ .)  
Note that 9 is the same as  $9^1$ .

#### WRITE

a  $7^5 \times 7^3$   
 $= 7^8$

b  $9^4 \times 9$   
 $= 9^5$

- 5 Use an index law to simplify:
  - a  $3^3 \times 3^2$
  - b  $4^5 \times 4^3$
  - c  $6^2 \times 6^4$
  - d  $5^3 \times 5^6$
  - e  $8^2 \times 8^1$
  - f  $2 \times 2^4$
- 6 Calculate the basic numeral for each result in question 5.

**EXAMPLE 11-2****Dividing two numbers in index form**

Use an index law to simplify  $4^{11} \div 4^6$ .

**THINK**

- 1 Check the bases are the same. (yes)
- 2 Write the base and subtract the indices. ( $11 - 6 = 5$ .)

**WRITE**

$$\begin{aligned} 4^{11} \div 4^6 \\ = 4^5 \end{aligned}$$

- 7 Use an index law to simplify these.

a  $7^9 \div 7^6$

b  $3^8 \div 3^3$

c  $5^{10} \div 5^8$

d  $9^{14} \div 9^{11}$

e  $4^6 \div 4^5$

f  $6^4 \div 6^1$

- 8 Calculate the basic numeral for each result in question 7.

- 9 Use an index law to first simplify and then calculate these.

a  $2^5 \times 2^6$

b  $5^2 \times 5^4$

c  $3^4 \times 3^3$

d  $4^6 \times 4^2$

e  $5^7 \div 5^4$

f  $3^9 \div 3^3$

g  $9^6 \div 9^3$

h  $2^{12} \div 2^5$

i  $6^3 \times 6^2$

j  $8^5 \div 8^2$

k  $10^4 \times 10^5$

l  $7^8 \div 7^6$

- 10 Use an index law to simplify these. Leave your answer in index form.

a  $2^5 \times 2^4 \times 2^3$

b  $3^3 \times 3^1 \times 3^2$

c  $10^3 \times 10^2 \times 10^5$

d  $4^2 \times 4^4 \times 4^3$

e  $5^7 \times 5^3 \times 5^6$

f  $9^1 \times 9^5 \times 9^8$

- 11 Use an index law to simplify these.

a  $(1.5)^3 \times (1.5)^6$

b  $(0.7)^4 \times (0.7)^2$

c  $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4$

d  $\left(\frac{7}{5}\right)^6 \times \left(\frac{7}{5}\right)^8$

e  $(9.2)^8 \div (9.2)^5$

f  $(0.63)^{11} \div (0.63)^4$

g  $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^2$

h  $\left(\frac{5}{2}\right)^9 \div \left(\frac{5}{2}\right)^7$

**NOTE** The bases do not need to be whole numbers for the index laws to apply. They just need to be the same.

- 12 a Use an index law to simplify  $5^7 \div 5^3$ .

b Another way of writing  $5^7 \div 5^3$  is  $\frac{5^7}{5^3}$ .

i Copy and complete:  $\frac{5^7}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times \_}{5 \times \_ \times 5}$ .

ii Simplify by cancelling common factors in the numerator and denominator.

c Do you obtain the same result for parts a and b?

d Explain how an index law can be used to simplify calculations like  $\frac{5^7}{5^3}$ .

**NOTE** The vinculum in a fraction is equivalent to a division sign.

- 13 Use an index law to simplify these.

a  $\frac{3^6}{3^4}$

b  $\frac{8^5}{8^2}$

c  $\frac{2^9}{2^3}$

d  $\frac{4^7}{4^2}$

e  $\frac{10^8}{10^5}$

f  $\frac{6^{11}}{6^7}$

**EXAMPLE 11-3****Using more than one index law**

Simplify:

**a**  $\frac{3^7 \times 3^2}{3^5}$

**b**  $\frac{7^4 \times 7^6}{7^5 \times 7^3}$

**THINK**

- a** 1 The vinculum groups together the terms in the numerator. Use an index law to multiply them. Write the base and add the indices. ( $7 + 2 = 9$ .)
- 2 Use another index law to divide the terms. Write the base and subtract the indices. ( $9 - 5 = 4$ .)
- b** 1 Multiply the terms in the numerator. Add the indices. ( $4 + 6 = 10$ .) Multiply the terms in the denominator. Add the indices. ( $5 + 3 = 8$ .)
- 2 Divide the terms. Subtract the indices. ( $10 - 8 = 2$ .)

**WRITE**

$$\begin{aligned} \mathbf{a} \quad & \frac{3^7 \times 3^2}{3^5} \\ &= \frac{3^9}{3^5} \\ &= 3^4 \end{aligned}$$

$$\begin{aligned} \mathbf{a} \quad & \frac{7^4 \times 7^6}{7^5 \times 7^3} \\ &= \frac{7^{10}}{7^8} \\ &= 7^2 \end{aligned}$$

**14** Copy and complete:

$$\begin{aligned} \mathbf{a} \quad & \frac{2^6 \times 2^5}{2^3} \\ &= \frac{2^{6+5}}{2^3} \\ &= \frac{2^{11}}{2^3} \\ &= 2^{11-3} \\ &= 2^8 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & \frac{5^3 \times 5^7}{5^4} \\ &= \frac{5^{3+7}}{5^4} \\ &= \frac{5^{10}}{5^4} \\ &= 5^{10-4} \\ &= 5^6 \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad & \frac{3^7 \times 3^6}{3^5 \times 3^4} \\ &= \frac{3^{7+6}}{3^{5+4}} \\ &= \frac{3^{13}}{3^9} \\ &= 3^{13-9} \\ &= 3^4 \end{aligned}$$

$$\begin{aligned} \mathbf{d} \quad & \frac{9^8 \times 9^{11}}{9^9 \times 9^3} \\ &= \frac{9^{8+11}}{9^{9+3}} \\ &= \frac{9^{19}}{9^{12}} \\ &= 9^{19-12} \\ &= 9^7 \end{aligned}$$

**15** Simplify.

**a**  $\frac{4^5 \times 4^3}{4^6}$

**b**  $\frac{7^6 \times 7^6}{7^{10}}$

**c**  $\frac{6^4 \times 6^7}{6^8}$

**d**  $\frac{2^5 \times 2^9}{2^6}$

**e**  $\frac{11^8 \times 11^3}{11^5}$

**f**  $\frac{3^4 \times 3^5}{3^8}$

**g**  $\frac{5^6 \times 5^{10}}{5^9 \times 5^3}$

**h**  $\frac{8^7 \times 8^2}{8^1 \times 8^5}$

**i**  $\frac{2^8 \times 2^6}{2^3 \times 2^4}$

**j**  $\frac{9^7 \times 9^{12}}{9^6 \times 9^3}$

**k**  $\frac{3^5 \times 3^4}{3^3 \times 3^2}$

**l**  $\frac{4^{11} \times 4^6}{4^{12} \times 4^3}$

**16 a** **i** Copy and complete:  $\frac{5^7}{5^7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times \underline{\quad} \times \underline{\quad}}{5 \times \underline{\quad} \times 5 \times 5 \times \underline{\quad} \times 5 \times 5}$ **ii** Simplify by cancelling common factors in the numerator and denominator.**b** Use an index law to simplify  $\frac{5^7}{5^7}$ . What power is shown in the result?**c** As the answers to parts **a** and **b** should be the same, what does  $5^0$  equal?**d** Repeat parts **a** and **b** for  $\frac{3^4}{3^4}$ . What conclusion do you reach? Will this work for any base?**e** Copy and complete: A number in index form with a power of  $\underline{\quad}$  equals  $\underline{\quad}$ .



**17** Find the value of these.

**a**  $4^0$

**b**  $6 \times 5^0$

**c**  $9^3 \div 9^3$

**d**  $7^4 \times 7^0$

**e**  $\frac{8^5}{8^5}$

**f**  $(3.2)^0$

**18** Use the index laws to simplify these.

**a**  $\frac{3^5 \times 3^3}{3^8}$

**b**  $\frac{14^8 \times 14^6}{14^{14}}$

**c**  $\frac{9^7 \times 9^3}{9^9}$

**d**  $\frac{5^2 \times 5^2 \times 5^4}{5^8}$

**e**  $\frac{7^6 \times 7^4}{7^5 \times 7^5}$

**f**  $\frac{11^3 \times 11^9}{11^4 \times 11^7}$

**g**  $\frac{3^2 \times 3^2 \times 3^2}{3^3 \times 3^3}$

**h**  $\frac{6^4 \times 6^5 \times 6^6}{6^3 \times 6^2}$

**i**  $\frac{4^3 \times 4^2 \times 4^3}{4^5 \times 4^1 \times 4^2}$

**19 a** David and Natalie simplified  $(5^2)^3$  in two different ways. However, they both used the power of three to write the contents of the brackets three times as a repeated multiplication.

Can you see another way that is quicker? Describe the shortcut you could use.

**b** Try your shortcut to simplify  $(2^3)^4$ .

**c** Check that you obtain the same result using David and Natalie's working.

That is, simplify  $2^3 \times 2^3 \times 2^3 \times 2^3$  and  $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$ .

**d** This shortcut is another index law. Copy and complete: When raising a number in index form to a power, write the base and \_\_\_\_ the indices (or powers).

David

$$\begin{aligned}(5^2)^3 &= 5^2 \times 5^2 \times 5^2 \\ &= 5^{2+2+2} \\ &= 5^6\end{aligned}$$

Natalie

$$\begin{aligned}(5^2)^3 &= (5 \times 5) \times (5 \times 5) \times (5 \times 5) \\ &= 5^6\end{aligned}$$

### EXAMPLE 11-4

#### Raising a number in index form to a power

Use an index law to simplify  $(3^4)^5$ .

**THINK**

Write the base and multiply the powers. ( $4 \times 5 = 20$ .)

**WRITE**

$$(3^4)^5 = 3^{20}$$

**20** Use an index law to simplify these.

**a**  $(6^4)^5$

**b**  $(2^6)^2$

**c**  $(7^4)^3$

**d**  $(11^2)^4$

**e**  $(8^5)^3$

**f**  $(3^7)^2$

**21** A mistake has been made in each of these.

Explain what the mistake is and write the calculation correctly.

**a**  $5^8 \div 5^2 = 5^4$

**b**  $2^3 \times 2^3 = 2^9$

**c**  $(6^5)^2 = 6^7$

**d**  $9^{12} \div 9^{12} = 9^1$

**22** Find the missing powers in these calculations.

**a**  $7^5 \times 7^- = 7^{15}$

**b**  $(4^3)^- = 4^{18}$

**c**  $\frac{3^- \times 3^2}{3^6} = 1$

**d**  $\frac{6^7 \times 6^7}{6^-} = 6^2$

**23 a** Use the index laws to simplify:

**i**  $2^5 \times 2^4$

**ii**  $2^8 \div 2^2$

**iii**  $\frac{2^7 \times 2^6}{2^9}$

**iv**  $\frac{2^9 \times 2^5}{2^4 \times 2^3}$

**v**  $(2^2)^5$

**vi**  $\frac{2^{13} \times (2^4)^3 \times 2^5}{2^{22}}$

**b** Use the table of values on page 52 to write each result in part **a** as a basic numeral without using a calculator.

**c** Use the index laws to simplify these.

**i**  $3^3 \times 3^3$

**ii**  $3^7 \div 3^4$

**iii**  $\frac{3^{11} \times 3^6}{3^8}$

**iv**  $(3^4)^2$

**v**  $\frac{(3^2)^4 \times 3^5}{3^4 \times 3^3}$

**vi**  $\frac{3^5 \times 3^6 \times 3^7}{3^{11} \times 3^2}$

**d** Copy and complete this table.

**e** Use the table of values in part **d** to write each result in part **c** as a basic numeral without using a calculator.

Index form	Basic numeral
$3^1$	3
$3^2$	9
$3^3$	27
$3^4$	
$3^5$	
$3^6$	
$3^7$	
$3^8$	
$3^9$	
$3^{10}$	

**24 a** Use the tables of values above and on page 52 to write the numbers in each calculation in index form. See the example shown at right.

**i**  $32 \times 16$

**ii**  $1024 \div 256$

**iii**  $\frac{128 \times 32}{16}$

**iv**  $\frac{512 \times 64}{128 \times 16}$

**v**  $59\,049 \div 729$

**vi**  $243 \times 81$

**vii**  $\frac{27 \times 243}{2187}$

**viii**  $\frac{2187 \times 729}{6561 \times 27}$

**ix**  $\frac{128 \times 1024}{256 \times 64}$

**b** Use the index laws to simplify each calculation.

**c** Refer to the tables to write the results for part **b** as basic numerals.

**d** Without using a calculator, find the value of each of these.

**i**  $512 \times 32 \div 64$

**ii**  $9 \times 243 \div 27$

**iii**  $\frac{32 \times 256}{8 \times 16}$

**iv**  $\frac{6561 \times 19\,683}{729 \times 81}$

**Example**

$$\begin{aligned} & 64 \times 256 \\ & \quad 512 \\ & = \frac{2^6 \times 2^8}{2^9} \end{aligned}$$

**25** Two other index laws involve raising a product to a power and raising a quotient to a power.

**a** Show that  $(2 \times 3)^5$  is the same as  $2^5 \times 3^5$ .

**b** Show that  $\left(\frac{2}{3}\right)^5$  is the same as  $\frac{2^5}{3^5}$ .

**c** Write each of these in index form without brackets.

**i**  $(5 \times 9)^3$

**ii**  $\left(\frac{3}{4}\right)^6$

**iii**  $\left(\frac{9}{5}\right)^4$

**iv**  $(11 \times 4)^7$

**v**  $\left(\frac{7}{10}\right)^3$

**vi**  $(3 \times 8)^9$

**26** Write a summary of the index laws in this topic and provide an example of each.

**27** Write three calculations that simplify to  $2^5$ .

**Reflect**

Why is it useful to know the index laws?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

rounding	simplifying	non-terminating decimal	basic numeral
leading digit	LCD	recurring decimal	squaring
approximate	decimal place	rational number	square root
estimate	placeholder zero	irrational number	cube root
order of operations	trailing zero	index form	index laws
equivalent fractions	round up	power	
HCF	round down	base	
simplest form	terminating decimal		

## MULTIPLE-CHOICE

- 1A** 1 After rounding each number to its leading digit, what is the estimate for  $526 \times 68$ ?  
**A** 36 000 **B** 35 000  
**C** 33 664 **D** 31 20
- 1B** 2 What operation is performed first in  $6 \times (12 - 5) + 8^2 \div 4$ ?  
**A**  $12 - 5$  **B**  $8^2$   
**C**  $6 \times 12$  **D**  $64 \div 4$
- 1C** 3 What missing value in  $\frac{7}{9} = \frac{49}{\quad}$  will produce an equivalent fraction?  
**A** 51 **B** 63 **C** 81 **D** 441
- 1D** 4 What is the lowest common denominator for  $\frac{3}{8} - \frac{1}{6}$ ?  
**A** 48 **B** 24 **C** 16 **D** 2
- 1D** 5 What is  $\frac{9}{10} \div 1\frac{1}{5}$ ?  
**A**  $\frac{9}{2}$  **B**  $\frac{27}{25}$  **C**  $\frac{3}{4}$  **D**  $\frac{21}{10}$
- 1E** 6 What is 3.125 84 rounded to three decimal places?  
**A** 3.125 **B** 3.13  
**C** 3.128 **D** 3.126
- 1F** 7 How many decimal places are there in the answer to  $26.018 \times 15.4$ ?  
**A** 1 **B** 3 **C** 4 **D** 8
- 1G** 8 Which number is an example of a recurring decimal?  
**A** 32.85 **B** 6.666...  
**C** 7.757 575 **D** 3.654 864 321...
- 1H** 9 What is the square root of 36 equivalent to?  
**A**  $\sqrt{36}$  **B**  $\sqrt{6}$  **C**  $36^2$  **D**  $6^2$
- 1I** 10 Which of these is the simplest form of  $3^4 \times 3^5$ ?  
**A**  $3^1$  **B**  $9^9$  **C**  $3^{20}$  **D**  $3^9$

## SHORT ANSWER

- 1A ▶ 1** Round each number to its leading digit.
- a** 236                      **b** 67 145  
**c** 3890                    **d** 149 046
- 1A ▶ 2** Estimate the result by first rounding each number to its leading digit.
- a**  $12\,345 + 3648$    **b**  $94\,501 - 32\,566$   
**c**  $394 \times 338$         **d**  $18\,654 \div 425$
- 1B ▶ 3** Calculate:
- a**  $33 + 40 \times 3 \div 10$   
**b**  $75 - 4 \times 11 + 6^2 \div 4$   
**c**  $10 \times 7 - 5 \times (4^2 - 4)$   
**d**  $120 \div [9 + (4 \times 8 - 11)]$
- 1C ▶ 4** Write each fraction in its simplest form.
- a**  $\frac{35}{60}$    **b**  $\frac{30}{16}$    **c**  $3\frac{18}{32}$    **d**  $5\frac{48}{72}$
- 1D ▶ 5** Calculate:
- a**  $\frac{3}{7} + \frac{2}{5}$                       **b**  $\frac{11}{15} - \frac{3}{10}$   
**c**  $\frac{12}{25} \times \frac{5}{14}$                     **d**  $\frac{7}{18} \div \frac{7}{9}$
- 1D ▶ 6** Calculate:
- a**  $5\frac{2}{3} + \frac{4}{11}$                       **b**  $6\frac{2}{5} - 3\frac{4}{7}$   
**c**  $6\frac{1}{4} \times 3\frac{2}{5}$                       **d**  $3\frac{1}{2} \div 1\frac{5}{9}$
- 1E ▶ 7** Write in ascending order.
- a** 0.123, 0.120, 0.213, 0.0012, 0.231, 0.2  
**b** 8.916, 9.6, 8.16, 8.9, 9.16, 9.916
- 1E ▶ 8** Write in descending order.
- a** 0.017, 0.7, 0.71, 0.0701, 0.717, 0.107  
**b** 2.127, 2.1, 2.721, 2.27, 2.72, 2.7
- 1E ▶ 9** Round each decimal to the number of decimal places shown in brackets.
- a** 13.216 51 (2)   **b** 0.001 215 (4)  
**c** 54.545 545 (3)  
**d** 181.182 154 136 (1)
- 1E ▶ 10** Calculate:
- a**  $23.56 + 45.98 + 74.13$   
**b**  $53.15 - 29.67$   
**c**  $123.5 + 46.12 + 9.0142 + 87.15$   
**d**  $1284.068 - 294.4687$
- 1F ▶ 11** Calculate:
- a**  $6.3128 \times 4$                       **b**  $65.589 \div 6$   
**c**  $3.46 \times 5.7$                       **d**  $468.268 \times 0.89$
- 1G ▶ 12 a** Decide whether each decimal is terminating or non-terminating.
- i** 0.123 123 123  
**ii** 6.562 562...  
**iii** 5.5  
**iv** 3.645 841 235...
- b** Which are recurring decimals?
- 1G ▶ 13** Write each decimal in short form using dots or a horizontal line.
- a** 0.333 333...                      **b** 5.385 385...  
**c** 45.685 757 575...  
**d** 0.092 451 451...
- 1G ▶ 14** Write each to show 12 decimal places.
- a** 0.23                                  **b**  $3.\overline{27}$   
**c**  $46.358\dot{7}$                               **d**  $17.364\overline{85}$
- 1H ▶ 15** Find the value of these.
- a**  $6^3$                                       **b**  $2^7$   
**c** 8 squared                              **d**  $11^2$
- 1H ▶ 16** Find the value of these.
- a** square root of 4                      **b**  $\sqrt{81}$   
**c** cube root of 27                      **d**  $\sqrt[3]{125}$
- 1H ▶ 17 a** Use a calculator to find these.
- i**  $\sqrt{529}$                                   **ii**  $\sqrt{700}$   
**iii**  $\sqrt{156.25}$                               **iv**  $\sqrt[3]{42}$
- b** Classify each answer as rational or irrational.
- 1I ▶ 18** Use an index law to simplify these.
- a**  $3^5 \times 3^7$    **b**  $2^7 \times 2^3$    **c**  $6^8 \div 6^2$   
**d**  $5^7 \div 5^7$    **e**  $(9^5)^3$    **f**  $4^6 \times 4^2 \times 4^3$
- 1I ▶ 19** Simplify these using index laws and then calculate the result.
- a**  $\frac{7^6 \times 7^5}{7^9}$                                   **b**  $\frac{2^8 \times (2^4)^2 \times 2^3}{2^{14}}$

## NAPLAN-STYLE PRACTICE

- 1 There are 26 845 people at a soccer match. What approximate number could be used to describe the crowd if you round to the leading digit?
- 
- 2 There are approximately 20 000 000 cattle in Australia. Which of these could *not* be the exact number of cattle?
- ☐ 15 674 806      ☐ 19 058 233  
☐ 24 947 321      ☐ 25 004 784
- 3 After rounding each number to its leading digit, what is the estimate for  $33\,457 \div 167$ ?
- 150      200      300      400  
☐      ☐      ☐      ☐
- 4 What is  $4 \times 15 - 5 \times (3^2 + 2)$ ?
- 
- 5 What is the missing number in these equivalent fractions?  $\frac{40}{72} = \frac{\quad}{9}$
- 
- 6 What is  $\frac{36}{22}$  written in simplest form?
- $\frac{11}{18}$        $\frac{9}{11}$        $1\frac{7}{11}$        $2\frac{4}{7}$   
☐      ☐      ☐      ☐
- 7 What is the lowest common denominator for  $\frac{13}{15} + \frac{3}{5}$ ?
- 
- 8 What is  $3\frac{3}{5} \times 4\frac{4}{9}$ ?
- $3\frac{1}{15}$        $\frac{4}{15}$        $12\frac{4}{15}$       16  
☐      ☐      ☐      ☐
- 9 What is  $\frac{5}{21} \div 1\frac{11}{14}$ ?
- $\frac{125}{294}$        $\frac{2}{15}$        $\frac{70}{121}$        $2\frac{1}{42}$   
☐      ☐      ☐      ☐
- 10 How many decimal places does 125.036 04 have?
- 
- 11 Which list is in descending order?
- ☐ 9.209, 9.82, 8.902, 8.029, 8.29, 8.2  
☐ 9.82, 9.209, 8.902, 8.29, 8.2, 8.029  
☐ 9.209, 8.902, 8.029, 9.82, 8.29, 8.2  
☐ 8.029, 8.2, 8.29, 8.902, 9.209, 9.82
- 12 What is 39.4951 correct to two decimal places?
- ☐ 39.50      ☐ 39.40  
☐ 39.410      ☐ 39.49
- 13 What is  $26.48 + 348.8 + 31.457$ ?
- ☐ 405.513      ☐ 406.017  
☐ 406.737      ☐ 91.593
- Questions 14–16 refer to this information.  
Chocolate bars cost \$2.95. Maria has \$30.
- 14 How many chocolate bars could Maria buy?
- 8      9      10      11  
☐      ☐      ☐      ☐
- 15 What is the cost of eight chocolate bars?
- 
- 16 How much change would Maria receive if she bought eight chocolate bars?
- 
- 17 What is  $3.84 \times 6.7$ ?
- ☐ 49.92      ☐ 25.728  
☐ 4.992      ☐ 257.28
- 18 Which is the best buy: a 2-L bottle of soft drink for \$5.30 or a 1.25-L bottle for \$3.25?
- 
- Questions 19–22 refer to these numbers:  
 $\frac{1}{4}$ , 0.763 491...,  $\frac{2}{3}$ ,  $\frac{3}{5}$ ,  $\frac{1}{8}$ , 8.26,  $\frac{9}{11}$ .
- 19 Which can be written as terminating decimals?
- 
- 20 Which can be written as non-terminating decimals?
- 
- 21 Which can be expressed as recurring decimals?
- 
- 22 Which are irrational?
-

- 23 What is 8 squared?

16      2.8       $\sqrt{8}$       64  
                 

- 24 What is the square root of 144?

- 25 What is 6 cubed?

6      18      36      216  
                 

- 26 What is the cube root of 64?

- 27 If a square piece of paper has area  $100 \text{ cm}^2$ , how long is the piece of paper?

- 28 Which of these does *not* simplify to  $5^6$ ?

☐  $\frac{5^9 \times 5^5}{5^8}$       ☐  $\frac{5^4 \times 5^8}{5^5 \times 5^7}$   
☐  $(5^2)^3$       ☐  $5^2 \times 5^3 \times 5^1$

Questions 29 and 30 refer to this table.

$4^2$	$4^3$	$4^4$	$4^5$	$4^6$	$4^7$
16	64	256	1024	4096	16 384

- 29 Find  $16\,384 \div 256$  without using a calculator.

- 30 Find  $\frac{1024 \times 64}{4096}$  without using a calculator.

- 31 Which of these is an irrational number?

$\frac{1}{3}$        $\sqrt{9}$        $\sqrt{3}$        $3^7$   
☐      ☐      ☐      ☐

## ANALYSIS

- 1 The population figures (rounded to the nearest 100) for each state and territory of Australia at the end of December 2010 are displayed in this table.

State or territory	Population
New South Wales	7 272 200
Victoria	5 585 600
Queensland	4 548 700
Western Australia	2 317 100
South Australia	1 650 400
Tasmania	509 300
Australian Capital Territory	361 900
Northern Territory	229 900

- a Determine the actual difference in population between NSW and Victoria.  
 b Round each population figure to the leading digit.  
 Use your answers to part b for these parts.  
 c Estimate the difference in population between NSW and Victoria. Compare this to part a.  
 d Estimate the population of Australia.

- e Write these populations as fractions in simplest form.  
 i Victoria as a fraction of NSW  
 ii Tasmania as a fraction of Queensland  
 iii NSW as a fraction of WA  
 iv Queensland as a fraction of Victoria  
 f Write each answer to part e as a decimal number then round to three decimal places.  
 g Which of your answers to part f are:  
 i terminating decimals?  
 ii non-terminating decimals?  
 iii recurring decimals?
- 2 The population of a town is 10 000 and is expected to double every 5 years.  
 a The town fits within a square of area  $9 \text{ km}^2$ . How long is the square?  
 b Use indices to find the population after 30 years.  
 c It is predicted the town will fit within a square of area  $85 \text{ km}^2$  after 30 years, how far is it along one side of this square?  
 d Classify each of the answers as rational or irrational. Explain your decision.



# CONNECT

## Owning a pet

Owning a pet is very rewarding, but owners have the responsibility of caring for the animal. Pet owners need to consider the type and quantity of food their pet needs, the amount of exercise, sleeping arrangements and health care. Your task is to investigate the costs involved.

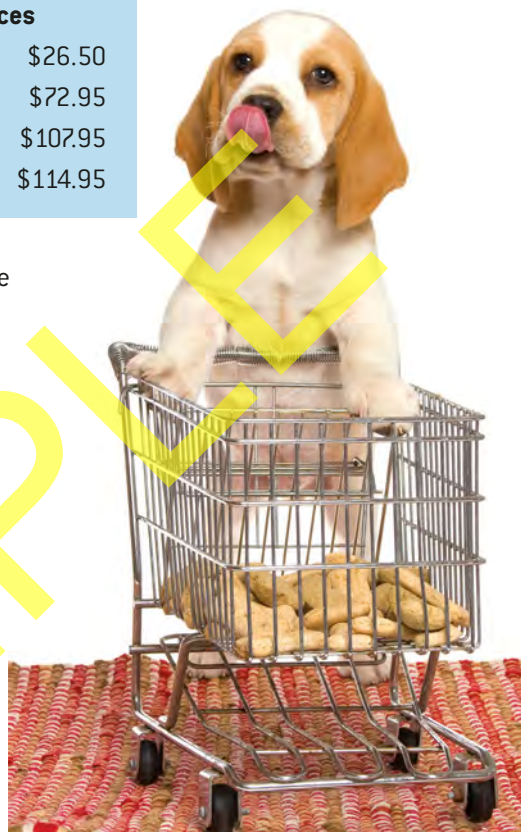
If you don't have a pet, you can refer to the information provided. Choose the type and size of pet you would like. (Even if you do have a pet, you may like to use this information as a starting point for your investigation.)

Here are some recommended food requirements for dogs and cats; however, these can vary depending on how active your pet is.

### Dog food prices

2.25 kg	\$26.50
7.5 kg	\$72.95
13.6 kg	\$107.95
16 kg	\$114.95

Weight of dog (kg)	Examples of dog breeds	Amount of dry food per day (g)
0–5	Chihuahua, Yorkshire terrier, Toy poodle	50–100
5–10	Miniature poodle, Scottish terrier	100–200
10–25	Cocker spaniel, Beagle	200–400
25–35	Collie, Boxer, Labrador, Golden Retriever	400–500
over 35	Great Dane, St Bernard	500–850



### Your task

To investigate the costs of owning a pet, carry out these steps.

- Work out the appropriate amount and type of food for your pet
- Investigate the cost of feeding your pet
- Explain how you can decide which is the best buy when buying food
- Calculate the size of bedding (or cage or aquarium) required
- Investigate the amount of exercise your pet requires
- Calculate the average amount of exercise your pet has in a week
- Consider the costs in health care for your pet, including preventative measures (such as worm, tick and flea control)
- Estimate the yearly cost of caring for a pet.

Include all necessary working to justify your answers and identify whether you have obtained exact or approximate results.

Weight of cat (kg)	Amount of dry food per day (g)
2	30–45
3	40–60
4	50–70
5	60–85
6	70–100
7	80–100

**Dry cat-food prices**

1.5 kg	\$22.95
6 kg	\$64.95
9 kg	\$84.95



The number of fleas on a pet can double in about one week!



Annual check up \$70

Complete the **1 CONNECT** worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a guide to owning a pet
- a slideshow or video
- a short presentation
- other (check with your teacher).

