



# Drawing and building solids

**This chapter deals with three-dimensional solids.**

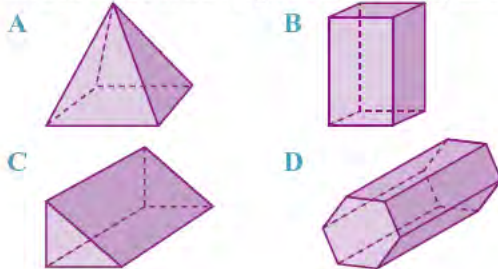
**At the end of this chapter you should be able to:**

- ▶ represent three-dimensional objects in two dimensions from different views
- ▶ identify and draw the cross-section of a prism
- ▶ determine whether a solid has a uniform cross-section
- ▶ visualise, construct and draw various prisms from a given cross-sectional diagram.



# Diagnostic test

- 1 Which of the following solids is *not* a prism?



- 2 The geometrical name that best describes the shape of a matchbox is:

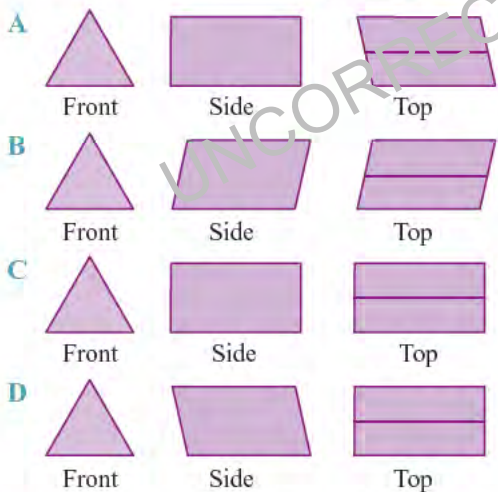
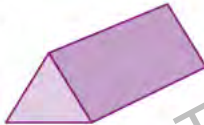
A rectangular pyramid B square prism  
C sphere D rectangular prism

- 3 The solids that have been combined to make this object are:

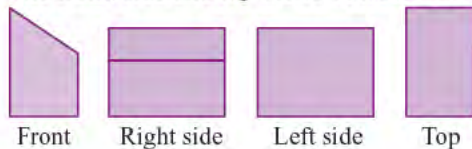
A a cone and a rectangular prism  
B a pyramid and a hemisphere  
C a pyramid and a cylinder  
D a cone and a cylinder



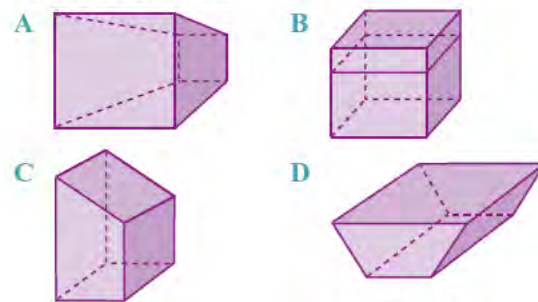
- 4 The view of this solid from the front, side and top is:



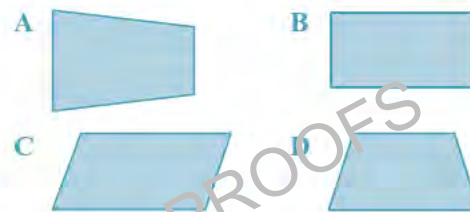
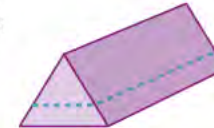
- 5 The front, side and top views of a solid are shown.



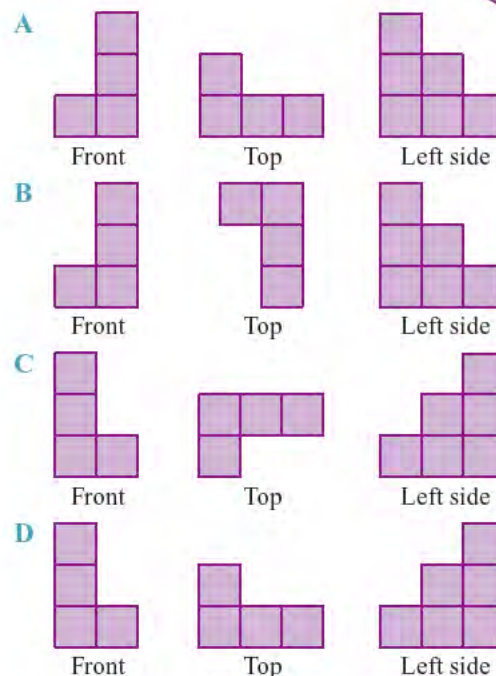
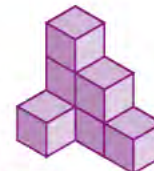
This solid could be:



- 6 The cross-section when this solid is sliced along the dashed line is:



- 7 The views of this solid from the front, top and left side are:



The Diagnostic Test questions refer to the sections of text listed in the table below.

Question	1–3	4, 5	6	7
Section	A	B	C	D





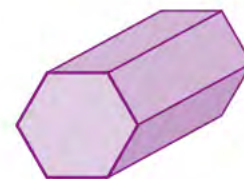
## Identifying solids

A **plane** (flat) shape with all sides straight is called a **polygon**. A solid that has only polygonal surfaces is called a **polyhedron** (plural is **polyhedra**).

- Solids that have identical polygonal ends joined by rectangular faces are known as **prisms**. The flat end is sometimes called the **base**.

Prisms are named according to the shape of their flat ends.

Shape 1 is called a hexagonal prism because its ends are identical hexagons, which are joined by rectangular faces.

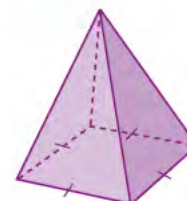


Shape 1

- Solids that have a polygon for the base and triangular faces meeting at a point are called **pyramids**.

Pyramids are named according to the shape of their base.

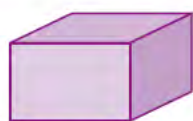
Shape 2 is called a square pyramid because its base is a square.



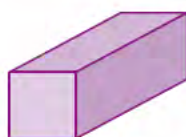
Shape 2

### Summary of common solids and their names

#### Prisms



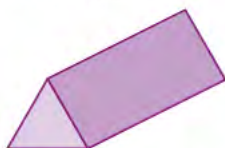
Rectangular prism



Square prism



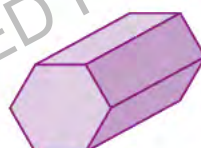
Cube



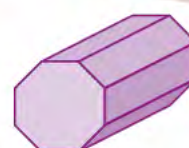
Triangular prism



Pentagonal prism



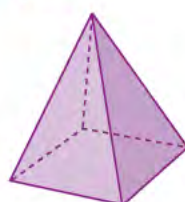
Hexagonal prism



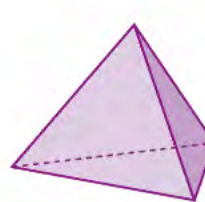
Octagonal prism



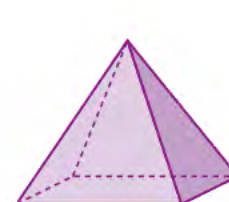
#### Pyramids



Square pyramid



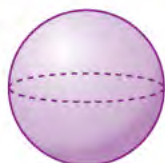
Triangular pyramid



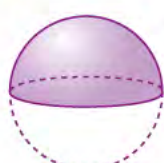
Rectangular pyramid



#### Solids with curved surfaces



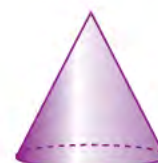
Sphere



Hemisphere



Cylinder



Cone





## Exercise 6A

1 What is the geometrical name of the shape that best describes a:

a netball?

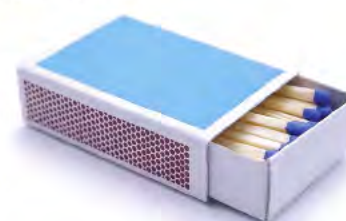
b soup can?

c matchbox?

d small tent?

e funnel?

f tetra juice pack?



2 Give an example of a solid with a shape resembling a:

a rectangular prism

b cone

c cylinder

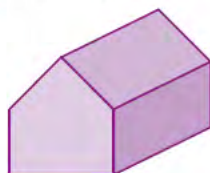
d cube

e pyramid

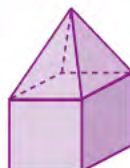
f hemisphere

3 List the solids that have been combined to make each of the following objects. They are sometimes referred to as **composite shapes**.

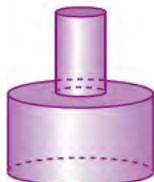
a



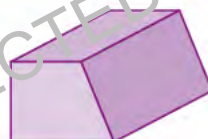
b



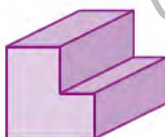
c



d



e



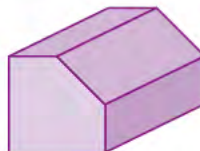
f



g



h



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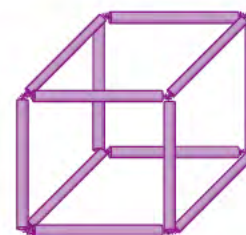
4 a **Skeletal models** of solids, like that of the cube shown, can be made using straws and pipe cleaners. Use this method to make a:

i rectangular prism

ii triangular prism

iii square pyramid

b Can you make a cylinder or sphere using this method? Explain your answer.





## Sketching solids

Here are some simple methods that can be used to draw three-dimensional shapes.

### EXAMPLE 1

Sketch a triangular prism on ordinary grid paper.

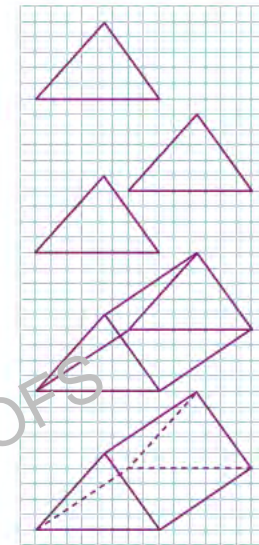
On ordinary grid paper the lines are at right angles. !

Step 1: Draw a triangle.

Step 2: Draw an identical triangle a short distance to the right of and up from the first.

Step 3: Join the corresponding corners of these triangles with straight lines.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.



### Exercise 6B

1 Using the method in Example 1, sketch a:

a triangular prism

b cube

c square prism

d rectangular prism

e pentagonal prism

f octagonal prism

2 Use the same method to copy and complete the following prisms whose front face is given.

a



b



c



d



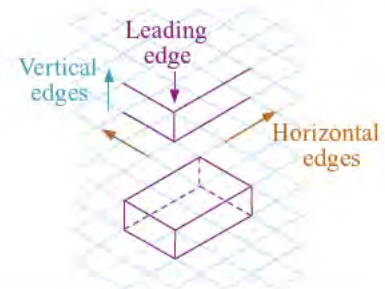
### EXAMPLE 2

Sketch a rectangular prism on **isometric** grid paper.

On isometric grid paper the lines are at  $60^\circ$  to each other. !

Step 1: Draw one vertical edge to be the leading edge. Then draw horizontal edges along the grid lines, away from the leading edge.

Step 2: Draw the further-away vertical edges and horizontal edges.





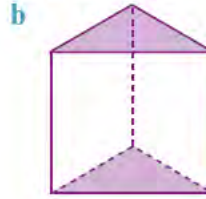
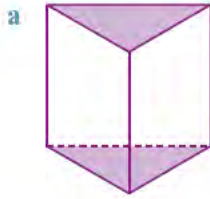
3 Using the method in Example 2, sketch a:

a rectangular prism

b cube

c square prism

4 Copy these sketches of a triangular prism onto isometric grid paper.



### EXAMPLE 3

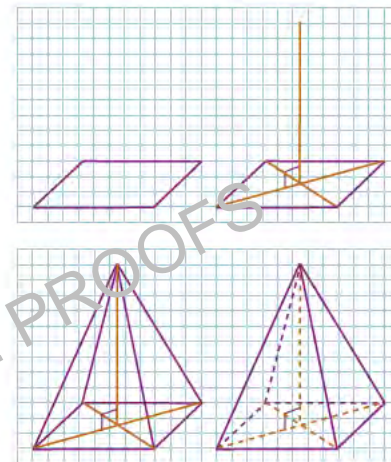
Sketch a rectangular pyramid on ordinary grid paper.

Step 1: Draw a parallelogram to represent the rectangular base.

Step 2: Lightly draw the diagonals of the parallelogram and find their point of intersection. Sketch a perpendicular line upward from the base at that point.

Step 3: Choose a point on the perpendicular line and join the corners of the parallelogram to that point.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.



5 Using the method in Example 3, sketch a square pyramid.

6 Follow the steps below to sketch a triangular pyramid on ordinary grid paper.

Step 1: Draw a triangle.

Step 2: Find the centre of the triangle and mark a point above it.

Step 3: Join this point to the three corners of the triangle.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.

You may like to use shading to emphasise the 3-dimensional nature of the solid.



### EXAMPLE 4

Sketch a cylinder on ordinary grid paper.

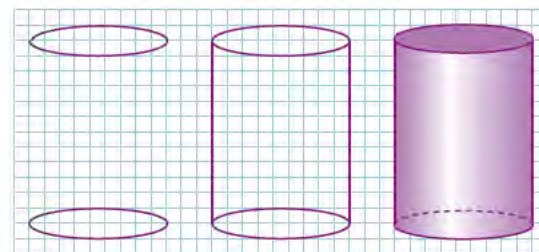
When drawing solids, circular faces are usually drawn as ellipses (ovals).



Step 1: Draw two identical ellipses, one directly above the other.

Step 2: Join the corresponding ends of the ellipses.

Step 3: Use dashed lines for all the edges of the solid that are hidden from view.



7 Using the method in Example 4, sketch a cylinder that is wider than it is high.

- 8 Follow the steps below to sketch a cone on ordinary grid paper.

Step 1: Draw an ellipse.

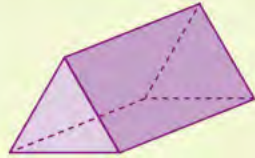
Step 2: Find the centre of the ellipse and mark a point above it.

Step 3: Join this point to the ends of the ellipse.

Step 4: Use dashed lines for all the edges of the solid that are hidden from view.



### EXAMPLE 5



For this solid draw the:

- a front view
- b side view
- c top view

Be careful as you orientate the top view. The top view should be as wide as the front view.

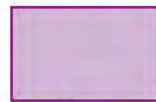


a



Front

b



Side

c



Top

- 9 For each solid given, draw the:

i front view

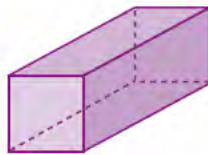
ii side view(s)

iii top view

If the right-side view and the left-side view are different, draw both.



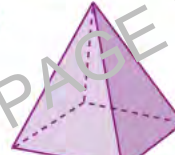
a



b



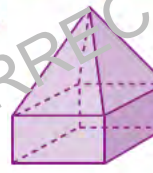
c



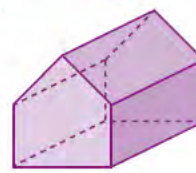
d



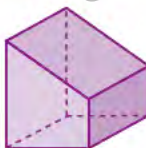
e



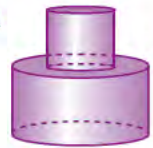
f



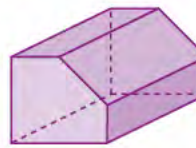
g



h



i

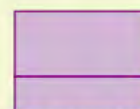


### EXAMPLE 6

Given the views of a solid, make a neat sketch of the solid.



Front



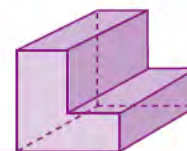
Right side



Left side

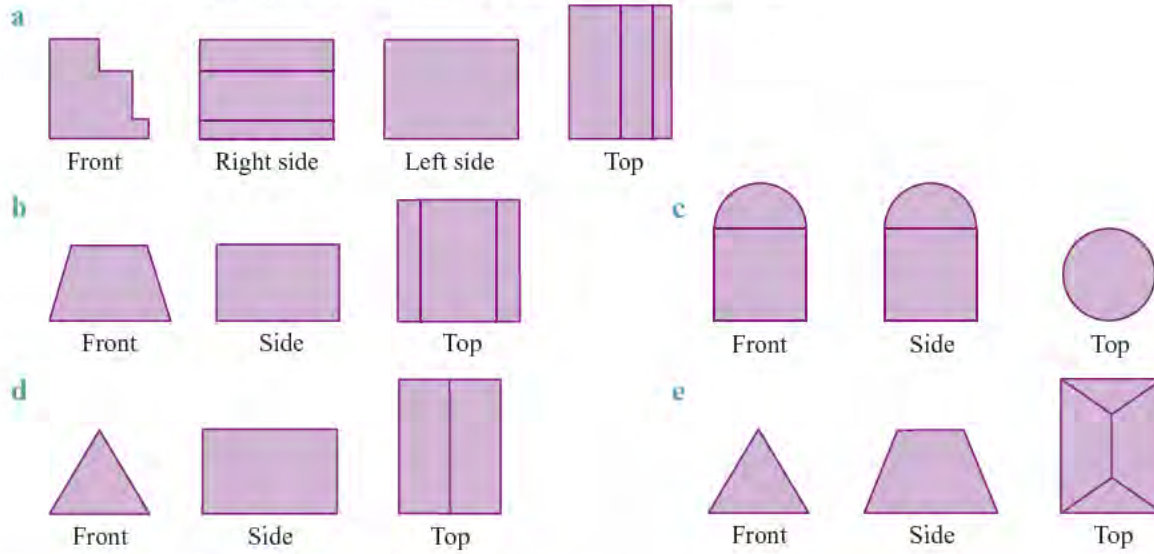


Top



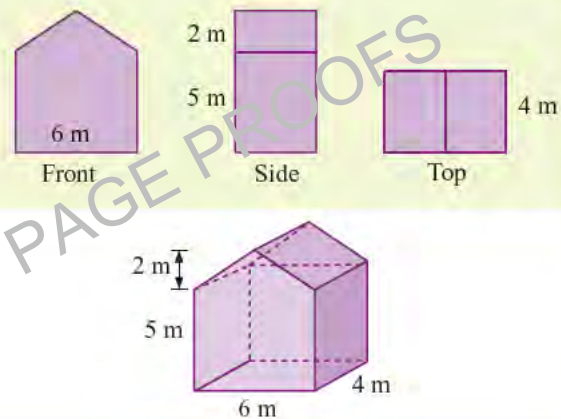


10 Given the following views of solids, make a neat sketch of each solid.

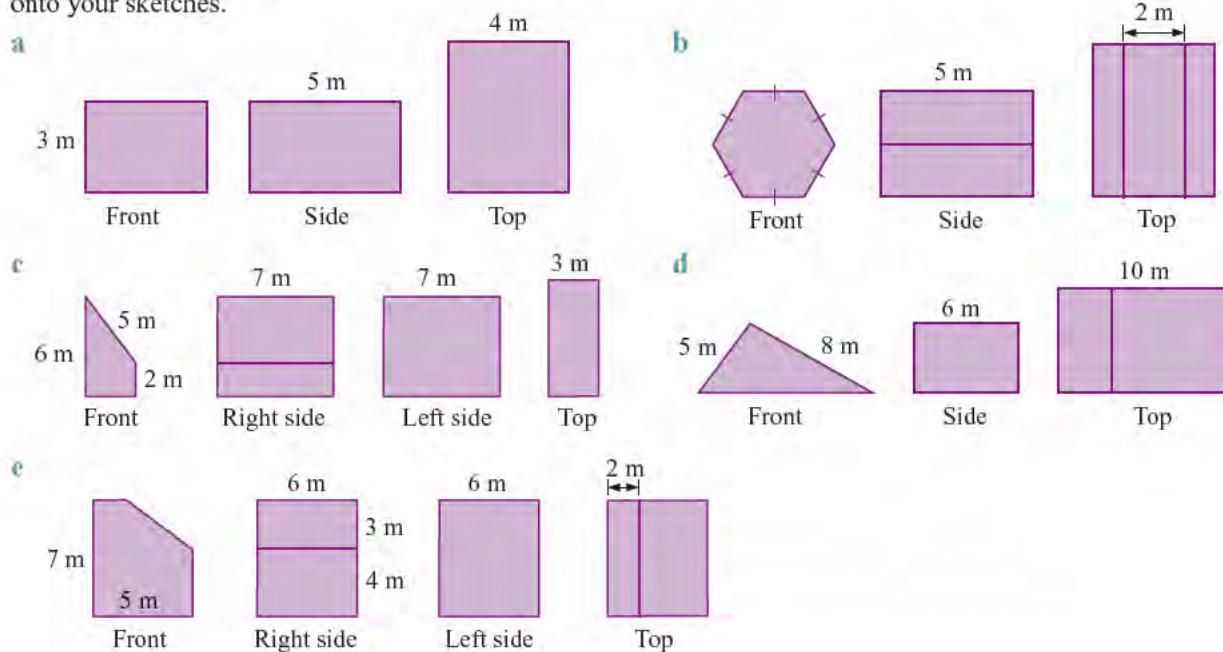


### EXAMPLE 7

Given the following views of a solid, make a neat sketch of the solid, transferring the dimensions given onto your sketch.



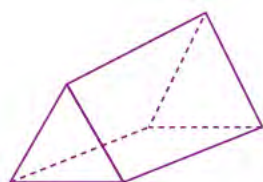
11 Given the following views of solids, make a neat sketch of each solid, transferring the dimensions given onto your sketches.







## Solids and cross-sections



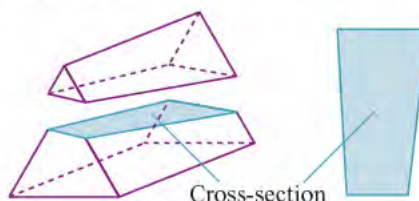
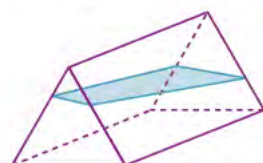
The **cross-section** of a solid is the shape of the surface formed when the solid is sliced through.

For example, consider this triangular prism.

Parallel flat faces  
never meet.

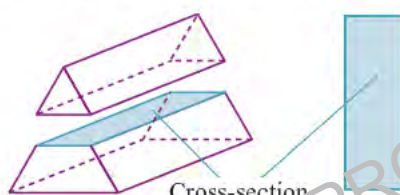
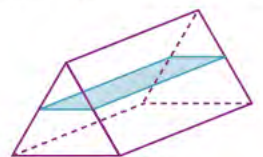


- When you take an oblique slice as shown, the resulting cross-section is a trapezium.



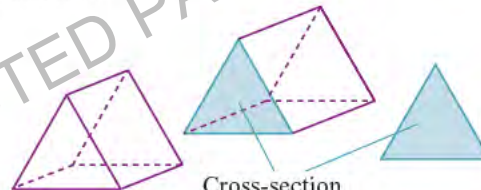
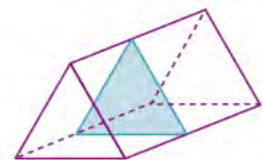
Cross-section

- When you take slices parallel to a side face, the cross-sections are rectangles of different sizes.



Cross-section

- When you take slices parallel to the front face (base), the cross-sections are identical triangles. This is called a **uniform (constant) cross-section**.



Cross-section

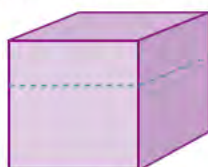
In all prisms, a face can be found such that all slices taken parallel to this face (the base) result in a uniform (the same) cross-section.

The prism is named according to the shape of the uniform cross-section.

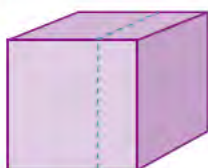
### Exercise 6C

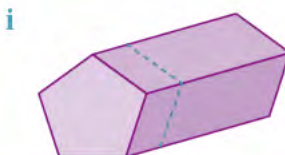
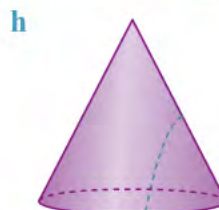
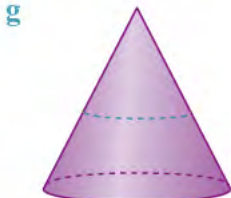
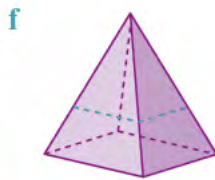
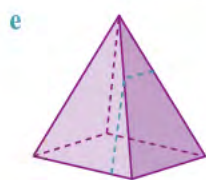
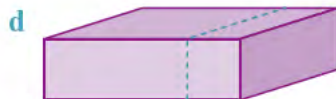
- Sketch the cross-sections when the following solids are sliced as shown.

a



b





- 2 a Which of the solids in question 1 have a uniform cross-section? That is, which are prisms?  
 b Name these prisms.
- 3 a Does a cylinder have a uniform cross-section? If so, sketch it.  
 b Is a cylinder a prism? Explain your answer.

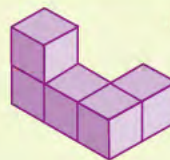


## D Building solids from cubes

Using cubes enables us to build solids without too much difficulty.

### EXAMPLE 1

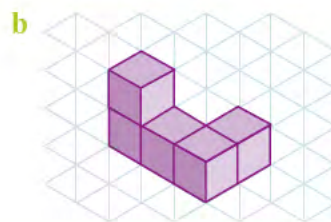
- a How many cubes are needed to build this solid?  
 b Sketch the solid on isometric grid paper.



It may be helpful to build a model of the solid.



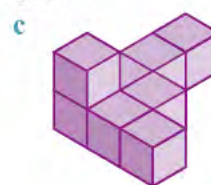
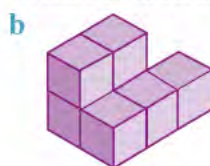
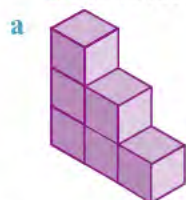
- a 5 cubes



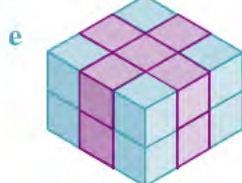
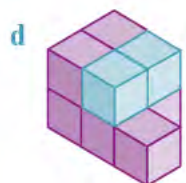
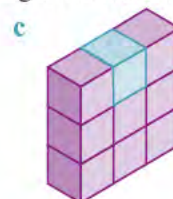
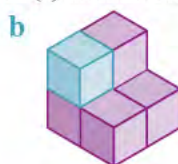
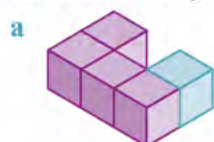


## Exercise 6D

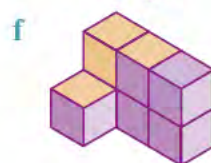
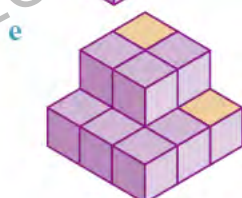
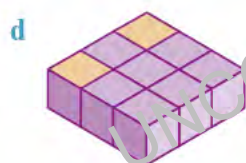
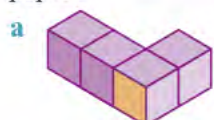
- 1 i How many cubes are needed to build each of the following solids?  
 ii Build the solid using cubes, then sketch the solid on isometric grid paper.



- 2 Build each solid, then *remove* the cube(s) shaded blue. Sketch the resulting solid on isometric grid paper.

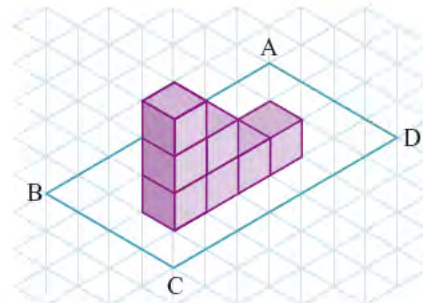
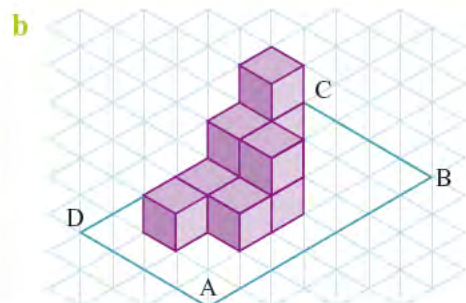
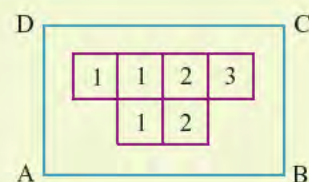


- 3 Build each solid, then *add* a cube to each face shaded orange. Sketch the resulting solid on isometric grid paper.

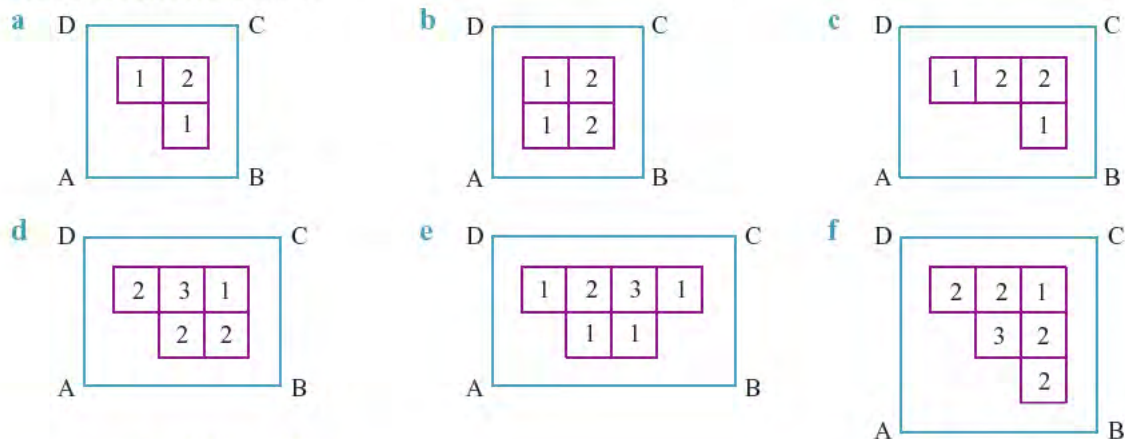


### EXAMPLE 2

- a Build a solid, using cubes, given the plan shown. The numbers indicate how many cubes there should be in each stack.  
 b Sketch, on isometric grid paper, the views of the solid from corners A and C.



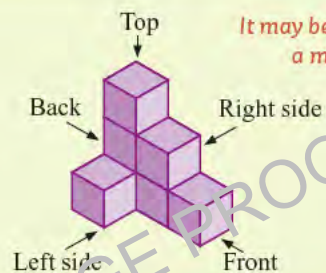
- 4 Build a solid, using cubes, for each of the following plans. Sketch, on isometric grid paper, the views of the solid from corners A and C.



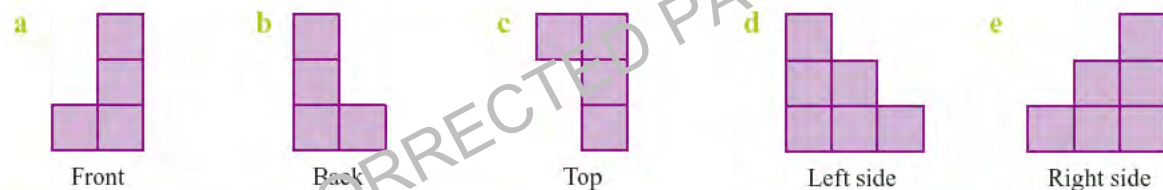
### EXAMPLE 3

Sketch the view of the solid shown from the:

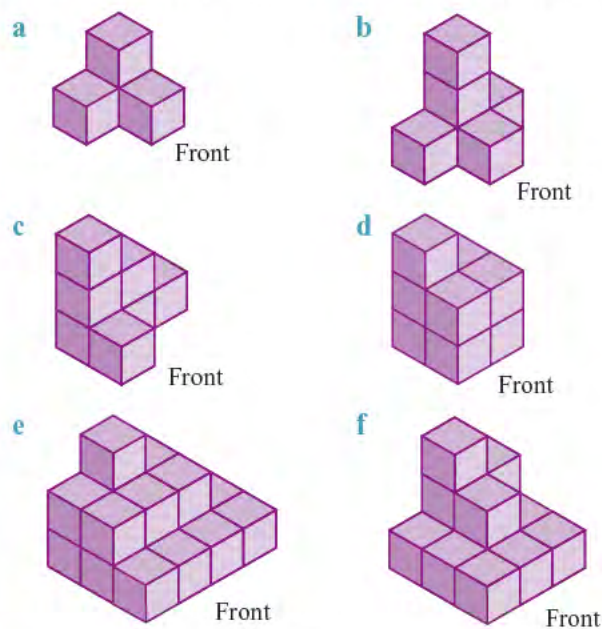
- a** front      **b** back      **c** top  
**d** left side      **e** right side



It may be helpful to build a model of the solid. !



- 5 For each of the solids drawn below, build the solid and sketch the view from the front, back, top, left side and right side.

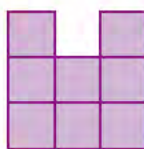




- 6 A solid was built from unit cubes. From the views given below, build the solid, then sketch the solid and mark the front.



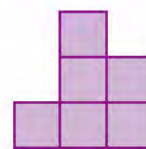
Front



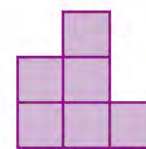
Back



Top



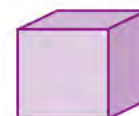
Left side



Right side

## Investigation 1 Cubes and cones

- 1 The Humble House factory manufactures cubic living quarters for use in central Australia where the conditions most of the year are dry and hot. Heat enters every roof and exposed wall by exactly the same amount. For a house made of one cube, heat enters in equal amounts from five sides, but not from the floor.



There are two possible house designs made from two cubes placed together:



Design 1



Second level

First level

Design 2



- How many exposed faces can be seen in Design 1 and Design 2? Which design would be more suitable for hot conditions?
  - Draw the four possible house designs using three cubes placed together. This investigation is best done with actual cubes. Remember, the blocks must touch face to face, and the building must be free-standing. (Columns for support are not acceptable; and in any case, you do not want heat to come in through the floor.)
  - Determine, from your models, the 'best' three-cube design to minimise heat intake.
  - Investigate possible four-cube buildings and determine the design that would take in the least amount of heat.
  - Are you game to try five-cube buildings? How many different five-cube designs are possible, and which one is 'best'?
- 2 Investigate the various shapes of the cross-sections of a cone, known as the **conic sections**.



## Language in mathematics

- 1 In the find-a-word puzzle below, find all the words in the following list:

circular, combination, composite, cone, constant, cube, cylinder, edge, ellipse, fold, hexagon, identical, isometric, link, pattern, pentagonal, plane, polygon, polyhedra, polyhedron, prism, pyramid, rectangular, right, sketch, solid, sphere, square, stack, straight, triangular, uniform, vertex, vertices, view.

S	P	P	G	H	A	N	O	I	T	A	N	I	B	M	O	C
P	O	L	Y	H	E	D	R	A	V	R	S	Q	U	A	R	E
H	L	A	L	K	C	E	X	W	E	I	V	F	A	P	E	V
E	Y	N	V	I	Y	N	J	I	R	G	G	Z	R	Y	C	Y
R	G	E	E	D	L	O	F	E	T	H	G	I	A	R	T	S
E	O	Z	R	S	I	C	U	B	E	T	S	Z	E	A	A	K
T	N	A	T	S	N	O	C	V	X	M	T	R	U	M	N	E
X	J	A	I	H	D	L	A	C	I	T	N	E	D	I	G	T
K	C	F	C	P	E	N	T	A	G	O	N	A	L	D	U	C
K	L	X	E	X	R	A	L	U	C	R	I	C	E	B	L	H
X	Y	B	S	E	N	O	R	D	E	H	Y	L	O	P	A	A
D	E	T	I	S	O	P	M	O	C	U	N	F	O	R	M	
Z	I	S	O	M	E	T	R	I	C	N	O	G	A	X	E	H
J	M	L	O	X	E	T	N	R	E	T	T	A	P	T	D	E
R	S	T	O	R	Q	P	R	A	L	U	G	N	A	I	R	T
S	T	V	E	S	P	I	L	L	E	L	O	N	O	G	O	N

- 2 Complete the given sentences using words from the following list:

polygon, point, identical, cross section, prism, triangular, uniform.

- a A solid that has \_\_\_\_\_ polygonal ends joined by rectangular faces is called a \_\_\_\_\_.
- b Prisms are named according to the shape of their \_\_\_\_\_.
- c Solids that have a \_\_\_\_\_ for the base and \_\_\_\_\_ faces meeting at a \_\_\_\_\_ are called pyramids.

- 3 a Look up 'isometric' in a dictionary. What does 'iso' mean?

- b Look up 'hemisphere' in a dictionary. What does 'hemi' mean?

- 4 Use a dictionary to find two different meanings for each of the following words:

- a solid                      b uniform                      c square                      d base

- 5 Write down the plural of polyhedron.

- 6 Three of the words in the following list are spelt incorrectly. Rewrite them with the correct spelling:  
cylinder, straght, circlar, view, right, skech.

- 7 Describe in your own words a:

- a cube                      b sphere                      c cone                      d square pyramid

- 8 Describe in your own words how to draw a triangular prism.

- 9 Explain the difference between a cube and a square prism.

- 10 How many words of three or more letters can you make from RECTANGULAR? (No proper nouns allowed.)



## Terms

cone	conic	composite	cube	cross-section	cylinder	ellipse
hemisphere	hexagon	horizontal	isometric	octagon	parallel	parallelogram
pentagon	plane	polygon	polyhedra	polyhedron	prism	pyramid
rectangle	solid	sphere	trapezium	uniform	vertical	view

## Check your skills

- 1 Which of the following solids is *not* a prism?



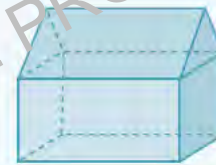
- 2 The geometrical name of this solid is a:

- A triangular prism      B triangular pyramid  
C square pyramid      D square prism

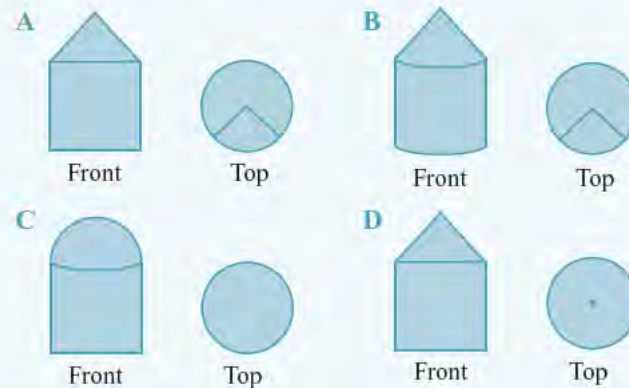


- 3 The solids that have been combined to make this object are:

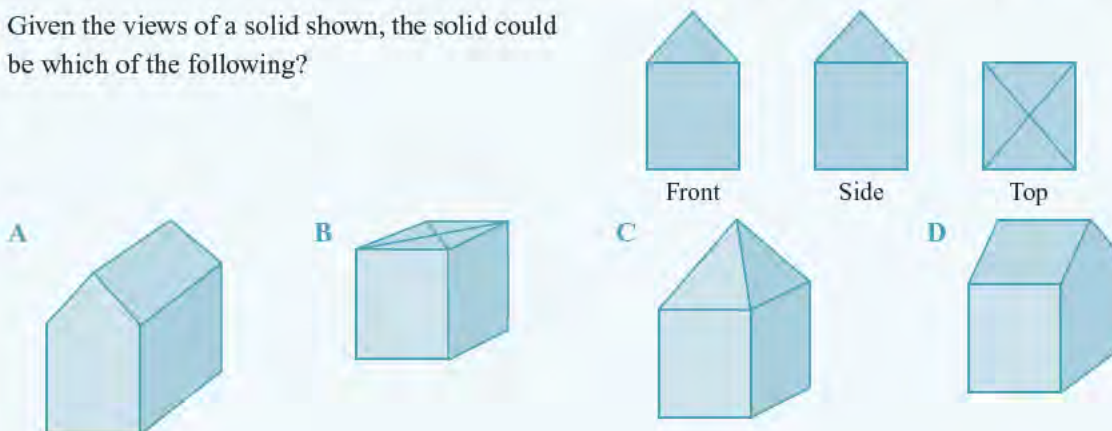
- A a triangular prism and a rectangular prism  
B a pyramid and a rectangular prism  
C a pyramid and a cube  
D a triangular prism and a rectangular pyramid



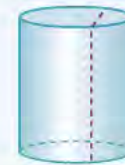
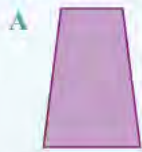
- 4 The views of the solid below from the front and top are:



- 5 Given the views of a solid shown, the solid could be which of the following?



6 The cross-section when this solid is sliced along the dashed line is:



7 The views of this solid from the front, top and left side are:



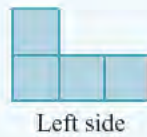
Front



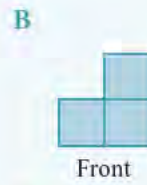
Front



Top



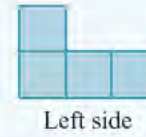
Left side



Front



Top



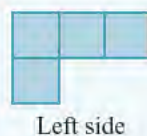
Left side



Front



Top



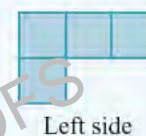
Left side



Front



Top



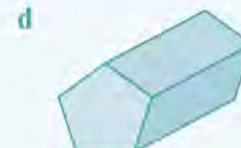
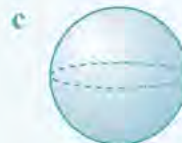
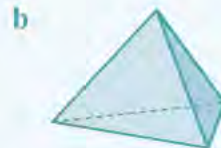
Left side

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4, 5	6	7
Section	A	B	C	D

## 6A Review set

1 Name the solids below.



2 Sketch a hexagonal prism.

3 Sketch the views of the following solids from the front, side(s) and top.



4 Given the following views of solids, make a neat sketch of each solid.



Front



Side



Top



Front



Side

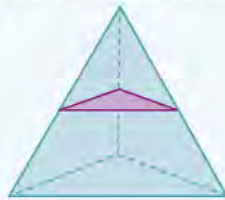


Top



- 5 Sketch the cross-section when each solid is sliced as shown.

a

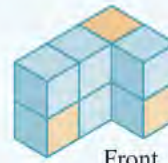


b



- 6 a How many cubes are needed to build the given solid?

b Build the solid and sketch it on isometric paper.



Front

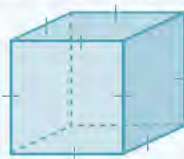
- 7 a For the solid in question 6, add a cube to each of the faces shaded orange and sketch the resulting solid on isometric paper.

b Sketch the views of the resulting solid from the front, back, top, left side and right side.

## 6B Review set

- 1 Name the solids below.

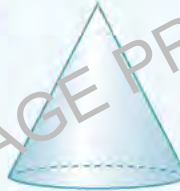
a



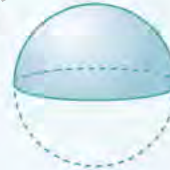
b



c



d



- 2 Sketch a cylinder that is lying with its end faces vertical.

- 3 Sketch the views of the following solids from the front, side and top.

a



b



- 4 Given the following views of solids, make a neat sketch of each solid.

a



Front



Side



Top

b



Front



Side

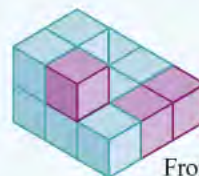


Top

- 5 a How many cubes are needed to build the given solid?

b Build the solid and sketch it on isometric grid paper.

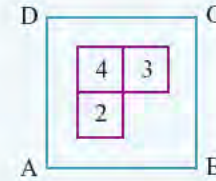
c Sketch the views of the solid from the front, back, top, left side and right side.



Front

- 6 For the solid in question 5, remove the cubes shaded purple and sketch the resulting solid on isometric grid paper.

- 7 The numbers on the plan give the number of cubes in each stack of a solid. Build the solid and sketch, on isometric grid paper, the view of the solid from corner B.



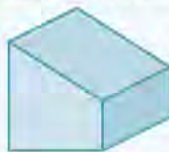
## 6C Review set

- 1 Name the common solids that have been combined to make each of the following solids.

a



b



c



d



- 2 Explain the differences between:

a a cylinder and a prism

b a cone and a pyramid

- 3 Sketch the views of each solid in question 1 from the front, side(s) and top.

- 4 Given the following views of solids, make a neat sketch of each solid.

a



Front



Right side



Left side



Top

b



Front



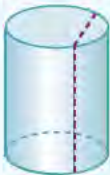
Side



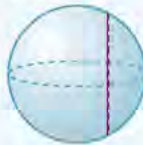
Top

- 5 Sketch the cross-section when each solid is sliced as shown.

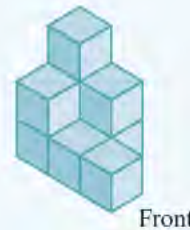
a



b

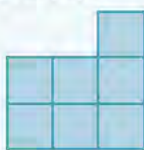


- 6 Build the given solid and then sketch the views from the front, back, top, left side and right side.



Front

- 7 A solid was built from cubes. From the views given below, build the solid and sketch it on isometric grid paper.



Front



Back



Top



Left side



Right side