

# OXFORD Study Buddy

## Revision and Exam Guide

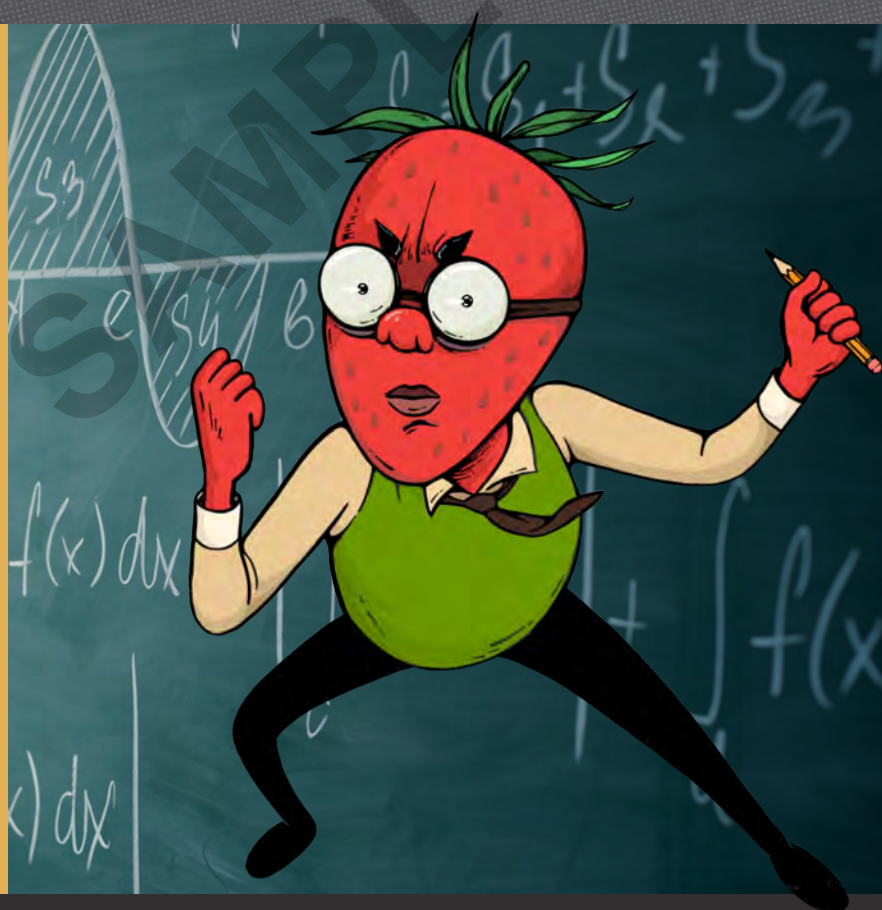
# QCE MATHEMATICAL METHODS

UNITS 3 & 4

VOLUME 1

WRITTEN BY  
PETER ANTROBUS

ILLUSTRATED BY  
JOSHUA SCOTT



OXFORD

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means

book  
essess

**OXFORD**  
UNIVERSITY PRESS

Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trademark of Oxford University Press in the UK and in certain other countries.

Published in Australia by  
Oxford University Press  
Level 8, 737 Bourke Street, Docklands, Victoria 3008, Australia.

© Oxford University Press 2021

The moral rights of the author/s have been asserted

First published 2021

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, by licence, or under terms agreed with the reprographics rights organisation. Enquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above.

You must not circulate this work in any other form and you must impose this same condition on any acquirer.



A catalogue record for this  
book is available from the  
National Library of Australia

ISBN 9780190326555

### **Reproduction and communication for educational purposes**

The Australian *Copyright Act 1968* (the Act) allows educational institutions that are covered by remuneration arrangements with Copyright Agency to reproduce and communicate certain material for educational purposes. For more information, see [copyright.com.au](http://copyright.com.au).



Edited by Michael Assis  
Illustrated by Joshua Scott Designs  
Logo design by Tuumik Ststudio OÜ  
Typeset by Newgen KnowledgeWorks Pvt. Ltd., Chennai, India  
Proofread by Monica Schaak  
Printed in Singapore by Markono Print Media Pte Ltd

*Links to third party websites are provided by Oxford in good faith and for information only. Oxford disclaims any responsibility for the materials contained in any third party website referenced in this work.*



# CONTENTS

<i>Welcome to Oxford Study Buddy.....</i>	<i>ix</i>
<i>Introducing your Study Buddy: Professor Ray Díaz .....</i>	<i>xv</i>
<i>Ray's berry bad day: The origin story of Professor Ray Díaz .....</i>	<i>xvi</i>

## CHAPTER 1 STUDYING FOR SUCCESS ..... 3

1.1 Overview of QCE Mathematical Methods Units 3 & 4.....	4
1.2 Everything you need to know about the summative external assessment.....	7
1.3 How to maximise your marks on the external assessment .....	18
1.4 How to get the most out of your revision and exam preparation time.....	21
1.5 Countdown to success.....	26

## CHAPTER 2 REVISION ..... 29

### Unit 3: Further calculus

2.1 Topic 1: The logarithmic function 2.....	30
• Logarithmic laws and logarithmic functions.....	30
2.2 Topic 2: Further differentiation and applications 2.....	38
• Calculus of exponential functions .....	38
• Calculus of logarithmic functions .....	43
• Calculus of trigonometric functions .....	45
• Differentiation rules .....	48
2.3 Topic 3: Integrals .....	51
• Anti-differentiation.....	51
• Fundamental theorem of calculus and definite integrals.....	54
• Applications of integration.....	58

### Unit 4: Further functions and statistics

2.4 Topic 1: Further differentiation and applications 3.....	63
• The second derivative and applications of differentiation.....	63

2.5	Topic 2: Trigonometric functions 2.....	70
	• Cosine and sine rules.....	70
2.6	Topic 3: Discrete random variables 2.....	78
	• Bernoulli distributions.....	78
	• Binomial distributions.....	79
2.7	Topic 4: Continuous random variables and the normal distribution.....	82
	• General continuous random variables.....	82
	• Normal distributions.....	85
2.8	Topic 5: Interval estimates for proportions.....	90
	• Random sampling.....	90
	• Sample proportions.....	92
	• Confidence intervals for proportions.....	93

## CHAPTER 3 PRACTICE QUESTIONS..... 97

### Unit 3: Further calculus

3.1	Topic 1: The logarithmic function 2.....	98
	Technology free.....	98
	• Multiple choice questions.....	98
	• Short response questions.....	101
	Technology active.....	103
	• Multiple choice questions.....	103
	• Short response questions.....	105
3.2	Topic 2: Further differentiation and applications 2.....	108
	Technology free.....	108
	• Multiple choice questions.....	108
	• Short response questions.....	112
	Technology active.....	115
	• Multiple choice questions.....	115
	• Short response questions.....	118
3.3	Topic 3: Integrals.....	122
	Technology free.....	122
	• Multiple choice questions.....	122
	• Short response questions.....	127
	Technology active.....	130
	• Multiple choice questions.....	130
	• Short response questions.....	132

### Unit 4: Further functions and statistics

3.4	Topic 1: Further differentiation and applications 3.....	138
	Technology free.....	138
	• Multiple choice questions.....	138
	• Short response questions.....	141
	Technology active.....	145
	• Multiple choice questions.....	145
	• Short response questions.....	147

3.5	Topic 2: Trigonometric functions 2.....	150
	Technology free.....	150
	• Multiple choice questions .....	150
	• Short response questions .....	153
	Technology active.....	155
	• Multiple choice questions .....	155
	• Short response questions .....	157
3.6	Topic 3: Discrete random variables 2.....	160
	Technology free.....	160
	• Multiple choice questions .....	160
	• Short response questions .....	162
	Technology active.....	164
	• Multiple choice questions .....	164
	• Short response questions .....	166
3.7	Topic 4: Continuous random variables and the normal distribution .....	169
	Technology free.....	169
	• Multiple choice questions .....	169
	• Short response questions .....	172
	Technology active.....	177
	• Multiple choice questions .....	177
	• Short response questions .....	180
3.8	Topic 5: Interval estimates for proportions.....	185
	Technology free.....	185
	• Multiple choice questions .....	185
	• Short response questions .....	187
	Technology active.....	189
	• Multiple choice questions .....	189
	• Short response questions .....	192

## CHAPTER 4 OFFICIAL PAST PAPERS ..... 197

4.1	<b>External assessment 2020: Mathematical Methods Paper 1 – Technology free</b>	
	• 10 multiple choice questions (10 marks), 10 short response questions (50 marks) .....	198
4.2	<b>External assessment 2020: Mathematical Methods Paper 2 – Technology active</b>	
	• 10 multiple choice questions (10 marks), 10 short response questions (50 marks) .....	208

## CHAPTER 5 ANSWERS ..... 219

### Unit 3: Further calculus

5.1	Topic 1: The logarithmic function 2.....	220
	Technology free.....	220
	• Multiple choice answers .....	220
	• Short response answers .....	221

	Technology active.....	223
	• Multiple choice answers .....	223
	• Short response answers .....	224
5.2	Topic 2: Further differentiation and applications 2.....	225
	Technology free.....	225
	• Multiple choice answers .....	225
	• Short response answers .....	230
	Technology active.....	232
	• Multiple choice answers .....	232
	• Short response answers .....	234
5.3	Topic 3: Integrals.....	235
	Technology free.....	235
	• Multiple choice answers .....	235
	• Short response answers .....	238
	Technology active.....	240
	• Multiple choice answers .....	240
	• Short response answers .....	241
<b>Unit 4: Further functions and statistics</b>		
5.4	Topic 1: Further differentiation and applications 3.....	244
	Technology free.....	244
	• Multiple choice answers .....	244
	• Short response answers .....	247
	Technology active.....	251
	• Multiple choice answers .....	251
	• Short response answers .....	253
5.5	Topic 2: Trigonometric functions 2.....	255
	Technology free.....	255
	• Multiple choice answers .....	255
	• Short response answers .....	257
	Technology active.....	258
	• Multiple choice answers .....	258
	• Short response answers .....	260
5.6	Topic 3: Discrete random variables 2.....	262
	Technology free.....	262
	• Multiple choice answers .....	262
	• Short response answers .....	264
	Technology active.....	265
	• Multiple choice answers .....	265
	• Short response answers .....	266
5.7	Topic 4: Continuous random variables and the normal distribution .....	267
	Technology free.....	267
	• Multiple choice answers .....	267
	• Short response answers .....	270

Technology active.....	273
• Multiple choice answers .....	273
• Short response answers .....	274
5.8 Topic 5: Interval estimates for proportions.....	277
Technology free.....	277
• Multiple choice answers .....	277
• Short response answers .....	278
Technology active.....	279
• Multiple choice answers .....	279
• Short response answers .....	282
5.9 <b>External assessment 2020 answers: Mathematical Methods Paper 1 – Technology free</b> .....	284
5.10 <b>External assessment 2020 answers: Mathematical Methods Paper 2 – Technology active</b> .....	289
<i>Ray's berry bad day: The redemption of Professor Ray Diaz</i> .....	294
<b>Appendix</b>	
<i>Mathematical Methods – Formula sheet</i> .....	297

## ACKNOWLEDGEMENTS

The author and the publisher wish to thank the following copyright holders for reproduction of their material.

Shutterstock, p.24, figure 1.

QCAA material reproduced from material from Mathematical Methods General Senior Syllabus 2019: Assessment, <https://www.qcaa.qld.edu.au/senior/senior-subjects/mathematics/mathematics-methods/assessment>, © State of Queensland ([qcaa.qld.edu.au](https://www.qcaa.qld.edu.au)) 2019, licensed under CC BY 4.0 <https://creativecommons.org/licenses/by/4.0/>

Every effort has been made to trace the original source of copyright material contained in this book. The publisher will be pleased to hear from copyright holders to rectify any errors or omissions.

SO THERE ARE A FEW PROBLEMS  
WITH MATHS PUNS... CALCULUS JOKES ARE  
ALL **DERIVATIVE**, TRIGONOMETRY JOKES ARE  
TOO **GRAPHIC**, ALGEBRA JOKES ARE USUALLY  
**FORMULAIC**!

NO TIME TO WORRY ABOUT THAT NOW  
THOUGH... THERE'S PLANNING TO BE DONE!



## CHAPTER

# 1

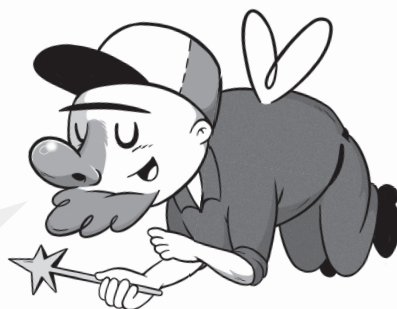
# STUDYING FOR SUCCESS

Before you start studying for your QCE Mathematical Methods exams, it's important to set yourself up for success. That's exactly what this chapter is designed to do, so thanks for stopping by!

As soon as it's time to start studying for the external assessment, we recommend that you work through this whole chapter before doing anything else!

You might do this at the start of the school year or at the start of your exam study period, but whatever you do, don't skip this chapter; it contains a bunch of really important information and tips that might just give you the edge you're looking for.

YOUR THIRST FOR  
KNOWLEDGE IS **CRYSTAL  
CLEAR!** COMPLETE THIS  
CHAPTER TO SCORE YOUR  
FIRST KNOWLEDGE CRYSTAL!  
GOOD LUCK!



# 1.1

## OVERVIEW OF QCE MATHEMATICAL METHODS UNITS 3 & 4

In this section, we will:

- provide a brief overview of how the QCE Mathematical Methods Units 3 & 4 course is structured
- list all of the concepts and topics that you will need to learn and understand
- explain how you will be assessed.



1.1.1

Resource:  
Mathematical Methods  
General Senior  
syllabus

### Study tip

The QCE Mathematical Methods General Senior Syllabus sets out all of the information you are expected to learn and also provides important information on how you will be assessed.

In this chapter, we have summarised all of the key information relating to external assessment you need to know, but the QCAA may update the syllabus from time to time, so it's important that you make sure you are using the most current version!

Make sure you visit the QCAA website and download a copy of the Mathematical Methods General Senior Syllabus and read the key information carefully before you sit your external assessment. To save you time, we've also included a link to it on your [obook assess!](#)

## UNDERSTANDING THE QCE MATHEMATICAL METHODS UNITS 3 & 4 COURSE STRUCTURE

The Mathematical Methods General Senior Syllabus is the most important document supporting the QCE Mathematical Methods course. It sets out all the content – known as subject matter – that you will be expected to learn and provides important information about how you will be assessed.

QCE Mathematical Methods is a course of study consisting of four units (i.e. Units 1 & 2 and Units 3 & 4) taught over 2 years, but in this revision and exam guide, we will only be focusing on information relating to Units 3 & 4 of the course. The topics you will be learning about in Units 3 & 4 are summarised in Table 1.

### Study tip

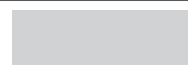
The notional hours shown in Table 1 are provided by the QCAA to help teachers with their planning and give them an estimate of how long to spend teaching the subject matter in each topic.

Notional hours can be a handy way to help you to structure and allocate your revision and preparation time for the external assessment because – as a general rule – there are likely to be more questions on subject matter with higher notional hours.





TIME FOR SOME  
REVISION! THERE'S  
PLENTY TO GET  
THROUGH, SO TRY TO  
STAY FOCUSED AND  
DON'T GO OFF ON  
A TANGENT!



## CHAPTER

# 2

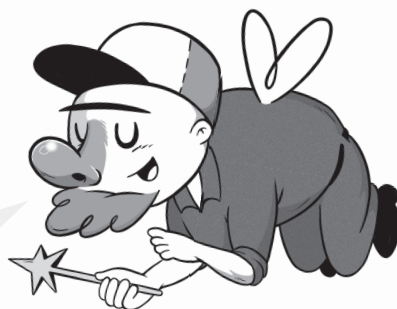
# REVISION

In this chapter, we provide a clear, concise summary of all examinable content from QCE Mathematical Methods Units 3 & 4. Everything has been organised by unit, topic and sub-topic in the General Senior Syllabus to help you focus your time and attention where it is needed most.

The revision notes are not designed to replace your teacher or your textbook. Instead, they have been designed to help you gauge your level of understanding and confidence of the subject matter before the exam. You can use them to identify those topics you know inside out and those that still require some extra attention.

The revision notes are also supported by a bunch of handy features, tips and icons designed to help you get the very best result on the day.

WE'RE ON THE SEARCH FOR  
YOUR SECOND KNOWLEDGE  
CRYSTAL. DON'T DOUBT  
YOURSELF, OF QUARTZ  
YOU CAN DO IT!



# 2.1

## UNIT 3 TOPIC 1 – THE LOGARITHMIC FUNCTION 2



**Questions**  
on pages 98–107

### LOGARITHMIC LAWS AND LOGARITHMIC FUNCTIONS

#### SUBJECT MATTER

By the end of this topic, you should be able to:

- establish and use logarithmic laws and definitions
- interpret and use logarithmic scales such as decibels in acoustics, the Richter scale for earthquake magnitude, octaves in music, pH in chemistry
- solve equations involving indices with and without technology
- recognise the qualitative features of the graph of  $\log_a(x)$  ( $a > 1$ ), including asymptotes, and of its translations  $y = \log_a(x) + b$  and  $y = \log_a(x + c)$
- solve equations involving logarithmic functions with and without technology
- identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

Modified from *Mathematical Methods General Senior Syllabus* 2019 v1.2,  
© State of Queensland (QCAA) 2019, licensed under CC BY 4.0

#### BACK TO BASICS

Logarithmic functions are the inverses of exponential functions:

$$a^x = b \text{ is equivalent to } x = \log_a(b)$$

The index laws

- $a^x \times a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $(a^m)^n = a^{mn}$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$



## KEY CONCEPT

## APPLYING LOGARITHMIC LAWS

Use the logarithmic laws to simplify logarithmic expressions and find the exact solution to indicial and logarithmic equations.

## Study tip

Remember that you can only apply Laws 1 and 2 to add and subtract logarithmic functions if they have the same base.

- $\log_a(x) + \log_a(y) = \log_a(xy)$
- $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$
- $\log_a(x^n) = n\log_a(x)$
- $\log_a(1) = 0$
- $\log_a(a) = 1$
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$




## WORKED EXAMPLE 2.1A

## SIMPLIFYING USING LOGARITHMIC LAWS

Questions	Breaking down the questions
<p><b>Simplify:</b></p> <p>a) <math>\log_2(144) - 2\log_2(6)</math> [1 mark]</p> <p>b) <math>\frac{\log_a(27)}{\log_a(243)}</math> [2 marks]</p>	<ul style="list-style-type: none"> <li><b>Simplify</b> requires you to use algebra to manipulate the expressions into a simpler form</li> <li>The presence of the <b>logarithmic</b> function in these expressions indicates that logarithmic laws are required to perform the simplification</li> </ul>
Solutions	Marking advice and tips
<p>a) <math>\log_2(144) - 2\log_2(6) = \log_2(144) - \log_2(6^2)</math>  <math>= \log_2(144) - \log_2(36)</math>  <math>= \log_2\left(\frac{144}{36}\right)</math>  <math>= \log_2(4)</math>  <math>= \log_2(2^2)</math>  <math>= 2\log_2(2)</math>  <math>= 2</math></p> <p>b) <math>\frac{\log_a(27)}{\log_a(243)} = \frac{\log_a(3^3)}{\log_a(3^5)}</math>  <math>= \frac{3\log_a(3)}{5\log_a(3)}</math>  <math>= \frac{3}{5}</math></p>	<p>a) This is a difference of two logarithms with the same base, so use Law 2 to combine the two logarithms into a single fraction. Before you can apply Law 2, move the coefficient of <math>\log_2(6)</math> into the argument using Law 3. Finally, express the argument in index form to eliminate the logarithm (Law 5) for the answer mark. [1 mark]</p> <p>b) Recognise that both 27 and 243 can be expressed in index form with a base of 3. [1 mark]          Use Law 3 to bring the indices down and cancel out <math>\log_a(3)</math> for the answer mark. [1 mark]</p>





I'D TELL YOU A  
JOKE ABOUT AN  
INFINITE LINE, BUT IT  
DOESN'T HAVE AN ENDPOINT!  
IN ANY CASE, IT'S REALLY  
TIME WE JUMPED INTO  
SOME PRACTICE  
QUESTIONS ... LET'S  
DO THIS!



## CHAPTER

# 3

# PRACTICE QUESTIONS

In this chapter, we provide a range of practice questions for all of the examinable content from QCE Mathematical Methods Units 3 & 4. What a surprise... not! I have a sneaking suspicion the title gave it away!

No fancy tricks here, we just provide over 300 questions organised by unit and topic so you can move effortlessly between revision and practice as you study. This will help you get the practice you need and build up your confidence!

To help you direct your time and effort where it's needed most, we've grouped the questions by type. Multiple choice and short response questions are also clearly labelled **Technology free** or **Technology active** so that you can prepare for Paper 1 and Paper 2 effectively.

You'll notice that we've provided a small amount of space under each question for you to jot down your answers or do some working out. In most cases it won't be as much space as you'll be given on the exam itself, but we know you wouldn't want to waste your money on a book full of empty pages.

You're here for the questions, so that's what we've given you.

If you want to practise under exam conditions, just write your answers on a separate piece of paper.



COMPLETE THIS CHAPTER  
TO MINE YOUR THIRD  
KNOWLEDGE CRYSTAL!  
YOU (BIG SHINY) ROCK!

# 3.1

## UNIT 3 TOPIC 1 – THE LOGARITHMIC FUNCTION 2



### TECHNOLOGY FREE



**Answers**  
on pages 220–221

### MULTIPLE CHOICE QUESTIONS

#### QUESTION 1

$\ln(7)$  is equivalent to

- (A)  $\frac{\ln(14)}{\ln(2)}$
- (B)  $\ln(4) + \ln(3)$
- (C)  $\int_0^7 \ln(x) dx$
- (D)  $2 \ln(14) - \ln(28)$

#### QUESTION 2

Given  $y = 5e^{2x}$ , rearrange the function to make  $x$  the subject.

- (A)  $x = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$
- (B)  $x = \sqrt{\ln\left(\frac{y}{5}\right)}$
- (C)  $x = \ln(\sqrt{5y})$
- (D)  $x = \ln\left(\left(\frac{y}{5}\right)^2\right)$

© State of Queensland (QCAA) Sample assessment 2020, Mathematics Methods,  
Paper 1 — Technology-free Question 8

#### QUESTION 3

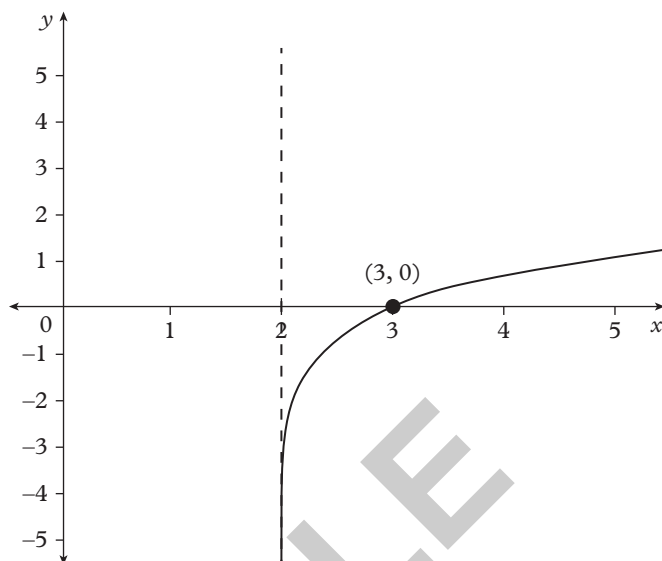
If  $\log_2(y) = 2 + \log_2(x) - a\log_2(4)$  for  $y > 0$  and  $x > 0$  then  $y$  is equal to

- (A)  $\frac{4x}{4a}$
- (B)  $\frac{x}{4^{a-1}}$
- (C)  $\frac{2x}{4^a}$
- (D)  $4x$



**QUESTION 4**

The graph below represents a translation of the graph of  $y = \log_3(x)$ .



The equation of the translation is

- (A)  $y = \log_3(x + 2)$
- (B)  $y = \log_3(x - 2)$
- (C)  $y = \log_3(x) + 2$
- (D)  $y = \log_3(x) - 2$

**QUESTION 5**

The point  $(b, \log_{10}(2.5))$  lies on the function  $y = 3 \log_{10}(x) - \log_{10}(50)$ .

The value of  $b$  is

- (A)  $\log_{10}(25)$
- (B) 5
- (C) 25
- (D) 4

**QUESTION 6**

Simplify  $\frac{\log_{10}(32)}{\log_{10}(4)}$

- (A)  $\log(28)$
- (B)  $\log(8)$
- (C) 8
- (D)  $\frac{5}{2}$



### QUESTION 13 (3 marks)

Solve the following

a)  $3^x = \frac{1}{81}$

[1 mark]

MY MARK

/1

b)  $\log_{10}(x + 3) + \log_{10}(x) = 1$

[2 marks]

MY MARK

/2

### QUESTION 14 (3 marks)

Solve the following

a)  $\log_6(3x - 1) = 1 + \log_6(4)$

[1 mark]

MY MARK

/1

b)  $6 + 11e^{2-5x} = 39$

[2 marks]

MY MARK

/2

### QUESTION 15 (2 marks)

The points  $A(4, a)$  and  $B(b, \ln(e))$  lie on the graph of the function  $y = 2 \ln(x) - \ln(5)$ .

Determine  $a$  and  $b$ .

MY MARK

/2



**QUESTION 16 (3 marks)**

The following function  $P(t)$  represents the population of fire ants (in hundreds) in a particular location at a time,  $t$ , in days

$$P(t) = a + e^{-bt} + 4bt$$

where  $t \geq 0$  and  $a$  and  $b$  are non-zero real numbers.

Determine the values of  $a$  and  $b$  if the initial population of fire ants is 9 and the minimum number of ants occurs at  $t = \ln(2)$  days.

MY MARK

/3

**TECHNOLOGY ACTIVE****MULTIPLE CHOICE QUESTIONS****Answers**

on pages 223–224

**QUESTION 1**

The approximate value of  $3\log_5(4) - 4\log_7(2)$  is

- (A) 0.602
- (B) 0.861
- (C) 1.159
- (D) 1.386


**QUESTION 2**

Determine the asymptote of the graph of the function  $f(x) = \log_3(x + 5)$ .

- (A)  $x = -5$
- (B)  $x = 5$
- (C)  $y = -5$
- (D)  $y = 5$

© State of Queensland (QCAA) Sample assessment 2020, Mathematics Methods,  
Paper 2 — Technology-active Question 1





I SEE YOU HAVE  
SOME GRAPH PAPER  
THERE... I HOPE  
YOU'RE NOT **PLOTTING**  
SOMETHING, BECAUSE  
IT'S TIME TO DO SOME  
PRACTICE EXAMS!



## CHAPTER

# 4

# OFFICIAL PAST PAPERS

In this chapter, things get serious! It's now time for you to put your revision and practice to the test – literally – by completing the official QCE Mathematical Methods external assessment from 2020!

We recommend you:

- don't look at this chapter until you've finished with your revision and completed all of the practice questions in Chapter 3.
- complete these papers under exam conditions (i.e. follow the instructions regarding perusal time and working time, don't refer to any notes or other materials that will not be allowed during the real exams)
- refer to the answers in Chapter 5 and use the marking advice to self-assess your responses once you've finished.

Remember... these are the QCE Mathematical Methods papers from 2020, so – if you complete them under exam conditions – they are arguably the best indicator of how well you're likely to perform on the day! Good luck!

SHINE ON! ACE THIS  
PRACTICE EXAM TO BAG  
YOUR FOURTH KNOWLEDGE  
CRYSTAL!



# 4.1

## EXTERNAL ASSESSMENT 2020: MATHEMATICAL METHODS PAPER 1 – TECHNOLOGY FREE

### Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

### General instructions

- Answer all questions in this question and response book.
- Calculators are not permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

### Section 1 (10 marks)

- 10 multiple choice questions

### Section 2 (50 marks)

- 10 short response questions

© State of Queensland (QCAA) External assessment 2020, Mathematics Methods,  
Paper 1 — Technology-free

For more information visit: <https://www.qcaa.qld.edu.au/copyright>

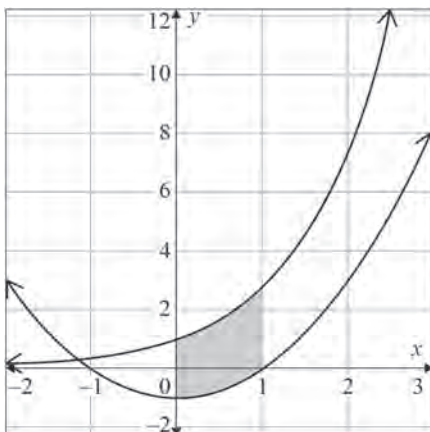
## Section 1 (10 marks)



**Answers**  
on pages 284–285

### QUESTION 1

The graphs of  $f(x) = e^x$  and  $g(x) = x - 1$  are shown.



The area of the shaded section bounded by these graphs between the lines  $x = 0$  and  $x = 1$  is

- (A)  $1 - e$
- (B)  $e - 2$
- (C)  $e - \frac{5}{3}$
- (D)  $e - \frac{1}{3}$

### QUESTION 2

Determine  $\int \frac{e^x + 1}{e^x} dx$

- (A)  $x - e^{-x} + c$
- (B)  $x + e^{-x} + c$
- (C)  $1 + xe^{-x} + c$
- (D)  $x + xe^{-x} + c$

### QUESTION 3

Determine  $2 \int (4x + 6)^3 dx$

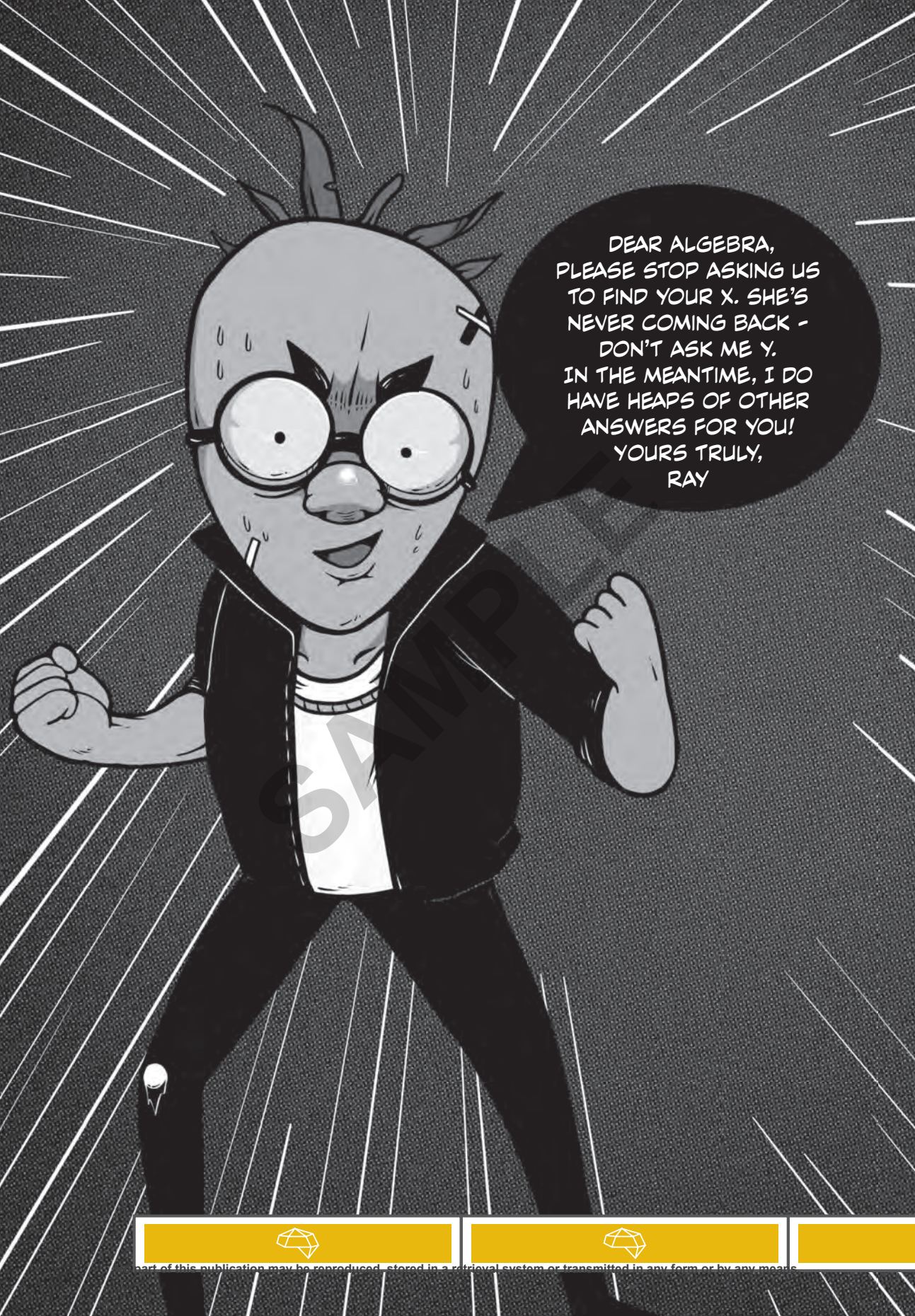
- (A)  $16(4x + 6)^4 + c$
- (B)  $8(4x + 6)^4 + c$
- (C)  $\frac{(4x + 6)^4}{2} + c$
- (D)  $\frac{(4x + 6)^4}{8} + c$

### QUESTION 4

Pulse rates of adult men are approximately normally distributed with a mean of 70 and a standard deviation of 8. Which of the following choices correctly describes how to determine the proportion of men that have a pulse rate greater than 78?

- (A) Determine the area to the left of  $z = 1$  under the standard normal curve.
- (B) Determine the area to the right of  $z = 1$  under the standard normal curve.
- (C) Determine the area to the right of  $z = -1$  under the standard normal curve.
- (D) Determine the area between  $z = -1$  and  $z = 1$  under the standard normal curve.





DEAR ALGEBRA,  
PLEASE STOP ASKING US  
TO FIND YOUR X. SHE'S  
NEVER COMING BACK -  
DON'T ASK ME Y.  
IN THE MEANTIME, I DO  
HAVE HEAPS OF OTHER  
ANSWERS FOR YOU!  
YOURS TRULY,  
RAY



## CHAPTER

# 5

# ANSWERS

OMG, another cliff hanger... what on Earth could be in this chapter I wonder?

You guessed it, in this chapter we provide the answers to absolutely everything! Sounds simple, I know, but to get the most out of this chapter, don't just cast an eye over the answers provided and move on.

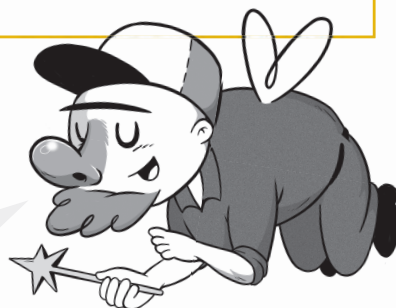
If you really want to increase your chances of excelling on the exam, we recommend you look carefully over each of your answers in Chapters 3 and 4 and compare them with the answers in this chapter.

Be sure to use the '**My mark**' box under each short response question to self-assess your answers. This will help to get you into the habit of structuring your responses in order to receive maximum marks and show you what the exam marker will be looking for.

### Notice to students

The answers and marking advice provided in this chapter are provided for practice purposes only. Unless specifically credited, the QCAA has not written this material and does not endorse the content.

A KNOWLEDGE CRYSTAL  
IS JUST A PIECE OF COAL  
THAT HANDLED PRESSURE  
REALLY WELL! COMPLETE THIS  
CHAPTER TO MINE YOUR FINAL  
ONE!  
YOU'VE GOT THIS!



# 5.1

## UNIT 3 TOPIC 1 – THE LOGARITHMIC FUNCTION 2

### TECHNOLOGY FREE

### MULTIPLE CHOICE ANSWERS

Question	Correct answer	Explanation
QUESTION 1	D	$2 \ln(14) - \ln(28) = \ln\left(\frac{14^2}{28}\right)$ $= \ln(7)$
QUESTION 2	A	$y = 5e^{2x}$ $\frac{y}{5} = e^{2x}$ $e^x = \frac{\sqrt{y}}{\sqrt{5}}$ $\ln(e^x) = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$ $x \ln(e) = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$ $x = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$
QUESTION 3	B	$\log_2(y) = 2 + \log_2(x) - a \log_2(4)$ $= \log_2(2^2) + \log_2(x) - \log_2(4^a)$ $= \log_2\left(\frac{4x}{4^a}\right)$ $= \log_2\left(\frac{x}{4^{a-1}}\right)$ $y = \frac{x}{4^{a-1}}$
QUESTION 4	B	The graph of $y = \log_3(x)$ has been translated 2 units in the positive $x$ direction to produce an asymptote at $x = 2$ .
QUESTION 5	B	$\log_{10}(2.5) = 3 \log_{10}(x) - \log_{10}(50)$ $= \log_{10}\left(\frac{x^3}{50}\right)$ $\frac{x^3}{50} = 2.5$ $x^3 = 125$ $x = 5$
QUESTION 6	D	$\frac{\log_{10}(32)}{\log_{10}(4)} = \frac{\log_{10}(2^5)}{\log_{10}(2^2)}$ $= \frac{5 \log_{10}(2)}{2 \log_{10}(2)}$ $= \frac{5}{2}$



Question	Correct answer	Explanation
<b>QUESTION 7</b>	C	<ul style="list-style-type: none"> <li>The asymptote of the translated graph is <math>x = 0</math>, so no horizontal translation has taken place.</li> <li>Substituting <math>x = 1</math> into option C, <math>y = \log_4(x) + 4</math>:  <math>y = \log_4(1) + 4</math>  <math>= 0 + 4</math>  <math>= 4</math>  This gives the labelled coordinate (1, 4).</li> </ul>
<b>QUESTION 8</b>	D	$2\log_5(6) - 2\log_5(3) - \log_5(20)$ $= \log_5\left(\frac{6^2}{3^2 \times 20}\right)$ $= \log_5\left(\frac{1}{5}\right)$ $= -1$
<b>QUESTION 9</b>	B	$\log_2(x + 4) + \log_2(x - 2) = 4$ $(x + 4)(x - 2) = 2^4$ $x^2 + 2x - 8 = 16$ $x^2 + 2x - 24 = 0$ $(x - 4)(x + 6) = 0$ $x = 4, -6$ $x = 4$ as the left side of the equation is undefined when $x = -6$ .
<b>QUESTION 10</b>	C	$\log_a(y) = 4 - \log_a(x) + b\log_a(4)$ $= \log_a\left(\frac{a^4 \times 4^b}{x}\right)$ $y = \frac{a^4 \times 4^b}{x}$ $xy = a^4 \times 4^b$

• 1 mark for each correct multiple choice answer

## SHORT RESPONSE ANSWERS

### QUESTION 11 (3 marks)

a)  $\log_6(9) + 2\log_6(2)$   
 $= \log_6(9) + \log_6(2^2)$   
 $= \log_6(9 \times 4)$   
 $= \log_6(36)$   
 $= \log_6(6^2)$   
 $= 2\log_6(6)$   
 $= 2$

• 1 mark for the correct answer

b)  $\frac{\log_3(16)}{\log_3(64)}$   
 $= \frac{\log_3(4^2)}{\log_3(4^3)}$   
 $= \frac{2\log_3(4)}{3\log_3(4)}$   
 $= \frac{2}{3}$

- 1 mark for the correct application of the logarithmic laws
- 1 mark for the correct answer

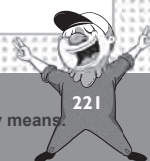
### QUESTION 12 (4 marks)

a)  $\log_5(2x + 4) = 2$   
 $= 2\log_5(5)$   
 $\log_5(2x + 4) = \log_5(5^2)$   
 $2x + 4 = 5^2$   
 $x = \frac{21}{2}$

- 1 mark for the correct application of the logarithmic laws
- 1 mark for the correct answer

b)  $\ln(7 - x) - \ln(10) = \ln(x)$   
 $\ln(7 - x) = \ln(x) + \ln(10)$   
 $\ln(7 - x) = \ln(10x)$   
 $7 - x = 10x$   
 $x = \frac{7}{11}$

- 1 mark for the correct application of the logarithmic laws
- 1 mark for the correct answer



YOU'VE REACHED  
THE PART OF THE BOOK  
MOST LIKELY TO BE USELESS  
OR BURST... THE APPENDIX!  
LUCKY FOR YOU, THIS  
APPENDIX ISN'T USELESS  
AT ALL. IT CONTAINS ALL  
THE GOOD STUFF THAT  
WILL HELP YOU ACE  
YOUR EXAM!



# APPENDIX

## MATHEMATICAL METHODS

### FORMULA SHEET

The QCAA has developed a formula sheet that will be provided for you to use during both examination papers. It provides a selection of useful formulas for you to refer to during the exam.

We want you to have everything you need in one spot so that you can study effectively with this book whenever and wherever you are – on the bus, in the bath... anywhere! For that reason we've included the formula sheet here too. Shucks, that's what buddies are for!

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a + b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n - 1)d$ $S_n = \frac{n}{2}(2t_1 + (n - 1)d) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r - 1)}$ $S_\infty = \frac{t_1}{(1 - r)},  r  < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n \log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Mathematics Methods Formula sheet © State of Queensland (QCAA) 2019,  
 licensed under CC BY 4.0



## Calculus

$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
<b>chain rule</b>	<p>If <math>h(x) = f(g(x))</math> then  <math>h'(x) = f'(g(x))g'(x)</math></p> <p>If <math>y = f(u)</math> and <math>u = g(x)</math> then  <math>\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}</math></p>
<b>product rule</b>	<p>If <math>h(x) = f(x)g(x)</math> then  <math>h'(x) = f(x)g'(x) + f'(x)g(x)</math></p> <p><math>\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}</math></p>
<b>quotient rule</b>	<p>If <math>h(x) = \frac{f(x)}{g(x)}</math> then  <math>h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}</math></p> <p><math>\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}</math></p>

## Trigonometry

<b>cosine rule</b>	$c^2 = a^2 + b^2 - 2ab \cos(C)$
<b>sine rule</b>	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
<b>area of a triangle</b>	$\text{area} = \frac{1}{2} bc \sin(A)$
<b>Pythagorean identity</b>	$\sin^2(A) + \cos^2(A) = 1$

Mathematics Methods Formula sheet © State of Queensland (QCAA) 2019,  
 licensed under CC BY 4.0

