OXFORD Study Buddy

QCE MATHEMATICAL METHODS

UNITS 3 & 4

VOLUME 1

Revision and Exam Guide

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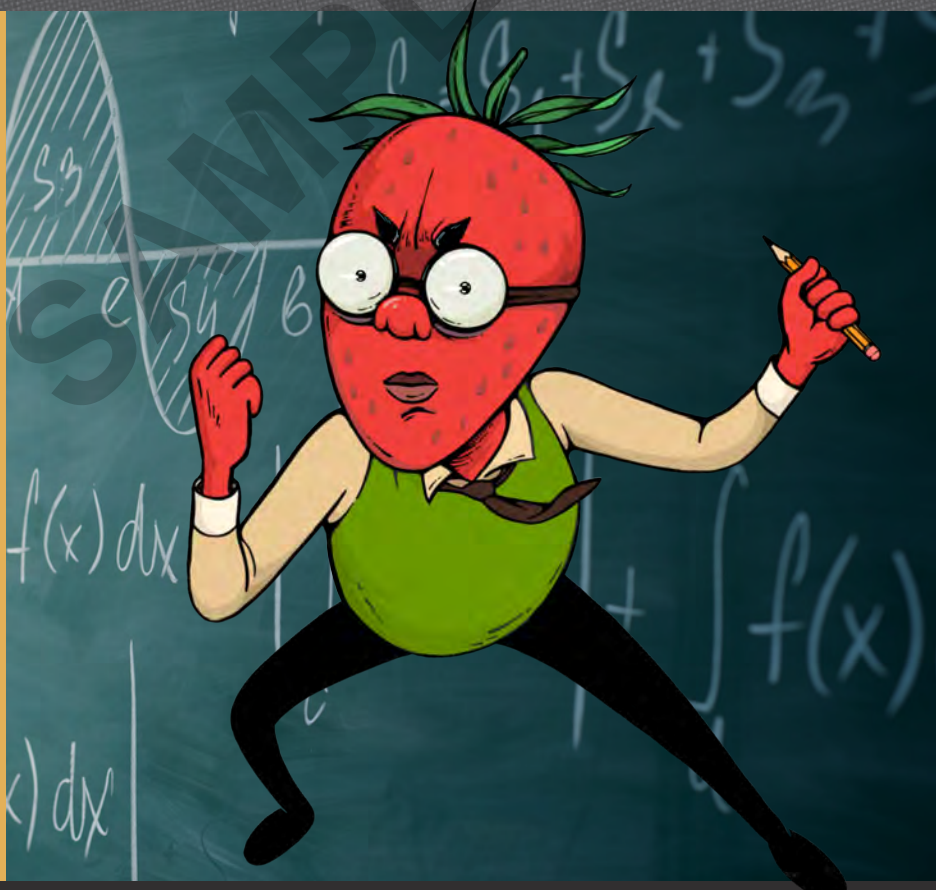

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Unit 4: Further functions and statistics

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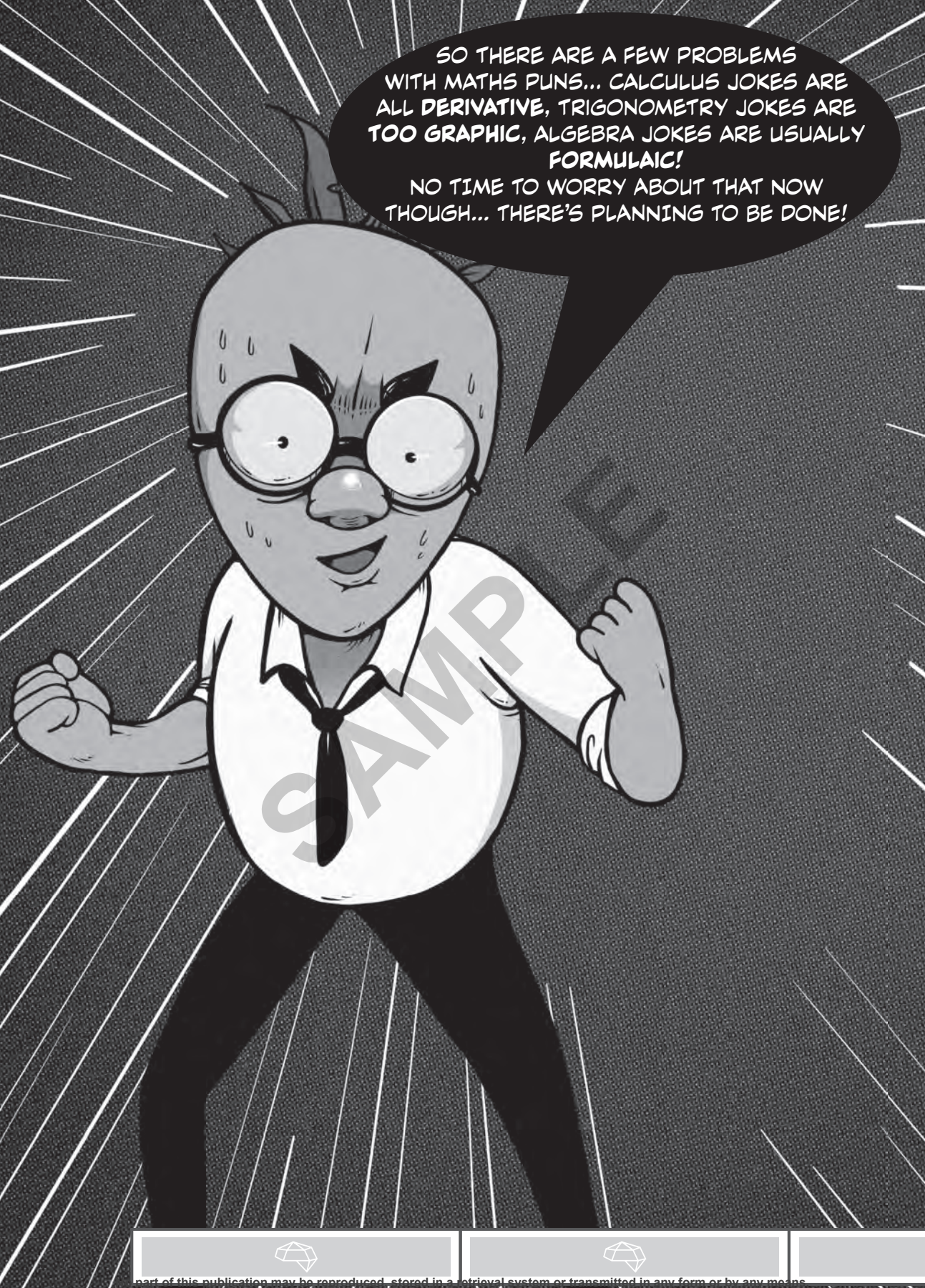
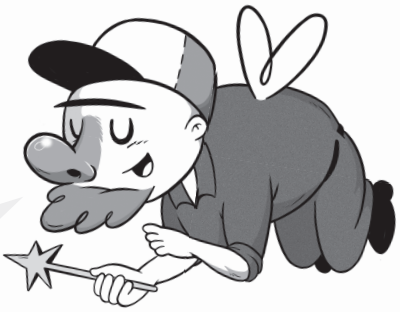


CHAPTER 1 STUDYING FOR SUCCESS

Before you start studying for your QCE Mathematical Methods exams, it's important to set yourself up for success. That's exactly what this chapter is designed to do, so thanks for stopping by!

As soon as it's time to start studying for the external assessment, we recommend that you work through this whole chapter before doing anything else!

You might do this at the start of the school year or at the start of your exam study period, but whatever you do, don't skip this chapter; it contains a bunch of really important information and tips that might just give you the edge you're looking for.



1.1 OVERVIEW OF QCE MATHEMATICAL METHODS UNITS 3 & 4

In this section, we will:

- provide a brief overview of how the QCE Mathematical Methods Units 3 & 4 course is structured
- list all of the concepts and topics that you will need to learn and understand
- explain how you will be assessed.

1.1.1
Resource:
Mathematical Methods
General Senior
syllabus

Study tip

The QCE Mathematical Methods General Senior Syllabus sets out all of the information you are expected to learn and also provides important information on how you will be assessed.

In this chapter, we have summarised all of the key information relating to external assessment you need to know, but the QCAA may update the syllabus from time to time, so it's important that you make sure you are using the most current version!

Make sure you visit the QCAA website and download a copy of the Mathematical Methods General Senior Syllabus and read the key information carefully before you sit your external assessment. To save you time, we've also included a link to it on your obook assess!

UNDERSTANDING THE QCE MATHEMATICAL METHODS UNITS 3 & 4 COURSE STRUCTURE

The Mathematical Methods General Senior Syllabus is the most important document supporting the QCE Mathematical Methods course. It sets out all the content – known as subject matter – that you will be expected to learn and provides important information about how you will be assessed.

QCE Mathematical Methods is a course of study consisting of four units (i.e. Units 1 & 2 and Units 3 & 4) taught over 2 years, but in this revision and exam guide, we will only be focusing on information relating to Units 3 & 4 of the course. The topics you will be learning about in Units 3 & 4 are summarised in Table 1.

Study tip

The notional hours shown in Table 1 are provided by the QCAA to help teachers with their planning and give them an estimate of how long to spend teaching the subject matter in each topic.

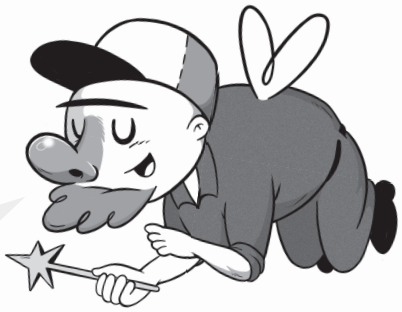
Notional hours can be a handy way to help you to structure and allocate your revision and preparation time for the external assessment because – as a general rule – there are likely to be more questions on subject matter with higher notional hours.

CHAPTER 2 REVISION

In this chapter, we provide a clear, concise summary of all examinable content from QCE Mathematical Methods Units 3 & 4. Everything has been organised by unit, topic and sub-topic in the General Senior Syllabus to help you focus your time and attention where it is needed most.

The revision notes are not designed to replace your teacher or your textbook. Instead, they have been designed to help you gauge your level of understanding and confidence of the subject matter before the exam. You can use them to identify those topics you know inside out and those that still require some extra attention.

The revision notes are also supported by a bunch of handy features, tips and icons designed to help you get the very best result on the day.



2.1 UNIT 3 TOPIC 1 – THE LOGARITHMIC FUNCTION 2

Questions
on pages 98–107

LOGARITHMIC LAWS AND LOGARITHMIC FUNCTIONS

SUBJECT MATTER

By the end of this topic, you should be able to:

- establish and use logarithmic laws and definitions
- interpret and use logarithmic scales such as decibels in acoustics, the Richter scale for earthquake magnitude, octaves in music, pH in chemistry
- solve equations involving indices with and without technology
- recognise the qualitative features of the graph of $\log_a(x)$ $(a > 1)$, including asymptotes, and of its translations $y = \log_a(x) + b$ and $y = \log_a(x + c)$
- solve equations involving logarithmic functions with and without technology
- identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

BACK TO BASICS

Logarithmic functions are the inverses of exponential functions:

$a^x = b$ is equivalent to $x = \log_a(b)$

The index laws

- $a^x \times a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $(a^m)^n = a^{mn}$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

Study tip

Remember that you can only apply Laws 1 and 2 to add and subtract logarithmic functions if they have the same base.

KEY CONCEPT

APPLYING LOGARITHMIC LAWS

Use the logarithmic laws to simplify logarithmic expressions and find the exact solution to indicial and logarithmic equations.

- $\log_a(x) + \log_a(y) = \log_a(xy)$
- $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$
- $\log_a(x^n) = n\log_a(x)$
- $\log_a(1) = 0$
- $\log_a(a) = 1$
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

WORKED EXAMPLE 2.1A

SIMPLIFYING USING LOGARITHMIC LAWS

Questions	Breaking down the questions
Simplify: a) $\log_2(144) - 2\log_2(6)$ *[1 mark]* b) $\frac{\log_a(27)}{\log_a(243)}$ *[2 marks]*	• Simplify requires you to use algebra to manipulate the expressions into a simpler form • The presence of the **logarithmic** function in these expressions indicates that logarithmic laws are required to perform the simplification

Solutions	Marking advice and tips
a) $\log_2(144) - 2\log_2(6) = \log_2(144) - \log_2(6^2)$ $= \log_2(144) - \log_2(36)$ $= \log_2\left(\frac{144}{36}\right)$ $= \log_2(4)$ $= \log_2(2^2)$ $= 2\log_2(2)$ $= 2$	a) This is a difference of two logarithms with the same base, so use Law 2 to combine the two logarithms into a single fraction. Before you can apply Law 2, move the coefficient of $\log_2(6)$ into the argument using Law 3. Finally, express the argument in index form to eliminate the logarithm (Law 5) for the answer mark. *[1 mark]*
b) $\frac{\log_a(27)}{\log_a(243)} = \frac{\log_a(3^3)}{\log_a(3^5)}$ $= \frac{3\log_a(3)}{5\log_a(3)}$ $= \frac{3}{5}$	b) Recognise that both 27 and 243 can be expressed in index form with a base of 3. *[1 mark]* Use Law 3 to bring the indices down and cancel out $\log_a(3)$ for the answer mark. *[1 mark]*

CHAPTER 3 PRACTICE QUESTIONS

In this chapter, we provide a range of practice questions for all of the examinable content from QCE Mathematical Methods Units 3 & 4. What a surprise… not! I have a sneaking suspicion the title gave it away!

No fancy tricks here, we just provide over 300 questions organised by unit and topic so you can move effortlessly between revision and practice as you study. This will help you get the practice you need and build up your confidence!

To help you direct your time and effort where it's needed most, we've grouped the questions by type. Multiple choice and short response questions are also clearly labelled **Technology free** or **Technology active** so that you can prepare for Paper 1 and Paper 2 effectively.

You'll notice that we've provided a small amount of space under each question for you to jot down your answers or do some working out. In most cases it won't be as much space as you'll be given on the exam itself, but we know you wouldn't want to waste your money on a book full of empty pages.
You're here for the questions, so that's what we've given you.
If you want to practise under exam conditions, just write your answers on a separate piece of paper.

3.1 UNIT 3 TOPIC 1 – THE LOGARITHMIC FUNCTION 2

TECHNOLOGY FREE

MULTIPLE CHOICE QUESTIONS

Answers
on pages 220–221

QUESTION 1

ln(7) is equivalent to

(A) $\frac{\ln(14)}{\ln(2)}$

(B) $\ln(4) + \ln(3)$

(C) $\int_0^7 \ln(x)\,dx$

(D) $2\ln(14) - \ln(28)$

QUESTION 2

Given $y = 5e^{2x}$, rearrange the function to make x the subject.

(A) $x = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$

(B) $x = \sqrt{\ln\left(\frac{y}{5}\right)}$

(C) $x = \ln\left(\sqrt{5y}\right)$

(D) $x = \ln\left(\left(\frac{y}{5}\right)^2\right)$

QUESTION 3

If $\log_2(y) = 2 + \log_2(x) - a\log_2(4)$ for $y > 0$ and $x > 0$ then y is equal to

(A) $\frac{4x}{4a}$

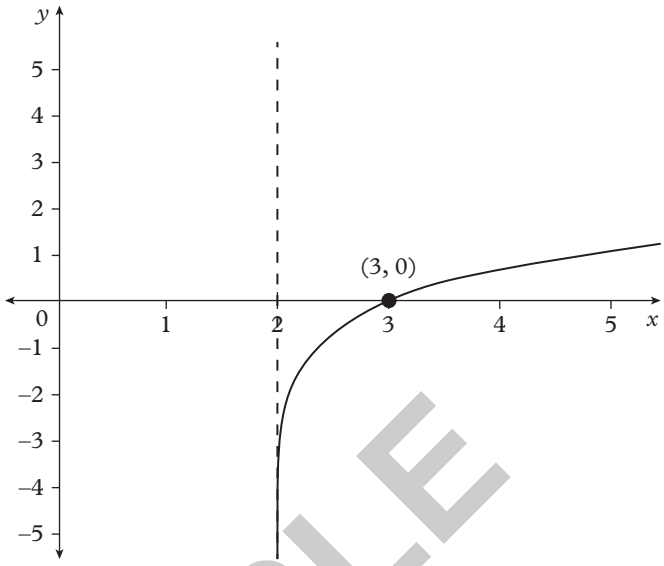
(B) $\frac{x}{4^{a-1}}$

(C) $\frac{2x}{4^a}$

(D) $4x$

QUESTION 4

The graph below represents a translation of the graph of $y = \log_3(x)$.



The equation of the translation is

(A) $y = \log_3(x + 2)$
(B) $y = \log_3(x - 2)$
(C) $y = \log_3(x) + 2$
(D) $y = \log_3(x) - 2$

QUESTION 5

The point $\left(b, \log_{10}(2.5)\right)$ lies on the function $y = 3\log_{10}(x) - \log_{10}(50)$. The value of b is

(A) $\log_{10}(25)$
(B) 5
(C) 25
(D) 4

QUESTION 6

Simplify $\dfrac{\log_{10}(32)}{\log_{10}(4)}$

(A) $\log(28)$
(B) $\log(8)$
(C) 8
(D) $\frac{5}{2}$

QUESTION 13 (3 marks)

Solve the following

a) $3^x = \frac{1}{81}$ *[1 mark]*

MY MARK /1

b) $\log_{10}(x + 3) + \log_{10}(x) = 1$ *[2 marks]*

MY MARK /2

QUESTION 14 (3 marks)

Solve the following

a) $\log_6(3x - 1) = 1 + \log_6(4)$ *[1 mark]*

MY MARK /1

b) $6 + 11e^{2-5x} = 39$ *[2 marks]*

MY MARK /2

QUESTION 15 (2 marks)

The points $A(4,a)$ and $B(b,\ln(e))$ lie on the graph of the function $y = 2\ln(x) - \ln(5)$.

Determine a and b.

MY MARK /2

QUESTION 16 (3 marks)

The following function $P(t)$ represents the population of fire ants (in hundreds) in a particular location at a time, t, in days

$$P(t) = a + e^{-bt} + 4bt$$

where $t \geq 0$ and a and b are non-zero real numbers.

Determine the values of a and b if the initial population of fire ants is 9 and the minimum number of ants occurs at $t = \ln(2)$ days.

MY MARK

/3

TECHNOLOGY ACTIVE

MULTIPLE CHOICE QUESTIONS

Answers
on pages 223–224

QUESTION 1

The approximate value of $3\log_5(4) - 4\log_7(2)$ is

(A) 0.602
(B) 0.861
(C) 1.159
(D) 1.386

QUESTION 2

Determine the asymptote of the graph of the function $f(x) = \log_3(x + 5)$.

(A) $x = -5$
(B) $x = 5$
(C) $y = -5$
(D) $y = 5$

CHAPTER 4 OFFICIAL PAST PAPERS

In this chapter, things get serious! It's now time for you to put your revision and practice to the test – literally – by completing the official QCE Mathematical Methods external assessment from 2020!

We recommend you:

- don't look at this chapter until you've finished with your revision and completed all of the practice questions in Chapter 3.
- complete these papers under exam conditions (i.e. follow the instructions regarding perusal time and working time, don't refer to any notes or other materials that will not be allowed during the real exams)
- refer to the answers in Chapter 5 and use the marking advice to self-assess your responses once you've finished.

Remember… these are the QCE Mathematical Methods papers from 2020, so – if you complete them under exam conditions – they are arguably the best indicator of how well you're likely to perform on the day! Good luck!

4.1 EXTERNAL ASSESSMENT 2020: MATHEMATICAL METHODS PAPER 1 – TECHNOLOGY FREE

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are not permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

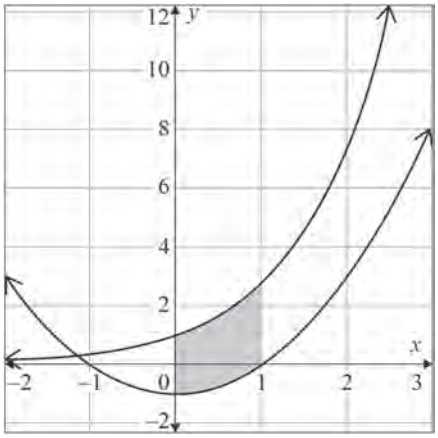
- 10 short response questions

Section 1 (10 marks)

Answers
on pages 284–285

QUESTION 1

The graphs of $f(x) = e^x$ and $g(x) = x - 1$ are shown.



The area of the shaded section bounded by these graphs between the lines $x = 0$ and $x = 1$ is

(A) $1 - e$

(B) $e - 2$

(C) $e - \frac{5}{3}$

(D) $e - \frac{1}{3}$

QUESTION 2

Determine $\int \frac{e^x + 1}{e^x} dx$

(A) $x - e^{-x} + c$

(B) $x + e^{-x} + c$

(C) $1 + xe^{-x} + c$

(D) $x + xe^{-x} + c$

QUESTION 3

Determine $2\int (4x + 6)^3 \, dx$

(A) $16(4x + 6)^4 + c$

(B) $8(4x + 6)^4 + c$

(C) $\frac{(4x + 6)^4}{2} + c$

(D) $\frac{(4x + 6)^4}{8} + c$

QUESTION 4

Pulse rates of adult men are approximately normally distributed with a mean of 70 and a standard deviation of 8. Which of the following choices correctly describes how to determine the proportion of men that have a pulse rate greater than 78?

(A) Determine the area to the left of $z = 1$ under the standard normal curve.

(B) Determine the area to the right of $z = 1$ under the standard normal curve.

(C) Determine the area to the right of $z = -1$ under the standard normal curve.

(D) Determine the area between $z = -1$ and $z = 1$ under the standard normal curve.

CHAPTER 5 ANSWERS

OMG, another cliff hanger… what on Earth could be in this chapter I wonder?

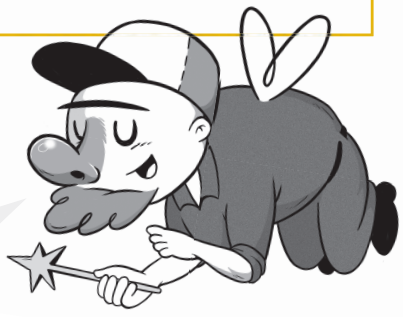
You guessed it, in this chapter we provide the answers to absolutely everything! Sounds simple, I know, but to get the most out of this chapter, don't just cast an eye over the answers provided and move on.

If you really want to increase your chances of excelling on the exam, we recommend you look carefully over each of your answers in Chapters 3 and 4 and compare them with the answers in this chapter.

Be sure to use the '**My mark**' box under each short response question to self-assess your answers. This will help to get you into the habit of structuring your responses in order to receive maximum marks and show you what the exam marker will be looking for.

Notice to students

The answers and marking advice provided in this chapter are provided for practice purposes only. Unless specifically credited, the QCAA has not written this material and does not endorse the content.



5.1 UNIT 3 TOPIC 1 – THE LOGARITHMIC FUNCTION 2

TECHNOLOGY FREE

MULTIPLE CHOICE ANSWERS

Question	Correct answer	Explanation
QUESTION 1	D	$2\ln(14) - \ln(28) = \ln\left(\frac{14^2}{28}\right)$ $= \ln(7)$
QUESTION 2	A	$y = 5e^{2x}$ $\frac{y}{5} = e^{2x}$ $e^x = \frac{\sqrt{y}}{\sqrt{5}}$ $\ln(e^x) = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$ $x\ln(e) = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$ $x = \ln\left(\frac{\sqrt{y}}{\sqrt{5}}\right)$
QUESTION 3	B	$\log_2(y) = 2 + \log_2(x) - a\log_2(4)$ $= \log_2(2^2) + \log_2(x) - \log_2(4^a)$ $= \log_2\left(\frac{4x}{4^a}\right)$ $= \log_2\left(\frac{x}{4^{a-1}}\right)$ $y = \frac{x}{4^{a-1}}$
QUESTION 4	B	The graph of $y = \log_3(x)$ has been translated 2 units in the positive x direction to produce an asymptote at $x = 2$.
QUESTION 5	B	$\log_{10}(2.5) = 3\log_{10}(x) - \log_{10}(50)$ $= \log_{10}\left(\frac{x^3}{50}\right)$ $\frac{x^3}{50} = 2.5$ $x^3 = 125$ $x = 5$
QUESTION 6	D	$\frac{\log_{10}(32)}{\log_{10}(4)} = \frac{\log_{10}(2^5)}{\log_{10}(2^2)}$ $= \frac{5\log_{10}(2)}{2\log_{10}(2)}$ $= \frac{5}{2}$

Question	Correct answer	Explanation
QUESTION 7	C	• The asymptote of the translated graph is $x = 0$, so no horizontal translation has taken place. • Substituting $x = 1$ into option C, $y = \log_4(x) + 4$: $y = \log_4(1) + 4$ $= 0 + 4$ $= 4$ This gives the labelled coordinate $(1,4)$.
QUESTION 8	D	$2\log_5(6) - 2\log_5(3) - \log_5(20)$ $= \log_5\left(\frac{6^2}{3^2 \times 20}\right)$ $= \log_5\left(\frac{1}{5}\right)$ $= -1$
QUESTION 9	B	$\log_2(x + 4) + \log_2(x - 2) = 4$ $(x + 4)(x - 2) = 2^4$ $x^2 + 2x - 8 = 16$ $x^2 + 2x - 24 = 0$ $(x - 4)(x + 6) = 0$ $x = 4, -6$ $x = 4$ as the left side of the equation is undefined when $x = -6$.
QUESTION 10	C	$\log_a(y) = 4 - \log_a(x) + b\log_a(4)$ $= \log_a\left(\frac{a^4 \times 4^b}{x}\right)$ $y = \frac{a^4 \times 4^b}{x}$ $xy = a^4 \times 4^b$

• *1 mark* for each correct multiple choice answer

SHORT RESPONSE ANSWERS

QUESTION 11 (3 marks)

a) $\log_6(9) + 2\log_6(2)$
$= \log_6(9) + \log_6(2^2)$
$= \log_6(9 \times 4)$
$= \log_6(36)$
$= \log_6(6^2)$
$= 2\log_6(6)$
$= 2$
• *1 mark* for the correct answer

b) $\frac{\log_3(16)}{\log_3(64)}$
$= \frac{\log_3(4^2)}{\log_3(4^3)}$
$= \frac{2\log_3(4)}{3\log_3(4)}$
$= \frac{2}{3}$
• *1 mark* for the correct application of the logarithmic laws
• *1 mark* for the correct answer

QUESTION 12 (4 marks)

a) $\log_5(2x + 4) = 2$
$= 2\log_5(5)$
$\log_5(2x + 4) = \log_5(5^2)$
$2x + 4 = 5^2$
$x = \frac{21}{2}$
• *1 mark* for the correct application of the logarithmic laws
• *1 mark* for the correct answer

b) $\ln(7 - x) - \ln(10) = \ln(x)$
$\ln(7 - x) = \ln(x) + \ln(10)$
$\ln(7 - x) = \ln(10x)$
$7 - x = 10x$
$x = \frac{7}{11}$
• *1 mark* for the correct application of the logarithmic laws
• *1 mark* for the correct answer

APPENDIX

MATHEMATICAL METHODS FORMULA SHEET

The QCAA has developed a formula sheet that will be provided for you to use during both examination papers. It provides a selection of useful formulas for you to refer to during the exam.

We want you to have everything you need in one spot so that you can study effectively with this book whenever and wherever you are – on the bus, in the bath… anywhere! For that reason we've included the formula sheet here too. Shucks, that's what buddies are for!

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}(2t_1 + (n-1)d) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1\frac{(r^n-1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \lvert r\rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x\,dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x}dx = \ln(x) + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x)\,dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x)\,dx = \sin(x) + c$

chain rule	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	area $= \frac{1}{2}bc\sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$