11 Surds and indices

This chapter deals with defining the system of real numbers, distinguishing between rational and irrational numbers, performing operations with surds, using integers and fractions for index notation, and converting between surd and index forms.

After completing this chapter you should be able to:

- define real numbers and distinguish between rational and irrational numbers
- simplify expressions involving surds
- expand expressions involving surds
- rationalise the denominators of simple surds
- use the index laws to define fractional indices
- translate expressions in surd form and index form
- evaluate numerical expressions involving fractional indices
- use the calculator to evaluate fractional powers of numbers
- evaluate a fraction raised to the power of –1
- prove general properties of real numbers.

Diagnostic test

1 Which of the following statements is *not* correct?

A the square root of 4 is 2 or −2

B $\sqrt{4} = 2$

C $-\sqrt{4} = -2$

D $\sqrt{-4} = -2$

2 Convert $0.\dot{3}1\dot{2}$ to a fraction.

A $\frac{39}{125}$ **B** $\frac{104}{333}$ **C** $\frac{78}{25}$ **D** $\frac{26}{75}$

3 Which of the following is *not* a rational number?

A $\sqrt{11}$ **B** $\sqrt{\frac{9}{16}}$ **C** $2\frac{3}{4}$ **D** -4.7

4 $(3\sqrt{5})^2 =$

A 45 **B** 225 **C** 15 **D** $9\sqrt{5}$

5 In simplest form, $\sqrt{32} =$

A $2\sqrt{8}$ **B** $8\sqrt{2}$ **C** $16\sqrt{2}$ **D** $4\sqrt{2}$

6 Written in the form $\sqrt{n}$, $5\sqrt{6} =$

A $\sqrt{30}$ **B** $\sqrt{180}$ **C** $\sqrt{150}$ **D** $\sqrt{900}$

7 $\frac{\sqrt{40}}{\sqrt{5}} =$

A 8 **B** $\frac{1}{8}$ **C** $2\sqrt{2}$ **D** $\sqrt{35}$

8 $\sqrt{4\frac{1}{9}} =$

A $\sqrt{\frac{37}{3}}$ **B** $2\frac{1}{3}$ **C** $4\frac{1}{3}$ **D** $2\frac{1}{9}$

9 $4\sqrt{10} - 2\sqrt{5} + 6\sqrt{10} =$

A $8\sqrt{15}$ **B** $10\sqrt{10} - 2\sqrt{5}$

C $8\sqrt{5}$ **D** $2\sqrt{15} + 6\sqrt{10}$

10 $\sqrt{18} + \sqrt{2} =$

A $\sqrt{20}$ **B** $4\sqrt{2}$

C $2\sqrt{3} + \sqrt{2}$ **D** $2\sqrt{5}$

11 $2\sqrt{3} \times 5\sqrt{6} =$

A $30\sqrt{6}$ **B** $30\sqrt{2}$ **C** $10\sqrt{30}$ **D** $\sqrt{180}$

12 $\sqrt{2}(\sqrt{5} - 2\sqrt{3}) =$

A $\sqrt{10} - 2\sqrt{3}$ **B** $\sqrt{10} - 2\sqrt{6}$

C $\sqrt{10} - 4\sqrt{3}$ **D** $10 - 2\sqrt{6}$

13 $(2\sqrt{3} + \sqrt{5})(2\sqrt{3} - \sqrt{5}) =$

A $4\sqrt{2} - 2\sqrt{5}$ **B** 9

C 7 **D** $7 + 4\sqrt{5}$

14 $(\sqrt{5} + \sqrt{2})^2 =$

A 7 **B** $7 + 2\sqrt{10}$

C $\sqrt{14}$ **D** $2\sqrt{7}$

15 Expressed with a rational denominator, $\frac{\sqrt{5}}{2\sqrt{3}} =$

A $\frac{\sqrt{5}}{6}$ **B** $\frac{\sqrt{15}}{6}$ **C** $\frac{\sqrt{5}}{3}$ **D** $\frac{\sqrt{10}}{6}$

16 $10^{\frac{1}{2}} =$

A 5 **B** $\frac{1}{5}$ **C** $\sqrt{10}$ **D** $\frac{1}{\sqrt{10}}$

17 In index form, $\sqrt[4]{k^3} =$

A k^{12} **B** $k^{\frac{4}{3}}$ **C** $k^{\frac{3}{4}}$ **D** k^7

18 When evaluated, $27^{\frac{2}{3}} =$

A 18 **B** 9 **C** 6 **D** 40.5

19 $\left(\frac{3}{5}\right)^{-1} =$

A $1\frac{2}{3}$ **B** $-\frac{3}{5}$ **C** $-\frac{5}{3}$ **D** $\frac{1}{15}$

The Diagnostic Test questions refer to the sections of text listed in the table below.

Question	1	2	3	4–8	9, 10	11–14	15	16–18	19
Section	A	B	C	D	E	F	G	H	I

A Square root of a number

The **square root** of a number, x, is the number that when multiplied by itself is equal to x. For example, the square root of 9 is 3 or -3, since $3^2 = 9$ and $(-3)^2 = 9$.

$\sqrt{x}$ is the positive square root of x.
For example, $\sqrt{9} = 3$ and $-\sqrt{9} = -3$. $-\sqrt{x}$ is then the negative square root of x.

11004 An aerial photo of a square house block (say 900 square metres). Needs to be square.

EXAMPLE 1

Find the following.

a the square root of 81 **b** $\sqrt{81}$ **c** $-\sqrt{81}$

a the square root of $81 = 9$ or -9 **b** $\sqrt{81} = 9$ **c** $-\sqrt{81} = -9$

Exercise 11A

1 Find the following.

a	**i** the square root of 4	**ii** $\sqrt{4}$	**iii** $-\sqrt{4}$
b	**i** the square root of 25	**ii** $\sqrt{25}$	**iii** $-\sqrt{25}$
c	**i** the square root of 49	**ii** $\sqrt{49}$	**iii** $-\sqrt{49}$
d	**i** the square root of 64	**ii** $\sqrt{64}$	**iii** $-\sqrt{64}$

Since there is no number that when multiplied by itself is equal to -9, it is not possible to find $\sqrt{-9}$.
We say that $\sqrt{-9}$ is **undefined**.
The square root of 0 is 0, since $0\sqrt{0} = 0$. Zero is neither positive nor negative but we define $\sqrt{0} = 0$.

In general:
- $\sqrt{x}$ is undefined for $x < 0$
- $\sqrt{x} = 0$ for $x = 0$
- $\sqrt{x}$ is the positive square root of x when $x > 0$.
- $-\sqrt{x}$ is the negative square root of x when $x > 0$.

EXAMPLE 2

Find the following, where possible.

a $\sqrt{36}$ **b** $-\sqrt{36}$ **c** $\sqrt{-36}$ **d** $\sqrt{0}$

a $\sqrt{36} = 6$ **b** $-\sqrt{36} = -6$ **c** undefined **d** 0

2 Find the following, where possible.

a $\sqrt{100}$	**b** $\sqrt{-100}$	**c** $\sqrt{-4}$	**d** $\sqrt{0}$	**e** $\sqrt{16}$
f $-\sqrt{16}$	**g** $\sqrt{-16}$	**h** $\sqrt{-49}$	**i** $-\sqrt{1}$	**j** $\sqrt{1}$
k $\sqrt{-25}$	**l** $\sqrt{-81}$	**m** $\sqrt{-64}$	**n** $-\sqrt{100}$	**o** $\sqrt{-1}$

B Recurring decimals

As a decimal, $\frac{3}{8} = 0.375$ and $\frac{1}{3} = 0.333\ 33\ \ldots$

- When converted to a decimal, the fraction $\frac{3}{8}$ terminates. That is, the digits after the decimal point stop after 3 places have been filled. We call this a **terminating decimal**.

When converted to a decimal, all fractions either terminate or recur.

- When the fraction $\frac{1}{3}$ is converted to a decimal, the digits after the decimal point keep repeating or recurring. We call this a recurring decimal.

  $0.3333\ldots$ is written $0.\dot{3}$.

  The dot above the 3 indicates that this digit recurs.

*We call this **dot notation**.*

EXAMPLE 1

Write the following recurring decimals using dot notation.

a $0.4444\ \ldots$	**b** $0.411\ 11\ \ldots$	**c** $0.414\ 141\ \ldots$
d $0.415\ 415\ 415\ \ldots$	**e** $0.415\ 341\ 534\ 153\ \ldots$	

a $0.\dot{4}$	**b** $0.4\dot{1}$	**c** $0.\dot{4}\dot{1}$
d $0.\dot{4}1\dot{5}$	**e** $0.\dot{4}15\dot{3}$	

The dots are put above the first and last digits of the group of digits that repeat.

Exercise 11B

1 Write the following recurring decimals using dot notation.

a $0.7777\ \ldots$	**b** $0.355\ 55\ \ldots$	**c** $0.282\ 828\ \ldots$	**d** $0.325\ 325\ 325\ \ldots$
e $0.678\ 467\ 846\ 784\ \ldots$	**f** $1.4444\ \ldots$	**g** $6.922\ 22\ \ldots$	**h** $0.494\ 949\ \ldots$
i $0.234\ 234\ 234\ \ldots$	**j** $0.033\ 33\ \ldots$	**k** $0.909\ 090\ \ldots$	**l** $0.536\ 666\ \ldots$
m $0.217\ 77\ \ldots$			

11005 Photo of an aerial view of an iron ore train or similar (say Pilbra)

EXAMPLE 2

Use your calculator to convert the following fractions to decimals.

a $\frac{5}{8}$ **b** $\frac{2}{3}$

a By calculating $5 \div 8$ or using the fraction key, $\frac{5}{8} = 0.625$.

b By calculating $2 \div 3$ or using the fraction key, the display could show 0.666666666 or 0.666666667, depending on the calculator used.

Both are approximations for the recurring decimal $0.\dot{6}$. In the first case the calculator has truncated the answer (because of the limitations of the display), and in the second case the calculator has automatically rounded up to the last decimal place.

Hence $\frac{2}{3} = 0.\dot{6}$.

2 Convert the following fractions to decimals.

a $\frac{7}{8}$ **b** $\frac{5}{9}$ **c** $\frac{1}{6}$ **d** $\frac{2}{11}$ **e** $1\frac{5}{12}$

f $1\frac{2}{3}$ **g** $\frac{11}{18}$ **h** $\frac{22}{33}$ **i** $1\frac{13}{22}$ **j** $\frac{11}{24}$

EXAMPLE 3

Convert the following decimals to fractions.

a 0.8 **b** 0.63 **c** 0.148

a $0.8 = \frac{8}{10} = \frac{4}{5}$

b $0.63 = \frac{63}{100}$

c $0.148 = \frac{148}{1000} = \frac{37}{250}$

3 Convert the following decimals to fractions.

a 0.6 **b** 0.78 **c** 0.125 **d** 0.08 **e** 0.256

EXAMPLE 4

Convert the following recurring decimals to fractions.

a $0.\dot{4}$ **b** $0.\dot{5}\dot{7}$

Before subtracting, multiply by the power of 10 that makes the decimal parts the same.

a

Let $n = 0.\dot{4}$.

$n = 0.4444\ldots$

Then $10n = 4.4444\ldots$

By subtraction, $9n = 4$

Hence $n = \frac{4}{9}$

$\therefore 0.\dot{4} = \frac{4}{9}$

b

Let $n = 0.\dot{5}\dot{7}$

$n = 0.575757\ldots$

Then $100n = 57.575757\ldots$

By subtraction, $99n = 57$

Hence $n = \frac{57}{99} = \frac{19}{33}$

$\therefore 0.\dot{5}\dot{7} = \frac{19}{33}$

4 Convert the following recurring decimals to fractions.

a $0.\dot{2}$ **b** $0.\dot{3}$ **c** $0.\dot{5}$ **d** $0.\dot{8}$ **e** $0.\dot{7}$

5 Convert $0.\dot{9}$ to a fraction. Discuss the result with your class.

6 Convert the following recurring decimals to fractions.

a $0.\dot{4}\dot{6}$ **b** $0.\dot{9}\dot{1}$ **c** $0.\dot{3}\dot{0}$ **d** $0.\dot{6}\dot{3}$ **e** $0.\dot{9}\dot{8}$

7 Convert the following recurring decimals to fractions.

a $0.\dot{5}8\dot{6}$ **b** $0.\dot{2}3\dot{9}$ **c** $0.\dot{8}5\dot{2}$

d $0.\dot{4}2\dot{3}$ **e** $0.\dot{6}1\dot{5}$

Hint: Before subtracting multiply by 1000.

EXAMPLE 5

Convert the following recurring decimals to fractions.

a $0.3\dot{5}$ **b** $0.51\dot{2}$

Make the decimal parts the same before subtraction.

a

$$\begin{aligned} \text{Let } n &= 0.3\dot{5}. \\ n &= 0.35555\ldots \\ \text{Then} \quad 10n &= 3.5555\ldots \\ \text{and} \quad 100n &= 35.5555\ldots \\ \text{By subtraction,} \quad 90n &= 32 \\ \text{Hence} \quad n &= \frac{32}{90} = \frac{16}{45} \\ \therefore 0.3\dot{5} &= \tfrac{16}{45} \end{aligned}$$

b

$$\begin{aligned} \text{Let } n &= 0.51\dot{2} \\ n &= 0.512222\ldots \\ \text{Then} \quad 100n &= 51.2222\ldots \\ \text{and} \quad 1000n &= 512.2222\ldots \\ \text{By subtraction,} \quad 900n &= 461 \\ \text{Hence} \quad n &= \frac{461}{900} \\ \therefore 0.51\dot{2} &= \tfrac{461}{900} \end{aligned}$$

8 Convert the following recurring decimals to fractions.

a $0.3\dot{8}$ **b** $0.6\dot{5}$ **c** $0.9\dot{2}$ **d** $0.1\dot{6}$ **e** $0.0\dot{9}$

9 Convert the following recurring decimals to fractions.

a $0.54\dot{6}$ **b** $0.72\dot{3}$ **c** $0.76\dot{2}$ **d** $0.90\dot{5}$ **e** $0.04\dot{9}$

In general:

Step 1: Let n equal the recurring decimal.

Step 2: Multiply n by the positive power of the decimal place *before* the first repeating digit.

Step 3: Multiply n by the positive power of the decimal place of the *last* repeating digit.

Step 4: Subtract Step 2 from Step 3.

Step 5: Solve for n as a fraction.

10 Convert the following recurring decimals to fractions, using the multiples of n given.

a $0.0\dot{5}\dot{7}$ ($10n$ and $1000n$) **b** $0.30\dot{4}\dot{5}$ ($100n$ and $10\,000n$)

c $0.\dot{2}20\dot{5}$ (n and $10\,000n$) **d** $0.1\dot{2}7\dot{5}$ ($10n$ and $10\,000n$)

C Real number system

Real numbers are those that can be represented by points on a number line. Real numbers are either rational or irrational.

- A **rational number** is a real number that can be expressed as the ratio $\frac{a}{b}$ of two integers, where $b \neq 0$.
- An **irrational number** is a real number that is not rational.

Think: rational means 'ratio-nal'.

EXAMPLE 1

Show that the following are rational numbers.

a $2\frac{3}{4}$ **b** 0.637 **c** 3 **d** $0.\dot{4}$ **e** -3.1

Any terminating or recurring decimal is a rational number.

a $2\frac{3}{4} = \frac{11}{4}$

This is in the form $\frac{a}{b}$, where a and b are integesr; hence $2\frac{3}{4}$ is a rational number.

b $0.637 = \frac{637}{1000}$

This is in the form $\frac{a}{b}$, where a and b are integers; hence 0.637 is a rational number.

c $3 = \frac{3}{1}$

This is in the form $\frac{a}{b}$, where a and b are integers; hence 3 is a rational number.

d $0.\dot{4} = \frac{4}{9}$

This is in the form $\frac{a}{b}$, where a and b are integers; hence $0.\dot{4}$ is a rational number.

e $-3.1 = -3\frac{1}{10}$

$= -\frac{31}{10}$

This is in the form $\frac{a}{b}$, where a and b are integers; hence -3.1 is a rational number.

Exercise 11C

1 Show that the following are rational numbers by expressing them in the form $\frac{a}{b}$.

a $4\frac{2}{3}$ **b** 0.91 **c** 5 **d** $0.\dot{7}$ **e** $-5\frac{1}{2}$ **f** 2.84

g $0.\dot{5}\dot{3}$ **h** -2.6 **i** $\sqrt{16}$ **j** $\sqrt{\frac{4}{9}}$ **k** 30% **l** 7.3%

2 Convert the following rational numbers to decimals.

When a rational number is converted to a decimal, the decimal either terminates or recurs.

a $\frac{3}{5}$ **b** $1\frac{5}{8}$

c $4\frac{2}{3}$ **d** $\frac{5}{12}$

e $\frac{1}{7}$ **f** 69% **g** 6.5% **h** $17\frac{2}{3}\%$

EXAMPLE 2

Convert the following real numbers to decimals and discuss whether they are rational or irrational.

a $\sqrt{2}$ **b** $\sqrt{5}$

Using a calculator:

a $\sqrt{2} = 1.414\,213\,562\ \ldots$ **b** $\sqrt{5} = 2.236\,067\,978\ \ldots$

Since neither decimal terminates or recurs (although we can show answers to only 9 decimal places, the limit of the calculator display) these numbers cannot be expressed as the ratio of two integers and hence are not rational. They are irrational numbers.

EXAMPLE 3

Determine whether the following real numbers are rational or irrational.

a $\sqrt{6}$ **b** $\sqrt{\frac{16}{49}}$

a $\sqrt{6} = 2.449\,897\,43\ldots$

Since the decimal neither terminates nor recurs, it cannot be expressed as the ratio of two integers, so $\sqrt{6}$ is an irrational number.

b $\sqrt{\frac{16}{49}} = \frac{4}{7}$ (since $\frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$)

This is in the form $\frac{a}{b}$, where a and b are integers, so $\sqrt{\frac{16}{49}}$ is a rational number.

3 Determine whether the following real numbers are rational or irrational

a $\sqrt{8}$ **b** $\sqrt{9}$ **c** $\sqrt{11}$ **d** $\sqrt{\frac{4}{25}}$ **e** $\sqrt{\frac{5}{16}}$

EXAMPLE 4

Using a calculator $\sqrt{6} = 2.449\,489\,743\ldots$ Write true or false for the following statements and discuss.

a $\sqrt{6} = 2.44$ **b** $\sqrt{6} = 2.449$ **c** $\sqrt{6} = 2.4494$

a $2.44^2 = 5.9536$ The statement is false.

b $2.449^2 = 5.997\,601$ The statement is false.

c $2.4494^2 = 5.999\,560\,36$ The statement is false.

Because $\sqrt{6}$ is irrational, its exact value cannot be written as a decimal.

The values given in parts **a**, **b** and **c** are rational approximations for $\sqrt{6}$.

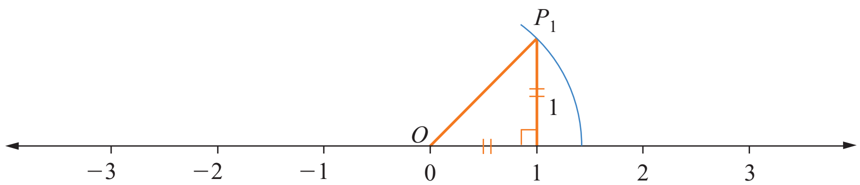
4 Using a calculator, $\sqrt{2} = 1.414\,213\,562\ldots$ Write true or false for the following statements.

a $\sqrt{2} = 1.41$ **b** $\sqrt{2} = 1.414$ **c** $\sqrt{2} = 1.4142$

5 Write rational approximations, correct to 3 decimal places, for the following.

a $\sqrt{11}$ **b** $\sqrt{15}$ **c** $\sqrt{37}$ **d** $\sqrt{99}$ **e** $\sqrt{151}$

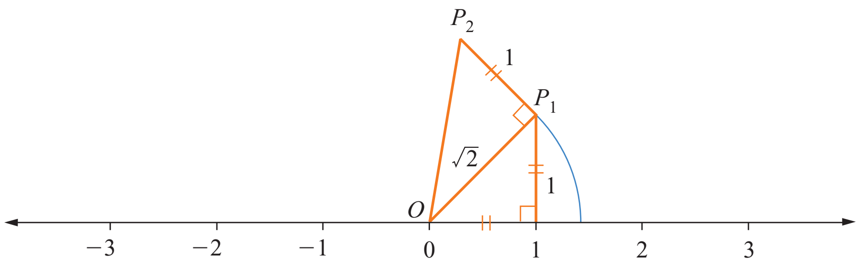
6 **a** Using a ruler and set square, copy the diagram.



b Use Pythagoras's rule to calculate the length of the interval OP_1.

c Using a pair of compasses, with point at O, accurately mark the position of $\sqrt{2}$ on the number line.

7 **a** Extend the diagram in question **6** as shown.



b Calculate the length of OP_2.

c Mark the position of $\sqrt{3}$ on the number line.

8 Extend the diagram in question **7** to show the positions on the number line of $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, …

Extension

9 **Proof that $\sqrt{2}$ is irrational**

In Example 2 we cannot be certain that the decimal form of $\sqrt{2}$ does not terminate or recur after some large number of decimal places; hence it is not a proof that $\sqrt{2}$ is irrational. Work through thesg proof with your teacher.

Assume that $\sqrt{2}$ is rational. That is, assume that $\sqrt{2}$ can be written in the form $\frac{a}{b}$, where a and b are integers and the fraction is written in its simplest form (that is, a and b have no common factors).

If $\qquad \sqrt{2} = \frac{a}{b}$

then, squaring both sides, $\qquad 2 = \frac{a^2}{b^2}$

and $\qquad a^2 = 2b^2 \quad$ …. (1)

Hence a^2 is even (any multiple of 2 is even) and therefore a is even.

If a is even, then a may be written in the form $2k$, where k is an integer.

$$a = 2k$$

$$a^2 = 4k^2$$

Substituting $a^2 = 4k^2$ into (1), $4k^2 = 2b^2$

$$b^2 = 2k^2$$

Hence b^2 is even and therefore b is even.

But if a and b are both even, the fraction $\frac{a}{b}$ cannot be in its simplest form, which contradicts our original statement.

Therefore $\sqrt{2}$ cannot be written in the form $\frac{a}{b}$, where a and b are integers with no common factor.

Hence $\sqrt{2}$ is not rational; it is irrational.

D Properties of surds

In section C we distinguished between rational and irrational numbers.

The set of irrational numbers contains numbers such as $\sqrt{2}$, $\sqrt[3]{2}$, π, etc.

Irrational numbers that contain the **radical sign** $\sqrt{\ }$ are called **surds**.

*A **set** is a group of objects (numbers, letters, names, etc.).*

When working with surds we may use the following properties:

If $x > 0$ and $y > 0$,

$$(\sqrt{x})^2 = x = \sqrt{x^2} \qquad \sqrt{xy} = \sqrt{x} \times \sqrt{y} \qquad \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Exercise 11D

1 Simplify the following.

a $(\sqrt{11})^2$ **b** $\sqrt{11^2}$ **c** $(\sqrt{8})^2$ **d** $\sqrt{8^2}$ **e** $(\sqrt{6})^2$

f $(3\sqrt{2})^2$ **g** $(2\sqrt{3})^2$ **h** $(5\sqrt{2})^2$ **i** $(10\sqrt{7})^2$ **j** $(4\sqrt{5})^2$

EXAMPLE 2

Simplify the following.

a $\sqrt{5} \times \sqrt{3}$ **b** $\sqrt{6} \times \sqrt{7}$

Use the property $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$.

a $\sqrt{5} \times \sqrt{3} = \sqrt{5 \times 3}$
$= \sqrt{15}$

b $\sqrt{6} \times \sqrt{7} = \sqrt{6 \times 7}$
$= \sqrt{42}$

2 Simplify the following.

a $\sqrt{2} \times \sqrt{7}$ **b** $\sqrt{3} \times \sqrt{10}$ **c** $\sqrt{5} \times \sqrt{2}$ **d** $\sqrt{7} \times \sqrt{11}$

e $\sqrt{13} \times \sqrt{17}$ **f** $\sqrt{3} \times \sqrt{2} \times \sqrt{5}$ **g** $\sqrt{7} \times \sqrt{5} \times \sqrt{10}$ **h** $\sqrt{3} \times \sqrt{11} \times \sqrt{5}$

EXAMPLE 3

Simplify the following.

a $\sqrt{28}$ **b** $\sqrt{45}$

Use the property $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

a $\sqrt{28} = \sqrt{2 \times 14}$ or $\sqrt{4 \times 7}$
$= \sqrt{2} \times \sqrt{14}$ or $\sqrt{4} \times \sqrt{7}$

Since 4 is a perfect square, $\sqrt{4}$ simplifies to 2, so choose the second product.

$\sqrt{28} = \sqrt{4} \times \sqrt{7}$
$= 2 \times \sqrt{7}$
$= 2\sqrt{7}$

b Look for factors of 45, one of which is a perfect square.

$\sqrt{45} = \sqrt{9} \times \sqrt{5}$
$= 3 \times \sqrt{5}$
$= 3\sqrt{5}$

3 Simplify the following.

a $\sqrt{12}$ **b** $\sqrt{20}$ **c** $\sqrt{18}$ **d** $\sqrt{27}$ **e** $\sqrt{8}$

f $\sqrt{90}$ **g** $\sqrt{50}$ **h** $\sqrt{75}$ **i** $\sqrt{200}$ **j** $\sqrt{98}$

k $\sqrt{24}$ **l** $\sqrt{32}$ **m** $\sqrt{48}$ **n** $\sqrt{72}$ **o** $\sqrt{128}$

EXAMPLE 4

Simplify the following.

a $\sqrt{3} \times \sqrt{12}$ **b** $\sqrt{2} \times \sqrt{6}$

a $\sqrt{3} \times \sqrt{12} = \sqrt{36}$
$= 6$

b $\sqrt{2} \times \sqrt{6} = \sqrt{12}$
$= \sqrt{4} \times \sqrt{3}$
$= 2\sqrt{3}$

4 Simplify the following.

a $\sqrt{8} \times \sqrt{2}$ **b** $\sqrt{2} \times \sqrt{32}$ **c** $\sqrt{5} \times \sqrt{20}$ **d** $\sqrt{2} \times \sqrt{10}$ **e** $\sqrt{10} \times \sqrt{5}$
f $\sqrt{14} \times \sqrt{2}$ **g** $\sqrt{8} \times \sqrt{5}$ **h** $\sqrt{15} \times \sqrt{3}$ **i** $\sqrt{15} \times \sqrt{5}$ **j** $\sqrt{3} \times \sqrt{8}$

EXAMPLE 5

Express $3\sqrt{5}$ in the form $\sqrt{n}$.

$3\sqrt{5} = 3 \times \sqrt{5}$
$= \sqrt{9} \times \sqrt{5}$
$= \sqrt{45}$

5 Express in the form $\sqrt{n}$.

a $3\sqrt{2}$ **b** $2\sqrt{3}$ **c** $4\sqrt{5}$ **d** $5\sqrt{2}$ **e** $10\sqrt{7}$

EXAMPLE 6

Simplify the following.

a $\sqrt{\frac{16}{25}}$ **b** $\sqrt{\frac{11}{25}}$ **c** $\sqrt{2\frac{1}{4}}$

Use the property $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$.

a $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}}$
$= \frac{4}{5}$

b $\sqrt{\frac{11}{25}} = \frac{\sqrt{11}}{\sqrt{25}}$
$= \frac{\sqrt{11}}{5}$

c $\sqrt{2\frac{1}{4}} = \sqrt{\frac{9}{4}}$
$= \frac{\sqrt{9}}{\sqrt{4}}$
$= \frac{3}{2} = 1\frac{1}{2}$

6 Simplify the following.

a $\sqrt{\frac{9}{16}}$ **b** $\sqrt{\frac{9}{25}}$ **c** $\sqrt{\frac{17}{25}}$ **d** $\sqrt{\frac{5}{16}}$ **e** $\sqrt{\frac{11}{16}}$
f $\sqrt{\frac{21}{9}}$ **g** $\sqrt{6\frac{1}{4}}$ **h** $\sqrt{1\frac{7}{9}}$ **i** $\sqrt{1\frac{3}{4}}$ **j** $\sqrt{2\frac{5}{9}}$

EXAMPLE 7

Simplify the following.

a $\dfrac{\sqrt{12}}{\sqrt{3}}$ **b** $\dfrac{\sqrt{15}}{\sqrt{5}}$ **c** $\dfrac{\sqrt{40}}{\sqrt{5}}$

Use the property $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$.

a $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}}$
$= \sqrt{4}$
$= 2$

b $\dfrac{\sqrt{15}}{\sqrt{5}} = \sqrt{\dfrac{15}{5}}$
$= \sqrt{3}$

c $\dfrac{\sqrt{40}}{\sqrt{5}} = \sqrt{\dfrac{40}{5}}$
$= \sqrt{8}$
$= 2\sqrt{2}$

7 Simplify the following.

a $\dfrac{\sqrt{24}}{\sqrt{6}}$ **b** $\dfrac{\sqrt{18}}{\sqrt{2}}$ **c** $\dfrac{\sqrt{18}}{\sqrt{3}}$ **d** $\dfrac{\sqrt{20}}{\sqrt{10}}$ **e** $\dfrac{\sqrt{24}}{\sqrt{8}}$

f $\dfrac{\sqrt{30}}{\sqrt{6}}$ **g** $\left(\dfrac{\sqrt{24}}{\sqrt{3}}\right)$ **h** $\dfrac{\sqrt{24}}{\sqrt{2}}$ **i** $\dfrac{\sqrt{32}}{\sqrt{2}}$ **j** $\dfrac{\sqrt{54}}{\sqrt{3}}$

8 Determine whether the following statements are true or false. Give reasons.

a $(\sqrt{5})^2 = 5$ **b** $3 \times \sqrt{7} = \sqrt{21}$ **c** $(4\sqrt{2})^2 = 8$ **d** $\sqrt{18} = 2\sqrt{3}$

e $\sqrt{20} \times \sqrt{5} = 10$ **f** $\dfrac{\sqrt{12}}{2} = \sqrt{6}$ **g** $\sqrt{1\tfrac{1}{4}} = 1\tfrac{1}{2}$ **h** $3\sqrt{\dfrac{81}{9}} = 3\sqrt{3}$

E Addition and subtraction of surds

The term surd traces back to the Arab mathematician al-Khwarizmi (about 825 AD) who referred to rational and irrational numbers as 'audible' and 'inaudible' respetively. This eventually led to the Arabic *asamm* (deaf, dumb) for irrational numbers being translated as *surdus* (deaf, mute) in Latin.

11006 Photo of an Arabic or Latin scholar or of a person with hand to ear or a hearing aid or old-fashion hearing trumpert

Surds can be added or subtracted only if they are like surds (that is, if they have the same value under the radical sign).

EXAMPLE 1

Simplify the following.

a $3\sqrt{2} + 5\sqrt{2}$ **b** $8\sqrt{5} - 2\sqrt{5}$

a $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$ **b** $8\sqrt{5} - 2\sqrt{5} = 6\sqrt{5}$

Exercise 11E

1 Simplify the following.

a $5\sqrt{3} + 4\sqrt{3}$
b $7\sqrt{11} + 6\sqrt{11}$
c $7\sqrt{5} - 3\sqrt{5}$
d $5\sqrt{10} - 7\sqrt{10}$
e $6\sqrt{3} + 4\sqrt{3} + 5\sqrt{3}$
f $8\sqrt{3} + 5\sqrt{3} - 7\sqrt{3}$
g $15\sqrt{6} - 3\sqrt{6} - 4\sqrt{6}$
h $8\sqrt{5} - 3\sqrt{5} + 2\sqrt{5}$
i $3\sqrt{5} - 8\sqrt{5} + 2\sqrt{5}$

EXAMPLE 2

Simplify the following.

a $2\sqrt{2} + 5\sqrt{3} + 3\sqrt{2}$
b $7\sqrt{6} + 4\sqrt{7} - 3\sqrt{6} - 5\sqrt{7}$

Collect like surds.

a $4\sqrt{2} + 5\sqrt{3} + 3\sqrt{2} = 4\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}$
$= 7\sqrt{2} + 5\sqrt{3}$

b $7\sqrt{6} + 4\sqrt{7} - 3\sqrt{6} - 5\sqrt{7} = 7\sqrt{6} - 3\sqrt{6} + 4\sqrt{7} - 5\sqrt{7}$
$= 4\sqrt{6} - \sqrt{7}$

2 Simplify the following.

a $5\sqrt{2} + 4\sqrt{3} + 6\sqrt{2}$
b $7\sqrt{3} + 4\sqrt{5} + 3\sqrt{5}$
c $5\sqrt{7} - 2\sqrt{10} + 3\sqrt{7}$
d $6\sqrt{5} + 2\sqrt{11} - 3\sqrt{5}$
e $7\sqrt{10} - 4\sqrt{6} - 6\sqrt{10}$
f $5\sqrt{2} + 6\sqrt{3} - 3\sqrt{2} + \sqrt{3}$
g $6\sqrt{3} + 2\sqrt{7} - 5\sqrt{3} - 4\sqrt{7}$
h $10\sqrt{5} - 4\sqrt{3} - 5\sqrt{3} + 2\sqrt{5}$
i $4\sqrt{11} - 3\sqrt{10} - 6\sqrt{11} - 2\sqrt{10}$

EXAMPLE 3

Simplify the following.

a $\sqrt{18} + \sqrt{2}$
b $\sqrt{50} - \sqrt{18}$

Convert to like surds before adding or subtracting.

a $\sqrt{18} + \sqrt{2} = \sqrt{9} \times \sqrt{2} + \sqrt{2}$
$= 3\sqrt{2} + \sqrt{2}$
$= 4\sqrt{2}$

b $\sqrt{50} - \sqrt{18} = \sqrt{25} \times \sqrt{2} - \sqrt{9} \times \sqrt{2}$
$= 5\sqrt{2} - 3\sqrt{2}$
$= 2\sqrt{2}$

3 Simplify the following.

a $\sqrt{18} + 4\sqrt{2}$
b $\sqrt{12} + 5\sqrt{3}$
c $\sqrt{20} - 2\sqrt{5}$
d $6\sqrt{2} - \sqrt{8}$
e $\sqrt{45} + \sqrt{20}$
f $\sqrt{54} - \sqrt{24}$
g $\sqrt{24} + 3\sqrt{6} - 4\sqrt{6}$
h $6\sqrt{2} - \sqrt{18} - 2\sqrt{2}$
i $\sqrt{75} - \sqrt{48} - \sqrt{27}$

4 Simplify the following.

a $5\sqrt{6} + \sqrt{24} - 3\sqrt{5}$
b $\sqrt{50} + 6\sqrt{3} - \sqrt{8}$
c $8\sqrt{10} + 4\sqrt{5} - \sqrt{90}$
d $\sqrt{28} + \sqrt{27} + \sqrt{63} + \sqrt{12}$
e $7\sqrt{5} - \sqrt{20} + \sqrt{45} - 5\sqrt{6}$
f $\sqrt{72} - 3\sqrt{8} + 6\sqrt{3} - \sqrt{108}$

5 Determine whether the following statements are true or false. Give reasons.

a $\sqrt{5} + \sqrt{3} = \sqrt{8}$
b $3\sqrt{5} - 2\sqrt{5} = 1$
c $\sqrt{24} - \sqrt{6} = \sqrt{6}$
d $4\sqrt{3} + 2\sqrt{7} = 6\sqrt{10}$
e $6\sqrt{5} - 5\sqrt{6} = 0$
f $\sqrt{75} - \sqrt{25} = \sqrt{50}$

F Multiplication of surds

Surds can be multiplied using the properties $\sqrt{x} \times \sqrt{x} = x$

and $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$.

EXAMPLE 1

Simplify the following.

a $4 \times 3\sqrt{7}$ **b** $3\sqrt{5} \times 2\sqrt{3}$ **c** $6\sqrt{2} \times 3\sqrt{8}$ **d** $5\sqrt{8} \times 4\sqrt{3}$

a $4 \times 3\sqrt{7} = 4 \times 3 \times \sqrt{7}$
$= 12\sqrt{7}$

b $3\sqrt{5} \times 2\sqrt{3} = (2 \times 3) \times (\sqrt{5} \times \sqrt{3})$
$= 6\sqrt{15}$

c $6\sqrt{2} \times 3\sqrt{8} = (6 \times 3) \times (\sqrt{2} \times \sqrt{8})$
$= 18 \times \sqrt{16}$
$= 18 \times 4$
$= 72$

d $5\sqrt{8} \times 4\sqrt{3} = (5 \times 4) \times (\sqrt{8} \times \sqrt{3})$
$= 20 \times \sqrt{24}$
$= 20 \times \sqrt{4} \times \sqrt{6}$
$= 20 \times 2\sqrt{6}$
$= 40\sqrt{6}$

Exercise 11F

1 Simplify the following.

a $5 \times 2\sqrt{3}$ **b** $2 \times 6\sqrt{2}$ **c** $4\sqrt{7} \times 10$
d $3\sqrt{3} \times 6\sqrt{2}$ **e** $10\sqrt{5} \times 3\sqrt{7}$ **f** $6\sqrt{7} \times 7\sqrt{6}$
g $6\sqrt{2} \times 3\sqrt{18}$ **h** $2\sqrt{32} \times 5\sqrt{2}$ **i** $\sqrt{5} \times 3\sqrt{20}$
j $5\sqrt{3} \times 2\sqrt{8}$ **k** $6\sqrt{6} \times 2\sqrt{2}$ **l** $3\sqrt{10} \times \sqrt{2}$

EXAMPLE 2

Simplify the following.

a $3(\sqrt{7} + 2\sqrt{6})$ **b** $\sqrt{2}(\sqrt{7} - 3\sqrt{5})$ **c** $4\sqrt{3}(2\sqrt{5} + 3\sqrt{2})$

a $3(\sqrt{7} + 2\sqrt{6}) = 3 \times \sqrt{7} + 3 \times 2\sqrt{6}$
$= 3\sqrt{7} + 6\sqrt{6}$

b $\sqrt{2}(\sqrt{7} - 3\sqrt{5}) = \sqrt{2} \times \sqrt{7} + \sqrt{2} \times (-3\sqrt{5})$
$= \sqrt{14} - 3\sqrt{10}$

c $4\sqrt{3}(2\sqrt{5} + 3\sqrt{2}) = 4\sqrt{3} \times 2\sqrt{5} + 4\sqrt{3} \times 3\sqrt{2}$
$= 8\sqrt{15} + 12\sqrt{6}$

*Use the **distributive law** to remove grouping symbols: a(b + c) = ab + ac*

2 Simplify the following.

a $5(\sqrt{6} + \sqrt{3})$ **b** $2(\sqrt{5} + 4\sqrt{3})$ **c** $4(2\sqrt{10} - 3\sqrt{2})$
d $\sqrt{3}(\sqrt{5} + \sqrt{2})$ **e** $\sqrt{5}(\sqrt{6} - 4\sqrt{3})$ **f** $\sqrt{2}(3\sqrt{5} + 2\sqrt{7})$
g $2\sqrt{10}(6\sqrt{2} - 3\sqrt{10})$ **h** $5\sqrt{2}(3\sqrt{5} - 2\sqrt{6})$ **i** $3\sqrt{6}(4\sqrt{3} + 2\sqrt{8})$

EXAMPLE 3

Simplify the following.

a $(\sqrt{5} + 3)(\sqrt{5} - 4)$ **b** $(2\sqrt{3} - 2\sqrt{5})(4\sqrt{3} + \sqrt{5})$

a $(\sqrt{5} + 3)(\sqrt{5} - 4) = \sqrt{5}(\sqrt{5} - 4) + 3(\sqrt{5} - 4)$

$= 5 - 4\sqrt{5} + 3\sqrt{5} - 12$

$= -7 - \sqrt{5}$

b $(2\sqrt{3} - 2\sqrt{5})(4\sqrt{3} + \sqrt{5}) = 2\sqrt{3}(4\sqrt{3} + \sqrt{5}) - 2\sqrt{5}(4\sqrt{3} + \sqrt{5})$

$= 24 + 2\sqrt{15} - 8\sqrt{15} - 10$

$= 14 - 6\sqrt{15}$

Use **binomial expression** to remove grouping symbols:
$(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$

3 Simplify the following.

a $(\sqrt{3} + 5)(\sqrt{3} + 2)$ **b** $(\sqrt{7} + 3)(\sqrt{7} - 4)$ **c** $(\sqrt{10} - 6)(\sqrt{10} - 1)$

d $(2\sqrt{3} + 5)(\sqrt{3} + 1)$ **e** $(3\sqrt{2} + 4)(2\sqrt{2} - 7)$ **f** $(2\sqrt{5} - 3)(4\sqrt{5} + 3)$

g $(5\sqrt{2} - 3\sqrt{3})(3\sqrt{2} + 4\sqrt{3})$ **h** $(3\sqrt{7} + 5\sqrt{2})(\sqrt{7} - 4\sqrt{2})$ **i** $(2\sqrt{6} - 4\sqrt{3})(3\sqrt{6} - \sqrt{3})$

EXAMPLE 4

Simplify the following.

a $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$ **b** $(8\sqrt{2} + 3\sqrt{5})(8\sqrt{2} - 3\sqrt{5})$

a $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3}) = (\sqrt{2})^2 - (\sqrt{3})^2$

$= 2 - 3 = -1$

b $(8\sqrt{2} + 3\sqrt{5})(8\sqrt{2} - 3\sqrt{5}) = (8\sqrt{2})^2 - (3\sqrt{5})^2$

$= 64 \times 2 - 9 \times 5$

$= 128 - 45 = 63$

Use the special binomial expansions:
$(a + b)(a - b) = a^2 - b^2$
$(a - b)(a + b) = a^2 - b^2$

4 Simplify the following.

a $(\sqrt{5} - 3)(\sqrt{5} + 3)$ **b** $(\sqrt{10} + \sqrt{7})(\sqrt{10} - \sqrt{7})$ **c** $(2\sqrt{7} + \sqrt{5})(2\sqrt{7} - \sqrt{5})$

d $(\sqrt{6} - 3\sqrt{8})(\sqrt{6} + 3\sqrt{8})$ **e** $(4\sqrt{5} - 3\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$ **f** $(5\sqrt{2} + 3\sqrt{10})(5\sqrt{2} - 3\sqrt{10})$

EXAMPLE 5

Simplify the following.

a $(\sqrt{7} + \sqrt{3})^2$ **b** $(3\sqrt{5} - 2\sqrt{5})^2$

a $(\sqrt{7} + \sqrt{3})^2 = (\sqrt{7})^2 + 2 \times \sqrt{7} \times \sqrt{3} + (\sqrt{3})^2$

$= 7 + 2\sqrt{21} + 3$

b $(3\sqrt{5} - 2\sqrt{5})^2 = (3\sqrt{5})^2 - 2 \times 3\sqrt{5} \times 2\sqrt{2} + (2\sqrt{2})^2$

$= 9 \times 5 - 12\sqrt{10} + 4 \times 2$

$= 45 - 12\sqrt{10} + 8$

$= 53 - 12\sqrt{10}$

Use the binomial expansions of **perfect squares**:
$(a + b)^2 = a^2 + 2ab + b^2$
$(a - b)^2 = a^2 - 2ab + b^2$

5 Simplify the following.

a $(\sqrt{3} + 6)^2$ **b** $(\sqrt{5} + \sqrt{2})^2$ **c** $(\sqrt{10} - \sqrt{5})^2$

d $(2\sqrt{5} + 3)^2$ **e** $(3\sqrt{7} - 2\sqrt{5})^2$ **f** $(\sqrt{11} - 5\sqrt{5})^2$

6 Determine whether the following statements are true or false. Give reasons.

a $4 \times 3\sqrt{5} = 12\sqrt{60}$

b $\sqrt{7}(\sqrt{2} - 3) = \sqrt{14} - 3$

c $2\sqrt{3}(3\sqrt{2} + \sqrt{3}) = 6\sqrt{6} + 6$

d $(3\sqrt{2} + 5\sqrt{3})(\sqrt{2} - \sqrt{3}) = -9 + 2\sqrt{6}$

e $(\sqrt{6} + 3)(2\sqrt{6} - 5) = 3\sqrt{6} - 2$

f $(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3}) = 17$

g $(\sqrt{5} + \sqrt{7})(\sqrt{5} - \sqrt{7}) = -24$

h $(\sqrt{5} + \sqrt{3})^2 = 8$

i $(2\sqrt{7} - \sqrt{10})^2 = 38 - 4\sqrt{70}$

G Rationalising the denominator

To **rationalise** the denominator of a fraction means to convert it to an equivalent fraction with a rational (non-surd) denominator.

EXAMPLE 1

Rationalise the denominator of each of the following.

a $\frac{1}{\sqrt{7}}$ **b** $\frac{5}{\sqrt{7}}$ **c** $\frac{\sqrt{3}}{\sqrt{7}}$ **d** $\frac{5\sqrt{3}}{\sqrt{7}}$

a $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{\sqrt{7}}{7}$

b $\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{5\sqrt{7}}{7}$

$\frac{\sqrt{7}}{\sqrt{7}} = 1$!

c $\frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{\sqrt{21}}{7}$

d $\frac{5\sqrt{3}}{\sqrt{7}} = \frac{5\sqrt{3}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{5\sqrt{21}}{7}$

Remember: $\sqrt{x} \times \sqrt{x} = x$!

Exercise 11G

1 Rationalise the denominator of each of the following.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{10}}$

d $\frac{5}{\sqrt{2}}$ **e** $\frac{3}{\sqrt{6}}$ **f** $\frac{8}{\sqrt{7}}$

g $\frac{\sqrt{3}}{\sqrt{5}}$ **h** $\frac{\sqrt{7}}{\sqrt{3}}$ **i** $\frac{\sqrt{11}}{\sqrt{6}}$

j $\frac{\sqrt{2}}{\sqrt{7}}$ **k** $\frac{3\sqrt{2}}{\sqrt{5}}$ **l** $\frac{4\sqrt{3}}{\sqrt{10}}$

m $\frac{3\sqrt{7}}{\sqrt{6}}$ **n** $\frac{5\sqrt{5}}{\sqrt{2}}$ **o** $\frac{4\sqrt{10}}{\sqrt{3}}$

11007 Photo of a person or hands trimming and loosening the roots of a root-bound plant lifted from a pot

EXAMPLE 2

Rationalise the denominator of each of the following.

a $\frac{1}{4\sqrt{7}}$ **b** $\frac{5}{4\sqrt{7}}$ **c** $\frac{5\sqrt{3}}{4\sqrt{7}}$

a $\frac{1}{4\sqrt{7}} = \frac{1}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{4 \times 7}$

$= \frac{\sqrt{7}}{28}$

b $\frac{5}{4\sqrt{7}} = \frac{5}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{5\sqrt{7}}{28}$

c $\frac{5\sqrt{3}}{4\sqrt{7}} = \frac{5\sqrt{3}}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{5\sqrt{21}}{28}$

2 Rationalise the denominator of each of the following.

a $\frac{1}{2\sqrt{3}}$ **b** $\frac{1}{3\sqrt{5}}$ **c** $\frac{4}{5\sqrt{2}}$ **d** $\frac{8}{3\sqrt{7}}$ **e** $\frac{\sqrt{5}}{3\sqrt{2}}$

f $\frac{\sqrt{10}}{4\sqrt{3}}$ **g** $\frac{5\sqrt{7}}{3\sqrt{10}}$ **h** $\frac{6\sqrt{2}}{5\sqrt{3}}$ **i** $\frac{3\sqrt{5}}{2\sqrt{11}}$ **j** $\frac{2\sqrt{7}}{5\sqrt{6}}$

Extension

EXAMPLE 3

a Expand and simplify $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$.

b Hence rationalise the denominator of each of the following.

i $\frac{1}{\sqrt{5} + \sqrt{3}}$ **ii** $\frac{1}{\sqrt{5} - \sqrt{3}}$

a $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = 5 - \sqrt{15} + \sqrt{15} - 3$

$= 2$

Remember: $(a + b)(a - b) = a^2 - b^2$.

b **i** $\frac{1}{\sqrt{5} + \sqrt{3}} = \frac{1}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

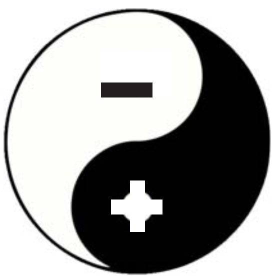
$= \frac{\sqrt{5} - \sqrt{3}}{2}$

$(a - b)$ is called the **conjugate** of $(a + b)$, and $(a + b)$ is the conjugate of $(a - b)$.

ii $\frac{1}{\sqrt{5} - \sqrt{3}} = \frac{1}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$= \frac{\sqrt{5} + \sqrt{3}}{2}$

11008 Photo or drawing of Yin-yang symbol with + and – sign



3 **a** Expand and simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$.

b Hence rationalise the denominator of each of the following.

i $\frac{1}{\sqrt{7} + \sqrt{2}}$ **ii** $\frac{1}{\sqrt{7} - \sqrt{2}}$

4 **a** Expand and simplify $(\sqrt{10} + \sqrt{3})(\sqrt{10} - \sqrt{3})$.

b Hence rationalise the denominator of the following.

i $\frac{1}{\sqrt{10}} + \sqrt{3}$ **ii** $\frac{1}{\sqrt{10} - \sqrt{3}}$

5 a Expand and simplify $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$.

b Hence rationalise the denominator of each of the following.

i $\frac{1}{\sqrt{5} + 2}$ ii $\frac{1}{\sqrt{5} - 2}$

6 a Expand and simplify $(6 + \sqrt{3})(6 - \sqrt{3})$.

b Hence rationalise the denominator of each of the following.

i $\frac{5}{6 + \sqrt{3}}$ ii $\frac{10}{6 - \sqrt{3}}$

EXAMPLE 4

a Expand and simplify $(5\sqrt{2} + \sqrt{3})(5\sqrt{2} - \sqrt{3})$.

b Hence rationalise the denominator of each of the following.

i $\frac{\sqrt{6}}{5\sqrt{2} + \sqrt{3}}$ ii $\frac{2\sqrt{2} - \sqrt{3}}{5\sqrt{2} - \sqrt{3}}$

a $(5\sqrt{2} + \sqrt{3})(5\sqrt{2} - \sqrt{3}) = (5\sqrt{2})^2 - (\sqrt{3})^2$
$= 25 \times 2 - 3$
$= 47$

b i $\frac{\sqrt{6}}{5\sqrt{2} + \sqrt{3}} = \frac{\sqrt{6}}{5\sqrt{2} + \sqrt{3}} \times \frac{5\sqrt{2} - \sqrt{3}}{5\sqrt{2} - \sqrt{3}}$

$= \frac{5\sqrt{12} - \sqrt{18}}{47}$

$= \frac{5\sqrt{4 \times 3} - \sqrt{9 \times 2}}{47}$

$= \frac{10\sqrt{3} - 3\sqrt{2}}{47}$

ii $\frac{2\sqrt{2} - \sqrt{3}}{5\sqrt{2} - \sqrt{3}} = \frac{2\sqrt{2} - \sqrt{3}}{5\sqrt{2} - \sqrt{3}} \times \frac{5\sqrt{2} + \sqrt{3}}{5\sqrt{2} + \sqrt{3}}$

$= \frac{10 \times 2 + 2\sqrt{6} - 5\sqrt{6} - 3}{47}$

$= \frac{17 - 3\sqrt{6}}{47}$

Simplify the numerator as much as possible.

7 a Expand and simplify $(2\sqrt{3} + \sqrt{2})(2\sqrt{3} - \sqrt{2})$.

b Hence rationalise the denominator of each of the following.

i $\frac{\sqrt{5}}{2\sqrt{3} + \sqrt{2}}$ ii $\frac{\sqrt{7}}{2\sqrt{3} - \sqrt{2}}$

8 a Expand and simplify $(3\sqrt{2} + 5)(3\sqrt{2} - 5)$.

b Hence rationalise the denominator of each of the following.

i $\frac{\sqrt{2} + 3}{3\sqrt{2} + 5}$ ii $\frac{3\sqrt{2} - 4}{3\sqrt{2} - 5}$

9 Rationalise the denominator of each of the following.

a $\frac{1}{\sqrt{6} - 2}$ b $\frac{3}{\sqrt{10} + \sqrt{2}}$ c $\frac{\sqrt{5} + 1}{\sqrt{3} - \sqrt{2}}$

d $\frac{\sqrt{10} - 2}{\sqrt{10} + 2}$ e $\frac{2\sqrt{2} - \sqrt{3}}{2\sqrt{2} + \sqrt{3}}$ f $\frac{2\sqrt{5} + 3\sqrt{2}}{2\sqrt{5} - 3\sqrt{2}}$

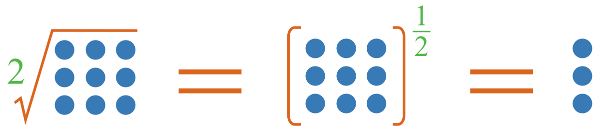
H Fractional indices

A surd can be expressed in index form as a **fractional index**.

In general, $\sqrt[n]{a} = a^{\frac{1}{n}}$

where n is the nth root of a.

We say that: $\sqrt[4]{a}$ is in **surd form**. $a^{\frac{1}{4}}$ is in **index form**



EXAMPLE 1

a Use the index laws to simplify $(a^{\frac{1}{4}})^4$.

b Simplify $(\sqrt[4]{a})^4$.

c Hence show that $a^{\frac{1}{4}} = \sqrt[4]{a}$.

a $(a^{\frac{1}{4}})^4 = a^{\frac{1}{4} \times 4}$
$= a^1$
$= a$

b $(\sqrt[4]{a})^4 = \sqrt[4]{a} \times \sqrt[4]{a} \times \sqrt[4]{a} \times \sqrt[4]{a}$
$= a$

c Since $(a^{\frac{1}{4}})^4 = (\sqrt[4]{a})^4 = a$, then $a^{\frac{1}{4}} = \sqrt[4]{a}$.

Exercise 11H

1 **a** Use the index laws to simplify $(a^{\frac{1}{2}})^2$.

b Simplify $(\sqrt{a})^2$.

c Hence show that $a^{\frac{1}{2}} = \sqrt{a}$.

2 **a** Use the index laws to simplify $(a^{\frac{1}{3}})^3$.

b Simplify $(\sqrt[3]{a})^3$.

c Hence show that $a^{\frac{1}{3}} = \sqrt[3]{a}$.

EXAMPLE 2

Write the following in index form.

a $\sqrt[3]{7}$

b $\sqrt[5]{m}$

a $\sqrt[3]{7} = 7^{\frac{1}{3}}$

b $\sqrt[5]{m} = m^{\frac{1}{5}}$

3 Write the following in index form.

a $\sqrt{10}$ **b** $\sqrt[3]{7}$ **c** $\sqrt[4]{5}$ **d** $\sqrt[5]{38}$

e $\sqrt[6]{56}$ **f** $\sqrt[3]{y}$ **g** $\sqrt[5]{k}$ **h** $\sqrt[n]{x}$

EXAMPLE 3

Write the following in surd form.

a $k^{\frac{1}{5}}$ **b** $z^{\frac{1}{10}}$

a $k^{\frac{1}{5}} = \sqrt[5]{a}$ **b** $z^{\frac{1}{10}} = \sqrt[10]{z}$

4 Write the following in surd form.

a $m^{\frac{1}{4}}$ **b** $y^{\frac{1}{2}}$ **c** $p^{\frac{1}{3}}$ **d** $t^{\frac{1}{8}}$

e $17^{\frac{1}{6}}$ **f** $25^{\frac{1}{4}}$ **g** $62^{\frac{1}{3}}$ **h** $x^{\frac{1}{n}}$

EXAMPLE 4

Evaluate the following.

a $25^{\frac{1}{2}}$ **b** $256^{\frac{1}{4}}$

a $25^{\frac{1}{2}} = \sqrt{25}$

$= 5$

b $256^{\frac{1}{4}} = \sqrt[4]{256}$

$= 4$ (since $4 \times 4 \times 4 \times 4 = 256$)

5 Evaluate the following.

a $49^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $625^{\frac{1}{4}}$ **d** $32^{\frac{1}{5}}$ **e** $1\,000\,000^{\frac{1}{6}}$

f $8^{\frac{1}{3}}$ **g** $121^{\frac{1}{2}}$ **h** $81^{\frac{1}{4}}$ **i** $64^{\frac{1}{3}}$ **j** $1^{\frac{1}{10}}$

EXAMPLE 5

Use your calculator to evaluate $12^{\frac{1}{5}}$ correct to 2 decimal places.

$12^{\frac{1}{5}} \approx 1.64$ 12 (1 ÷ 5) =

Check:

$\sqrt[5]{12} \approx 1.64$ 5 SHIFT $x^{\square}$ (12) =

Some calculators have different keys for these operations.

6 Use your calculator to evaluate the following, correct to 2 decimal places.

a $5^{\frac{1}{3}}$ **b** $2^{\frac{1}{4}}$ **c** $298^{\frac{1}{5}}$ **d** $41^{\frac{1}{2}}$ **e** $831^{\frac{1}{6}}$

EXAMPLE 6

Write the following in index form.

a $(\sqrt{m})^3$ **b** $\sqrt{m^3}$

a $(\sqrt{m})^3 = (m^{\frac{1}{2}})^3$

$= m^{\frac{1}{2} \times 3}$

$= m^{\frac{3}{2}}$

b $\sqrt{m^3} = (m^3)^{\frac{1}{2}}$

$= m^{3 \times \frac{1}{2}}$

$= m^{\frac{3}{2}}$

7 Write the following in index form.

a $(\sqrt{m})^5$ **b** $\sqrt{m^5}$ **c** $(\sqrt{a})^5$ **d** $\sqrt{a^5}$

e $\sqrt[4]{10^3}$ **f** $(\sqrt[4]{10})^3$ **g** $(\sqrt[7]{x})^n$ **h** $(\sqrt[n]{x})^7$

EXAMPLE 7

Write the following in surd form.

a $k^{\frac{3}{4}}$ **b** $w^{\frac{5}{3}}$

a $k^{\frac{3}{4}} = (k^{\frac{1}{4}})^3$ or $(k^3)^{\frac{1}{4}}$

$= (\sqrt[4]{k})^3$ or $\sqrt[4]{k^3}$

b $w^{\frac{5}{3}} = (w^{\frac{1}{3}})^5$ or $(w^5)^{\frac{1}{3}}$

$= (\sqrt[3]{w})^5$ or $\sqrt[3]{w^5}$

8 Write the following in surd form.

a $k^{\frac{2}{3}}$ **b** $m^{\frac{4}{3}}$ **c** $t^{\frac{3}{2}}$ **d** $a^{\frac{3}{5}}$

e $17^{\frac{5}{6}}$ **f** $25^{\frac{3}{4}}$ **g** $x^{\frac{n}{5}}$ **h** $x^{\frac{5}{n}}$

EXAMPLE 8

Evaluate the following.

a $8^{\frac{5}{3}}$ **b** $25^{\frac{3}{2}}$

a $8^{\frac{5}{3}} = (\sqrt[3]{8})^5$

$= 2^5$

$= 32$

b $25^{\frac{3}{2}} = (\sqrt{25})^3$

$= 5^3$

$= 125$

9 Evaluate the following.

a $9^{\frac{3}{2}}$ **b** $8^{\frac{2}{3}}$ **c** $16^{\frac{3}{4}}$ **d** $27^{\frac{4}{3}}$ **e** $4^{\frac{5}{2}}$

f $8^{\frac{4}{3}}$ **g** $49^{\frac{3}{2}}$ **h** $32^{\frac{3}{5}}$ **i** $10\,000^{\frac{3}{4}}$ **j** $125^{\frac{2}{3}}$

10 Use your calculator to evaluate the following, correct to 2 decimal places where necessary.

a $61^{\frac{3}{2}}$ **b** $729^{\frac{4}{3}}$ **c** $16\,807^{\frac{3}{5}}$ **d** $298^{\frac{3}{4}}$ **e** $1024^{\frac{13}{10}}$

11009 Photo of students using a calculator to work out fractional indices

I Some properties of real numbers

Exercise 11I

1 Use the fraction key on your calculator to evaluate (as fractions) the following.

a $\left(\frac{5}{2}\right)^{-1}$ **b** $\left(\frac{6}{5}\right)^{-1}$ **c** $\left(\frac{9}{7}\right)^{-1}$ **d** $\left(\frac{2}{3}\right)^{-1}$ **e** $\left(\frac{3}{4}\right)^{-1}$

2 Show that $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

3 If a and b are real numbers, determine whether the following statements are true or false. If false, give a **counter example**. (*Hint:* Try several different pairs of real numbers to test the truth of the statements.)

A counter example is an example that demonstrates the statement is false.

a **i** $a + b$ is a real number. **ii** $a - b$ is a real number.
iii $a \times b$ is a real number. **iv** $a \div b$ is a real number.

b **i** $a + b = b + a$ **ii** $a - b = b - a$
iii $a \times b = b \times a$ **iv** $a \div b = b \div a$

c **i** $(a + b) + c = a + (b + c)$ **ii** $(a - b) - c = a - (b - c)$
iii $(a \times b) \times c = a \times (b \times c)$ **iv** $(a \div b) \div c = a \div (b \div c)$

d **i** $a \times 0 = 0$ **ii** $a + 0 = a$

e **i** $a \times 1 = a$ **ii** $a \div a = 1$

4 If m and n are rational numbers, determine whether the following statements are true or false. If false give a counter example.

a $m + n$ is always rational. **b** $m - n$ is always rational.
c $m \times n$ is always rational. **d** $m \div n$ is always rational.

5 Find a pair of surds that satisfy each condition.

Remember: $\times$ gives product. $\div$ gives quotient.

a The product of the surds is irrational.
b The product of the surds is rational.
c The quotient of the surds is irrational.
d The quotient of the surds is rational.

6 **a** Write three **consecutive** integers starting with y.
b Hence show that the sum of any three consecutive integers is divisible by 3.

Consecutive means in order and without gaps.

7 **a** Show that any even real number can be written in the form $2k$, where k is an integer.
b Show that any odd real number can be written in the form $2k + 1$, where k is an integer.
c Hence prove the following properties of real numbers.
i The sum of any two even numbers is even.
ii The sum of any two odd numbers is even.
iii The sum of an even number and an odd number is odd.
iv The product of two even numbers is even.
v The product of an odd number and an even number is even.
vi The product of two odd numbers is odd.

Language in mathematics

1 **a** Explain, in words, how to calculate:

  i the square of a number **ii** the square root of a number.

b Write down the:

  i square of 9 **ii** square root(s) of 9.

c Which of the following numbers are perfect squares: 1, 4, 12, 25, 27, 200?

d Write in words:

  i m^2 **ii** $\sqrt{m}$ **iii** $-\sqrt{m}$ **iv** $m^{\frac{1}{2}}$

2 What is a:

a real number? **b** rational number? **c** irrational number?

3 Use words from the list below to complete the following statements.

radical, terminating, false, rationalising, recurring

a Every rational number can be expressed as a _____ or _____ decimal.

b Converting the denominator of a fraction into a rational number is called _____ the denominator.

c The mathematical sign $\sqrt{}$ is called the _____ sign.

d A counter example is an example that demonstrates that a statement is _____.

4 Write down an example of three consecutive integers.

5 **a** Find the sum and the product of 5 and 9.

b What is the quotient when 13 is divided by 5?

6 Use a dictionary to write down two meanings of each word.

a general **b** property **c** index

7 Three of the words in the following list have been spelt incorrectly. Rewrite them with the correct spelling.

ratio, convurt, indixes, recur, fractional, intejer

Terms

approximation	binomial expansion		conjugate	consecutive	convert
counter	example	denominator	distributive law	dot notation	expansion
fractional	general	index law	indices	integer	irrational
like surds	perfect square	power	product	property	quotient
radical sign	ratio	rational	rationalise	real number	recurring
square	square root	surd	terminating	undefined	

Check your skills

1 Which of the following statements is *not* correct?

A the square root of 25 is 5 or -5 **B** $\sqrt{25} = 5$ **C** $-\sqrt{25} = -5$ **D** $\sqrt{-25} = -5$

2 Convert $0.\dot{3}1\dot{2}$ to a fraction.

A $\frac{78}{25}$ **B** $\frac{103}{300}$ **C** $\frac{39}{125}$ **D** $\frac{103}{330}$

3 Which of the following is *not* a rational number?

A $\sqrt{17}$ **B** $\sqrt{\frac{9}{16}}$ **C** $3\frac{3}{4}$ **D** -21

4 $(2\sqrt{3})^2 =$

A 12 **B** 36 **C** 6 **D** $\sqrt{6}$

5 In simplest form, $\sqrt{80} =$

A $10\sqrt{8}$ **B** $8\sqrt{10}$ **C** $2\sqrt{20}$ **D** $4\sqrt{5}$

6 Written in the form $\sqrt{n}$, $2\sqrt{7} =$

A $\sqrt{14}$ **B** $\sqrt{28}$ **C** $\sqrt{98}$ **D** $\sqrt{196}$

7 $\dfrac{\sqrt{50}}{\sqrt{5}} =$

A 10 **B** $\sqrt{10}$ **C** $2\sqrt{5}$ **D** $5\sqrt{2}$

8 $\sqrt{9\frac{1}{4}} =$

A $\dfrac{\sqrt{37}}{2}$ **B** $3\frac{1}{2}$ **C** $3\frac{1}{4}$ **D** $9\frac{1}{2}$

9 $6\sqrt{5} - 2\sqrt{3} + 3\sqrt{5} =$

A $7\sqrt{7}$ **B** $9\sqrt{5} - 2\sqrt{3}$ **C** $7\sqrt{2}$ **D** $4\sqrt{2} + 3\sqrt{5}$

10 $\sqrt{12} + \sqrt{27} =$

A $\sqrt{39}$ **B** $5\sqrt{3}$ **C** $5\sqrt{6}$ **D** $3\sqrt{5}$

11 $4\sqrt{3} \times 5\sqrt{2} =$

A $20\sqrt{6}$ **B** $60\sqrt{2}$ **C** $5\sqrt{24}$ **D** 120

12 $\sqrt{5}(3\sqrt{2} - 3) =$

A $3\sqrt{10} - 3$ **B** $\sqrt{30} - \sqrt{15}$ **C** $3\sqrt{10} - \sqrt{15}$ **D** $3\sqrt{10} - 3\sqrt{5}$

13 $(\sqrt{6} - 2\sqrt{6})(\sqrt{6} + 2\sqrt{2}) =$

A 20 **B** $2\sqrt{6} - 4\sqrt{2}$ **C** $-2 - 8\sqrt{3}$ **D** -2

14 $(\sqrt{7} - \sqrt{2})^2 =$

A 5 **B** 25 **C** $5 - 2\sqrt{14}$ **D** $9 - 2\sqrt{14}$

15 Expressed with a rational denominator, $\dfrac{3\sqrt{2}}{2\sqrt{5}} =$

A $\dfrac{3\sqrt{2}}{10}$ **B** $\dfrac{3\sqrt{2}}{5}$ **C** $\dfrac{3\sqrt{10}}{10}$ **D** $\dfrac{6}{2\sqrt{10}}$

16 $12^{\frac{1}{2}} =$

A 6 **B** $\frac{1}{6}$ **C** $\sqrt{12}$ **D** $\frac{1}{144}$

17 In index form, $\sqrt[3]{k^4} =$

A k^{12} **B** $k^{\frac{4}{3}}$ **C** $k^{\frac{3}{4}}$ **D** k^{-1}

18 When evaluated, $16^{\frac{3}{4}} =$

A 12 **B** $21\frac{1}{3}$ **C** 8 **D** $\frac{1}{8}$

19 $\left(\frac{4}{3}\right)^{-1} =$

A $-\frac{4}{3}$ **B** $\frac{1}{12}$ **C** $\frac{3}{4}$ **D** $-\frac{3}{4}$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1	2	3	4–8	9, 10	11–14	15	16–18	19
Section	A	B	C	D	E	F	G	H	I

11A Review set

1 Find the following, where possible.

a Square root of 16 **b** $\sqrt{16}$ **c** $-\sqrt{16}$ **d** $\sqrt{-16}$ **e** $\sqrt{0}$

2 Convert the following rational numbers to decimals.

a $\frac{3}{8}$ **b** 23% **c** $\frac{1}{3}$ **d** $3\frac{1}{2}$

3 Show that the following are rational numbers by expressing them in the form $\frac{a}{b}$.

a $3\frac{1}{4}$ **b** 5 **c** 0.83 **d** $0.\dot{5}$

4 Determine whether the following real numbers are rational or irrational.

a -3 **b** $\sqrt{5}$ **c** $\sqrt{9}$ **d** $\sqrt{\frac{3}{4}}$ **e** $\sqrt{\frac{16}{9}}$

5 Simplify the following.

a $(\sqrt{2})^2$ **b** $(5\sqrt{2})^2$ **c** $\sqrt{6} \times \sqrt{7}$ **d** $3\sqrt{5} \times 2\sqrt{7}$

6 Express the following in simplest form.

a $\sqrt{18}$ **b** $\sqrt{20}$ **c** $\sqrt{3} \times \sqrt{12}$ **d** $\sqrt{8} \times \sqrt{3}$

7 Express the following in the form $\sqrt{n}$.

a $2\sqrt{5}$ **b** $5\sqrt{2}$

8 Simplify the following.

a $\sqrt{\frac{16}{25}}$ **b** $\sqrt{\frac{17}{25}}$ **c** $\sqrt{2\frac{1}{4}}$ **d** $\sqrt{1\frac{4}{9}}$

e $\frac{\sqrt{15}}{\sqrt{3}}$ **f** $\frac{\sqrt{20}}{\sqrt{5}}$ **g** $\frac{\sqrt{24}}{\sqrt{3}}$ **h** $\frac{\sqrt{21}}{\sqrt{3}}$

9 Write true or false.

a $\sqrt{7^2} = 7$ **b** $3\sqrt{5} = \sqrt{15}$ **c** $\sqrt{45} = 5\sqrt{3}$

d $\sqrt{2} \times \sqrt{32} = 8$ **e** $\frac{\sqrt{20}}{2} = \sqrt{5}$ **f** $\sqrt{4\frac{1}{4}} = 2\frac{1}{2}$

10 Simplify the following.

a $5\sqrt{3} + 7\sqrt{3} - 2\sqrt{3}$ **b** $8\sqrt{2} - 6\sqrt{6} + 4\sqrt{6}$ **c** $\sqrt{50} - \sqrt{18}$ **d** $2\sqrt{5} + \sqrt{20}$

11 Write true or false.

a $\sqrt{6} + \sqrt{3} = \sqrt{9}$ **b** $\sqrt{27} - \sqrt{6} = \sqrt{21}$ **c** $5\sqrt{7} - 4\sqrt{7} = \sqrt{7}$

12 Simplify the following.

a $2\sqrt{8} \times 5\sqrt{2}$ b $6\sqrt{8} \times 2\sqrt{3}$ c $4 \times 5\sqrt{10}$

d $\sqrt{5}(\sqrt{7} + \sqrt{2})$ e $3\sqrt{2}(2\sqrt{5} - 3\sqrt{3})$ f $(\sqrt{3} + 2\sqrt{2})(2\sqrt{3} - \sqrt{2})$

g $(\sqrt{6} + 3)(\sqrt{6} - 3)$ h $(3\sqrt{7} + 4)^2$

13 Rationalise the denominator of:

a $\frac{1}{\sqrt{10}}$ b $\frac{\sqrt{3}}{2\sqrt{10}}$

14 a Expand and simplify $(\sqrt{2} + 1)(\sqrt{2} - 1)$.

b Hence rationalise the denominator of $\frac{1}{\sqrt{2} + 1}$.

15 Write each of the following as a surd.

a $k^{\frac{1}{3}}$ b $k^{\frac{1}{5}}$ c $k^{\frac{3}{4}}$ d $k^{\frac{2}{3}}$

16 Write the following in index form.

a $\sqrt{z}$ b $\sqrt[3]{y}$ c $\sqrt{m^5}$ d $\sqrt[3]{y^2}$

17 Evaluate the following.

a $4^{\frac{5}{2}}$ b $25^{\frac{1}{2}}$ c $8^{\frac{4}{3}}$ d $343^{\frac{2}{3}}$

18 Write $\left(\frac{5}{4}\right)^{-1}$ as a fraction.

11B Review set

1 Find the following where possible.

a Square root of 36 b $\sqrt{36}$ c $-\sqrt{36}$ d $\sqrt{-36}$ e $\sqrt{0}$

2 Convert the following rational numbers to decimals.

a $\frac{5}{8}$ b 49% c $\frac{2}{3}$ d $4\frac{1}{2}$

3 Show that the following are rational numbers, by expressing them in the form $\frac{a}{b}$.

a $3\frac{1}{3}$ b 4 c 0.27 d $0.\dot{3}$

4 Determine whether the following real numbers are rational or irrational.

a -5.2 b $\sqrt{7}$ c $\sqrt{36}$ d $\sqrt{\frac{5}{8}}$ e $\sqrt{\frac{16}{9}}$

5 Simplify the following.

a $(\sqrt{3})^2$ b $(5\sqrt{3})^2$ c $\sqrt{5} \times \sqrt{6}$ d $2\sqrt{7} \times 6\sqrt{2}$

6 Express the following in simplest form.

a $\sqrt{8}$ b $\sqrt{45}$ c $\sqrt{8} \times \sqrt{2}$ d $\sqrt{8} \times \sqrt{6}$

7 Express the following in the form $\sqrt{n}$.

a $2\sqrt{7}$ b $4\sqrt{3}$

8 Simplify the following.

a $\sqrt{\frac{9}{4}}$ b $\sqrt{\frac{11}{4}}$ c $\sqrt{1\frac{7}{9}}$ d $\sqrt{3\frac{1}{4}}$

e $\frac{\sqrt{18}}{\sqrt{6}}$ f $\frac{\sqrt{24}}{\sqrt{6}}$ g $\frac{\sqrt{24}}{\sqrt{2}}$ h $\frac{\sqrt{45}}{\sqrt{5}}$

9 Write true or false.

a $\sqrt{10^2} = 10$ **b** $5\sqrt{3} = \sqrt{15}$ **c** $\sqrt{54} = 6\sqrt{3}$

d $\sqrt{2} \times \sqrt{18} = 6$ **e** $\frac{\sqrt{20}}{4} = \sqrt{5}$ **f** $\sqrt{9\frac{1}{4}} = 3\frac{1}{2}$

10 Simplify the following.

a $4\sqrt{2} + 5\sqrt{2} - 10\sqrt{2}$ **b** $5\sqrt{3} - 6\sqrt{5} - 4\sqrt{5}$

c $\sqrt{45} - \sqrt{20}$ **d** $3\sqrt{6} + \sqrt{24}$

11 Write true or false.

a $\sqrt{7} + \sqrt{5} = \sqrt{12}$ **b** $\sqrt{27} - \sqrt{12} = \sqrt{15}$

c $6\sqrt{6} - 5\sqrt{6} = 1$ **d** $\sqrt{20} - \sqrt{10} = \sqrt{10}$

12 Simplify the following.

a $2\sqrt{12} \times 5\sqrt{3}$ **b** $4\sqrt{6} \times 2\sqrt{3}$ **c** $3 \times 7\sqrt{2}$

d $\sqrt{3}(\sqrt{10} - \sqrt{3})$ **e** $2\sqrt{5}(2\sqrt{3} - 3\sqrt{5})$ **f** $(\sqrt{7} + \sqrt{3})(\sqrt{7} - 2\sqrt{3})$

g $(\sqrt{10} - \sqrt{6})(\sqrt{10} + \sqrt{6})$ **h** $(2\sqrt{6} - 5)^2$

13 Rationalise the denominator of:

a $\frac{1}{\sqrt{5}}$ **b** $\frac{\sqrt{2}}{3\sqrt{5}}$

14 **a** Expand and simplify $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$.

b Hence rationalise the denominator of $\frac{1}{\sqrt{5} - \sqrt{2}}$.

15 Write each of the following as a surd.

a $n^{\frac{1}{2}}$ **b** $n^{\frac{1}{4}}$ **c** $n^{\frac{4}{3}}$ **d** $n^{\frac{3}{5}}$

16 Write the following in index form.

a $\sqrt{m}$ **b** $\sqrt[3]{w}$ **c** $\sqrt{t^3}$ **d** $\sqrt[4]{y^3}$

17 Evaluate the following.

a $8^{\frac{1}{3}}$ **b** $49^{\frac{1}{2}}$ **c** $16^{\frac{3}{4}}$ **d** $324^{\frac{3}{2}}$

18 Write $\left(\frac{3}{5}\right)^{-1}$ as a fraction.

11C Review set

1 Find the following where possible.

a Square root of 9 **b** $\sqrt{9}$ **c** $-\sqrt{9}$ **d** $\sqrt{-9}$ **e** $\sqrt{0}$

2 Convert the following rational numbers to decimals.

a $\frac{1}{8}$ **b** 137% **c** $\frac{5}{9}$ **d** $5\frac{2}{3}$

3 Show that the following are rational numbers by expressing them in the form $\frac{a}{b}$.

a $1\frac{7}{8}$ **b** 2 **c** 0.314 **d** $0.\dot{6}$

4 Determine whether the following real numbers are rational or irrational.

a -17 **b** $\sqrt{11}$ **c** $\sqrt{100}$ **d** $\sqrt{\frac{7}{9}}$ **e** $\sqrt{\frac{16}{25}}$

5 Simplify the following.

a $(\sqrt{7})^2$ b $(2\sqrt{7})^2$ c $\sqrt{5} \times \sqrt{11}$ d $3\sqrt{6} \times 2\sqrt{5}$

6 Express the following in simplest form.

a $\sqrt{60}$ b $\sqrt{54}$ c $\sqrt{20} \times \sqrt{5}$ d $\sqrt{24} \times \sqrt{2}$

7 Express the following in the form $\sqrt{n}$.

a $10\sqrt{2}$ b $3\sqrt{7}$

8 Simplify the following.

a $\sqrt{\frac{9}{16}}$ b $\sqrt{\frac{17}{16}}$ c $\sqrt{1\frac{9}{16}}$ d $\sqrt{1\frac{5}{16}}$

e $\frac{\sqrt{30}}{\sqrt{10}}$ f $\frac{\sqrt{27}}{\sqrt{3}}$ g $\frac{\sqrt{48}}{\sqrt{6}}$ h $\frac{\sqrt{60}}{\sqrt{5}}$

9 Write true or false.

a $\sqrt{3^2} = 3$ b $4\sqrt{3} = \sqrt{12}$ c $\sqrt{63} = 7\sqrt{3}$

d $\sqrt{27} \times \sqrt{3} = 9$ e $\frac{\sqrt{40}}{4} = \sqrt{10}$ f $\sqrt{1\frac{4}{9}} = 1\frac{2}{3}$

10 Simplify the following.

a $2\sqrt{5} + 8\sqrt{5} - \sqrt{5}$ b $6\sqrt{3} - 5\sqrt{3} + 2\sqrt{7}$ c $\sqrt{32} - \sqrt{18}$ d $\sqrt{3} + \sqrt{27}$

11 Write true or false.

a $\sqrt{10} + \sqrt{10} = \sqrt{20}$ b $\sqrt{12} - \sqrt{3} = 3$ c $9\sqrt{5} - 8\sqrt{5} = 1$ d $\sqrt{28} - 2\sqrt{7} = 2\sqrt{7}$

12 Simplify the following.

a $2\sqrt{8} \times 5\sqrt{8}$ b $4\sqrt{8} \times 2\sqrt{6}$ c $3 \times 2\sqrt{7}$

d $\sqrt{3}(\sqrt{6} + \sqrt{5})$ e $5\sqrt{2}(3\sqrt{5} - 2\sqrt{2})$ f $(\sqrt{10} + 5)(\sqrt{10} - 6)$

g $(\sqrt{5} + 3\sqrt{7})(\sqrt{5} - 3\sqrt{7})$ h $(\sqrt{5} + 2\sqrt{3})2$

13 Rationalise the denominator of the following

a $\frac{1}{\sqrt{7}}$ b $\frac{\sqrt{2}}{3\sqrt{7}}$

14 a Expand and simplify $(5 + 2\sqrt{3})(5 - 2\sqrt{3})$.

b Hence rationalise the denominator of $\frac{1}{5 - 2\sqrt{3}}$.

15 Write each of the following as a surd.

a $w^{\frac{1}{3}}$ b $w^{\frac{1}{6}}$ c $w^{\frac{3}{2}}$ d $w^{\frac{2}{3}}$

16 Write the following in index form.

a $\sqrt{n}$ b $\sqrt[4]{z}$ c $\sqrt[3]{n^2}$ d $(\sqrt{m^2})^2$

17 Evaluate the following.

a $16^{\frac{1}{2}}$ b $25^{\frac{3}{2}}$ c $9261^{\frac{4}{3}}$ d $1296^{\frac{3}{4}}$

18 Write $\left(\frac{3}{8}\right)^{-1}$ as a fraction.