



Probability

This chapter deals with chance.

At the end of this chapter you should be able to:

- ▶ list all possible outcomes of a simple event
- ▶ find the sample space
- ▶ find the probability of simple events
- ▶ understand that probabilities take values from 0 and 1
- ▶ order words used to informally describe chance
- ▶ recognise that the sum of all possible outcomes of a simple event is 1
- ▶ identify complementary events
- ▶ calculate the probability of a complementary event

Diagnostic test

- 1 A normal die is thrown once. The number of outcomes in the sample space is:
A 1 B 3 C 5 D 6
- 2 A ticket is selected at random from a hat containing 1 blue, 2 red and 1 green tickets. The sample space is:
A {blue, red, green} B {red}
C {blue, red, red, green} D {not red}
- 3 A bag contains 1 pink, 1 purple and 1 orange ball. The probability of randomly selecting a purple ball is:
A 1 B 3 C $\frac{1}{3}$ D $\frac{2}{3}$
- 4 The probability of an event is $\frac{2}{5}$. As a percentage this is:
A 20% B 40%
C 50% D none of these
- 5 A hat contains 3 blue and 5 black tickets. One ticket is chosen from the hat at random. The probability that it is blue is:
A $\frac{1}{2}$ B $\frac{3}{5}$ C 3 D $\frac{3}{8}$
- 6 A letter is chosen randomly from the word PROBABILITY. The probability that the letter is B is:
A $\frac{1}{11}$ B $\frac{2}{11}$ C 1 D $\frac{2}{9}$
- 7 A term describing a probability of about 80% is:
A certain B highly probable
C evens D low probability
- 8 A coin is tossed once. An impossible event would be getting:
A heads B tails
C head or tails D a six
- 9 When a normal six-sided die is rolled, the probability of getting 7 is:
A $\frac{7}{6}$ B $\frac{6}{7}$ C $\frac{1}{6}$ D 0
- 10 If an event is certain to happen, its probability is:
A 0 B $\frac{1}{2}$ C 1 D 2
- 11 A bag contains red, blue and white marbles. If a marble is chosen at random from this bag, the probability that it is red is $\frac{2}{5}$ and the probability that it is blue is $\frac{1}{3}$. The probability that it is white is therefore:
A $\frac{3}{5}$ B $\frac{4}{15}$ C $\frac{2}{3}$ D $\frac{11}{15}$
- 12 When a normal six-sided die is rolled, the complement of the event 'rolling a number less than 3' is:
A rolling a number greater than 3
B rolling 1 or 2
C rolling a number greater than 2
D rolling 1 or 2 or 3
- 13 When three coins are tossed the probability of getting 3 heads is $\frac{1}{8}$. The probability of *not* getting 3 heads is therefore:
A $\frac{1}{8}$ B $\frac{3}{8}$ C $\frac{5}{8}$ D $\frac{7}{8}$

The Diagnostic Test questions refer to the sections of text listed in the table below.

Question	1, 2	3, 4	5, 6	7–10	11	12	13
Section	A	B	C	D	E	F	G

PROBABILITY

We use probability every day in our lives. People often say things like:

- 'It is likely to rain today.'
- 'I will probably come to your party.'
- 'I am unlikely to pass this exam.'
- 'The Suns will certainly beat the Eagles this game.'

These statements talk about the **chance** of something occurring. Many terms are used to discuss the likelihood of something happening, such as: certain, very probable, highly likely, not likely, no chance, small chance, even chance.



A The sample space

One of the most common examples of chance is the toss of a coin at the start of a sporting event. When the coin is tossed, there are two possible **outcomes**: head or tail. If the coin is unbiased, these outcomes are equally likely. This means there is an equal chance of each occurring.

The **sample space** is a list of all the possible outcomes of an experiment. For the toss of a coin, the sample space is head and tail. This is written as {head, tail}.

One outcome in the sample space is called an **element** of the sample space.

EXAMPLE 1

A six-sided die has the numbers 1 to 6 on its faces.
List the sample space when the die is thrown.



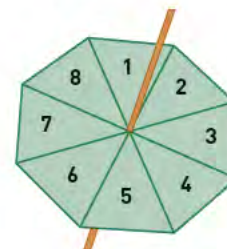
The sample space is a list of all possible outcomes.

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

No other outcomes are possible. For a normal die, these outcomes are equally likely.

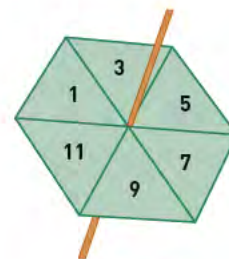
Exercise 10A

- 1 A coin is tossed. The sample space is {_____}.
- 2 A ball is selected from a bag containing 1 black, 1 white and 1 green ball. The sample space is {_____}.
- 3 A spinner with eight edges has the numbers 1 to 8 listed on it.
 - a What is the sample space when it is spun?
 - b Are these outcomes equally likely?
- 4 A hat contains 1 red, 1 white and 1 blue ticket.
 - a List the sample space when a ticket is drawn at random.
 - b Are these outcomes equally likely?

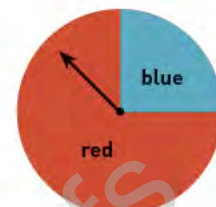




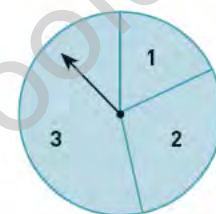
- 5 a** List the sample space when this spinner is spun.
b Are these outcomes equally likely?



- 6 a** List the sample space when this spinner is spun.
b Are these outcomes equally likely?



- 7 a** List the sample space when this spinner is spun.
b Are these outcomes equally likely?



- 8 a** List the sample space when a child is born.
b Are these outcomes equally likely?
- 9 a** List the sample space for the colour of the traffic light as a car approaches it.
b Are these outcomes equally likely?
- 10 a** List the sample space for the result (not the score) of a soccer match.
b Are these outcomes equally likely?
- 11** A ticket is selected from a bag containing red, green and blue tickets.
a Is a red ticket in the sample space?
b Is a pink ticket in the sample space?
c List the sample space.
- 12** A ten-sided die with the numbers 1 to 10 is tossed.
a Is 6 in the sample space?
b Is 12 an element of the sample space?



B Probability

EXAMPLE 1

What is the probability of getting 5 when a normal six-sided die is thrown?



The sample space is $\{1, 2, 3, 4, 5, 6\}$. There are 6 equally likely outcomes.

There is only one 5, so the probability of getting 5 is 1 out of 6. This is written as $P(5) = \frac{1}{6}$.

Exercise 10B

- 1 A normal six-sided die is thrown. Complete the following.
 - a Sample space $S = \{\text{_____}\}$
 - b Number of equally likely outcomes = _____
 - c i $P(1) = \text{_____}$ ii $P(3) = \text{_____}$ iii $P(4) = \text{_____}$ iv $P(6) = \text{_____}$
- 2 A bag contains 1 blue, 1 red and 1 green ball. A ball is selected at **random**.

'At random' or 'randomly' means without any bias. 

 - a List the sample space.
 - b How many equally likely outcomes are there?
 - c What is the probability of selecting a red ball?
- 3 This spinner is spun once.
 
 - a List the sample space.
 - b How many equally likely outcomes are there?
 - c What is the probability that the colour spun is green?
- 4 Ten cards with the numbers 0 to 9 written on them are shuffled, then one card is selected at random.
 - a List the sample space.
 - b How many outcomes are in the sample space?
 - c What is the probability that the number selected is:
 - i 5? ii 8?
- 5 The letters of the alphabet are written on cards and the cards shuffled. A card is chosen at random.
 - a How many equally likely outcomes are there?
 - b What is the probability that the letter chosen is:
 - i T? ii M? iii Z?

Investigation 1 Fractions, decimals and percentages

Fractions, decimals and percentages are different ways of showing the same probability. For example, a probability of $\frac{1}{4}$ may be written as 0.25 or as 25%. Copy and complete this table. Which method of showing probability is easiest? Explain.

Fraction	Decimal	Percentage
$\frac{1}{10}$		
	0.2	
		30%
$\frac{2}{5}$		
		35%
	0.43	
$\frac{1}{2}$		
$\frac{5}{8}$		
		60%
$\frac{3}{4}$		
	0.85	
		100%

C Theoretical probability

In probability, an **event** is a collection of outcomes. (A **simple event** has outcomes that are equally likely.) The possible outcomes that make up an event are said to be **favourable** for that event. We formalise the chance of an event happening by defining its **theoretical probability** as:

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

EXAMPLE 1

A normal six-sided die is rolled. Calculate the probability of getting:

- a 4
- b 5 or 6
- c an even number



$S = \{1, 2, 3, 4, 5, 6\}$, so number of possible outcomes = 6

Remember to simplify fractions. !

- a Favourable outcome = $\{4\}$, so number of favourable outcomes = 1. $P(4) = \frac{1}{6}$
- b Favourable outcomes = $\{5, 6\}$, so number of favourable outcomes = 2. $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$
- c Favourable outcomes = $\{2, 4, 6\}$, so number of favourable outcomes = 3. $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

Exercise 10C

1 A normal six-sided die is rolled. Complete the following to calculate the probability of getting:

- a 6
- b 2 or 3
- c an odd number
- d a number < 3
- e a number > 2

$S = \{\underline{\hspace{1cm}}\}$, so number of possible outcomes = $\underline{\hspace{1cm}}$

- a Favourable outcome = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(6) = \frac{\square}{\square}$
- b Favourable outcomes = $\{2, 3\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(2 \text{ or } 3) = \frac{\square}{\square} = \frac{\square}{\square}$
- c Favourable outcomes = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(\text{odd number}) = \frac{\square}{\square} = \frac{\square}{\square}$
- d Favourable outcomes = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(\text{number} < 3) = \frac{\square}{\square} = \frac{\square}{\square}$
- e Favourable outcomes = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(\text{number} > 2) = \frac{\square}{\square} = \frac{\square}{\square}$

2 The numbers 0 to 9 are written on cards. The cards are shuffled and one is chosen at random. Complete the following to find the probability (as a decimal) of getting:

- a 5
- b 0 or 9
- c an odd number
- d an even number
- e a number < 5
- f a number > 7

$S = \{\underline{\hspace{1cm}}\}$, so number of possible outcomes = $\underline{\hspace{1cm}}$

- a Favourable outcome = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(5) = \frac{\square}{\square} = \underline{\hspace{1cm}}$
- b Favourable outcomes = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(0 \text{ or } 9) = \frac{\square}{\square} = \underline{\hspace{1cm}}$
- c Favourable outcomes = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$. $P(\text{odd number}) = \frac{\square}{\square} = \underline{\hspace{1cm}}$
- d Favourable outcomes = $\{\underline{\hspace{1cm}}\}$, so number of favourable outcomes = $\underline{\hspace{1cm}}$.
 $P(\text{even number}) = \frac{\square}{\square} = \underline{\hspace{1cm}}$

e Favourable outcomes = {____}, so number of favourable outcomes = ____.

$$P(\text{number} < 5) = \frac{\square}{\square} = \text{____}$$

f Favourable outcomes = {____}, so number of favourable outcomes = ____.

$$P(\text{number} > 7) = \frac{\square}{\square} = \text{____}$$

- 3 The letters of the word RANDOM are written on cards, then one is selected at random. Find the probability that it is:

a N

b N or M

c a vowel

d a consonant

- 4 A bag of lollies contains 5 red and 2 green lollies. One lolly is chosen at random from the bag. Complete the following to find the probability that it is:

a red

b green

$S = \{r, r, r, r, r, g, g\}$, so number of possible outcomes (= number of lollies in the bag) = ____.

a Favourable outcomes = {____},
so number of favourable outcomes = ____.

$$P(\text{red}) = \frac{\square}{\square}$$

b Favourable outcomes = {____},
so number of favourable outcomes = ____.

$$P(\text{green}) = \frac{\square}{\square}$$



- 5 A hat contains 4 black and 5 green tickets. One ticket is chosen at random from the hat. Complete the following to find the probability that it is:

a black

b green

$S = \{b, b, b, b, g, g, g, g, g\}$, so number of possible outcomes (= number of tickets in hat) = ____.

a Favourable outcomes = {____}, so number of favourable outcomes = ____.

$$P(\text{black}) = \frac{\square}{\square}$$

b Favourable outcomes = {____}, so number of favourable outcomes = ____.

$$P(\text{green}) = \frac{\square}{\square}$$

- 6 A bag contains 5 green and 8 purple lollies. One lolly is chosen at random from the bag. Find the probability that it is:

a green

b purple



- 7 A hat contains 4 green, 6 red and 5 blue tickets. If one ticket is chosen at random from the hat, find the probability of getting:

a a red ticket

b a blue ticket

c a green or red ticket

- 8 A box of ice-blocks contains 6 green, 5 red and 3 blue ice-blocks. One ice-block is chosen at random from the box. Find the probability that it is:

a blue

b red or green

c blue or green

- 9 A purse contains seven 10-cent coins, eight 20-cent coins and nine dollar coins. A coin is chosen at random from the purse. Find:

a P(20-cent coin)

b P(10-cent coin)

c P(value of coin > 10 cents)

- 10 A hat contains 3 red, 4 white and 5 blue tickets. A ticket is selected at random from the hat. Find the probability that the ticket is:

a red

b white

c blue

d green

e not red

f not white

g red or blue

h not red or white

i red, white or blue

- 11 A letter is chosen at random from the word MATHEMATICS. Find the probability that the letter is:

a H b I c M
d A e a vowel f not repeated

- 12 Mark is holding 13 cards, from a normal playing pack. They are all the same suit. One card is selected at random. Find the probability that it is:

a an ace b 3, 4 or 5
c a king or queen d a number > 5



- 13 Adam has spread out a pack of playing cards. He picks a card at random. Hearts and diamonds are red. Clubs and spades are black. Picture cards are king, queen and jack. Find:

a P(spade) b P(4 of hearts)
c P(club) d P(black 10)
e P(ace) f P(picture card)
g P(5 or 6) h P(green card)
i P(not a picture card) j P(number < 10)



- 14 In Year 7, 40% of students walk to school, 25% travel by bus, 20% travel by car and 15% ride a bicycle. A student is chosen at random. As a percentage, find:

a P(travels by bus) b P(rides a bicycle) c P(walks) d P(does not travel by bus)

- 15 Rewrite your percentage probabilities in question 14 as decimals.

- 16 Year 7 were surveyed on the amount of pocket money they received. 35 students received less than \$10, 58 received from \$10 to \$20, and 7 received more than \$20. A student is chosen at random. As a decimal, find the probability that the student received as pocket money:

a less than \$10 b more than \$20 c \$20 or less

- 17 Rewrite your decimal probabilities in question 16 as percentages.

- 18 a A normal die is rolled once. List the sample space.
b How many times does a 7 appear in the sample space?
c Write down the probability of getting a 7.
d How many times does a number less than 7 appear in the sample space?
e Write down the probability of getting a number less than 7.

- 19 a This spinner is spun once. List the sample space.
b How many times does the outcome purple appear in the sample space?
c What is the probability of getting purple?
d How many elements of the sample space are either red or green or blue or gold?
e Write down the probability of getting red or green or blue or gold.



- 20 A set of 9 cards have the numbers 1 to 9 written on them. One card is selected at random. What is the probability that the number on the card is:

a 15? b odd or even?

- 21 Discuss the results of questions 18, 19 and 20 and complete the following.

a If an event is **impossible**, its probability is _____.
b If an event is **certain**, its probability is _____.



Informal concept of chance

Everyday conversations include many references to probability and chance, in statements like 'I am unlikely to win a prize in the raffle'. They give an indication of whether or not something is going to happen.

As you have seen, probabilities are often written as fractions. Probabilities always take values from 0 to 1.

The probability of an event E occurring can be written as $P(E)$.

- A probability of 0 means that the event cannot occur. It is impossible. $P(E) = 0$.
- A probability of 1 means that the event must happen. It is certain. $P(E) = 1$.
- The closer a probability is to 1, the more likely the event is to occur.

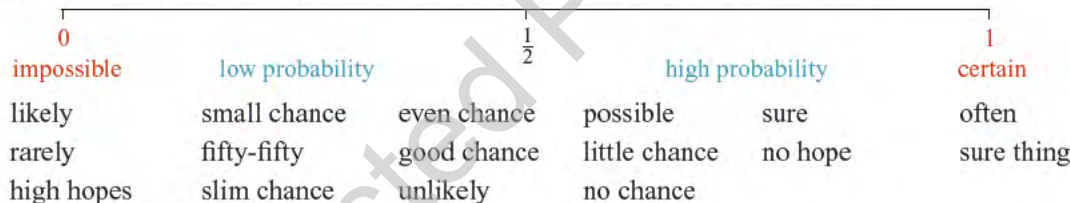
Exercise 10D

- 1 A bag contains 99 black and 1 white marbles. A marble is selected at random from the bag.

- Is it likely that the marble is black?
- Is the chance of selecting a black marble the same as the chance of selecting a white one?
- Is it likely that the marble will be white?
- Is it impossible to select a white marble?
- Is it impossible to select a green marble? Explain.



- 2 Copy this number line, then locate each of the terms below on the line.



- 3 Use one of these terms to describe each event described below:

impossible low probability
even chance high probability
certain

- 25 December being a holiday
- snow falling in Darwin
- your family going on an overseas trip this year
- the sun rising tomorrow
- you being struck by lightning
- living to 200 years of age
- getting Mathematics homework tonight
- winning Lotto
- winning the toss next time you try
- getting 6 if you throw a normal six-sided die once



4 Use one of the percentages 0%, 10%, 50%, 80%, 100% to describe each event in question 3.

5 Write your own statement to describe an event that is:

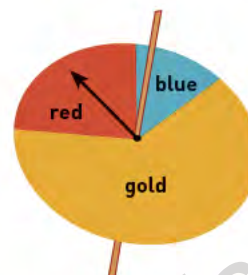
- a impossible b low probability c even chance d high probability e certain

6 For this spinner, describe each event below using one of the terms:

- certain high probability even chance
low probability impossible

The spinner is spun and the colour is:

- a blue b red c gold
d green e red or gold f red, gold or blue



7 Describe these percentage probabilities using the same terms as in question 6.

- a 10% b 50% c 100% d 80%
e 30% f 49% g 0% h 5%

EXAMPLE 1

A normal six-sided die is thrown once. Describe an event that would be:

- a certain
b impossible
c even chance



- a Getting 1 or 2 or 3 or 4 or 5 or 6 would be certain. $P(1, 2, 3, 4, 5 \text{ or } 6) = 1$
b Getting tails would be impossible. $P(\text{tails}) = 0$.
c The chance of getting 1 or 2 or 3 would be even. $P(1, 2 \text{ or } 3) = \frac{1}{2}$.

8 A coin is tossed once. Describe an event that would be:

- a certain b impossible c even chance

9 A hat contains 10 tickets numbered 1 to 10. Describe an event that would be:

- a certain b impossible c even chance d probability $\frac{1}{10}$

10 A tennis racquet with sides labelled rough and smooth is spun once.

Describe an event with a probability of:

- a 1 b 0 c $\frac{1}{2}$

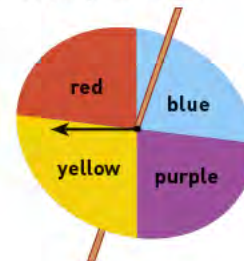
11 A spinner has 4 equal sections coloured red, blue, green and yellow. The spinner is spun once. Describe an event with a probability of:

- a 1 b 0 c $\frac{1}{2}$
d $\frac{1}{4}$ e $\frac{3}{4}$



Investigation 2 Probability certainties

- 1 A coin is tossed once.
 - a List the sample space.
 - b What is the probability of heads?
 - c What is the probability of tails?
 - d Add the probability of heads and the probability of tails.
 - e Describe an event that results in a certainty.
 - f What is the probability of the event in part e?
- 2 A normal six-sided die is thrown once.
 - a List the sample space.
 - b What is:
 - i $P(6)$? ii $P(5)$? iii $P(4)$?
 - iv $P(3)$? v $P(2)$? vi $P(1)$?
 - c Add the probabilities of getting 6, 5, 4, 3, 2 and 1.
 - d What is the probability of getting 6 or 5 or 4 or 3 or 2 or 1 when throwing a die once?
 - e Why is this a certainty?
- 3 A spinner has 4 equal-sized segments coloured blue, purple, red and yellow. The spinner is spun once.
 - a List the sample space.
 - b What is the probability of the outcome blue, purple, red or yellow?
 - c What is:
 - i $P(\text{blue})$? ii $P(\text{purple})$? iii $P(\text{red})$? iv $P(\text{yellow})$?
 - d Add the probabilities from part c. What do you notice? Explain.
- 4 Complete:
The sum of the probabilities of all possible outcomes of a single experiment is _____.
- 5 A normal six-sided die is thrown once.
 - a Find $P(6)$.
 - b Find $P(1, 2, 3, 4 \text{ or } 5)$.
 - c Find $P(\text{not } 6)$. (*Hint: see part b.*)
 - d Add the probability of getting 6 to the probability of *not* getting 6. What do you notice? Explain.



E Sum of probabilities

In Investigation 2 you found that the sum of the probabilities of all possible outcomes of an experiment is 1.

EXAMPLE 1

A bag contains red, blue and green marbles. If one marble is chosen at random from this bag, the probability that it is red is $\frac{6}{13}$ and the probability that it is blue is $\frac{3}{13}$. Calculate the probability that it is green.

$$\begin{aligned}
 P(\text{red}) + P(\text{blue}) + P(\text{green}) &= 1 \\
 \frac{6}{13} + \frac{3}{13} + P(\text{green}) &= 1 \\
 \frac{9}{13} + P(\text{green}) &= 1 \\
 P(\text{green}) &= 1 - \frac{9}{13} = \frac{4}{13}
 \end{aligned}$$

Exercise 10E

- 1 A bag contains black, white and orange counters. If a counter is chosen at random from this bag, the probability that it is black is $\frac{7}{19}$ and the probability that it is white is $\frac{4}{19}$. Complete the following to calculate the probability that it is orange.

$$\begin{aligned} P(\text{black}) + P(\text{white}) + P(\text{orange}) &= ______ \\ \frac{7}{19} + \frac{\square}{\square} + P(\text{orange}) &= ______ \\ \frac{\square}{\square} + P(\text{orange}) &= ______ \\ P(\text{orange}) &= ______ - \frac{\square}{\square} = \frac{\square}{\square} \end{aligned}$$

- 2 A European roulette wheel has red, black and green numbers. When the wheel is spun, the probability that the ball will stop on a red number is $\frac{18}{37}$, and the probability that it will stop on the green number (0) is $\frac{1}{37}$. What is the probability that it will stop on a black number?
- 3 A spinner consists of pink, blue and green sections. If the spinner is spun, the probability that it will stop on blue is $\frac{1}{2}$ and the probability that it will stop on green is $\frac{1}{3}$. Find the probability that it will stop on pink.
- 4 When three coins are tossed the possible outcomes are 3 heads, exactly 2 heads, exactly 1 head and no heads. The probability of getting 3 heads is $\frac{1}{8}$, the probability of getting exactly 1 head is $\frac{3}{8}$ and the probability of getting no heads is $\frac{1}{8}$. Calculate the probability of getting exactly 2 heads.
- 5 A purse contains 5-cent, 10-cent, 20-cent and 50-cent coins. If a coin is chosen at random from this purse, the probability of getting a 5-cent coin is $\frac{1}{5}$, the probability of getting a 10-cent coin is $\frac{1}{3}$ and the probability of getting a 50-cent coin is $\frac{1}{10}$. Calculate the probability of getting a 20-cent coin.



F Identifying complements of events

In an experiment with sample space S , the **complement** of an event A is the set of outcomes, from the sample space S , that are not in A .

EXAMPLE 1

When a normal six-sided die is thrown, find the complement of the event:

- a throwing 2
- b throwing 1 or 2
- c throwing an even number



- a $S = \{1, 2, 3, 4, 5, 6\}$. Let A be the event 'throwing a 2'; then $A = \{2\}$.

The complement of A is the set of outcomes, from S , that are not in A .

Complement of $A = \{1, 3, 4, 5, 6\}$

The complement of A is the event 'throwing a 1 or 3 or 4 or 5 or 6' or 'not throwing a 2'.

Write the event as a set of outcomes, even if there is only one outcome in the set.





- b** $S = \{1, 2, 3, 4, 5, 6\}$. Let B be the event 'throwing a 1 or 2'; then $B = \{1, 2\}$.
The complement of B is the set of outcomes, from S , that are not in B .
Complement of $B = \{3, 4, 5, 6\}$
The complement of B is the event 'throwing a 3 or 4 or 5 or 6' or 'not throwing a 1 or 2' or 'throwing a number greater than 2'.
- c** $S = \{1, 2, 3, 4, 5, 6\}$. Let C be the event 'throwing an even number'; then $C = \{2, 4, 6\}$.
The complement of C is the set of outcomes, from S , that are not in C .
Complement of $C = \{1, 3, 5\}$
The complement of C is the event 'throwing a 1 or 3 or 5' or 'not throwing an even number' or 'throwing an odd number'.

Exercise 10F

- 1** A normal die is thrown. Complete the following to find the complement of each event given.
- a** throwing a 6
 $S = \{\underline{\hspace{1cm}}\}$. Let A be the event 'throwing a 6'; then $A = \{\underline{\hspace{1cm}}\}$.
The complement of A is the set of outcomes, from S , that are not in A .
Complement of $A = \{\underline{\hspace{1cm}}\}$
The complement of A is the event ' ' or ' ' or ' '.
- b** throwing a 5 or 6
 $S = \{\underline{\hspace{1cm}}\}$. Let B be the event 'throwing a 5 or 6'; then $B = \{\underline{\hspace{1cm}}\}$.
The complement of B is the set of outcomes, from S , that are not in B .
Complement of $B = \{\underline{\hspace{1cm}}\}$
The complement of B is the event ' ' or ' ' or ' '.
- c** throwing an odd number
 $S = \{\underline{\hspace{1cm}}\}$. Let C be the event 'throwing an odd number'; then $C = \{\underline{\hspace{1cm}}\}$.
The complement of C is the set of outcomes, from S , that are not in C .
Complement of $C = \{\underline{\hspace{1cm}}\}$
The complement of C is the event ' ' or ' ' or ' '.
- 2** A normal die is thrown. Find the complement of the event:
- a** throwing 3 **b** throwing 3 or 5 **c** throwing a number greater than 2
- 3** The numbers 1 to 9 are written on cards and one card is chosen at random. Find the complement of the event:
- a** choosing 7 **b** choosing 1 or 7
c choosing an even number **d** choosing a number greater than 4

EXAMPLE 2

- a** If a student is chosen from a class consisting of boys and girls, what is the complement of the event 'choosing a boy'.
- b** If a marble is chosen from a bag containing black and yellow marbles, what is the complement of the event 'selecting a black marble'.
- a** The complement of the event 'choosing a boy' is 'not choosing a boy' or 'choosing a girl'.
- b** The complement of the event 'selecting a black marble' is 'not selecting a black marble' or 'selecting a yellow marble'.

4 Complete the following.

- If a student is chosen from a class consisting of boys and girls, the complement of the event 'choosing a girl' is '_____ choosing a girl' or 'choosing a _____'.
- If a marble is chosen from a bag containing red and blue marbles, the complement of the event 'choosing a red marble' is '_____ choosing a red marble' or 'choosing a _____ marble'.
- If a marble is chosen from a bag containing red, blue and green marbles, the complement of the event 'choosing a red marble' is '_____ choosing a red marble' or 'choosing a _____ marble'.
- If a marble is chosen from a bag containing red, blue and green marbles, the complement of the event 'choosing a red or blue marble' is '_____ choosing a red or blue marble' or 'choosing a _____ marble'.

5 A coin is tossed. What is the complement of the event 'getting a head'?

6 A spinner has four segments coloured red, blue, green and yellow. The spinner is spun and the colour on which it stops is noted. What is the complement of the event:

- stops on red?
- stops on red or blue?
- stops on blue or green?
- stops on red or blue or green?

7 A card is selected from a normal playing pack. What is the complement of selecting:

- a red card?
- a spade?
- a spade or a club?
- a picture card?

8 Sydney plays Melbourne in a soccer final. What is the complement of the event:

- Sydney wins or draws?
- Sydney wins?
- a draw?
- Melbourne loses or draws?



Investigation 3 Sum of probabilities of complementary events

1 A coin is tossed once. Find:

- $P(\text{head})$
- $P(\text{not head})$
- $P(\text{head}) + P(\text{not head})$

2 A normal six-sided die is rolled. Find:

- i $P(2)$ ii $P(\text{not } 2)$ iii $P(2) + P(\text{not } 2)$
- i $P(6)$ ii $P(\text{not } 6)$ iii $P(6) + P(\text{not } 6)$
- i $P(\text{even number})$ ii $P(\text{not even number})$ iii $P(\text{even number}) + P(\text{not even number})$
- i $P(\text{number} < 5)$ ii $P(\text{number not} < 5)$ iii $P(\text{number} < 5) + P(\text{number not} < 5)$

3 A bag contains 2 red, 5 blue and 4 white counters. A counter is chosen at random from the bag. Find:

- i $P(\text{red})$ ii $P(\text{not red})$ iii $P(\text{red}) + P(\text{not red})$
- i $P(\text{blue})$ ii $P(\text{not blue})$ iii $P(\text{blue}) + P(\text{not blue})$
- i $P(\text{white})$ ii $P(\text{not white})$ iii $P(\text{white}) + P(\text{not white})$

The pairs of events investigated above:

- getting a head, not getting a head
- getting 2, not getting 2
- getting a white counter, not getting a white counter

are called **complementary events**. Each event is the complement of the other.

The complement of the set E , called **not E** , can be written $\bar{E}$ or E' .

From the investigation we see that: $P(E) + P(\text{not } E) = 1$ or $P(E) + P(\bar{E}) = 1$





Probabilities of complementary events

EXAMPLE 1

When two coins are tossed the probability of getting 2 heads is $\frac{1}{4}$. What is the probability of *not* getting 2 heads when two coins are tossed?

$$P(2H) + P(\text{not } 2H) = 1$$

$$\frac{1}{4} + P(\text{not } 2H) = 1 \quad \therefore P(\text{not } 2H) = 1 - \frac{1}{4} = \frac{3}{4}$$

From Example 1 we can see that $P(E) + P(\text{not } E) = 1$ is equivalent to $P(\text{not } E) = 1 - P(E)$.

Exercise 10G

- On any weekday, the probability of a particular bus arriving at its destination on time is $\frac{9}{11}$. Complete the following to find the probability that the bus does *not* arrive on time.
 $P(\text{on time}) + P(\text{not on time}) = \underline{\hspace{2cm}}$
 $\frac{9}{11} + P(\text{not on time}) = \underline{\hspace{2cm}} \quad \therefore P(\text{not on time}) = \underline{\hspace{2cm}} - \frac{\square}{\square} = \frac{\square}{\square}$
- The probability of choosing a 20-cent coin from a particular purse is $\frac{3}{10}$. If a coin is chosen randomly from this purse, what is the probability that it is not a 20-cent coin?
- When a lolly is selected at random from a bag, the probability that it is red is $\frac{5}{13}$. If a lolly is chosen at random from this bag, what is the probability that it is not red?
- The letters of a word are written on cards and placed in a hat. When a card is chosen at random from this hat, the probability that the letter is m is $\frac{2}{7}$. What is the probability that the letter is not m ?
- On approaching a set of traffic lights, the probability that they will be red is $\frac{30}{67}$. What is the probability that the lights will not be red?
- On a European roulette wheel, the probability that the ball will stop on a red number is $\frac{18}{37}$. What is the probability that it will *not* stop on a red number?
- There are 1000 tickets sold in a raffle. If Angela buys 1 ticket, what is the probability that:
 - she will win the raffle?
 - she will not win the raffle?
- The numbers 1 to 9 are written on cards. The cards are shuffled and one is selected at random. Find:

a $P(9)$	b $P(\text{not } 9)$	c $P(\text{even number})$
d $P(\text{not even number})$	e $P(\text{number} < 3)$	f $P(\text{number not} < 3)$
- The numbers 0 to 9 are written on cards. The cards are shuffled and one is selected at random. Find:

a $P(6)$	b $P(\text{not } 6)$	c $P(\text{odd number})$
d $P(\text{not odd number})$	e $P(\text{number} > 7)$	f $P(\text{number not} > 7)$
- A bag contains 6 red, 9 blue and 8 white marbles. One marble is chosen at random from the bag. Find:

a $P(\text{red})$	b $P(\text{not red})$	c $P(\text{not blue})$	d $P(\text{not white})$
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Calculator activities

- Lotto is a game where six numbers are selected from 44.
 - The number of ways of choosing six numbers from 44 is $\frac{44 \times 43 \times 42 \times 41 \times 40 \times 39}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = \underline{\hspace{2cm}}$.
 - Write the probability of winning Lotto as a fraction.
- Year 7 students could buy one item from the canteen each day for a week. Each student could choose from a pie, hot-dog or pizza. The canteen sold 320 pies, 170 hot-dogs and 410 pizzas in the week.
 - Find the total number of food items.
 - Find the probability that a given student would have purchased a:
 - hot-dog
 - pie
 - pizza
- A bag contains 480 red marbles and 270 black marbles. What is the probability of selecting at random a:
 - a red marble?
 - a black marble?
- In a raffle there are 280 white, 150 blue and 320 green tickets. What is the probability of drawing a:
 - white ticket?
 - blue ticket?
 - green ticket?
- A game requires you to throw a 6 to start. A normal six-sided die is thrown.
 - What is the probability of starting on the first attempt?
 - What is the probability of *not* starting on the first attempt?
 - Calculate $\frac{5}{6} \times \frac{5}{6}$, which is the probability of not starting after 2 throws of the die.
 - The probability of not starting after 4 throws of the die is $\left(\frac{5}{6}\right)^4$. Calculate this probability.

Language in mathematics

Here is a list of some of the terms used in this chapter:

certainty, chance, equally likely, even chance, event, good chance, high probability, impossible, likely, low probability, no chance, no hope, often, outcome, probability, rarely, sample space, unlikely

- Find a term from the word list that is the opposite to:
 - unlikely
 - high probability
 - certain
 - good chance
 - rarely
- Complete these terms by filling in the missing vowels.

pr _ b _ b _ l _ ty	_ v _ n ch _ nc _	_ v _ nt	_ _ tc _ m _
_ ft _ n	_ mp _ ss _ bl _	r _ r _ ly	s _ mpl _ sp _ c _
- Find a term from the list that describes each probability.

a $\frac{1}{100}$	b $\frac{25}{50}$	c $\frac{2}{10}$	d $\frac{99}{100}$	e 0	f 1
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- Rearrange this statement to make sense. The first word begins with a capital letter. The last is followed by a full stop.

all probabilities The of a outcomes of possible experiment 1. is sum of the single

Terms

certain	chance	complement	complementary	element	event
experiment	favourable	impossible	occur	outcome	probability
random	sample space	set	theoretical		

Check your skills

- 1 A coin is tossed once. The number of outcomes in the sample space is:
A 0 B 1 C 2 D 3
- 2 A ticket is selected from a hat containing 4 tickets numbered from 1 to 4. The sample space is:
A {tickets} B {1, 4} C {1, 2, 3, 4} D {4}
- 3 The probability of randomly selecting ticket number 3 from the hat in question 2 is:
A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D 1
- 4 The probability of an event is 0.3. As a fraction this is:
A 3 B $\frac{3}{10}$ C $\frac{1}{3}$ D 30
- 5 A hat contains 8 red and 5 blue tickets. One ticket is chosen at random. The probability that it is red is:
A $\frac{1}{2}$ B $\frac{5}{8}$ C $\frac{5}{13}$ D $\frac{8}{13}$
- 6 A letter is chosen at random from the word ALGEBRA. The probability that it is A is:
A $\frac{1}{7}$ B $\frac{1}{6}$ C $\frac{2}{7}$ D $\frac{2}{6}$
- 7 Which percentage probability approximates an event that is unlikely?
A 0% B 10% C 50% D 80%
- 8 A normal six-sided die is thrown. A certain event is:
A {1} B {6} C {7} D {1, 2, 3, 4, 5 or 6}
- 9 When a coin is tossed, the probability of getting 6 is:
A 0 B $\frac{1}{6}$ C $\frac{2}{6}$ D 6
- 10 When a normal six-sided die is rolled, the probability of getting a number less than 7 is:
A $\frac{6}{7}$ B $\frac{7}{6}$ C 1 D 6
- 11 A spinner has three sections coloured yellow, black and pink. When it is spun, the probability that it will stop on yellow is $\frac{3}{10}$ and the probability that it will stop on black is $\frac{1}{2}$. The probability that it will stop on pink is therefore:
A $\frac{1}{5}$ B $\frac{3}{5}$ C $\frac{4}{5}$ D $\frac{3}{20}$
- 12 A marble is selected from a bag containing red, blue and green marbles. The complement of the event 'selecting a red or blue marble' is selecting a:
A red marble B blue marble C green marble D blue or green marble
- 13 During peak hour, the probability that a particular train leaves the city on time is $\frac{89}{100}$. The probability that it does not leave on time is:
A $\frac{11}{100}$ B $\frac{89}{200}$ C $\frac{11}{200}$ D $\frac{11}{89}$

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3, 4	5, 6	7–10	11	12	13
Section	A	B	C	D	E	F	G

10A Review set

- 1 A seven-sided spinner with the numbers 1 to 7 is spun once.

- a List the sample space.
b What is the probability of getting 5?



- 2 A letter is chosen at random from the word NUMBER.

- a List the sample space.
b What is the probability of choosing B?

- 3 Complete the table.

	Fraction	Decimal	Percentage
a	$\frac{1}{5}$		
b		0.4	
c			50%

- 4 A bag contains 8 orange, 12 black and 5 gold marbles. One marble is selected at random.

- a How many marbles are in the bag?
b What is the probability of selecting a black marble?

- 5 A letter is chosen from the word STUDENT. What is:

- a $P(E)$? b $P(T)$? c $P(\text{consonant})$?

- 6 a Write a statement describing an event with a probability of 1.

- b Estimate a percentage probability for the phrase 'very likely'.

- c Write a term for an event with a probability of about 15%.

- 7 A spinner has four equal sections coloured pink, purple, blue and brown. The spinner is spun once.

Describe an event that would be:

- a certain b impossible c even chance d probability of $\frac{1}{4}$

- 8 A bag contains red, blue and green marbles. If one marble is chosen at random from the bag, the probability that it is red is $\frac{2}{5}$ and the probability that it is green is $\frac{1}{10}$. Calculate the probability that it is blue.

- 9 A card is selected from a normal playing pack. What is the complement of the event 'selecting a heart or diamond'?

- 10 The probability of selecting a \$1 coin from a particular purse is $\frac{2}{11}$. If a coin is selected at random from this purse, what is the probability that it is not a \$1 coin?

10B Review set

- 1 A hat contains 1 red, 1 blue and 1 green ticket. One ticket is chosen at random.

- a List the sample space.
b What is the probability of selecting the red ticket?

- 2 Ten cards with the numbers 1 to 10 written on them are shuffled, and one card is chosen at random.

- a List the sample space.
b What is the probability that the chosen card has 5 written on it?

- 3 Complete the table.

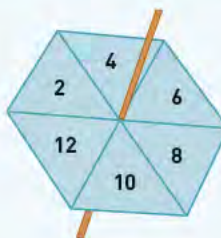
	Fraction	Decimal	Percentage
a		0.6	
b			70%
c	$\frac{3}{8}$		

- 4 A bag contains 5 green, 7 red and 8 blue marbles. One marble is chosen at random.
- How many marbles are in the bag?
 - What is the probability of choosing a red marble?
- 5 One card is selected at random from a normal deck of 52 cards. What is:
- P(heart)?
 - P(red card)?
 - P(ace)?
- 6
- Write a statement describing an event with a probability of 0.
 - Estimate a percentage probability for the phrase 'even chance'.
 - Write a phrase for an event with a probability of about 90%.
- 7 A coin with sides coloured red and blue is tossed once. Describe an event that would be:
- certain
 - impossible
 - even chance
- 8 The Roadside Assistance Association claims that the probability that it will respond to a breakdown in less than 30 min is $\frac{137}{250}$ and the probability that it will respond in from 30 to 60 min is $\frac{98}{250}$. Calculate the claimed probability that it will respond to a breakdown in more than 60 min.
- 9 A die is rolled. What is the complement of the event 'rolling a number less than 4'?
- 10 There are 1000 tickets sold in a raffle. If Jack buys 10 tickets, what is the probability that:
- Jack will win?
 - Jack will not win?



10C Review set

- 1
- List the sample space of this spinner.
 - What is the probability of spinning 10?
- 2 This spinner is spun once.
- List the sample space.
 - What is the probability that the colour spun is gold?



- 3 Complete the table.

	Fraction	Decimal	Percentage
a			80%
b		0.35	
c	$\frac{1}{25}$		

- 4 A hat contains 7 orange and 8 black tickets. One ticket is selected at random.

- a How many tickets are in the hat?
b What is the probability of selecting an orange ticket?

- 5 In Year 7, 55% of students walk to school, 25% catch the bus, 15% come by car and 5% bicycle. A student is chosen at random. Find the probability that the student:

- a walks b comes by car
c does not walk d walks or bicycles

- 6 a Write a statement describing an event with a probability of $\frac{1}{2}$.
b State the percentage probability for the phrase 'no chance'.
c Write a phrase for an event with a probability of 100%.



- 7 A bag contains 8 tickets labelled 1 to 8. Describe an event with a probability of:
a 1 b 0 c $\frac{1}{2}$ d $\frac{1}{4}$
- 8 A bag contains green, yellow and black jubes. If a jube is chosen at random from this bag, the probability that it is green is $\frac{1}{3}$ and the probability that it is yellow is $\frac{2}{5}$. Calculate the probability that it is black.
- 9 The numbers 0 to 9 are written on cards and one card is selected. Find the complement of the event 'selecting a 0 or an odd number'.
- 10 A card is selected at random from a normal playing pack. Calculate the probability that the card is not a queen or a king.