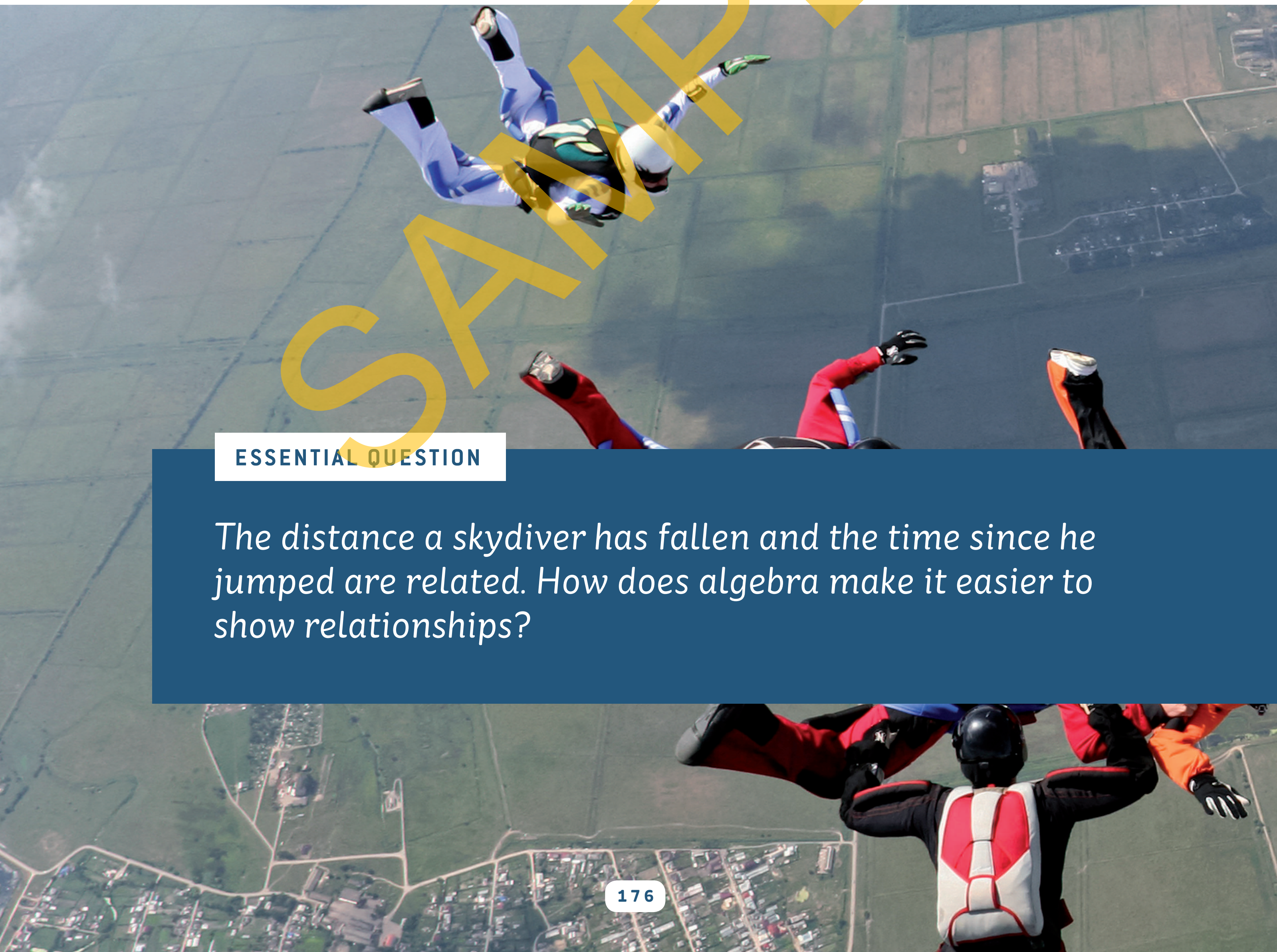
4 ALGEBRA



ESSENTIAL QUESTION

The distance a skydiver has fallen and the time since he jumped are related. How does algebra make it easier to show relationships?

4A 1 Look at the algebraic expression $4x + 7$.

a How many terms does it have?

b What is the variable?

A $4x$ B 7 C x D 4

c What is the coefficient of the term containing the pronumeral?

A $4x$ B 7 C x D 4

d What is the constant term?

A $4x$ B 7 C x D 4

4A 2 How many terms does $ab + 3a + abc - 8 + 5ac$ have?

A 3 B 4 C 5 D 6

4A 3 Which of these is an equation?

A $4x + 5y + 7$ B $5k - 2p = 12$

C $9a - 5b - 8$ D $20 - c^2 + 2cd$

4B 4 What is $-8 + 13$?

A -21 B -5 C 5 D 21

4B 5 Calculate:

a $15 - 19$ b $-3 - 5$.

4B 6 What is -6×4?

A -24 B -10 C 10 D 24

4B 7 Calculate:

a -2×-5 b $18 \div (-6)$.

4B 8 What is $5 + 3 \times 2$?

A 11 B 13 C 16 D 36

4B 9 Calculate:

a $(10 + 25) \div 5$ b $3(7 - 2)$.

4B 10 a What is 8^2?

A 4 B 10 C 16 D 64

b What is 4^3 written in expanded form?

A $4 + 4 + 4$ B $3 + 3 + 3 + 3$

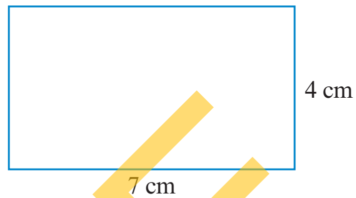
C 4×3 D $4 \times 4 \times 4$

4B 11 Calculate $7^2 - 5^2$.

4C 12 Calculate $3 + 5 - 9 + 2 - 7$.

A -26 B -6 C -4 D 26

Questions 13 and 14 refer to this rectangle.



4C 13 What is its perimeter?

4D 14 What is its area?

4D 15 a How is 5^4 written in expanded form?

A $5 + 5 + 5 + 5$

B $5 \times 5 \times 5 \times 5$

C $4 + 4 + 4 + 4 + 4$

D $4 \times 4 \times 4 \times 4 \times 4$

b Write $3 \times 3 \times 3 \times 3 \times 3$ in index form.

4E 16 a What are the factors of 24?

A 1, 2, 3, 4, 6, 8, 12, 24

B 1, 2, 3, 4, 5, 6, 8, 12, 24

C 1, 2, 4, 6, 12, 24

D 1, 3, 6, 8, 10, 12, 14, 16, 24

b Which factors are common to 18 and 48?

A 2, 3, 4, 6, 8, 12, 18

B 1, 2, 3, 4, 6, 8

C 2, 3, 6, 9, 12, 18, 48

D 1, 2, 3, 6

4E 17 a What is $\frac{18}{24}$ in simplest form?

A $\frac{18}{24}$ B $\frac{9}{12}$ C $\frac{6}{8}$ D $\frac{3}{4}$

b Write $\frac{45}{72}$ in simplest form.

4A Using pronumerals

Start thinking!

An art teacher orders paint brushes online for her class. There is a delivery charge of $12.

1 **a** What is the total cost if she orders:
  i 10 brushes? **ii** 25 brushes?
  b Explain how you worked this out.

2 The cost varies depending on how many paint brushes are ordered. How many **variables** are in this relationship? Describe them.

You can use a **pronumeral** to stand for or represent a variable.
A pronumeral is a letter or symbol that takes the place of a number.

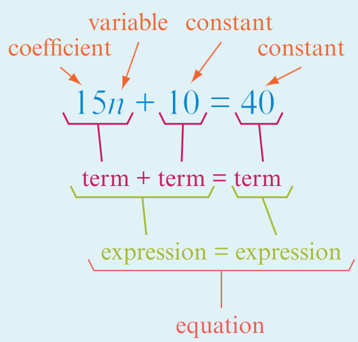
3 The teacher orders n brushes. Why can the cost be written as the algebraic **expression** $8n + 12$?

4 **a** How many terms are in the expression in question **3**?
  b What is the **coefficient** of the term containing the variable?
  c Is there a **constant term**? Explain.

5 If the teacher spends $140 on paint brushes, the relationship can be written as $8n + 2 = 140$. Is this an expression, an **equation** or a **formula**? Explain.

KEY IDEAS

- A variable is a quantity that can have different values.
- A pronumeral is a letter or symbol used in place of a number. It can represent a variable or an unknown number.
- In algebra, you use pronumerals to make it easier to represent information or a relationship.
- The meanings of the algebraic words expression, equation, formula, term, coefficient and constant are shown in the diagram.



- Equations always have an equals sign. Expressions do not. For example, $5x + 2 = 17$ is an equation and $5x + 2$ is an expression.
- A formula is an equation that has more than one variable; for example, $k = 3m + 7$.

EXERCISE 4A Using pronumerals

UNDERSTANDING AND FLUENCY

1 Answer true or false to each statement. If false, provide your reasoning.

a $5x + y$ is an expression
b $2k + 4$ is an equation
c the constant in $7 + 9p$ is 9
d $a = b + c$ is a formula
e the coefficient of $6m$ is m
f $8y + 3 - x$ has three terms

2 State whether each of these is an expression or an equation.

a $2a + 3$
b $x - 6 = 1$
c $7 = 5 + h$
d $x + y + z = 9$
e $3a + b - 7$
f $k = 4m + 1$

> **NOTE** Pronumerals are printed in italics so they are not confused with abbreviations such as units of measurement. For example, m is for metre but m is a pronumeral representing a variable.

3 Look at the relationships in question **2**.

a Which of the equations is also a formula?
b Which expression contains a term with a coefficient of 2?
c Which expression contains the constant term -7?

EXAMPLE 4A-1 Writing simple expressions

Write an expression for each statement. Use a to represent the unknown number.

a 8 more than a number
b 5 times a number
c a number divided by 9

THINK	WRITE
a '8 more' means to add 8. Write a plus 8.	a Expression is $a + 8$.
b Multiply a by 5. Simplify by writing the number before the pronumeral and leaving out the × sign.	b $a \times 5 = 5a$ Expression is $5a$.
c Write a divided by 9 as a fraction. Remember that the ÷ sign is equivalent to the **vinculum** in a fraction.	c $a \div 9 = \frac{a}{9}$ Expression is $\frac{a}{9}$.

4 Write an expression for each statement. Use a to represent the unknown number.

a 5 more than a number
b a number divided by 3
c 4 less than a number
d 3 times a number
e 8 is subtracted from a number
f 2 is added to a number
g a number is subtracted from 10
h 6 divided by a number
i 50 times a number
j the sum of a number and 19
k twice the value of a number
l the sum of a number, another number x and 8

EXAMPLE 4A-2 Writing more complex expressions

Write an expression for each statement. Use x to represent the unknown number.

a a number is multiplied by 3 and then 5 is added
b a number is divided by 4 and then 7 is subtracted

THINK	WRITE
a 1 Multiply the number (x) by 3 and simplify.	a $x \times 3 = 3x$
2 Add 5 to the result and write your answer.	Expression is $3x + 5$.
b 1 Divide the number (x) by 4 and simplify.	b $x \div 4 = \frac{x}{4}$
2 Subtract 7 from the result and write your answer.	Expression is $\frac{x}{4} - 7$.

5 Write an expression for each statement. Use x to represent the unknown number.

a a number is multiplied by 7 and then 2 is added
b a number is divided by 3 and then 6 is subtracted
c a number is multiplied by 2 and then 5 is subtracted
d a number is divided by 10 and then 1 is added
e a number is multiplied by 15 and then 28 is added
f a number is divided by 20 and then 11 is subtracted

EXAMPLE 4A-3 Simplifying expressions

Write each expression more simply.

a $4 \times (x + 2)$ b $(x + 3) \div 5$

THINK	WRITE
a The × sign can be left out. This is the same as when you write $4 \times a$ more simply as $4a$.	a $4 \times (x + 2)$ $= 4(x + 2)$
b The brackets and ÷ sign can be replaced with a vinculum.	b $(x + 3) \div 5$ $= \frac{x + 3}{5}$

NOTE Remember that a pair of brackets and a vinculum are examples of grouping symbols, as they group terms together.

6 Write each expression more simply.

a $3 \times (x + 1)$ b $5 \times (x - 9)$ c $(x + 2) \times 7$
d $(x - 6) \times 4$ e $(x + 8) \times 11$ f $(4 - x) \times 9$
g $(x + 5) \div 2$ h $(x - 9) \div 8$ i $(x + 1) \div 4$
j $(7 + x) \div 12$ k $(3 - x) \div 5$ l $(x - 10) \div 21$

UNDERSTANDING AND FLUENCY

7 Ingrid starts with an unknown number, x.

a She adds 2 to the number. What expression does she make?

b She wants to multiply the result by 5 so she uses a pair of brackets to group the result before multiplying. Copy and complete: $(x + \_\_) \times 5$ or $\_\_ \times (x + \_\_)$.

c The expression in part **b** can be written more simply.
Copy and complete: $\_\_(x + \_\_)$.

8 Jake starts with an unknown number, m.

a He subtracts 3 from the number. What expression does he make?

b He wants to divide the result by 4 so he uses brackets to group the result before dividing. Copy and complete: $(m - \_\_) \div \_\_$.

c This can be written as a fraction. Copy and complete: $\frac{m - \_\_}{\_\_}$.

EXAMPLE 4A-4 Writing expressions using grouping symbols

Write an expression for each statement. Use x to represent the unknown number.

a add 3 to a number and then multiply by 7

b subtract 5 from a number and then divide by 2

THINK	WRITE
a **1** Add 3 to the number (x).	**a** $x + 3$
2 Multiply by 7. Show brackets around $x + 3$ so the multiplication applies to the whole result.	$7 \times (x + 3)$
3 Simplify the expression and write your answer.	Expression is $7(x + 3)$.
b **1** Subtract 5 from the number (x).	**b** $x - 5$
2 Divide by 2. Show brackets around $x - 5$ so the division applies to the whole result.	$(x - 5) \div 2$
3 Simplify the expression using a vinculum and write your answer.	Expression is $\frac{x - 5}{2}$.

9 Write an expression for each statement. Use x to represent the unknown number.

a add 5 to a number and then multiply by 2

b subtract 9 from a number and then multiply by 3

c add 2 to a number and then divide by 4

d subtract 10 from a number and then divide by 7

e subtract a number from 20 and then multiply by 5

f subtract a number from 8 and then divide by 6

10 How is an equation different from an expression?

EXAMPLE 4A-5 Writing equations

Write an equation for each statement. Use a to represent the unknown number.

a 4 more than a number is equal to 15

b the result of multiplying a number by 3 and then subtracting 8 equals 9

THINK	WRITE
a **1** Add 4 to the number (a).	**a** $a + 4$
2 Expression equals 15 so use = to write an equation.	Equation is $a + 4 = 15$.
b **1** Multiply the number (a) by 3 and then subtract 8.	**b** $3a - 8$
2 Expression equals 9 so use = to write an equation.	Equation is $3a - 8 = 9$.

UNDERSTANDING AND FLUENCY

11 Write an equation for each statement. Use a to represent the unknown number.

a 6 more than a number is equal to 18

b 2 less than a number is equal to 9

c a number multiplied by 3 is equal to 24

d a number divided by 5 is equal to 2

12 Write an equation for each statement. Use x for the unknown number.

a the result of multiplying a number by 4 then adding 3 is equal to 11

b the result of dividing a number by 2 then adding 1 is equal to 6

c the result of multiplying a number by 7 then subtracting 5 is equal to 12

d the result of dividing a number by 3 then subtracting 6 is equal to 21

e the result of adding 6 to a number then multiplying by 3 is equal to 24

f the result of adding 11 to a number then dividing by 2 is equal to 9

g the result of subtracting 2 from a number then multiplying by 10 is equal to 60

h the result of subtracting 5 from a number then dividing by 8 is equal to 100

PROBLEM SOLVING AND REASONING

13 Mac owns a bee hive. Use n to represent the number of bees in his hive.

a Write an expression for the number of bees in the hive if 40 young bees hatch.

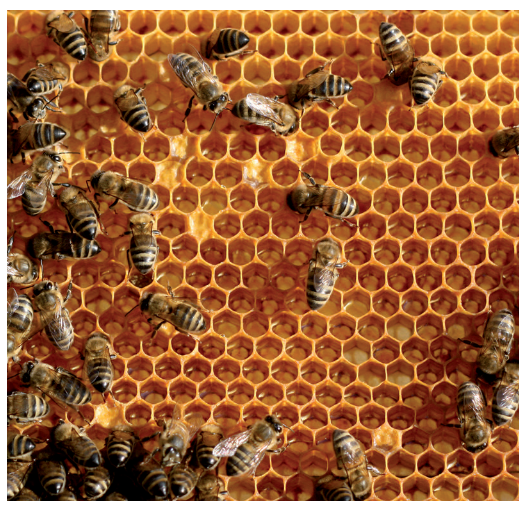
b What could each expression represent?

i $n + 300$ **ii** $n - 200$ **iii** $2n$

Mac's friend Kyna also owns a bee hive.

c Kyna estimates she has $n - 500$ bees. Does she have more or fewer bees than Mac?

d Using n, write an expression to represent another hive which has more bees than either Mac's or Kyna's.



PROBLEM SOLVING AND REASONING

14 Hayden is y years old. Write the answer to each of these using algebra.

a How old was he:

i 6 years ago? **ii** 10 years ago?

b How old will he be in:

i 5 years time? **ii** 23 years time?

c How old is his sister if she is x years younger than him?

d How old is his uncle if he is twice as old?

15 Tickets to a concert cost \$35 for an adult and \$25 for a child. Write an expression for each of these.

a the cost of m adult tickets

b the cost of k child tickets

c the total cost of m adult tickets and k child tickets

16 Shalini is washing up c cups and p plates.

a How many items does she have to wash?

b If she breaks a plate, how many plates are left?

c How many cups and plates does she now have to dry?

17 Kasey earns \$300 each month. The formula $y = 300 - x$ represents a relationship between how much Kasey spends and how much she saves.

a What does x represent?

b What does y represent?

c Write five sets of possible values for x and y that would fit this formula.

d Why are x and y variables?

18 What do expressions, equations and formulas have in common? How are they different? Provide examples to support your answer.

19 What are the advantages of using a pronumeral in a relationship between variables?

CHALLENGE

20 Will has many songs on his iPod. He downloads a further 18 songs. Write a formula to represent the relationship between the number of songs he originally had and the number he now has. Remember to define the pronumerals that represent the two variables.

21 Suggest a real-life situation that could be represented by each expression.

a $a + 6$ **b** $b - 1$ **c** $\frac{c}{4}$

d $2d$ **e** $50 - e$ **f** $3f + 2$

22 If each expression in question **21** is equal to 20, write an equation for each.

Reflect

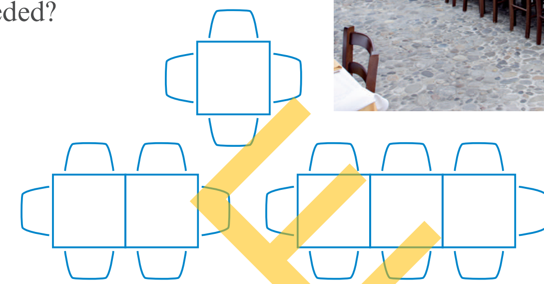
Why are pronumerals used to represent variables?

4B Evaluating expressions

Start thinking!

When tables are joined in a line, will four chairs be needed for every table? What's an easy way to calculate the number of chairs needed?

Consider these table and chair plans.



1 How many chairs fit around two tables?
Why isn't it double the number that fits at one table?

2 How many chairs can be placed around:
a three tables? **b** four tables? **c** five tables?

3 Copy and complete this table.

Number of tables	1	2	3	4	5
Number of chairs	4				

4 If t represents the number of tables and c represents the number of chairs, explain why the formula for the relationship between them is $c = 2t + 2$.

This formula quickly tells you the number of chairs needed if you know the number of tables.

5 If the number of tables is five, replace t with 5 in the formula. This is called **substitution**.
Copy and complete: $c = 2 \times \_\_ + \_\_$.

6 Calculate the value of the expression: multiply 2 by 5 then add 2. This is called **evaluating** the expression.

7 Use the formula to calculate the number of chairs needed for:
a 25 tables **b** 50 tables **c** 117 tables **d** 162 tables.

8 What is the advantage of substituting into a formula rather than drawing a diagram?

KEY IDEAS

- To evaluate an expression, substitute (or replace) each pronumeral with a number and then work out the value.

Remember to use the correct order of operations:
1st **B**rackets (operations inside grouping symbols always calculated first)
2nd **I**ndices (powers and square roots)
3rd **D**ivision and **M**ultiplication (work from left to right)
4th **A**ddition and **S**ubtraction (work from left to right)

- An algebraic term can be written in expanded form, where the multiplication signs are shown, or in simplified form, where the multiplication signs are left out.

$$\underbrace{4 \times a \times b}_{\text{expanded form}} = \underbrace{4ab}_{\text{simplified form}}$$

EXERCISE 4B Evaluating expressions

EXAMPLE 4B-1 Substituting a value into an expression

Evaluate each expression by substituting 3 for x.

a $x + 4$ **b** $2x - 1$

THINK	WRITE
a 1 Replace x with 3.	**a** $x + 4$
2 Evaluate the expression.	$= 3 + 4$ $= 7$
b 1 Remember that $2x$ means $2 \times x$. Replace x with 3.	**b** $2x - 1$
2 Evaluate the expression. Follow the correct order of operations (multiply then subtract).	$= 2 \times 3 - 1$ $= 6 - 1$ $= 5$

UNDERSTANDING AND FLUENCY

1 Evaluate each expression by substituting 8 for x.

a $x + 9$ **b** $x - 1$ **c** $7x$ **d** $\frac{x}{2}$
e $3x + 5$ **f** $2x - 6$ **g** $10 + x$ **h** $12 - x$

2 Evaluate $3a + 5$ by substituting each value for a.

a $a = 2$ **b** $a = 7$ **c** $a = 1$ **d** $a = 0$
e $a = -1$ **f** $a = -3$ **g** $a = \frac{1}{3}$ **h** $a = -\frac{2}{3}$

3 Evaluate $6y - 2$ by substituting each value for y.

a $y = 1$ **b** $y = 3$ **c** $y = -1$ **d** $y = -4$
e $y = 0$ **f** $y = \frac{1}{2}$ **g** $y = \frac{1}{3}$ **h** $y = 2.5$

NOTE Hint: when you substitute into a term that is a product of a number and one or more pronumerals, show the × signs as a reminder to multiply.

EXAMPLE 4B-2 Substituting two values into an expression

Evaluate $5a + 3b$ by substituting $a = 3$ and $b = 2$.

THINK	WRITE
Replace a with 3 and b with 2. Follow the correct order of operations (multiply then add).	$5a + 3b$ $= 5 \times 3 + 3 \times 2$ $= 15 + 6$ $= 21$

EXAMPLE 4B-3 Substituting more than two values into an expression

Evaluate each expression by substituting $a = 3$, $b = 2$ and $c = -4$.

a $2(c - ab)$ **b** $\frac{b + 2c}{3}$

THINK	WRITE
a Replace c with -4, a with 3 and b with 2. Calculate inside the brackets first (multiply then subtract). Remember that $2(-10)$ means $2 \times (-10)$.	**a** $2(c - ab)$ $= 2(-4 - 3 \times 2)$ $= 2(-4 - 6)$ $= 2 \times (-10)$ $= -20$
b Replace b with 2 and c with -4. Calculate the numerator first and then divide by the denominator.	**b** $\frac{b + 2c}{3}$ $= \frac{2 + 2 \times (-4)}{3}$ $= \frac{2 - 8}{3}$ $= \frac{-6}{3}$ $= -2$

4 Evaluate each expression by substituting $a = 5$, $b = 4$ and $c = -2$.

a $a + 4$ **b** $2b + c$ **c** $7a + 2b$ **d** $3b + 6c$

e $4ab$ **f** $\frac{b}{c}$ **g** $5b - 2a$ **h** $3(a + bc)$

i $\frac{2a + b}{7}$ **j** $8(3a - 4b)$ **k** $\frac{abc - 5}{9}$ **l** $\frac{b + 2c}{a}$

EXAMPLE 4B-4 Substituting a value into a formula

Find the value of y for each x value, using the formula $y = 3x + 5$.

a $x = -4$ **b** $x = \frac{2}{3}$

THINK	WRITE
a **1** Replace x with -4.	**a** $y = 3x + 5$ $= 3 \times (-4) + 5$
2 Evaluate the RS of the formula to find y. Take care with negative signs.	$= -12 + 5$ $= -7$
b **1** Replace x with $\frac{2}{3}$.	**b** $y = 3x + 5$ $= 3 \times \frac{2}{3} + 5$
2 Evaluate the RS of the formula to find y.	$= \frac{\cancel{3}^{1}}{1} \times \frac{2}{\cancel{3}_{1}} + 5$ $= 2 + 5$ $= 7$

UNDERSTANDING AND FLUENCY

5 Find the value of y for each x value, using the formula $y = 2x + 7$.

a $x = 3$ b $x = 8$ c $x = -6$ d $x = -1$
e $x = 4.5$ f $x = 0$ g $x = \frac{1}{2}$ h $x = -\frac{3}{2}$

6 Find the value of k for each m value, using the formula $k = 4m - 3$.

a $m = 1$ b $m = 6$ c $m = -2$ d $m = 50$
e $m = 1.5$ f $m = \frac{1}{4}$ g $m = 0$ h $m = -\frac{3}{4}$

7 a The term 3^2 means 3×3. What does a^2 mean?
b If $a = 5$, evaluate:
i a^2 ii $3a^2$ iii $a^2 + 7$.

8 a The term 2^3 means $2 \times 2 \times 2$. What does p^3 mean?
b If $p = 4$, evaluate:
i p^3 ii $2p^3$ iii $p^3 - 12$.

EXAMPLE 4B-5 Substituting values into expressions containing powers

Evaluate each expression by substituting $x = 3$ and $y = 2$.

a $x^2 + 2$ b $5y^3 - 7$ c $\frac{x^2 - y^2}{10}$

THINK	WRITE
a 1 Replace x with 3. 2 Follow the correct order of operations (square 3 then add 2).	a $x^2 + 2$ $= 3^2 + 2$ $= 3 \times 3 + 2$ $= 9 + 2$ $= 11$
b 1 Replace y with 2. 2 Follow the correct order of operations (raise 2 to the power of 3, multiply by 5 and then subtract 7).	b $5y^3 - 7$ $= 5 \times 2^3 - 7$ $= 5 \times (2 \times 2 \times 2) - 7$ $= 5 \times 8 - 7$ $= 40 - 7$ $= 33$
c 1 Replace x with 3 and y with 2. 2 Calculate each power and then subtract. Write the answer as a simplified fraction.	c $\frac{x^2 - y^2}{10}$ $= \frac{3^2 - 2^2}{10}$ $= \frac{9 - 4}{10}$ $= \frac{5}{10}$ $= \frac{1}{2}$

UNDERSTANDING AND FLUENCY

9 Evaluate each expression by substituting $x = 4$ and $y = 3$.

a x^2 **b** y^2 **c** $x^2 + 7$ **d** $y^2 - 5$

e $3x^2$ **f** $2y^2$ **g** $5x^2 - 2$ **h** $8y^2 + 11$

i $2x^3 + 6$ **j** $4y^3 - 1$ **k** $y^4 + 9$ **l** $3x^2 + 5y^2$

m $\dfrac{x^2}{2}$ **n** $\dfrac{y^2 - 1}{4}$ **o** $\dfrac{x^2 + y^2}{5}$ **p** $\dfrac{x^2 - y^2}{7}$

q $\dfrac{x^2 + 2y^3}{80}$ **r** $\dfrac{64}{2x^3}$ **s** $\dfrac{1}{x^3 + y^3}$ **t** $\dfrac{3}{2x^2 - y^3}$

10 Evaluate each expression by substituting $a = -6$ and $b = 2$.

a $a - b$ **b** $a^2 + b^2$ **c** $7ab$ **d** $8a + 7b$

e a^2b **f** ab^2 **g** $5ab - b^2$ **h** $100 - 2a^2b^2$

i $-3ab^2 + 1$ **j** $2(5ab - 3)$ **k** $\dfrac{4a^2b + b}{10}$ **l** $\dfrac{9(a - b^2)}{6}$

11 Complete the table of values for each formula.

a $y = 2x + 5$

x	0	1	2	3	4
y					

b $y = 4x - 3$

x	−2	−1	0	1	2
y					

c $y = 3x^2 + 1$

x	0	1	2	3	4
y					

d $y = 2x^3 - 7$

x	−2	−1	0	1	2
y					

PROBLEM SOLVING AND REASONING

12 The number of legs on n grasshoppers can be represented by l.

a Write a formula using l and n to describe the relationship between them.

b Find the value of l for each of these n values.

i 3 **ii** 7

iii 50 **iv** 101

c Use the formula to calculate how many legs in total there would be on 950 grasshoppers.

d If the number of legs on an unknown number of grasshoppers was estimated to be between 500 and 600, give three possible answers for the number of grasshoppers there could be. Explain your reasoning.

e Grasshoppers are unusual as they have five eyes. Write a formula to describe the relationship between the number of grasshoppers and the corresponding number of eyes. Remember to define the pronumeral you use for each variable.

f Use your formula to calculate the number of eyes on:

i 456 grasshoppers

ii 2035 grasshoppers.

PROBLEM SOLVING AND REASONING

13 The relationship between average speed (s), distance (d) and time (t) can be represented by the formula $s = \frac{d}{t}$.

a Calculate the value of s for these d and t values.

i $d = 8, t = 2$ **ii** $d = 48, t = 6$ **iii** $d = 6.5, t = 5$ **iv** $d = 82, t = 4$

b Find the average speed (in metres per second) of a bicycle that travels 120 m in 40 seconds.

c Find the average speed (in kilometres per hour) of a motorcyclist who travels 282 km in 3 hours.

14 The distance a skydiver falls through the air in metres (d) after an amount of time in seconds (t) can be calculated using the formula $d = 4.9t^2$.

a How far does the skydiver fall in the first second? (Hint: what value of t will you substitute into the formula?)

b How far does the skydiver fall:

i in the first 3 seconds? **ii** in the first 5 seconds?

c Is the skydiver's speed increasing or decreasing over the first 5 seconds? Explain.

15 A recent 'longest lunch' catered for 900 people. Use the formula in 4B Start thinking (page 184) to work out how many tables and chairs were needed.

16 The volume of a sphere can be calculated if you know its radius. If V represents the volume and r represents the radius, use the formula $V = 4.2r^3$ to find the approximate volume of a golf ball with a radius of 2.1 cm.

CHALLENGE

17 In most countries of the world, temperature is measured in degrees Celsius. Some countries, such as the USA, use degrees Fahrenheit. A formula can be used to convert from one temperature unit to the other. Let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit.

a Use the formula $C = \frac{5}{9}(F - 32)$ to convert these temperatures to degrees Celsius. Write each answer to the nearest degree.

i 77°F **ii** −4°F **iii** 41°F **iv** 100°F **v** −1°F **vi** 62°F

b Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures to degrees Fahrenheit. Write each answer to the nearest degree.

i 30°C **ii** −10°C **iii** 0°C **iv** 27°C **v** −8°C **vi** 42°C

c The temperature in Chicago is 23°F. What is it in degrees Celsius?

d Which temperature is lower: 86°F or 32°C? Justify your answer.

18 Choose a relationship between two (or more) variables and describe it with a formula. Remember to define the pronumeral used for each variable. Now write three questions involving substituting into your formula and give them to a classmate to work out.

Reflect

When is substitution useful in algebra?

4C Simplifying expressions containing like terms

Start thinking!

1 Each bag of Cherry Bites contains *a* chocolates. Melinda buys three bags of Cherry Bites. Explain why the total number of chocolates in the three bags would be $a + a + a$ or $3 \times a$ or $3a$.

2 Peter buys five bags of Cherry Bites. Write three expressions for the total number of chocolates in five bags.

3 The term $3a$ is the simplest form of the three expressions in question **1**. Which expression is the simplest in question **2**?

4 Peter adds his five bags to Melinda's three bags.
  - **a** Complete this new expression for the total number of chocolates: $3a$ + __a
  - **b** Can this expression be simplified further? If so, write your simplified expression.

5 Nick has bought two bags of chocolate bullets. Each bag contains *b* chocolates. Write a simple expression for the total number of chocolates in the two bags of chocolate bullets.

6 Melinda, Peter and Nick pool all their chocolates together.
  - **a** Complete this new expression for the total number of chocolates: __a + __b
  - **b** Can this expression be simplified further?

7 The terms $3a$ and $5a$ are called **like terms**. Are $8a$ and $2b$ like terms? Explain.

8 Explain when an expression can be simplified.

KEY IDEAS

- Terms containing exactly the same pronumerals are called like terms.
- Expressions can be simplified by adding or subtracting like terms.
- Like terms can be added (or subtracted) by adding (or subtracting) the coefficients of the terms.
- Rearranging an expression to group like terms is called 'collecting like terms'.

NOTE The order of the pronumerals can be different, but pronumerals are usually shown in alphabetical order.

EXERCISE 4C Simplifying expressions containing like terms

UNDERSTANDING AND FLUENCY

1 Answer true or false to each statement. For false statements, provide your reasoning.

a $5p$ and $9p$ are like terms
b $3k$ and k are like terms
c $7a$ and $7d$ are like terms.
d $-6xy$ and $2xy$ are like terms.
e $2a$ and $3ac$ are like terms.
f $8abc$ and $-4acb$ are like terms.

2 Write three more examples of terms that are like terms with each of these.

a $4y$
b $7a$
c $-9n$
d $2ab$
e $-10gh$
f $3kmn$
g $5a^2b$
h x

3 Answer true or false to each statement. For the statements that are false, write the correct simplified expression.

a $8x + 6x$ simplifies to $14x$
b $3p + 6q$ simplifies to $9pq$
c $5m + 2n + m$ simplifies to $6m + 2n$
d $11k - k$ simplifies to 11
e $2ab + 3a + 7ba$ simplifies to $12ab$
f $4a + 3c + 2a + c$ simplifies to $6a + 4c$
g $xy + 2x + y$ cannot be simplified
h $9p + 5q + 2p$ cannot be simplified

EXAMPLE 4C-1 Simplifying an expression with two terms

Simplify each expression.

a $9x + 7x$
b $5d - 3d$
c $6y - y$
d $5ab - 9ab$
e $10gh - 8h$

THINK	WRITE
a The expression contains like terms as the pronumeral is exactly the same. Simplify by adding the terms.	a $9x + 7x = 16x$
b The expression contains like terms, so simplify by subtracting the terms.	b $5d - 3d = 2d$
c The expression contains like terms, so the terms can be subtracted. Remember that y is the same as $1 \times y$ or $1y$.	c $6y - y = 5y$
d The expression contains like terms as the two pronumerals in each term are exactly the same. Simplify by subtracting the terms.	d $5ab - 9ab = -4ab$
e The two terms are not like terms as the pronumerals in each term are not exactly the same. The expression cannot be simplified.	e $10gh - 8h$ cannot be simplified.

4 Simplify each expression.

a $5x + 4x$	**b** $8y - 3y$	**c** $6mn + 9mn$
d $10kp - 2kp$	**e** $3a - 7a$	**f** $xy + 4$
g $3c + c$	**h** $5g - g$	**i** $2xy + 4y$
j $6ab - ab$	**k** $m - 8m$	**l** $3abc - 4abc$

EXAMPLE 4C-2 Simplifying expressions with more than two terms

Simplify:

a $4ab + 5ab - 2ab$ **b** $3xy + 4yx + xy$ **c** $2m + 3n + 4m + 5mn$

THINK	WRITE
a All three terms are like terms as they have exactly the same pronumerals, so the expression can be simplified.	**a** $4ab + 5ab - 2ab$ $= 9ab - 2ab$ $= 7ab$
b All three terms are like terms as they have exactly the same pronumerals, even though the order of the pronumerals is different in $4yx$. Simplify by adding the terms. Remember that xy is the same as $1xy$.	**b** $3xy + 4yx + xy$ $= 7xy + xy$ $= 8xy$
c Identify any like terms ($2m$ and $4m$) and group them together. Simplify the expression by adding the like terms.	**c** $2m + 3n + 4m + 5mn$ $= 2m + 4m + 3n + 5mn$ $= 6m + 3n + 5mn$

UNDERSTANDING AND FLUENCY

5 Simplify:

a $2k + 7k + 6k$	**b** $3ab + 5ab - 2ab$
c $8x + 4x + 3y$	**d** $4m + 3n + 9m$
e $8d - 6d + 1$	**f** $3x + 2xy + 4yx$
g $2x + 3y + 8y + 9x$	**h** $a + 7 + a + 2$
i $5rst + 2rst - 7rst$	**j** $3dy + 5y + yd + 8$
k $4gh + 3hg - 6gh$	**l** $a + 2ab + 3b + 5a$

6 Which expression (**A**, **B**, **C** or **D**) is the same as $7a^2b$?

A $7 \times a \times b$ **B** $7 + a + a + b$

C $7 \times a \times a \times b$ **D** $7 \times a \times b \times b$

7 Answer true or false to each statement. For false statements, provide your reasoning.

a $2a^2$ and $7a^2$ are like terms	**b** $8b$ and b^2 are like terms
c $3a^2b$ and $5a^2b$ are like terms	**d** $4ab$ and $6a^2b$ are like terms
e $-2a^2$ and $-7a^2b$ are like terms	**f** ab^2 and $2ab^2$ are like terms
g $8a^2bc$ and $-a^2bd$ are like terms	**h** $5ab^2$ and $-9a^2b$ are like terms

EXAMPLE 4C-3 Simplifying expressions with powers

Simplify:

a $5a^2b + 2a^2b$ b $4x^2 - 6x^2$ c $2c^2d + 3cd$ d $7xy^2 + 4x^2y + xy^2$

THINK	WRITE
a The expression contains like terms, as the pronumerals are exactly the same ($a \times a \times b$). Simplify by adding the terms.	a $5a^2b + 2a^2b$ $= 7a^2b$
b The expression contains like terms, so can subtract one from the other.	b $4x^2 - 6x^2$ $= -2x^2$
c The two terms are not like terms as the pronumerals are not exactly the same. $c \times c \times d$ is different from $c \times d$. The expression cannot be simplified.	c $2c^2d + 3cd$ cannot be simplified.
d Identify any like terms ($7xy^2$ and xy^2) and group them together. Simplify by adding like terms.	d $7xy^2 + 4x^2y + xy^2$ $= 7xy^2 + xy^2 + 4x^2y$ $= 8xy^2 + 4x^2y$

UNDERSTANDING AND FLUENCY

8 Simplify:

a $3x^2y + 11x^2y$ b $9ab^2 - 7ab^2$
c $4k^2 + 3k^2$ d $4mn + 2m^2n$
e $-5c^2 + 3c^2$ f $6pq - 5pq + 8p^2q$
g $ab^2 + 4a^2b + 3ab^2$ h $2x^2y + 10yx^2 - x^2y$
i $7g^3h - 9g^3h + 5g^2h$ j $4m^2n^4 + 6m^2n - 3m^2n^4$

9 Consider the expression $5m + 4k + 9m + k$.

a Evaluate the expression by substituting $m = 2$ and $k = 7$ into the four terms.
b Simplify the expression by adding like terms.
c Check your answer to part b by substituting $m = 2$ and $k = 7$ into the simplified expression. Do you obtain the same answer as part a?
d Which expression was easier to evaluate? Explain.

10 Simplify these expressions first, then evaluate each expression by substituting $a = 5$ and $b = 3$.

a $4a + 2a + 6b + 8b$ b $2a + 3b + 7a + 11b$
c $9a - 6a + 7b + 2b$ d $5ab + 4b + 2a + b$
e $6ab - ba + 3ab + a$ f $4b - b + 8a - a$
g $3a^2 + 2a^2 + b^2 - 4b^2$ h $7ab^2 + 2a^2b - 5ab^2$

11 Write three different expressions that simplify to $4p + 5t$.

12 Paree is deciding on the size of her garden bed, which is to be rectangular.

a Draw a diagram of a rectangle to represent Paree's garden bed.

b If l stands for the length and w stands for the width, label each of the four sides of the rectangle.

c Write an expression for the perimeter of the garden bed by completing this: $l + w + \_\_\_\_ + \_\_\_\_$.

d Simplify the expression.

e If P stands for the perimeter, write a formula for the perimeter of a rectangle.

f Use the formula to calculate the perimeter of rectangles with each of these length and width measurements.

i $l = 4$ m, $w = 3$ m **ii** $l = 6.4$ m, $w = 2.8$ m **iii** $l = 15\frac{1}{2}$ m, $w = 7\frac{1}{2}$ m

g Paree decides to plant box hedge around the edge of her garden bed. If she has enough plants to extend around a rectangle with perimeter 20 m, suggest three different sets of length and width measurements for her garden bed.

13 Consider the expression: $7a + 5b - 2a + 3b + 4a - 6b + c$. To simplify this expression, it is easier to rearrange the expression so that like terms are grouped together. This is called collecting like terms.

a Copy and complete the following to show how the expression can be rearranged.

$$\underbrace{7a - \_\_a + 4a}_{\text{like terms}} + \underbrace{5b + \_\_b - \_\_b}_{\text{like terms}} + c$$

b Notice that the minus sign (−) in front of $2a$ has moved with the term. Has this happened with any other term? Explain.

c Check that the addition or subtraction sign in front of each term has moved with that term. (That is, does each term still have the same sign in front of it?)

d Now that like terms have been collected, simplify the expression.

e Show how you can check your simplified expression is equivalent to the original expression. (Hint: substitute a value for each of a, b and c.)

f Which expression is easier to evaluate if you have a, b and c values? Explain.

14 Copy and complete each set of working to simplify the expression.

a $6x + 4y - 3x + 7y + 5x - 2y + y$

$= 6x - \_\_x + \_\_x + 4y + \_\_y - \_\_y + y$

$= \_\_x + \_\_y$

b $2mn + 5m + 8mn - 9m + 4n - 6mn + 2m$

$= 2mn + \_\_mn - \_\_mn + 5m - \_\_m + \_\_m + \_\_$

$= \_\_mn - \_\_m + \_\_\_$

15 When rearranging $3x - 2y + 7x - 4y$, Tania wrote $3x - 7x + 2y - 4y$.

a Explain her mistake.

b Show how to get the correct result for simplifying $3x - 2y + 7x - 4y$.

PROBLEM SOLVING AND REASONING

16 Simplify each expression by collecting like terms.

a $5x + 3y + 6y - 2x - 7y + 8x$ **b** $4y + 6 - 3y - 2 + 5y - y$

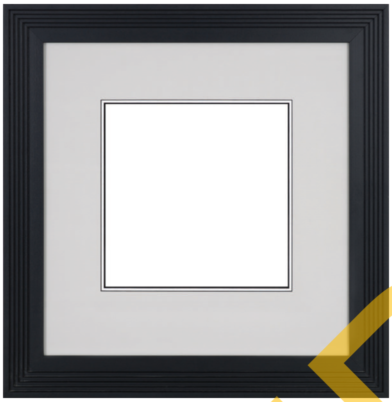
c $-2x + 5y + 4x - 8y + 2y - 3$ **d** $xy - x + 5y - 3xy + 9x - 5y$

e $8x^2 + 3x^2 - 6x + 4 + 5x - 9 + x$ **f** $3x + 7xy - 2y - 11x - xy + 8y - x$

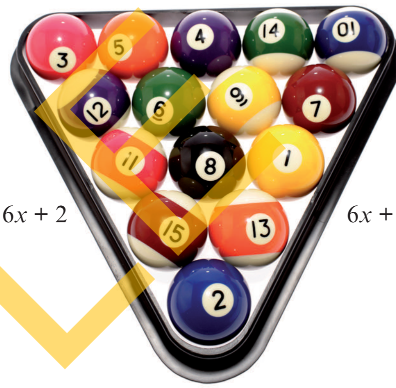
17 Evaluate each expression in question **16** by substituting $x = 2$ and $y = -5$.

18 Write an expression for the perimeter described in each situation.

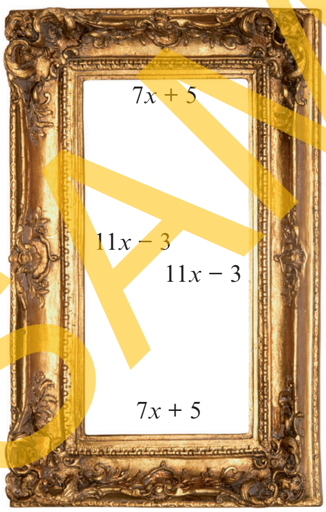
a length of outer edge of frame



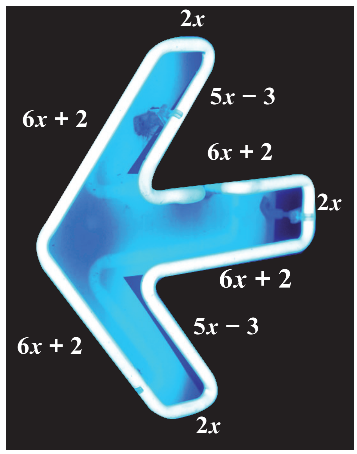
b perimeter of triangle



c perimeter of inner frame



d length of neon used



19 Calculate the perimeter for each shape in question **18** if x is 4 cm.

CHALLENGE

20 For each shape in question **18**, list three possible values of x that would give a perimeter of between 200 and 350 cm.

21 Simplify:

a $8a^2b + 2ab^2 + 3a^2b + 9ab^2$

b $4e^2fg + e^2fh + efgh - 6efgh$

c $x^2y + 3xy^2 + 5xy + xy^2 + 2xy$

Reflect

How can you recognise like terms?

4D Multiplying algebraic terms

Start thinking!

1 During a sale, a store offers DVDs for \$15 each.
  a How much would it cost to buy:
    i two DVDs? ii seven DVDs?
  b Which operation (+, −, × or ÷) did you use to obtain your answers?
  c How much would it cost to buy k DVDs? Write your answer as an algebraic term.
  d This term can be written in simplified form or expanded form. Copy and complete this statement. Notice that the number goes first in each case.

$$\underbrace{15 \times \_\_}_{\text{expanded form}} = \underbrace{\_\_k}_{\text{simplified form}}$$

  e How is the simplified form of the term different from the expanded form?

2 Before the sale, the price varied.
  a If the price for each DVD was p dollars, how much would it cost to buy:
    i two DVDs? ii seven DVDs?
  b How much would it cost to buy k DVDs?

3 When simplifying an algebraic term, what do you notice when you:
  a multiply numbers?
  b multiply pronumerals?

4 How could you simplify an algebraic term that contains numbers and pronumerals multiplied together?

KEY IDEAS

- An algebraic term can be written in expanded form, where the multiplication signs are shown, or in simplified form, where the multiplication signs are left out.

$$\underbrace{4 \times a \times b}_{\text{expanded form}} = \underbrace{4ab}_{\text{simplified form}}$$

- The product of any pronumerals that are the same can be simplified using index notation. For example, $a \times a \times a = a^3$.
- When multiplying algebraic terms, multiply the numbers (or coefficients) first and simplify the products of any pronumerals that are the same.
- The final simplified term is written with the number (or coefficient) first and the pronumerals in alphabetical order.
- The **index laws** apply to pronumerals in the same way as they do to numbers.
- To multiply terms in index form with the same base, write the base and add the indices (powers). For example, $a^5 \times a^3 = a^8$.

EXERCISE 4D Multiplying algebraic terms

EXAMPLE 4D-1 Writing algebraic terms in expanded form

Write each term in expanded form.

a $8abc$ b $-5m^2n$ c $2xy^3z$

THINK

a Write the term with multiplication signs.

b Write the term with multiplication signs.
As the term is negative, the coefficient is negative.
Remember that m^2 means $m \times m$.

c Write the term with multiplication signs.
Remember that y^3 means $y \times y \times y$.

WRITE

a $8abc = 8 \times a \times b \times c$

b $-5m^2n$
$= -5 \times m \times m \times n$

b $2xy^3z$
$= 2 \times x \times y \times y \times y \times z$

1 Write each term in expanded form.

a $5m$ b $-2a$ c $8xy$ d $-6cd$
e $7abc$ f $-10gkmn$ g $3p^2q$ h $-4x^3y^2z$

2 Match each term in simplified form (left column) with the equivalent term in expanded form (right column).

simplified form	expanded form
a $2a^2bc$	A $2 \times a \times a \times a \times b \times c$
b $2a^3bc$	B $2 \times a \times a \times b \times c$
c $2ab^2c^2$	C $2 \times a \times b \times b \times c \times c$

EXAMPLE 4D-2 Multiplying two simple terms

Simplify:

a $7 \times a$ b $b \times -6$ c $-1 \times -c$

THINK

a Simplify by leaving out the × sign.

b The product of a positive term and a negative term is negative.
Leave out the × sign and write the number before the pronumeral.

c The product of two negative terms is positive. Leave out the × sign.
Remember that if the coefficient is 1, it does not need to be shown.

WRITE

a $7 \times a = 7a$

b $b \times -6 = -6b$

c $-1 \times -c = 1c$
$= c$

3 Simplify:

a $4 \times x$	**b** $6 \times k$	**c** $d \times 7$	**d** $m \times 5$	**e** $1 \times p$	**f** $-3 \times h$
g $a \times -9$	**h** $-y \times 10$	**i** $8 \times -c$	**j** $-1 \times e$	**k** $-2 \times -f$	**l** $-m \times -3$

EXAMPLE 4D-3 Multiplying three simple terms

Simplify:

a $2 \times a \times d$ **b** $b \times -5 \times c$ **c** $3 \times k \times 6$ **d** $4 \times m \times m$

THINK	WRITE
a Simplify by leaving out the × signs.	**a** $2 \times a \times d$ $= 2ad$
b The terms can be multiplied in any order. The product of two positive terms and one negative term is negative. Simplify by leaving out the × signs and writing the number before the pronumerals.	**b** $b \times -5 \times c$ $= -5 \times b \times c$ $= -5bc$
c Multiply the numbers. ($3 \times 6 = 18$) Simplify by leaving out the × sign.	**c** $3 \times k \times 6$ $= 3 \times 6 \times k$ $= 18 \times k$ $= 18k$
d Write the product of any pronumerals that are the same in simplified form. ($m \times m = m^2$) Leave out the × sign.	**d** $4 \times m \times m$ $= 4 \times m^2$ $= 4m^2$

4 Simplify:

a $3 \times a \times b$	**b** $4 \times m \times n$	**c** $p \times 2 \times t$	**d** $d \times -8 \times c$
e $x \times y \times 9$	**f** $k \times f \times 5$	**g** $a \times b \times c$	**h** $h \times y \times e$
i $2 \times q \times 4$	**j** $m \times 3 \times -6$	**k** $7 \times a \times a$	**l** $n \times 2 \times n$

EXAMPLE 4D-4 Multiplying algebraic terms

Simplify $3ab \times 6bc$.

THINK	WRITE
1 Write each term in expanded form.	$3ab \times 6bc$ $= 3 \times a \times b \times 6 \times b \times c$
2 Multiply the numbers (or coefficients). ($3 \times 6 = 18$)	$= 18 \times a \times b \times b \times c$
3 Write the product of any pronumerals that are the same in simplified form. ($b \times b = b^2$)	$= 18 \times a \times b^2 \times c$
4 Simplify by leaving out the × signs.	$= 18ab^2c$

UNDERSTANDING AND FLUENCY

5 Simplify:

a $7a \times 3c$ **b** $-5p \times 6$ **c** $8n \times 3n$
d $2ef \times 5gh$ **e** $9xy \times -2xz$ **f** $3ab \times 2d \times 4f$
g $-2gh \times 5b \times 7k$ **h** $mn \times 6m \times 8q$ **i** $4ab \times 3ac \times 2a$
j $9m \times np \times 2m^2$ **k** $7gh \times -3gh \times fgh$ **l** $-2wx^2 \times 5wy \times -4u^2$

6 Copy and complete these multiplication tables. Multiply each term in the left column by each term in the top row.

a

×	$4a$	3	$6m$	$2abc$
$3d$				
ab^2				
$5mn$				
$2ac$				

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$				
xy^2				
$6xy$				
$-7yz$				

7 Copy and complete these multiplication problems. Each term is written in expanded form first.

a $2^3 \times 2^4$
$= 2 \times \_ \times \_ \times 2 \times \_ \times \_ \times \_$
$= 2^7$

b $a^3 \times a^4$
$= a \times \_ \times \_ \times a \times \_ \times \_ \times \_$
$= \_\_$

c $m^2 \times m^6$
$= m \times \_ \times m \times \_ \times \_ \times \_ \times \_ \times \_$
$= \_\_$

d $x^5 \times x^2 \times x^3$
$= x \times \_ \times \_ \times \_ \times \_ \times x \times \_ \times x \times \_ \times \_$
$= \_\_$

8 Look at your answers to question **7**. Can you see a shortcut that could be used? Explain. This shortcut is known as one of the index laws.

EXAMPLE 4D-5 Multiplying terms in index form using an index law

Use an index law to simplify:

a $a^4 \times a^7$ **b** $b^3 \times b \times b^5$.

THINK	WRITE
a 1 Check the bases are the same (yes).	**a** $a^4 \times a^7$
2 Write the base and add the indices. (4 + 7 = 11)	$= a^{11}$
b 1 Check the bases are the same (yes).	**b** $b^3 \times b \times b^5$
2 Write the base and add the indices. (3 + 1 + 5 = 9) Remember that b has an index or power of 1.	$= b^9$

9 Use an index law to simplify:

a $a^9 \times a^7$ **b** $x^5 \times x^8$ **c** $m^6 \times m^{11}$
d $k^4 \times k$ **e** $y^{10} \times y^{10}$ **f** $e \times e^5$
g $c^4 \times c^5 \times c^6$ **h** $p^2 \times p^7 \times p^3$ **i** $g^8 \times g^4 \times g^2$
j $n^5 \times n^9 \times n$ **k** $x \times x^7 \times x^8$ **l** $h^6 \times h \times h^6$

EXAMPLE 4D-6 Multiplying algebraic terms using an index law

Use an index law to simplify:

a $2a^3 \times a^6$ **b** $4b^3 \times 3b \times 2b^5$.

THINK	WRITE
a 1 Write the first term as the product of a number and a pronumeral in index form.	**a** $2a^3 \times a^6$ $= 2 \times a^3 \times a^6$
2 Simplify the terms in index form. Check that they have the same base (yes), write the base and add the indices. (3 + 6 = 9)	$= 2 \times a^9$
3 Simplify by leaving out the × sign.	$= 2a^9$
b 1 Write each term as the product of a number and a pronumeral in index form.	**b** $4b^3 \times 3b \times 2b^5$ $= 4 \times b^3 \times 3 \times b \times 2 \times b^5$
2 Reorder the terms so the numbers are grouped together and the pronumerals are grouped together.	$= 4 \times 3 \times 2 \times b^3 \times b \times b^5$
3 Multiply the numbers (4 × 3 × 2 = 24) and simplify the terms in index form by adding the indices. (3 + 1 + 5 = 9)	$= 24 \times b^9$
4 Simplify by leaving out the × sign.	$= 24b^9$

UNDERSTANDING AND FLUENCY

10 Use an index law to simplify:

a $3y^3 \times y^6$ **b** $g^2 \times 7g^5$ **c** $2b^8 \times 3b^3$
d $6k^5 \times 2k^8$ **e** $4w^5 \times 2w^3 \times 5w^9$ **f** $5g^5 \times 2g \times 8g^5$
g $3c \times 3c^7 \times 3c^6$ **h** $p^6 \times 3p^2 \times 5p^2$ **i** $7h^7 \times 2h \times h^4$

11 Jacob wrote that $a^6 \times b^5$ simplifies to ab^{11}. Is he correct? Explain.

12 In a few sentences, explain how and when the index law for multiplication of terms can be used.

13 Simplify:

a $a^2 \times h^3$ **b** $6x^4 \times 2y^5$ **c** $4t^3 \times 5b^8$
d $m^6 \times m^2 \times q^7$ **e** $x^2 \times y^5 \times x^6 \times y^2$ **f** $3g^4 \times 5h^3 \times 2g^6$
g $a^5b^4 \times a^3b^2$ **h** $5x^6y^5 \times 3x^2y^5$ **i** $9w^4x^8 \times 6x^5y^4$

PROBLEM SOLVING AND REASONING

14 Give three examples of two terms that can be multiplied to make each statement true.

a ___ × ___ = $abcd$ **b** ___ × ___ = $12x^2yz$ **c** ___ × ___ = $20mn^3p^2$

15 A rectangle has an area given by $24x^2$. One way of writing this is $8x \times 3x$. Describe three other rectangles that have an area of $24x^2$. That is, list the length and width for each of the three different rectangles.

16 The top of this box is rectangular, with a length three times its width. All measurements are in centimetres.

a If the width is x, write an algebraic term that would represent the length in centimetres.

b The area of a rectangle is length × width. Write the area of the top of the box as a multiplication problem using algebra and then simplify.

c Calculate the area of the top of the box if x is 8.

d How can you check your answer to part **c** is correct? Show there are two ways of obtaining the answer.

e Which way would be quicker if you were to calculate the area of the top using many different x values? Explain.

f The height of the box is five times the width. Write an algebraic term to represent the height.

g Draw a diagram of the box with the length, width and height labelled with their respective algebraic terms.

h Copy and complete this table to obtain the simplified algebraic term for the area of each of the six sides (or faces) of the box.

	area
top	$3x \times x = 3x^2$
bottom	__ × __ = __
front	__ × __ = __
back	__ × __ = __
left side	__ × __ = __
right side	__ × __ = __

i Use your answers to part **h** to write an expression for the surface area of the box. Simplify if possible.

j Calculate the surface area of the box if x is 4. Show two different ways of obtaining your answer.

k Calculate the surface area of the box for each x value.

i 5 **ii** 12 **iii** 0.5 **iv** 2.1

l Write an algebraic term to represent the volume of the box. (Hint: volume = length × width × height.)

m Use a calculator to work out the volume of the box for each x value given in part **k**.

CHALLENGE

17 Write two algebraic multiplication problems of your own that need to be simplified. Swap them with a classmate to complete. Check the answers together and discuss how any errors may have been made and how they can be corrected.

18 Simplify $a^m b^x \times a^n b^y$.

Reflect

What is important to remember when multiplying algebraic terms?

4E Dividing algebraic terms

Start thinking!

Dividing algebraic terms is very like dividing numbers written as fractions.

1 What is:

a $3 \div 3$? b $a \div a$?

2 These division problems can be written as fractions: $3 \div 3 = \frac{3}{3}$ and $a \div a = \frac{a}{a}$.

Show how dividing the numerator and denominator by the highest common factor (HCF) produces the same results you found in question **1**.

3 The division problem $10 \div 2$ can be written as $\frac{10}{2}$. Show how this simplifies to 5. (Hint: what is the HCF of the numerator and denominator?)

4 Similarly, $10a \div 2$ can be written as $\frac{10a}{2}$ or $\frac{10 \times a}{2}$. Show how $\frac{10a}{2}$ simplifies to $5a$.

5 Now look at simplifying $\frac{10a}{2a}$ in the same way.

a Write $\frac{10a}{2a}$ with both the numerator and denominator written in expanded form.

b Which two factors are common to the numerator and denominator?

c Show that $\frac{10a}{2a}$ simplifies to 5.

6 Repeat question **5** for $\frac{10ab}{2a}$ to show that the fraction simplifies to $5b$.

7 Explain how to divide algebraic terms.

KEY IDEAS

- Division problems that contain algebraic terms should be written as fractions.
- Algebraic fractions can be simplified like any other fractions:
  1. Look for common factors (numbers or pronumerals) in the numerator and denominator.
  2. Cancel (or divide by) the common factors.
  3. Write the remaining terms in simplified form.
- Writing algebraic terms in expanded form can help identify any common factors.
- The index laws apply to pronumerals in the same way as they do to numbers.
- To divide terms in index form that have the same base, write the base and subtract the indices (powers). For example, $a^5 \div a^3 = a^2$.
- A number in index form with a power of zero equals one. For example, $a^0 = 1$.

EXERCISE 4E Dividing algebraic terms

EXAMPLE 4E-1 Writing an algebraic fraction in expanded form then simplifying

For each algebraic fraction: **a** $\frac{8x}{x}$ **b** $\frac{4mn}{10n}$

i write the numerator and denominator in expanded form
ii identify any common factors
iii simplify.

THINK	WRITE
a 1 Write the numerator with a multiplication sign.	**a** i $\frac{8x}{x}$ $= \frac{8 \times x}{x}$
2 Look for common factors. x is in the numerator and denominator.	ii Common factor is x.
3 Cancel (or divide by) x in the numerator and denominator and simplify.	iii $= \frac{8 \times \cancel{x}^1}{\cancel{x}_1}$ $= \frac{8 \times 1}{1}$ $= 8$
b 1 Write the numerator and denominator with multiplication signs.	**b** i $\frac{4mn}{10n}$ $= \frac{4 \times m \times n}{10 \times n}$
2 Look for common factors in the numerator and the denominator. The HCF of 4 and 10 is 2, and n is also in both.	ii Common factors are 2 and n.
3 Cancel 2 and n in the numerator and denominator and simplify.	iii $= \frac{{}^2\cancel{4} \times m \times \cancel{n}^1}{{}_5\cancel{10} \times \cancel{n}_1}$ $= \frac{2 \times m \times 1}{5 \times 1}$ $= \frac{2m}{5}$

1 For each algebraic fraction:

i write the numerator and denominator in expanded form
ii identify any common factors
iii simplify.

a $\frac{5a}{5}$ **b** $\frac{9d}{d}$ **c** $\frac{4m}{m}$ **d** $\frac{18c}{6}$

e $\frac{10b}{15}$ **f** $\frac{4x}{4y}$ **g** $\frac{6ab}{a}$ **h** $\frac{5m}{mn}$

i $\frac{cd}{3c}$ **j** $\frac{14st}{7t}$ **k** $\frac{8hk}{10k}$ **l** $\frac{15ab}{12a}$

EXAMPLE 4E-2 Dividing algebraic terms

Simplify $7ab \div a$.

THINK	WRITE
1 Write the division problem as a fraction.	$7ab \div a$ $= \frac{7ab}{a}$
2 Write the numerator in expanded form and look for common factors. a is a common factor, so cancel it in the numerator and denominator.	$= \frac{7 \times {}^{1}\cancel{a} \times b}{{}_{1}\cancel{a}}$ $= \frac{7 \times 1 \times b}{1}$
3 Simplify.	$= 7b$

2 Simplify:

a $xy \div y$	**b** $cd \div c$	**c** $8k \div 4$	**d** $5p \div 30$	**e** $14f \div 12$	**f** $20gh \div 5$
g $18ab \div b$	**h** $9xy \div x$	**i** $3mn \div 12$	**j** $abcd \div c$	**k** $7ef \div g$	**l** $4kmn \div m$

EXAMPLE 4E-3 Simplifying an algebraic fraction

Simplify $\frac{15abc}{25bcd}$.

THINK	WRITE
1 Write the numerator and denominator in expanded form and look for common factors. HCF of 15 and 25 is 5. Common factors are 5, b and c.	$\frac{15abc}{25bcd}$ $= \frac{15 \times a \times b \times c}{25 \times b \times c \times d}$
2 Cancel (or divide by) 5, b and c in the numerator and denominator.	$= \frac{{}^{3}\cancel{15} \times a \times \cancel{b}^{1} \times \cancel{c}^{1}}{{}_{5}\cancel{25} \times \cancel{b}_{1} \times \cancel{c}_{1} \times d}$ $= \frac{3 \times a \times b \times c}{5 \times b \times c \times d}$
3 Simplify the numerator and denominator.	$= \frac{3a}{5d}$

3 Simplify:

a $\frac{8a}{2b}$	**b** $\frac{15x}{3y}$	**c** $\frac{24c}{10d}$	**d** $\frac{12w}{4w}$	**e** $\frac{6k}{16k}$	**f** $\frac{3m}{9m}$
g $\frac{4ab}{a}$	**h** $\frac{da}{2a}$	**i** $\frac{18bc}{6c}$	**j** $\frac{6rt}{8r}$	**k** $\frac{20k}{28kn}$	**l** $\frac{9rst}{18s}$
m $\frac{35wx}{21wxy}$	**n** $\frac{7bc}{bc}$	**o** $\frac{25mn}{10mn}$	**p** $\frac{8abc}{2ac}$	**q** $\frac{4mnp}{24mn}$	**r** $\frac{12wxy}{28xy}$

UNDERSTANDING AND FLUENCY

4 Simplify:

a $\frac{a^2b}{ab}$ **b** $\frac{6c^2e}{ce}$ **c** $\frac{4w^2x}{2wx}$ **d** $\frac{3km^2}{9km}$ **e** $\frac{8ef^2}{20ef}$ **f** $\frac{2rs}{r^2s}$

g $\frac{4tx^2}{5xy}$ **h** $\frac{a^2bc^2}{abc}$ **i** $\frac{18b^2d^2}{15bd}$ **j** $\frac{6m^3bc}{9mc}$ **k** $\frac{10k^3mn}{5k^2mn}$ **l** $\frac{7abc^2}{7ab^2c}$

5 Copy and complete these division problems.
Each term is written in expanded form first.

a $a^6 \div a^4$

$= \frac{a^6}{a^4}$

$= \frac{a \times a \times a \times a \times a \times a}{a \times a \times \_ \times \_}$

$= \frac{{}^1\cancel{a} \times {}^1\cancel{a} \times {}^1\cancel{a} \times {}^1\cancel{a} \times a \times a}{{}_1\cancel{a} \times {}_1\cancel{a} \times {}_1\cancel{a} \times {}_1\cancel{a}}$

$= \frac{1 \times 1 \times 1 \times 1 \times a \times a}{1 \times \_ \times \_ \times 1}$

$= \frac{a^2}{1}$

$= \_$

b $m^5 \div m^2$

$= \frac{m^5}{m^2}$

$= \frac{m \times m \times m \times m \times m}{\_ \times \_}$

$= \frac{{}^1\cancel{m} \times {}^1\cancel{m} \times m \times m \times m}{{}_1\cancel{m} \times {}_1\cancel{m}}$

$= \frac{1 \times \_ \times m \times m \times \_}{\_ \times \_}$

$= \frac{m^{-}}{1}$

$= \_$

6 Look at your answers to question **5**. Can you see a shortcut that could be used? Explain. This shortcut is known as one of the index laws.

EXAMPLE 4E-4 Dividing algebraic terms using an index law

Use an index law to simplify:

a $x^9 \div x^3$ **b** $5k^{11} \div k^7$

THINK	WRITE
a 1 Check the bases are the same (yes).	**a** $x^9 \div x^3$
2 Write the base and subtract the indices. $(9 - 3 = 6)$	$= x^6$
b 1 Write the first term as the product of a number and a pronumeral in index form.	**b** $5k^{11} \div k^7$ $= 5 \times k^{11} \div k^7$
2 Write the base and subtract the indices. $(11 - 7 = 4)$	$= 5 \times k^4$
3 Simplify.	$= 5k^4$

7 Use an index law to simplify:

a $p^{10} \div p^7$ **b** $a^8 \div a^3$ **c** $n^{14} \div n^{11}$

d $r^9 \div r$ **e** $8x^{17} \div x^6$ **f** $6m^8 \div m^2$

8 In a few sentences, explain how and when the index law for dividing terms is used.

9 a Simplify $a^3 \div a^3$ by first writing as a fraction with each term in expanded form.
b Simplify $a^3 \div a^3$ using an index law. Leave your answer in index form.
c Use your answers to parts a and b to explain why $a^0 = 1$.

10 Simplify each expression.
a $x^5 \div x^5$ b $b^8 \div b^8$ c $m^{11} \div m^{11}$ d $2p^9 \div p^9$ e $5y^9 \div y^9$ f $8g^2 \div g^2$

EXAMPLE 4E-5 Simplifying an algebraic fraction using index laws

Use index laws to simplify: a $\dfrac{6a^8}{2a^3}$ b $\dfrac{b^4 \times b^6}{b^7}$

THINK	WRITE
a 1 Write each term in the numerator and denominator as the product of a number and a pronumeral in index form.	a $\dfrac{6a^8}{2a^3}$ $= \dfrac{6 \times a^8}{2 \times a^3}$
2 Cancel 2 in the numerator and denominator. To divide a^8 by a^3, write the base and subtract the indices. $(8 - 3 = 5)$	$= 3 \times a^5$
3 Simplify.	$= 3a^5$
b 1 Simplify the numerator by multiplying b^4 and b^6: write the base and add the indices. $(4 + 6 = 10)$	b $\dfrac{b^4 \times b^6}{b^7}$ $= \dfrac{b^{10}}{b^7}$
2 Write the base and subtract the indices. $(10 - 7 = 3)$	$= b^3$

UNDERSTANDING AND FLUENCY

11 Use index laws to simplify:
a $\dfrac{7b^6}{7b^4}$ b $\dfrac{2q^7}{2q^3}$ c $\dfrac{10c^7}{2c^3}$ d $\dfrac{15y^{12}}{6y^5}$
e $\dfrac{ax^{13}}{ax^4}$ f $\dfrac{a^9b}{a^3}$ g $\dfrac{m^5n}{m^5}$ h $\dfrac{b^{20}d}{b^{14}d}$
i $\dfrac{x^4 \times x^3}{x^2}$ j $\dfrac{m^7 \times m^6}{m^9}$ k $\dfrac{6a^2 \times a^8}{a^4}$ l $\dfrac{n^5 \times n^7}{n^3 \times n^4}$
m $\dfrac{5d^6 \times d^3}{d^9}$ n $\dfrac{8t^2 \times t^3}{2t^5}$ o $\dfrac{4k \times 3k^9}{6k^{10}}$ p $\dfrac{15e^{13}}{3e^8 \times 5e^5}$

12 Simplify each of these, if possible.
a $\dfrac{15k}{12kn}$ b $\dfrac{2jk^{11}}{k^4}$ c $\dfrac{3xy^4}{y^4}$ d $\dfrac{m^7}{p^3}$
e $\dfrac{b^5c^4}{b^3c^2}$ f $\dfrac{4k^8}{2a^5}$ g $\dfrac{18x^4}{22y^4}$ h $\dfrac{6a^3b}{2c^3}$
i $\dfrac{x^2y}{4x}$ j $\dfrac{8mn^2}{2m}$ k $\dfrac{2m^{13}n^7}{2m^5n^2}$ l $\dfrac{6x^6y^5}{6x^2y^5}$
m $\dfrac{18w^4x^8}{3x^5y^4}$ n $\dfrac{16pq}{20p^2q}$ o $\dfrac{ab^2cd}{bc^2}$ p $\dfrac{24a^7c^6e^2}{32a^3c^2}$

PROBLEM SOLVING AND REASONING

13 The school canteen has a 10-kg box of apples delivered each week. As the price of apples can vary, the canteen manager must work out how much each apple costs so she knows how to price them for sale. She generally receives about 75 apples in the 10-kg box.

a One week apples cost \$4 per kg. Calculate the cost of one apple (to the nearest 5 cents). Clearly show your working.

b The next week apples cost \$5 per kg. Calculate the cost of one apple (to the nearest 5 cents). Clearly show your working.

c Write an expression to help the canteen manager calculate the cost of one apple if the cost in dollars per kilogram is represented by x. Write your answer as a fraction in simplest form.

d Calculate the cost per apple for these values of x. Round your answers to the nearest 5 cents.

i $x = 6$ **ii** $x = 8$ **iii** $x = 7$ **iv** $x = 3$

14 Fabian and his father are laying cement pavers to create a smooth surface for skateboarding. The pavers come in different sizes but are rectangular in shape and have a length twice their width.

a Let x represent the width of a paver in centimetres. Write the length in terms of x and hence write an algebraic term for the area of one paver.

b The ground to be paved is square with a length represented by $10x$ cm. Write an algebraic term for the area of the square to be paved.

c Divide the area of the square to be paved by the area of one paver to work out the number of pavers required for this task. Write your answer in simplest form.

d To check whether the answer you found in part **c** is correct, substitute a value for x; for example, let $x = 40$. Evaluate each of these using this x value.

i width of paver **ii** length of paver **iii** area of paver

iv length of square to be paved **v** area of square to be paved

e Use your answers to part **d iii** and **v** to calculate the number of pavers required. How does this compare to your answer to part **c**?

f Will the size of the pavers affect the number of pavers needed? Explain. (Hint: consider different values of x and perform the necessary calculations.)

CHALLENGE

15 Write two algebraic division problems of your own that can be simplified. Swap them with a classmate to complete. Check the answers together and discuss how any errors may have been made and how they can be corrected.

16 Simplify $a^m b^x \div (a^n b^y)$.

Reflect

What is important to remember when dividing algebraic terms?

4F Working with brackets

Start thinking!

A pair of brackets can be used to group particular numbers, pronumerals or operations together. It can change the order of the operations you perform.

1 **a** Calculate:

i $3 \times 2 + 5$ **ii** $3 \times (2 + 5)$.

b Do you obtain the same answer to each problem in part **a**? Explain.

2 The expression $3 \times (2 + 5)$ means 3 lots of $(2 + 5)$.
Copy and complete:

$$\begin{aligned} 3 \text{ lots of } (2+5) &= (2+5)+(2+5)+(2+5) \\ &= 2+2+2+5+5+5 \\ &= \_\_ \text{ lots of } 2 + \_\_ \text{ lots of } 5 \\ \text{so } 3 \times (2+5) &= \_\_ \times 2 + \_\_ \times 5 \end{aligned}$$

You have now written the expression in **expanded form** (that is, without brackets).

3 Use the same method to copy and complete the following working for $3 \times (a + 2)$.

$$\begin{aligned} 3 \text{ lots of } (a+2) &= (a+2)+(a+2)+(a+2) \\ &= a+a+a+2+\_\_+\_\_ \\ &= \_\_ \text{ lots of } a + \_\_ \text{ lots of } 2 \\ \text{so } 3 \times (a+2) &= \_\_ \times a + \_\_ \times 2 \end{aligned}$$

4 This method is called the **distributive law**. For example, $4 \times (m + n) = 4 \times m + 4 \times n$.
Explain how it works.

5 The example in question **4** can be simplified by leaving out the multiplication signs.
$4 \times (m + n) = 4 \times m + 4 \times n$ is the same as
$4(m + n) = 4m + 4n$
Can $4m + 4n$ be simplified further? Explain.

KEY IDEAS

- A pair of brackets can group numbers, pronumerals and operations together.
- An expression such as $a(b + c)$ can be written in expanded form (that is, without brackets) by using the distributive law.
- Using the distributive law means that each term inside the pair of brackets is multiplied by the term outside the brackets (see example).

Example

$$\begin{aligned} a(b+c) &= a \times b + a \times c \\ &= ab + ac \end{aligned}$$

EXERCISE 4F Working with brackets

UNDERSTANDING AND FLUENCY

1 Calculate the value of each expression. What do you notice about each pair?

a **i** $2(3 + 5)$ **ii** $2 \times 3 + 2 \times 5$
b **i** $6(8 + 1)$ **ii** $6 \times 8 + 6 \times 1$
c **i** $4(7 - 2)$ **ii** $4 \times 7 + 4 \times (-2)$
d **i** $5(9 - 6)$ **ii** $5 \times 9 + 5 \times (-6)$
e **i** $-3(5 + 4)$ **ii** $(-3) \times 5 + (-3) \times 4$
f **i** $-2(6 - 5)$ **ii** $(-2) \times 6 + (-2) \times (-5)$

2 Copy and complete each set of working to expand these expressions.

a $4(m + 3)$
$= 4 \times \_\_ + 4 \times \_\_$
$= \_\_ + \_\_$

b $a(c + 5b)$
$= \_\_ \times c + \_\_ \times 5b$
$= \_\_ + \_\_$

c $7(2p - 3)$
$= \_\_ \times 2p + \_\_ \times (-3)$
$= \_\_ - \_\_$

d $-2(3a + 1)$
$= (-2) \times \_\_ + (-2) \times \_\_$
$= \_\_ - \_\_$

EXAMPLE 4F-1 Writing expressions in expanded form

Use the distributive law to write each expression in expanded form and simplify.

a $6(k + 4)$ **b** $2(3x + y)$ **c** $4(a - 5)$

THINK	WRITE
a 1 Multiply each term inside the brackets (k and 4) by the term in front of the brackets (6).	**a** $6(k + 4)$ $= 6 \times k + 6 \times 4$
2 Simplify by performing the multiplications.	$= 6k + 24$
b 1 Multiply each term inside the brackets ($3x$ and y) by the term in front of the brackets (2).	**b** $2(3x + y)$ $= 2 \times 3x + 2 \times y$
2 Simplify by performing the multiplications.	$= 6x + 2y$
c 1 Multiply each term inside the brackets (a and -5) by the term in front of the brackets (4). Note the coefficient of the second term inside the brackets is negative.	**c** $4(a - 5)$ $= 4 \times a + 4 \times (-5)$
2 Simplify by performing the multiplications.	$= 4a - 20$

3 Use the distributive law to write each expression in expanded form and simplify.

a $3(y + 2)$ **b** $5(k + 7)$ **c** $2(5 + a)$ **d** $6(2 + d)$
e $8(a + b)$ **f** $3(p + t)$ **g** $4(2x + 6)$ **h** $2(1 + 8m)$
i $5(4a + b)$ **j** $6(m + 3n)$ **k** $7(5a + 3b)$ **l** $3(2x + 9y)$
m $3(m - 2)$ **n** $2(6 - x)$ **o** $4(1 - p)$ **p** $6(8c - 5d)$

EXAMPLE 4F-2 Expanding and simplifying expressions

Expand and simplify each expression.

a $7p(3q + 4e)$ **b** $2m(5m - 4)$

THINK	WRITE
a 1 Use the distributive law. Multiply each term inside the brackets ($3q$ and $4e$) by the term in front of the brackets ($7p$).	**a** $7p(3q + 4e)$ $= 7p \times 3q + 7p \times 4e$
2 Simplify by performing the multiplications. Write the pronumerals in each term in alphabetical order.	$= 21pq + 28ep$
b 1 Multiply each term inside the brackets ($5m$ and -4) by $2m$.	**b** $2m(5m - 4)$ $= 2m \times 5m + 2m \times (-4)$
2 Simplify the multiplications.	$= 10m^2 - 8m$

4 Expand and simplify each expression.

a $a(b + 4)$ **b** $x(3 + w)$ **c** $d(c - e)$ **d** $p(2m + q)$
e $k(b - 4c)$ **f** $t(3s + 5r)$ **g** $2x(4y + 3w)$ **h** $3a(2b - 4c)$
i $9p(3q + 5k)$ **j** $-8m(2n + 7w)$ **k** $-4t(5a - 2b)$ **l** $-5y(-3x - 4z)$
m $4a(3a + 2)$ **n** $6k(2k - 3)$ **o** $3x(1 - 4x)$ **p** $-2e(4e + 5)$
q $-7n(n - 6)$ **r** $-2b(2a - 3b)$ **s** $5a^2(2a - 3)$ **t** $-11y(x^2 + y^2)$

EXAMPLE 4F-3 Expanding and simplifying more complex expressions

Expand and simplify each expression.

a $4(2p + k - 3m)$ **b** $-3(4xy + 7a - 1)$

THINK	WRITE
a 1 Multiply each term inside the brackets ($2p$, k and $-3m$) by the term in front of the brackets (4). Write negative terms in brackets to see the operations clearly.	**a** $4(2p + k - 3m)$ $= 4 \times 2p + 4 \times k + 4 \times (-3m)$
2 Simplify the multiplications.	$= 8p + 4k - 12m$
b 1 Multiply each term inside the brackets ($4xy$, $7a$ and -1) by -3.	**b** $-3(4xy + 7a - 1)$ $= (-3) \times 4xy + (-3) \times 7a + (-3) \times (-1)$
2 Simplify the multiplications.	$= -12xy - 21a + 3$

5 Expand and simplify each expression.

a $2(3x + y + 7)$ **b** $5(2ab + 4d - 3)$ **c** $7(2c - d + 5ef)$
d $-6(3g - 2h + gh)$ **e** $-4(5k + 2km - 9)$ **f** $-3(2xy + 4 - 7w)$

UNDERSTANDING AND FLUENCY

6 Use the distributive law and then simplify any like terms to copy and complete the following.

a $5(a+4)+2(a+7)$
$= 5 \times \_ + 5 \times \_ + 2 \times \_ + 2 \times \_$
$= 5a + \_ + 2a + \_$
$= \_a + \_$

b $3b(c+2)+2c(a+5b)$
$= \_ \times c + \_ \times 2 + 2c \times \_ + 2c \times \_$
$= 3bc + \_ + \_ + \_$
$= \_ + \_ + \_$

c $d(d+8)+6(d-3)$
$= d \times \_ + d \times \_ + \_ \times d + \_ \times (-3)$
$= \_ + 8d + \_ - \_$
$= \_ + \_ - \_$

d $4k(2k+3)-2(k-5)$
$= 4k \times \_ + 4k \times \_ + (-2) \times \_ + (-2) \times (-5)$
$= 8k^2 + \_ + \_$
$= \_ + \_ + \_$

7 Expand and simplify each expression.

a $2(a+6)+5(a+3)$
b $4(b-3)+6(b+2)$
c $5m(n+4)+3n(k+2m)$
d $7x(3+y)+2y(8-3x)$
e $k(k-1)+5(k+4)$
f $c(c+4)-7(c-5)$
g $rst(2+5t)+9r(4t+st)$
h $h(h^2-2)-4h(h+3)$

8 Copy and complete the following to expand and simplify the expressions. (Hint: $(a+b)$ means $1 \times (a+b)$ and $-(a+b)$ means $-1 \times (a+b)$.)

a $4(m-2)+(m+6)$
$= 4(m-2)+1(m+6)$
$= 4 \times \_ + 4 \times \_ + 1 \times \_ + 1 \times \_$
$= 4m - \_ + m + \_$
$= \_ - \_$

b $2x(y+3)-(5+xy)$
$= 2x(y+3)-1(5+xy)$
$= 2x \times \_ + 2x \times \_ + (-1) \times \_ + (-1) \times xy$
$= \_ + \_ - 5 - \_$
$= \_ + \_ - \_$

c $k(k-1)-(k-7)$
$= k(k-1)-1(k-7)$
$= k \times \_ + k \times \_ + \_ \times k + \_ \times (-7)$
$= \_ - \_ - \_ + \_$
$= \_ - 2k + \_$

d $3w(w-2)+(4w-1)$
$= 3w(w-2)+1(4w-1)$
$= 3w \times \_ + 3w \times \_ + \_ \times 4w + 1 \times \_$
$= \_ - \_ + \_ - \_$
$= \_ - \_ - \_$

9 Expand and simplify each expression.

a $6(p+3)+(p+2)$
b $3(d-2)-(d+8)$
c $x(x+2)+(x+5)$
d $k(k+4)-(3k+1)$
e $2e(e+4)-(e-3)$
f $4a(b+1)-(6+ab)$
g $5w(y-3)-(wy+2)$
h $mp(n-m)+(n-3m^2p)$

10 In three years, Monette will be twice as old as Chanelle.

a Write an expression for Chanelle's age in three years.
(Hint: use a pronumeral to represent Chanelle's current age.)
b Use your answer to part **a** to write an expression for Monette's age.
c Use the distributive law to write the expression for Monette's age without brackets.
d Work out Monette's age if Chanelle's current age is: **i** 14 **ii** 18 **iii** 25.
e Which expression did you use for your calculations? Explain your choice.

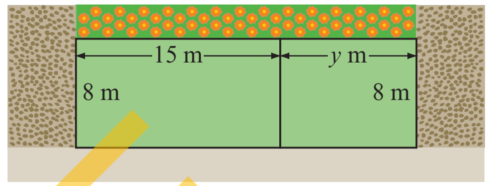
PROBLEM SOLVING AND REASONING

11 One of Andre's jobs is to mow the front lawn. It is rectangular and measures 15 m by 8 m.

a Calculate the area of lawn to be mown.

A neighbour notices Andre is about to start mowing and asks if he could mow her front lawn as well.

b Since the width of the neighbour's front yard is unknown, represent it with the pronumeral y. Write an expression for the area of lawn in the neighbour's front yard.



c Add your answers to parts **a** and **b** to write an expression for the total area of lawn to be mown.

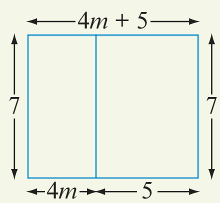
d Now consider both lawns as one large rectangle. Draw a diagram of this rectangle with the length and width measurements shown.

e Explain why the expression for the area of the large rectangle can be written as $8(15 + y)$.

f Explain why the two answers for the total area of lawn to be mown (parts **c** and **e**) are equivalent.

EXAMPLE 4F-4 Demonstrating the distributive law using areas of rectangles

Demonstrate that $7(4m + 5) = 28m + 35$ is true by finding the appropriate areas in the diagram.

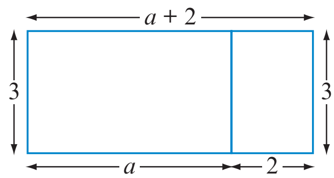


THINK	WRITE
1 Write an expression for the area of the large rectangle. Identify 7 as length and $4m + 5$ as width.	area of large rectangle = length × width $= 7 \times (4m + 5)$ $= 7(4m + 5)$
2 Write an expression for the area of the smaller rectangle on the left (within the large rectangle). Identify 7 as length and $4m$ as width.	area of smaller rectangle on left = length × width $= 7 \times 4m$ $= 28m$
3 Write an expression for the area of the smaller rectangle on the right (within the large rectangle). Identify 7 as length and 5 as width.	area of smaller rectangle on right = length × width $= 7 \times 5$ $= 35$
4 Use the fact that the area of the large rectangle is the same as the sum of the areas of the two smaller rectangles.	area of large rectangle = area of smaller rectangle on left + area of smaller rectangle on right so $7(4m + 5) = 28m + 35$

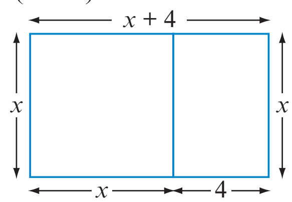
PROBLEM SOLVING AND REASONING

12 Demonstrate that each statement is true by finding the appropriate areas in the diagram.

a $3(a + 2) = 3a + 6$



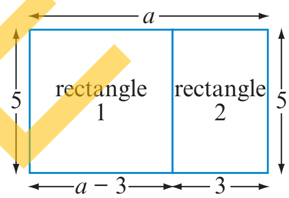
b $x(x + 4) = x^2 + 4x$



13 Draw a diagram to demonstrate that each statement is true.

a $6(k + 8) = 6k + 48$ **b** $4(2d + 9) = 8d + 36$ **c** $2p(p + 3) = 2p^2 + 6p$

14 This diagram can be used to demonstrate that $5(a - 3) = 5a - 15$.



a What is the length and width of rectangle 1?

b Use your answer to part **a** to write an expression for the area of rectangle 1 using brackets.

c Another way to work out the area of rectangle 1 is to subtract the area of rectangle 2 from the total area of the large rectangle.

i Write the length and width of the large rectangle.

ii Use these length and width measurements to write an expression for the area of the large rectangle.

iii Write the length and width of rectangle 2.

iv Use these length and width measurements to write an expression for the area of rectangle 2.

v Use your answers to parts **ii** and **iv** to write an expression for the area of rectangle 1.

d Explain how you were able to show that $5(a - 3) = 5a - 15$.

15 Draw a diagram and find appropriate areas of rectangles to demonstrate why each statement is true.

a $4(x - 2) = 4x - 8$ **b** $c(d - 5) = cd - 5c$ **c** $h(h - 4) = h^2 - 4h$

CHALLENGE

16 Expand and simplify each expression.

a $3x^2(4x^3 - 7) - 5x^4(x + 2)$

b $a^3(ab + c - 1) + a^2(3ac - a^2b + a)$

c $2m(3m^2np^2 + 5mn) - 6n(m^2 + m^2p^2 - 4)$

d $5w(4w^2x - 3y) + 2y(9w - 6xy) - 4x(5w^3 - 3y^2)$

17 Evaluate each expression in question **16** using these values:

$a = -3, b = 7, c = 4$

$m = 2, n = -4, p = -1$

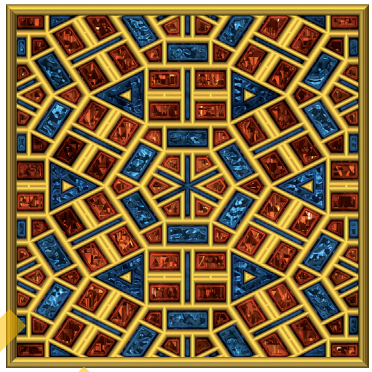
$w = -5, x = 2, y = -6$.

Reflect

What is the distributive law and how is it used?

4G Factorising expressions

Start thinking!



Each piece of glass in a leadlight panel must be edged with a strip of lead.

1 **a** Calculate the length of lead needed for this piece of red glass.

b Which one of these three ways did you use to obtain your answer?

5 cm

3 cm

$5 + 3 + 5 + 3$	length + width + length + width
or $2 \times (5 + 3)$	2 lots of (length + width)
or $2 \times 5 + 2 \times 3$	2 lots of length + 2 lots of width

c The expression $2 \times (5 + 3)$ is said to be in **factor form**, as it shows the product of factors. In this case, there are two factors. One factor is $(5 + 3)$. What is the other factor?

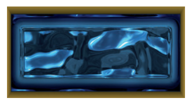
d The distributive law can also be used in the reverse direction. Explain how you could **factorise** (write in factor form) the expression $2 \times 5 + 2 \times 3$ to produce $2 \times (5 + 3)$. (Hint: which factor is common to 2×5 and 2×3?)

$$\underbrace{2 \times 5 + 2 \times 3}_{\text{expanded form}} = \underbrace{2 \times (5 + 3)}_{\text{factor form}}$$

e Factorise $4 \times 7 + 4 \times 11$. Explain the method you used.

2 **a** Copy and complete this expression for the perimeter of the blue glass in expanded form: $2 \times$ __ $+ 2 \times$ __.

l cm



w cm

b For the expression found in part **a**, circle any factors that are common to both terms.

c Copy and complete this expression for the perimeter of the glass in factor form: __ $\times$ (__ + __).

d **i** For the expression you wrote in factor form, list the two factors.

ii Which of these two factors is the same as the common factor you circled in part **b**?

3 Explain the difference between expanding an expression and factorising an expression.

KEY IDEAS

$$4(3a + 2) \xrightarrow{\text{expanding}} 12a + 8 \qquad 12a + 8 \xrightarrow{\text{factorising}} 4(3a + 2)$$

- Factorising an expression is the opposite to expanding an expression.
- The distributive law can be used to write expressions in factor form. One of the factors is the HCF of each term in the expression. Write the HCF in front of the brackets and put the other factor of each term inside the brackets.

$$\underbrace{a \times b + a \times c}_{\text{expanded form}} = \underbrace{a \times (b + c)}_{\text{factor form}} \quad \text{or} \quad ab + ac = a(b + c)$$

- To find the HCF of a set of algebraic terms, first identify the common factors (including numbers and pronumerals) of each term and then multiply them.

EXERCISE 4G Factorising expressions

1 Find the missing factor in each of these.

a $2 \times \_\_ = 6$
b $8 \times \_\_ = -16$
c $\_\_ \times k = 6k$
d $3 \times \_\_ = 3x$
e $x \times \_\_ = -7x$
f $a \times \_\_ = ab$
g $\_\_ \times c = cd$
h $4a \times \_\_ = 8a$
i $\_\_ \times 3m = 15m$
j $6e \times \_\_ = -12ef$
k $\_\_ \times 4k = 20kp$
l $5xy \times \_\_ = -5xy$

EXAMPLE 4G-1 Finding the HCF of a pair of terms

Find the highest common factor (HCF) of each pair of terms.

a $3d$ and 15 b $10fg$ and $-5gh$

THINK	WRITE
a 1 Write $3d$ as a product of factors.	a $3d = 3 \times d$
2 Write 15 as a product of factors.	$15 = 3 \times 5$
3 Look for factors that are common to each term. The only common factor is 3 so this is the HCF.	HCF = 3.
b 1 Write $10fg$ as a product of factors.	b $10fg = 2 \times 5 \times f \times g$
2 Write $-5gh$ as a product of factors. Show -5 as -1×5 since the other term has a factor of 5.	$-5gh = -5 \times g \times h$ $= -1 \times 5 \times g \times h$
3 Look for common factors.	Common factors are 5 and g.
4 Multiply the common factors to obtain the HCF.	HCF $= 5 \times g$ $= 5g$

UNDERSTANDING AND FLUENCY

2 Find the highest common factor (HCF) of each pair of terms.

a $4x$ and 12
b $10d$ and -5
c 6 and $8k$
d $3a$ and -3
e $9c$ and $9e$
f $7x$ and $14y$
g $6xy$ and $2y$
h $3c$ and $9bc$
i $30gh$ and $-24g$
j $4ab$ and $8bc$
k $2mn$ and $3mp$
l $15rs$ and $-20st$

3 Write each term in question 2 as a product of two factors where one of the factors is the HCF of the pair of terms. For example, for part a, $4x = 4 \times x$ and $12 = 4 \times 3$.

4 Copy and complete the following to write each expression in factor form.

a $4x + 12$
$= 4 \times \_\_ + 4 \times \_\_$
$= 4 \times (x + \_\_)$
$= 4(x + \_\_)$

b $10a - 35$
$= 5 \times \_\_ + 5 \times (-7)$
$= 5 \times (\_\_ - 7)$
$= 5(\_\_ - \_\_)$

c $24km + 16k$
$= 8k \times \_\_ + 8k \times \_\_$
$= 8k \times (\_\_ + \_\_)$
$= 8k(\_\_ + \_\_)$

d $18cd - 12de$
$= \_\_ \times 3c + \_\_ \times (-2e)$
$= \_\_ \times (\_\_ - \_\_)$
$= \_\_(\_\_ - \_\_)$

NOTE Remember that you are still using the distributive law but in the opposite direction.

EXAMPLE 4G-2 Factorising expressions

Factorise each expression.

a $8x + 28$ **b** $15mn - 20m$

THINK	WRITE
a 1 Identify the HCF of $8x$ and 28. (HCF = 4) Write each term as a product of the HCF and its other factor.	**a** $8x + 28$ $= 4 \times 2x + 4 \times 7$
2 Write the HCF at the front of a pair of brackets and the other factor for each term inside the brackets.	$= 4 \times (2x + 7)$ $= 4(2x + 7)$
b 1 Identify the HCF of $15mn$ and $-20m$. (HCF = $5m$) Write each term as a product of the HCF and its other factor.	**b** $15mn - 20m$ $= 5m \times 3n + 5m \times (-4)$
2 Write the HCF at the front of a pair of brackets and the other factor for each term inside the brackets.	$= 5m \times (3n - 4)$ $= 5m(3n - 4)$

UNDERSTANDING AND FLUENCY

5 Factorise each expression.

a $4a + 8$ **b** $7e + 21$ **c** $45 + 5k$
d $3 + 12y$ **e** $2d - 6$ **f** $27 - 9p$
g $8m + 20$ **h** $6h - 30$ **i** $24b + 18b$
j $12t - 14$ **k** $5ab + 10a$ **l** $9n + 3mn$
m $22xy - 4x$ **n** $3ab + 3bc$ **o** $16mn - 8mp$
p $27xy + 18wx$ **q** $12k + 15m$ **r** $21ef - 24gh$

6 Find the HCF of each pair of terms.

a abc and bcd **b** wxy and x **c** $9mn$ and $9kmn$
d $3bcd$ and $6def$ **e** $8pq$ and $-2pqr$ **f** $12agk$ and $18ak$
g $16gh$ and $-28mn$ **h** $10abcd$ and $14bcde$ **i** $6efgh$ and $-7abfk$

7 Write each term in question **6** as a product of two factors where one of the factors is the HCF of the pair of terms. For example, for part **a**: $abc = bc \times a$ and $bcd = bc \times d$.

UNDERSTANDING AND FLUENCY

8 Copy and complete the following to write each expression in factor form.

a $abc + bcd$
$= bc \times \_\_ + bc \times \_\_$
$= bc \times (a + \_\_)$
$= bc(a + \_\_)$

b $8pq - 2pqr$
$= 2pq \times \_\_ + 2pq \times \_\_$
$= 2pq \times (\_\_ - \_\_)$
$= 2pq(\_\_ - \_\_)$

c $12agk + 18ak$
$= \_\_ \times 2g + \_\_ \times 3$
$= \_\_ \times (\_\_ + 3)$
$= \_\_(\_\_ + 3)$

d $6efgh - 7abfk$
$= \_\_ \times 6egh + f \times \_\_$
$= \_\_ \times (6egh - \_\_)$
$= \_\_(6egh - \_\_)$

9 Factorise each expression.

a $mnp + kmn$ **b** $3ab - 9abc$ **c** $14rst + 21rt$
d $4cde + 5def$ **e** $2wxyz - 2rstx$ **f** $24apy + 40aqy$

10 Explain how to check your answers to question **9** are correct. (Hint: factorising is the opposite process to expanding.) Use this method to check your answers.

11 Find the missing factor in each of these.

a $a \times \_\_ = a^2$ **b** $3m \times \_\_ = 3m^2$ **c** $x \times \_\_ = 7x^2$
d $5b \times \_\_ = 10b^2$ **e** $\_\_ \times 4k = 12k^2$ **f** $\_\_ \times (-6y) = -42y^2$
g $4p \times \_\_ = -16p^2$ **h** $\_\_ \times 8c^2 = 24c^2$ **i** $\_\_ \times -3y = -15y^2$

12 Find the HCF of each pair of terms.

a x^2 and $8x$ **b** $2a^2$ and $3a$ **c** $7k^2$ and $7k$
d $2y$ and $4y^2$ **e** $6m$ and $8m^2$ **f** $12p^2$ and $-15p$
g $28t$ and $35t^2$ **h** $5a^2$ and a **i** $3n^2$ and $-6n$

13 Copy and complete the following to write each expression in factor form.

a $x^2 + 8x$
$= x \times \_\_ + x \times \_\_$
$= x \times (x + \_\_)$
$= x(x + \_\_)$

b $5m^2 - 5m$
$= 5m \times \_\_ + \_\_ \times (-1)$
$= 5m \times (\_\_ - 1)$
$= 5m(\_\_ - \_\_)$

c $3b + 6b^2$
$= \_\_ \times \_\_ + \_\_ \times 2b$
$= \_\_ \times (\_\_ + 2b)$
$= \_\_(\_\_ + \_\_)$

d $18k^2 - 21k$
$= 3k \times \_\_ + 3k \times \_\_$
$= 3k \times (\_\_ - \_\_)$
$= 3k(\_\_ - \_\_)$

14 Factorise each expression.

a $c^2 + 4c$ **b** $6k + k^2$ **c** $a^2 - 2a$
d $3m^2 + 3m$ **e** $10p^2 + 11p$ **f** $4h - 7h^2$
g $6x^2 + 3x$ **h** $5b^2 - 10b$ **i** $8n^2 + 36n$
j $27y^2 - 15y$ **k** $14a + 16a^2$ **l** $e^2 + e$

UNDERSTANDING AND FLUENCY

15 Use the process of expanding to check whether your answers to question **14** are correct.

16 In some cases, it can be best to use an HCF that is a term with a negative coefficient. Find the HCF that has a negative coefficient for each pair of terms.

a $-5a$ and -10 **b** $-2pq$ and $-6p$

c $-9k$ and $-15kx$ **d** $-14b^2$ and $-7b$

e $-4y^2$ and $4y$ **f** $-12m$ and $20mnp$

g $-x^2y - xy$ **h** $-8bc + 28abc$

17 Copy and complete the following to write each expression in factor form. In each case, the HCF is a term with a negative coefficient.

a $-5a - 10$
$= (-5) \times a + (-5) \times 2$
$= -5 \times (a + \_\_)$
$= -5(a + \_\_)$

b $-2pq - 6p$
$= \_\_ \times q + \_\_ \times 3$
$= \_\_ \times (q + 3)$
$= \_\_(q + 3)$

c $-4y^2 + 4y$
$= \_\_ \times y + \_\_ \times (-1)$
$= \_\_ \times (y - 1)$
$= \_\_(y - 1)$

d $-12m + 20mnp$
$= (-4m) \times \_\_ + (-4m) \times \_\_$
$= -4m \times (\_\_ - \_\_)$
$= -4m(\_\_ - \_\_)$

18 Factorise each expression by finding the HCF that has a negative coefficient.

a $-4a - 12$ **b** $-6 - 8k$ **c** $-7m + 14$

d $-15b + 35$ **e** $-3st - 9t$ **f** $-18c - 24cd$

g $-12mx + 21my$ **h** $-h^2 - 2h$ **i** $-16x^2 + 8x$

j $-45b - 10abc$ **k** $-x - x^2$ **l** $-11a^2b + 9ab$

19 Now that you have practised factorising, try writing the expression in factor form directly from the expanded form. Copy and complete the following to factorise each expression in one step.

a $2x + 6 = 2(\_\_ + \_\_)$ **b** $8a - 36 = \_\_(2a - 9)$

c $h^2 + 7h = h(\_\_ + \_\_)$ **d** $25mn + 30n = 5n(\_\_ + \_\_)$

e $kmn - mn = \_\_(k - \_\_)$ **f** $12ef - 18df = 6f(\_\_ - \_\_)$

g $-12 - 9y = -3(\_\_ + \_\_)$ **h** $-26b - 24c = -2(\_\_ + \_\_)$

i $-abc + acd = -ac(\_\_ - \_\_)$ **j** $6xy + 4x^2y = 2xy(\_\_ + \_\_)$

k $mn^2 + 5mn = \_\_(\_\_ + 5)$ **l** $24cd - 16abc = \_\_(3d - \_\_)$

20 Factorise each of these. If possible, use just one step to write the expression in factor form. Use the process of expanding to check your answers.

a $6g + 6$ **b** $10 + 18x$ **c** $kp + kq$

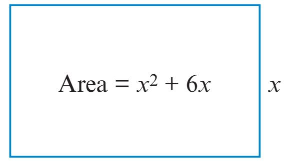
d $m^2 - m$ **e** $14 - 7ab$ **f** $3eh + 3gh$

g $8abcd + 4abde$ **h** $-5a^2 - 15a$ **i** $-24k + 12p$

j $3x^2 - xy$ **k** $16cde + 20e^2$ **l** $-ad - 6d$

PROBLEM SOLVING AND REASONING

21 Consider this rectangle, which has all measurements in centimetres. The expression for the width is x and the expression for the area is $x^2 + 6x$.



- **a** Factorise the expression for the area of the rectangle.
- **b** List the two factors that multiply to give the expression for the area.
- **c** What is the expression for the length of this rectangle?
- **d** Demonstrate you have the correct expression for the length of this rectangle by multiplying the length by the width to obtain the expression for its area.
- **e** Another way of showing that you have the correct expression for the length of this rectangle is to use substitution. Try this method with $x = 3$.

22 Write an expression for the missing side length for each rectangle.

a

area of photo is $(8x + 16)$ cm^2

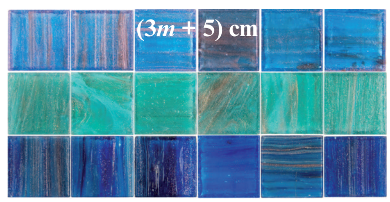
b

area of oriental rug is $(10x + 14)$ m^2

c

area of television screen is $(k^2 + 20k)$ cm^2

d



area of glass panel is $(6m^2 + 10m)$ cm^2

23 Demonstrate you have obtained the correct expressions in question **22**.

CHALLENGE

24 Write a pair of terms whose HCF is:

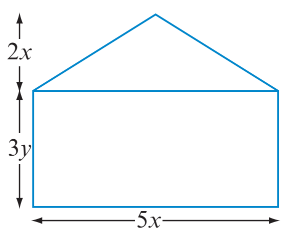
- **a** $2x$
- **b** $-3ab$.

25 A rectangle has an area of $8x - x^2$.

- **a** Write this in factor form.
- **b** List the expressions for the length and width of the rectangle.
- **c** Suggest values for the length and width of three different rectangles that would fit these expressions.

26 Write an expression for the area of this shape in:

- **a** expanded form
- **b** factor form.



Reflect

What is the difference between expanding and factorising algebraic expressions?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

pronumeral	expression	like terms	factor form
variable	equation	simplify	expand
algebraic term	formula	index laws	factorise
coefficient	substitution	distributive law	
constant term	evaluate	expanded form	

MULTIPLE-CHOICE

4A **1** How many terms does $2x^2 - 3xy + 4y^2 - 1$ have?

A 2 **B** 3 **C** 4 **D** 5

4A **2** What is the coefficient of $-6a^2b$?

A a **B** 6 **C** -6 **D** a^2b

4B **3** What is the result of substituting $x = 4$ and $y = -3$ into $2xy - 9y$?

A 3 **B** -3 **C** 51 **D** -51

4C **4** Which list shows a set of like terms?

A $3a, ab, -5a$ **B** $4mn, mn, 2nm$
C $-8p, 6p^2, 5p^3$ **D** $2cd, de, -7ce$

4C **5** Simplify $7m + 2n - 5m - 2n$.

A $2m$ **B** $12m + 4n$
C $2mn$ **D** $2m + 4n$

4C **6** What does $3x^2 - 2x^2$ simplify to?

A $5x^2$ **B** 1 **C** $-x^2$ **D** x^2

4D **6** What is the expanded form of $-5a^2bc$?

A $5 \times a \times b \times c$
B $-5 \times a \times a \times b \times c$
C $5 \times a \times a \times b \times c$
D $-5 \times a \times b \times c$

4D **8** Which of these is *not* equivalent to $24a^6b^3c^5$?

A $2a^2b^2 \times 3a^2c^3 \times 4a^2bc^2$
B $3a^4b \times 8b^2c \times a^2c^4$
C $3a^4c^5 \times 4a^2b \times 2ab^2$
D $bc^4 \times 6a^3 \times 4a^3b^2c$

4E **9** What does $\frac{4ab^2}{8ab}$ simplify to?

A $2b$ **B** $\frac{1}{2b}$ **C** $\frac{2}{b}$ **D** $\frac{b}{2}$

4F **10** What is $5m(3m + 2n - 6)$ in expanded form?

A $15m^2 + 10mn - 30m$
B $8m^2 + 7mn - 11m$
C $15m^2 + 10mn + 30m$
D $8m^2 + 10mn - 6$

4F **11** Which expression simplifies to $12mn + 5m$?

A $m(3n - 4) + 2m(5n + 4)$
B $4n(2m - 3) + 2m(2n + 4)$
C $m(2n - 3) + 2m(5n + 4)$
D $3m(3n - 3) + m(3n + 4)$

4G **12** What is the HCF of $24ab^2c$ and $18bc$?

A $2bc$ **B** $6c$ **C** $6bc$ **D** $8ab$

SHORT ANSWER

4A 1 Write an expression for each statement. Use m for the unknown number.

a a number is multiplied by 4 and then 3 is added

b a number is divided by 5 and then 8 is subtracted

c add 3 to a number and then divide the result by 7

d subtract 6 from a number and multiply the result by 2

4A 2 Zoo entry is \$25 per adult, \$13 per child and \$20 per pensioner. Write an expression for each of these (in dollars).

a the cost of entry for x adults

b the cost of entry for y children

c the cost of entry for p pensioners

d the total cost of entry for x adults, y children and p pensioners

4B 3 Evaluate each expression by substituting $a = 2$, $b = 5$ and $c = -3$.

a $7a + 2b$ b $5a^2 + 4c$

c $6(3ab - 2)$ d $\frac{b^2 - a^2}{c}$

4B 4 The number of days in a given number of weeks can be written as a formula.

a Choose a pronumeral to represent the number of days and another for the number of weeks.

b Write a formula using the pronumerals for the relationship between the two variables.

c Use your formula to find the number of days in:

i 13 weeks ii 29 weeks

iii 145 weeks iv 7.5 weeks.

4C 5 Simplify:

a $6a + 7a$ b $4b - b$

c $3c - 8c$ d $5de + de + 9de$

e $7g + 2g + 4h$ f $3x^2 + 8x + 2x^2$

4C 6 Simplify:

a $4m + 5n + 8m + 3n$

b $2k^2 + 6 + 9k^2 + 1$

c $3cd + 2c + 5dc + 4$

d $7x + 5y - 2x + y + 3x - 2y$

e $5ab^2 - 2a^2b + 4a^2b - 3ab^2$

f $6pq + 7p + 8q - p^2 - 5q$

4D 7 Simplify:

a $5d \times 4a$ b $3ab \times 8bc$

c $-6gk \times 2gh$ d $7ef \times bf \times 9cf$

4D 8 Simplify:

a $x^6 \times x^8$ b $4a^5 \times 7a^4$

c $h^7 \times 5h \times 2h^3$ d $2m^5n^8 \times 3m^7n^4$

4E 9 Simplify:

a $\frac{15a}{20}$ b $\frac{6cd}{2d}$

c $\frac{4xy}{x}$ d $\frac{6abcd}{10bc}$

e $\frac{18m^2n}{12mn}$ f $\frac{2e^2hk^2}{5ek}$

4E 10 Simplify:

a $k^{11} \div k^7$ b $8c^9 \div c$

c $\frac{6a^8b}{a^5}$ d $\frac{16x^{13}y^6}{28x^6y}$

e $\frac{3w^6 \times 2w^3}{4w^7}$ f $\frac{2t^4 \times 4t^5}{8t^9}$

4F 11 Expand:

a $5(a + 3)$ b $4(7b - 2)$

c $c(3d + 10a)$ d $-6m(4m + 1)$

4F 12 Expand and simplify:

a $3(k + 7) + 5(3k - 2)$

b $2y(y - 6) - (y + 4)$

4G 13 Factorise:

a $14h - 18$ b $20ab + 10ac$

c $6a - 4b$ d $8c^2 + 12c$

e $-5ef - 5f$ f $-kmn + 7np$

g $4a^2b - 14ab$ h $xy^2 + wx^2y$

NAPLAN-STYLE PRACTICE

1 What is the coefficient of $-8mnp$?

2 Caitlin buys x pens and y pencils. A pen costs \$2 and a pencil costs \$1. Which expression represents the total cost in dollars?

- $x + y$
- $2xy$
- $x + 2y$
- $2x + y$

3 Which of these is equivalent to $5ab$?

- $5 \times a \times b$
- $5 + a + b$
- $5 \times a + b$
- $5 + a \times b$

4 If $a = 4$ and $b = -3$, what is $7a + 2b$?

- -13
- 22
- 34
- 97

5 Which of these is equivalent to $-3m^2n$?

- $3 \times m \times n$
- $-3 \times m \times n$
- $3 \times m \times m \times n$
- $-3 \times m \times m \times n$

6 Simplify $5x^2 - 3x + x^2 + 2x - 4$.

7 What is the product of $4ab$ and $3bc$?

- $7abc$
- $12ab^2c$
- $7ab^2c$
- $12abc$

8 Which of these is equivalent to $h^4 \times h \times h^7$?

- h^3
- h^{11}
- h^{12}
- h^{28}

9 Which of these shows $\frac{24ef}{20e}$ in simplest form?

- $\frac{12f}{10}$
- $\frac{6f}{5}$
- $\frac{12ef}{10e}$
- $\frac{6ef}{5e}$

10 Write $b^8 \div b^2$ in simplest form.

11 Simplify $3mn \times 2m^2n \times n$.

12 Simplify $\frac{2x^3 \times 3x^2y^4}{8x^4y^2}$.

13 A rectangle is three times as long as it is wide. If the width is represented by w, write an expression for the area of the rectangle in terms of w.

14 Which expression is equivalent to $a(b + c)$?

- $a + (b + c)$
- $a \times b + c$
- $a \times b + a \times c$
- $a \times b \times c$

15 Which expression is equivalent to $5x(x - 2y)$?

- $5x^2 - 2y$
- $5x - 10xy$
- $5x^2 - 5xy$
- $5x^2 - 10xy$

16 If $m = 5$, $n = -2$ and $p = 3$, what is the value of $\frac{2m^2 - 5n}{4mp}$?

17 Which expression becomes $3ab + 6a$ when expanded?

- $3(ab + 2)$
- $3a(b + 2)$
- $3b(a + 2)$
- $3a(b + 6)$

18 Which expression is equivalent to $d(6 - d) + 2d(3d - 1)$?

- $8d$
- $4d + 5d^2$
- $8d + 7d^2$
- $4d - 7d^2$

19 Which expression becomes $4x^2y - 6xy^3$ when expanded?

- $2xy(2x - 3y^2)$
- $2xy(2x - 6y^2)$
- $-xy(4x - 6y^2)$
- $xy(4x + 6y)$

20 Which expression does *not* simplify to $2x - 3y$?

- $\frac{8x^2 - 12xy}{4x}$
- $5x + 3x(y - 1) - 3y(x + 1)$
- $5(x - y) + 3(2y - x)$
- $2(x - 3y) + 3y$

21 Which expression is equivalent to $4p^2 - 10pq$?

- $2p(2p - 5q)$
- $4p(p - 10q)$
- $2(2p - 5pq)$
- $2p^2(2 - 5q)$

22 Which expression is equivalent to $4(m + 3n) - (5m - 2n)$?

- $5n - m$
- $10n - m$
- $12n - m$
- $14n - m$

23 The area of this rectangle is represented by $c^2 + 5c$. What expression can be used to represent the unknown side length?

Questions **24** and **25** refer to a rug with an area given by $(6x^2 + 12x)$ m^2.

24 Which are possible measurements for the rug?

- length $(3x + 4)$ m, width $(2x)$ m
- length $(4x + 6)$ m, width $(2x)$ m
- length $(6x + 10)$ m, width (x) m
- length $(2x + 4)$ m, width $(3x)$ m

25 If $x = 2$, what is the length, width and area of the rug?

- 8 m, 6 m, 28 m^2
- 12 m, 4 m, 48 m^2
- 8 m, 6 m, 48 m^2
- 6 m, 4 m, 24 m^2

ANALYSIS

Jared buys a group of friends some boxes of hot chips to share. He wants a way to quickly calculate the cost for each person. The cost depends on whether he buys large or small boxes.

a Jared buys three large boxes of hot chips.
   i What is the total cost?
   ii If the cost is shared between five, how much will each person pay?

b Consider when Jared buys x large boxes of chips.
   i What is the total cost?
   ii If the cost is shared between five, how much will each person pay?

c Write an expression for how much each person pays if x small boxes of chips are bought to be shared between:
   i four people **ii** five people.

d Jared considers the cost of buying the same number of large and small boxes of chips.
   i Write an expression, in simplest form, for the total cost of buying x large boxes and x small boxes of chips.
   ii What values could x represent if Jared has \$50 to spend?

e Jared now considers the cost of buying x large boxes of chips and y small boxes of chips. Write an expression for the total cost in:
   i expanded form **ii** factorised form.

f For the general case of buying x large boxes and y small boxes of chips, write an expression for the cost per person if the chips are shared between:
   i six people **ii** p people.

g Use your expression from part **f ii** to calculate the cost per person if $x = 3$, $y = 2$ and $p = 6$. Write a sentence to explain this situation.

h For a group of six people, suggest at least three sets of values for x and y so that each person does not spend more than \$5.

i Suggest at least three sets of values for x, y and p so that each person does not spend more than \$8.

CONNECT

The magnificent mind reader!

Have you ever wished to be a mind reader? Here's a mind-reading test to try on your friends and family.

You will be the mind reader and one of your friends or family will be the test subject.

Ask the subject to think of a number and to really concentrate on that number for 5 seconds.

Make a show of closing your eyes to concentrate on 'receiving' the number.

Now get your subject to follow these three steps:

1 Add 5 to the number.
2 Double the result.
3 Subtract 10.

Ask them to say out loud the final result obtained after following these steps.

You can now reveal the original number they chose. (To do this, just halve the result.)

If your mind-reading skills are doubted, get them to try it again with a different number.

Does this mind-reading test work every time? Are you actually a mind reader or is it just a trick?

You may first like to practise the trick with a classmate. Repeat it a few times with a different starting number. Does the trick work every time?

One way to investigate how this trick works is to use your skills in algebra. Since the starting number can vary, use a pronumeral to represent it. Then follow the steps using this pronumeral. Remember to perform each operation on the result obtained from the previous step. Now can you explain how the trick works?

Trick 1

Think of a number.
1 Add 5 to the number.
2 Double the result.
3 Subtract 10.
What is the result? The result will be double the original number.

Trick 2

Think of a number.
1 Double the number.
2 Add the original number.
3 Divide by 3.
What is the result?

Trick 3

Think of a number.
1 Add 10 to the number.
2 Multiply by 10.
3 Subtract 100.
4 Divide by 10.
What is the result?

Your task

To investigate these mind-reading tricks, follow these steps:

- investigate each trick using algebra
- explain how you think each trick works
- create some of your own tricks
- demonstrate that each trick will work every time.

Include all necessary working to justify your answers.

Trick 4

Think of a number.

1 Add 3 to the number.
2 Multiply by 2.
3 Subtract twice the original number.

The result will be 6.

Trick 6

Think of a number.

1 Multiply the number by 3.
2 Add 6.
3 Double it.
4 Divide by 6.
5 Subtract the original number.

The result will be 2.

Trick 5

Think of a number.

1 Multiply the number by 10.
2 Add 10.
3 Divide by 10.
4 Subtract 1.

What is the result?

Trick 7

This mind-reading trick allows you to discover when a person's birthday is.

Ask the person to think of numbers to represent the month and day of their birthday. For example, if their birthday is June 17, then the month is 6 and the day is 17.

1 Multiply the month number by 10.
2 Add 20.
3 Multiply by 10.
4 Subtract 100.
5 Add the day number.
6 Subtract 100.

Ask the person to say out loud the result obtained.

The last two digits give the day and the other digits give the month of the birthday.

Complete the 4 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a story about a magnificent mind reader
- a demonstration of your mind-reading skills
- a short presentation to the class of one of your new tricks and how it works
- other (check with your teacher).