

6

The normal distribution

This chapter deals with the application of the properties of the standard normal distribution to the solution of real problems.

The main mathematical ideas investigated are:

- ▶ determining standardised scores (z-scores)
- ▶ using z-scores to compare data sets
- ▶ drawing frequency graphs
- ▶ identifying normal distributions
- ▶ using the properties of normally distributed data.

6A

Standardised score

An important consideration when analysing sets of data is the relative significance of each score. For example, suppose Kim scores 40 marks in a Mathematics test. If the mark is out of a total of 50 then it is likely that it is a good mark; however, if it is out of 100 then it is probably a poor mark.

These conclusions are by no means certain. If the test was out of 50 and most students in the class scored more than 40 out of 50 then it is a relatively poor mark. Similarly, if the test was out of 100 and most students scored less than 40 then it is a relatively good mark: it is the relative value of a score in a data set that indicates its significance.

There are a number of ways in which the relative significance of a score can be indicated. The rank position of the score, or the decile or quartile in which the score lies could be given. A convenient method, which is also useful for comparing scores from different data sets, is to indicate the distance of the score from the mean of the data in terms of the standard deviation of the data. For example, suppose the mean score in the Mathematics test above was 56 and the standard deviation was 8.

The difference between Kim's score and the mean is 16, which is two times the standard deviation; that is, the distance of Kim's score from the mean is twice that of the 'typical' distance of all the scores from the mean.

Further, as Kim's mark is below the mean, the relative significance of her mark could be given as a score -2 .

This is known as the **standardised score** or **z-score** for Kim's mark on this test.

Remember, the standard deviation is the 'typical' distance of all the scores from the mean.

$$\text{Standardised score} = \frac{\text{raw score} - \text{mean}}{\text{standard deviation}}$$

$$\text{or} \quad z = \frac{x - \bar{x}}{s}$$

where s is the standard deviation ($s = \sigma_n$ for population standard deviation).

Standardised scores from different tests can be compared to find the better relative performance, or added to find the better total performance.

WORKED EXAMPLE 1

In an English examination the mean was 60 and the standard deviation was 10.

a Calculate the z-score (standardised scores) for marks of:

i 50

ii 58

iii 60

iv 64

v 83

b Explain the meaning of these scores.

	Solve	Think	Apply
a i	$z = \frac{50 - 60}{10} = -1$	Subtract the mean of 60 from the score. Divide the answer by the standard deviation of 10.	If the score is less than the mean, the z-score will be negative. If the score is greater than the mean, the z-score will be positive.
ii	$z = \frac{58 - 60}{10} = -0.2$		
iii	$z = \frac{60 - 60}{10} = 0$		
iv	$z = \frac{64 - 60}{10} = 0.4$		
v	$z = \frac{83 - 60}{10} = 2.3$		

WORKED EXAMPLE 1 CONTINUED

	Solve/Think	Apply
b i	$z\text{-score} = -1$. This mark is 1 standard deviation below the mean.	Positive z -scores are above the mean, negative z -scores are below the mean. A z -score of 0 is equal to the mean. The larger the magnitude of the z -score, the greater the distance from the mean.
ii	$z\text{-score} = -0.2$. This mark is 0.2 standard deviations below the mean.	
iii	$z\text{-score} = 0$. This mark is equal to the mean.	
iv	$z\text{-score} = 0.4$. This mark is 0.4 standard deviations above the mean.	
v	$z\text{-score} = 2.3$. This mark is 2.3 standard deviations above the mean.	

EXERCISE 6A

- 1** In a Science test the mean was 63 and the standard deviation 10.
- a** Calculate the standardised scores for these marks.
- | | | | |
|-------------|--------------|---------------|----------------|
| i 43 | ii 53 | iii 60 | iv 63 |
| v 69 | vi 73 | vii 78 | viii 85 |
- b** Explain the meaning of these scores.

- 2** In a History examination the mean was 56 and the standard deviation 8. Complete the table, which shows the marks scored by some students.

Student	Mark	Standardised score
Julie	65	
Imran	48	
Sevvy	72	
Brad	52	
Katrina	71	
Sam	40	
Megumi	56	
Sasha	38	

WORKED EXAMPLE 2

Write down the z -score corresponding to a raw score that is:

- a** 0.8 standard deviations above the mean
b equal to the mean
c 1.3 standard deviations below the mean.

	Solve	Think	Apply
a	0.8	Above the mean is positive.	The sign of the z -score is very important as it describes whether the score is above (+) or below (-) the mean.
b	0	Equal to the mean is zero.	
c	-1.3	Below the mean is negative.	

3 Write the standardised score corresponding to a score that is:

- | | |
|---|--|
| a 1 standard deviation above the mean | b 1 standard deviation below the mean |
| c 2 standard deviations above the mean | d 2 standard deviations below the mean |
| e 3 standard deviations above the mean | f 3 standard deviations below the mean |
| g equal to the mean | h 1.5 standard deviations above the mean |
| i 0.8 standard deviations below the mean | j 2.6 standard deviations below the mean. |

WORKED EXAMPLE 3

Harry scored 55 in an English test for which the mean was 50 and the standard deviation was 6. He scored 64 in a Mathematics test for which the mean was 59 and the standard deviation 9.

- a** Calculate his standardised score for each subject.
b In which subject did he perform better, given that the classes are of equal ability?
c What mark would Harry have had to score in Mathematics for his performance to be equivalent to that for English?

	Solve	Think	Apply
a	<p><i>English</i></p> $z = \frac{55 - 50}{6} = 0.8 \text{ (1 decimal place)}$ <p><i>Mathematics</i></p> $z = \frac{64 - 59}{9} = 0.6 \text{ (1 decimal place)}$	Scores for both subjects are above the mean, so both are positive.	The z-score allows comparisons between results with different means and standard deviations. The number of students in each class does not have to be the same.
b	Harry performed better in English.	<p>Harry's score for English was 0.8 standard deviations above the mean.</p> <p>His score for Mathematics was 0.6 standard deviations above the mean.</p> <p>He performed slightly better in English than in Mathematics.</p>	
c	<p>Mathematics = $59 + 0.8 \times 9 = 66$</p> <p>or $\frac{M - 59}{9} = 0.8$</p> $M - 59 = 7.2$ $M = 66.2$ <p>He would have had to score 66.</p>	Harry's score would have to be 0.8 standard deviations above the mean for Mathematics.	

4 In an examination the mean was 65 and the standard deviation 10. Write the raw examination mark corresponding to these standardised scores:

- | | | | | | |
|------------|--------------|---------------|--------------|---------------|--------------|
| a 1 | b -1 | c 2 | d -2 | e 3 | f -3 |
| g 0 | h 1.5 | i -1.5 | j 0.8 | k -2.3 | l 2.7 |

5 The following are marks scored in a Geography test:

57, 58, 56, 47, 62, 74, 60, 55, 33, 85, 63, 71, 58, 40, 55

- a** Calculate the mean and standard deviation of the marks in this test.
b Calculate the z-score for each mark.
c How many of the marks are within 1 standard deviation of the mean? (That is, how many had a z-score between -1 and 1?)
d What percentage of the marks lie within 1 standard deviation of the mean?
e What percentage of the marks are within 2 standard deviations of the mean?
f What percentage of the marks are more than 2 standard deviations from the mean?

- 6** Jenny scored 62 in a History test for which the mean was 52 and the standard deviation was 8. She scored 67 in a Geography test for which the mean was 55 and the standard deviation 12.
- Calculate her standardised score in each subject.
 - In which subject did she perform better, given that the classes are of equal ability?
 - What mark would Jenny have to have scored in Geography for her performance to be equivalent to that for History?

- 7 a** Complete the following table, which shows Hassan's marks in the yearly examinations.

Subject	Mark	Mean	Standard deviation	z-score
English	51	55	5	
Mathematics	64	60	12	
Biology	62	58	8	
Art	76	56	9	
Economics	58	58	14	
History	51	60	11	

- In which subject did he perform best?
 - In which subject was his relative performance the worst?
 - List all his subjects in order of relative performance.
- 8 a** The table shows Melissa's marks in four Mathematics tests. In which term was her relative performance best?

Test	Mark	Mean	Standard deviation
Term 1	74	60	12
Term 2	71	58	10
Term 3	78	61	15
Term 4	72	60	8

- b** Did her performance improve from:
- term 1 to term 2?
 - term 2 to term 3?
 - term 3 to term 4?
- 9 a** The marks of five students in the half-yearly Physics examination for which the mean was 60 and standard deviation 16 are shown in the table. Complete the table.

Student	Mark	z-score
Phil	73	
Evie	41	
Tina	84	
Han Jin	54	
Kyla	62	

- b** The marks of the same five students in the yearly examination for which the mean was 58 and the standard deviation 12 are shown in this table. Complete this table.

Student	Mark	z-score
Phil	69	
Evie	40	
Tina	80	
Han Jin	59	
Kyla	52	

- c Which student(s) improved their relative performance from the first to the second examination?
- d Which student improved the most?
- e Which students' performances decreased in the second examination?
- f Which student's performance decreased the most?
- g Which student's performance was the most consistent over the two examinations?



WORKED EXAMPLE 4

In the yearly examination Kylie scored 60 in Mathematics and 71 in Science. Elena scored 68 in Mathematics and 63 in Science. If the mean and standard deviation for Mathematics were 50 and 8 respectively and 58 and 10 for Science, which student had the better total performance in these subjects?

Solve						Apply
Student	Mathematics		Science		Total	
	Mark	z-score	Mark	z-score	Mark	z-score
Kylie	60	1.25	71	1.3	131	2.55
Elena	68	2.25	63	0.5	131	2.75

Think

By completing the above table, we see that both students had the same total number of raw marks. However, by totalling the standardised scores for each subject, we can conclude that Elena's total performance was better than Kylie's.

z-scores can be added to obtain an overall result. The higher the total z-score, the better the result.

- 10 In an examination, Brian scored 65 in English and 71 in Mathematics. Trevor scored 72 in English and 64 in Mathematics. If the mean and standard deviation for English were 60 and 6 respectively and 65 and 8 for Mathematics, which student had the better total performance in these subjects?
- 11 The marks scored by five students in English, Mathematics, Science and Art are shown in the table below. Place the students in order of merit on the basis of their performance in all subjects.

Student	English	Mathematics	Science	Art
Melanie	74	71	84	52
Josh	67	70	75	76
Ming	53	85	83	64
Vera	76	67	64	78
Jake	71	70	71	69
Mean	64	66	59	64
Standard deviation	10	15	12	13

6B

Frequency graphs

Frequency distributions may be displayed using histograms and polygons. If instead of a frequency polygon we draw a smooth curve, we get a frequency graph whose general shape and properties can be used to make judgements about the data.

WORKED EXAMPLE 1

Draw a histogram and a frequency polygon for the data given.

Weight	Frequency
48	6
49	9
50	15
51	10
52	7
53	3

Solve	Think	Apply
	<p>Plot the points from the table. Leave a half column space from the axis. Columns are equal widths. The polygon passes through the middle of the top of each column. It joins to the horizontal axis.</p>	<p>The frequency polygon can be drawn by plotting the points and joining them, but usually both the columns and line are drawn.</p>

EXERCISE 6B

1 On the same diagram, draw a frequency histogram, polygon and curve for each set of data.

a

Height (cm)	Frequency
155	5
156	25
157	26
158	24
159	3

b

Mark	Frequency
0	1
1	2
2	4
3	5
4	8
5	10
6	7

c

Mark	Frequency
0	5
1	8
2	9
3	4
4	3
5	2
6	1

d

Shoe size	Frequency
3	4
4	5
5	9
6	6
7	7
8	9
9	5

e

Number of minutes late	Number of people
0	55
1	16
2	10
3	7
4	4
5	2

f

Age	Number of pedestrians killed
0–9	85
10–19	30
20–29	11
30–39	22
40–49	25
50–59	41
60–69	62

6C

The normal distribution

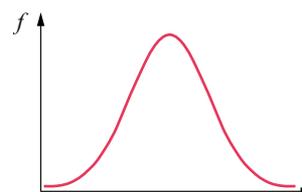
When the frequency polygon for a distribution of scores is drawn as a smooth curve, one curve in particular occurs many times, especially for large distributions of naturally occurring characteristics such as height or weight.

The shape of this particular curve is similar to the one drawn below and it is known as the **normal curve**.

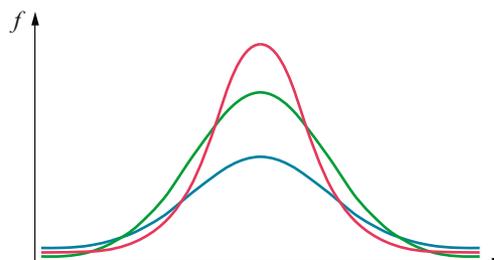
A distribution of scores that produces a curve with this shape is known as a **normal distribution**.

For a normal distribution:

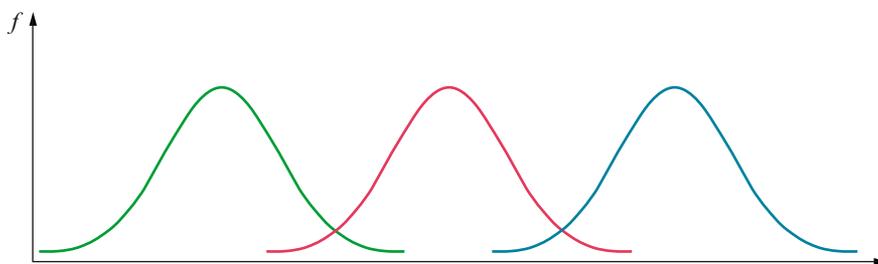
- The frequency curve is bell-shaped and symmetrical about the mean.
- The mean, median and mode are equal.



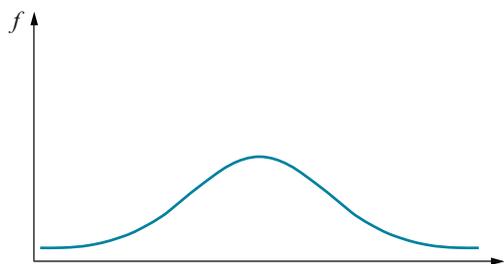
The size and shape of the bell are determined by the mean and standard deviation of the distribution. These three normal curves have the same mean but different standard deviations.



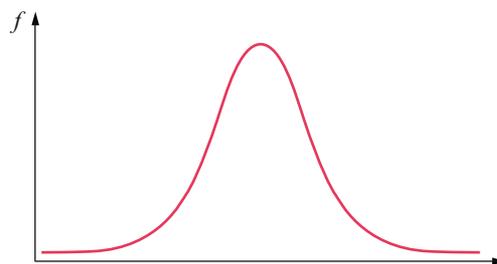
These three normal curves have the same standard deviation but different means.



If a normal distribution of scores has a large standard deviation, the scores are widely spread from the mean and the graph will be short and widely spread (below left). If the standard deviation of the scores is small, most of the scores are close to the mean and the graph will be tall and narrow (below right).



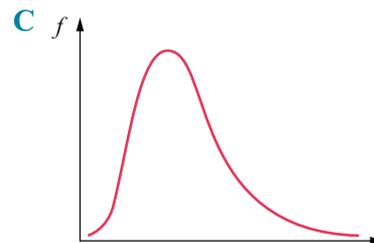
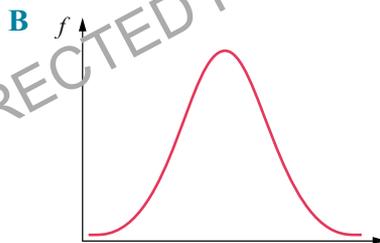
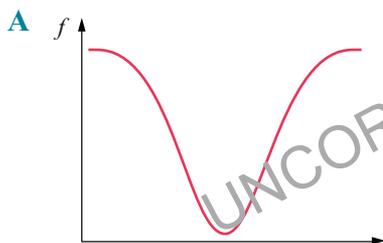
Normal distribution with large s



Normal distribution with small s

EXERCISE 6C

1 Which of the following is a normal curve?

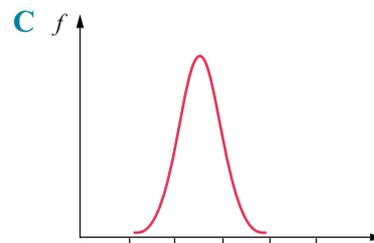
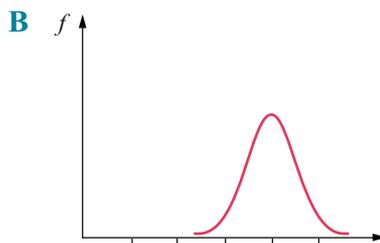
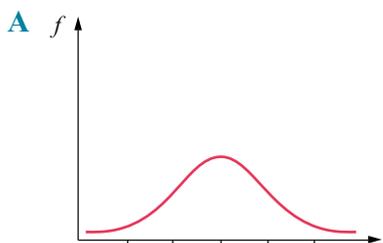


2 Draw two normal curves with:

- a the same means but different standard deviations
- b different means but the same standard deviation.

3 The normal curves below are drawn with the same scale on both axes. Which distribution has the:

- a largest standard deviation?
- b smallest standard deviation?
- c largest mean?
- d smallest mean?



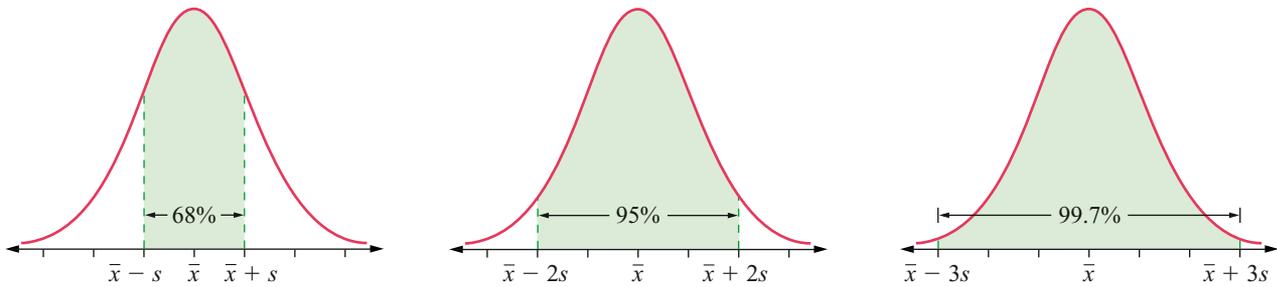
6D

Properties of the normal distribution

The normal distribution has been studied in some detail by mathematicians and it can be shown that for a normal distribution, approximately:

- 68% of the scores will lie within one standard deviation of the mean: $-1 < z\text{-score} < 1$
- 95% of the scores will lie within two standard deviations of the mean: $-2 < z\text{-score} < 2$
- 99.7% of the scores will lie within three standard deviations of the mean: $-3 < z\text{-score} < 3$

This may be illustrated as follows.



WORKED EXAMPLE 1

The marks on a test are normally distributed with a mean of 65 and a standard deviation of 8.

- a** What mark is one standard deviation:
- i** above the mean? **ii** below the mean?
- b** What proportion of students scored a mark between 57 and 73?
- c** What mark is two standard deviations:
- i** above the mean? **ii** below the mean?
- d** What proportion of students scored a mark between 49 and 81?
- e** What mark is three standard deviations:
- i** above the mean? **ii** below the mean?
- f** What proportion of students scored a mark between 41 and 89?

	Solve	Think	Apply
a	i $65 + 8 = 73$	Add and subtract the standard deviation.	One standard deviation above the mean is a z-score of +1. One standard deviation below the mean is a z-score of -1. Similarly for z-score of ± 2 , ± 3 .
	ii $65 - 8 = 57$		
b	Approximately 68% of the students scored a mark between 57 and 73.	68% of scores are between z-scores of ± 1 .	
c	i $65 + 2 \times 8 = 81$	Add and subtract 2 times standard deviation from the mean.	
	ii $65 - 2 \times 8 = 49$		
d	Approximately 95% of the students scored a mark between 49 and 81.	These are z-scores of ± 2 , $\therefore 95\%$.	
e	i $65 + 3 \times 8 = 89$	Add and subtract 3 times the standard deviation from the mean.	
	ii $65 - 3 \times 8 = 41$		
f	Approximately 99.7% of the students scored a mark between 41 and 89.	These are z-scores of ± 3 , $\therefore 99.7\%$.	

EXERCISE 6D

- 1** The marks on a test are normally distributed with a mean of 62 and a standard deviation of 5.
- What mark is one standard deviation:
 - above the mean?
 - below the mean?
 - Approximately what proportion of students scored a mark within ± 1 standard deviation?
 - What mark is two standard deviations:
 - above the mean?
 - below the mean?
 - Approximately what proportion of students scored a mark within ± 2 standard deviations?
 - What mark is three standard deviations:
 - above the mean?
 - below the mean?
 - Approximately what proportion of students scored a mark within ± 3 standard deviations?
- 2** The life of a certain brand of torch batteries is normally distributed with a mean of 56 hours and standard deviation of 2.6 hours.
- Complete the following table.

Life (hours)	48.2	50.8	53.4	56	58.6	61.2	63.8
z-score				0			

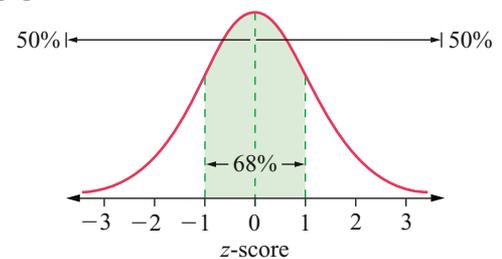
- Approximately what proportion of batteries have a life between:
 - 48.2 h and 63.8 h?
 - 50.8 h and 61.2 h?
 - 53.4 h and 58.6 h?



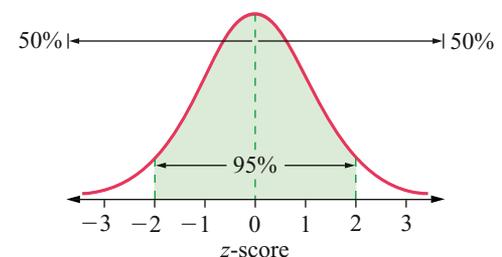
- 3** The heights of 1000 Year 12 males are approximately normally distributed with a mean of 168 cm and standard deviation of 9 cm. What percentage of students have heights between:
- 159 cm and 177 cm?
 - 150 cm and 186 cm?
 - 141 cm and 195 cm?

- 4** Use the given graphs of normal distributions to answer the following questions.

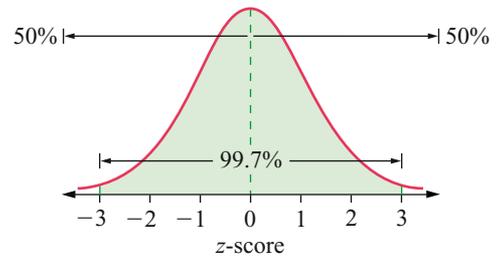
- What percentage of scores have a z-score:
 - between -1 and 1 ?
 - between 0 and 1 ?
 - between 0 and -1 ?
 - greater than 0 ?
 - less than 0 ?
 - greater than 1 ?
 - less than -1 ?
 - greater than -1 ?
 - less than 1 ?



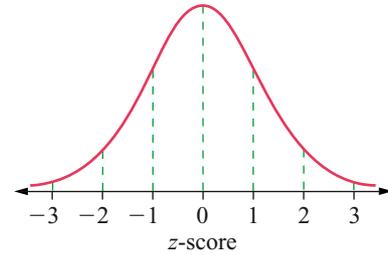
- What percentage of scores have a z-score:
 - between -2 and 2 ?
 - between 0 and 2 ?
 - between 0 and -2 ?
 - greater than 2 ?
 - less than -2 ?
 - greater than -2 ?
 - less than 2 ?



- c** What percentage of scores have a z-score:
- i** between -3 and 3 **ii** between 0 and 3 ?
 - iii** between 0 and -3 ? **iv** greater than 3 ?
 - v** less than -3 ? **vi** greater than -3 ?
 - vii** less than 3 ?



- d** What percentage of scores have a z-score between:
- i** 1 and 2? **ii** 1 and 3?
 - iii** 2 and 3? **iv** -1 and 2?
 - v** -1 and 3? **vi** -2 and 1?
 - vii** -2 and 3? **viii** -3 and 1?
 - ix** -3 and 2?



- 5** Packets of rice are filled and labelled as having a weight of 1 kg. The weight of these packets is normally distributed with a mean of 1.02 kg and a standard deviation of 0.01 kg.

a Complete the following table.

Weight (kg)	1.00	1.01	1.02	1.03	1.04
z-score			0		

- b** What percentage of packets will have a weight less than 1.02 kg?
- c** What percentage of packets will have a weight between 1.01 kg and 1.03 kg?
- d** What percentage of packets will have a weight between 1.02 and 1.03 kg?
- e** What percentage of packets will have a weight between 1.00 kg and 1.04 kg?
- f** What percentage of packets will have a weight greater than 1.03 kg?
- g** What percentage of packets will have a weight less than the labelled weight?



- 6** Tim scored 53 in a test for which the marks were normally distributed with a mean of 65 and a standard deviation of 12. What percentage of students scored higher marks than Tim?
- 7** The average life of a particular brand of car battery is 4 years and 4 months with a standard deviation of 8 months. The manufacturer guarantees to replace any battery that does not last 3 years. If the life of these batteries is normally distributed, approximately what percentage could the manufacturer expect to replace?
- 8** The heights of a large group of female university students is found to be normally distributed with a mean of 160 cm and a standard deviation of 6 cm. Between what limits would you expect the heights of:
- a** the middle 68% of these students to lie?
 - b** the middle 95% of these students to lie?
 - c** almost all, 99.7%, of these students to lie?
- 9** A machine is set to produce metal cylinders with a mean diameter of 5.00 cm and a standard deviation of 0.02 cm.
- a** If the diameters are normally distributed, within what interval will 99.7% of the diameters lie?
 - b** A cylinder is chosen at random and its diameter measured to be 5.07 cm. Discuss what conclusions could be drawn.

- 10** A machine fills packets of lollies. The mean number of lollies in each packet is 40 with a standard deviation of 1.5 lollies. The number of lollies is normally distributed. Rory claims that he bought a packet that had 48 lollies in it. Discuss Rory's claim.



WORKED EXAMPLE 2

Given the frequency tables below, determine whether or not the distributions are normal.

a	Score	5	6	7	8	9	10	11
	Frequency	1	7	14	18	13	5	2

b	Score	0	1	2	3	4	5	6
	Frequency	3	8	15	10	16	6	2

	Solve	Think	Apply
a	<p>Draw a frequency histogram and sketch the frequency curve for this distribution. The curve appears to be symmetrical and bell-shaped, but not perfectly so.</p>	<p>Calculate the mean, mode and median for the distribution.</p> <p>Mean = 7.97 (2 decimal places)</p> <p>Mode = 8</p> <p>Median = 8</p> <p>As the mean, mode and median are all almost exactly the same, the distribution is approximately normally distributed.</p>	<p>A normal distribution is symmetrical with mean, mode and median equal.</p> <p>In this case all three measures equal 8, so it is a normal distribution.</p>
b	<p>Draw a frequency histogram and sketch the frequency curve for this distribution.</p>	<p>The graph is not bell-shaped so the distribution is not normal. (There is no need to find the mean, mode and median.)</p>	<p>This distribution is bimodal and not a normal distribution.</p>

11 Given the frequency distributions below, determine whether or not each is normally distributed by:

- i** first drawing a frequency graph
- ii** then, if necessary, find the mean, mode and median.

a

Score	4	5	7	6	8	9	10
Frequency	7	9	21	14	20	2	1

b

Score	8	9	10	11	12	13	14
Frequency	2	9	14	25	16	7	2

c

Score	30	31	32	33	34	34
Frequency	10	28	6	4	2	1

d

Score	24	25	26	27	28	29	30	31
Frequency	3	12	18	31	30	19	13	1

12 The number of peas in each pod of a large sample of a particular species of pea is shown in the table below.

Number of peas/pod	Number of pods
2	10
3	16
4	38
5	71
6	80
7	73
8	35
9	19
10	8

- a** Determine whether or not the distribution is normal.
- b** Find the standard deviation.
- c** What percentage of the pods in this sample lie within:
 - i** one standard deviation of the mean?
 - ii** two standard deviations of the mean?
 - iii** three standard deviations of the mean?



REVIEW 6 THE NORMAL DISTRIBUTION

Language and terminology

Here is a list of terms used in this chapter. Explain each term in a sentence.

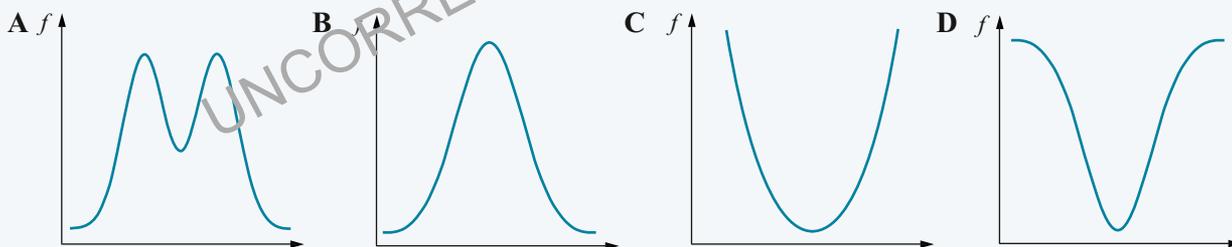
frequency curve, normal curve, normal distribution, standardised score, z-score

Having completed this chapter you should be able to:

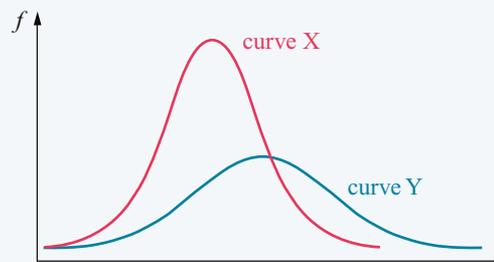
- determine z-scores
- use z-scores to compare different data sets
- draw frequency graphs
- make judgements about the nature of data based on the shape of the display
- identify and use the properties of normally distributed data.

6 REVIEW TEST

- 1 The z-score for a mark of 56 in a test with a mean of 60 and standard deviation of 8 is:
A 4 B -4 C 0.5 D -0.5
- 2 In an examination the mean was 65 and the standard deviation 15. The raw examination mark corresponding to a standardised score of 1.4 is:
A 86 B 79 C 80 D 66.4
- 3 Which of the graphs below could represent a normal distribution?



- 4 Compare the curves shown. Which statement is correct?
A X has a larger mean and larger standard deviation than Y.
B X has a larger mean and smaller standard deviation than Y.
C X has a smaller mean and larger standard deviation than Y.
D X has a smaller mean and smaller standard deviation than Y.



- 5 The marks on a test are normally distributed with a mean of 55 and a standard deviation of 9. The proportion of scores that lie between 37 and 73 is approximately:
A 68% B 86% C 95% D 99.7%

- 6 The number of matches in a box is approximately normally distributed with a mean of 50 and a standard deviation of 1. Approximately 95% of boxes will have between:
- A 49 and 51 matches B 48 and 52 matches
C 47 and 53 matches D 46 and 54 matches
- 7 For a normal distribution, the percentage of scores between $z = -2$ and $z = -1$ is:
- A 27 B 34 C 13.5 D 2.35
- 8 Which of the following distributions could be normal?

A

x	7	8	9	10	11	12	13
f	2	5	10	18	12	6	1

B

x	2	3	4	5	6	7	8
f	25	18	12	9	11	19	24

C

x	31	32	33	34	35	36	37
f	19	28	16	8	4	2	1

D

x	24	25	26	27	28	29	30
f	5	7	10	13	15	26	38

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1, 2	3, 4	5–8
Section	A	C	D

6A REVIEW SET

- 1 In an Art test the mean was 68 and the standard deviation was 8. Calculate the standardised score for a mark of:
- a 60 b 76 c 48 d 80
- 2 Skye scored 48 in a Geography test for which the mean was 58 and the standard deviation 4. She scored 49 in a History test, for which the mean was 61 and the standard deviation 8.
- a In which subject did she perform better, given that the classes are of equal ability?
b What mark would Skye have to have scored in Geography for her performance to be equivalent to that in History?
- 3 The following table shows Ben's marks in the term tests. In which test did he perform the best?

Test	Mark	Mean	Standard deviation
Term 1	65	55	8
Term 2	69	60	6
Term 3	66	58	10
Term 4	65	62	4

- 4 Draw a frequency histogram, polygon and curve for the height data given in the table.

Height (cm)	16	17	18	19	20	21	22
Frequency	4	9	15	21	16	10	3

- 5 Draw three normal curves with different means but the same standard deviation.
- 6 For a normal distribution, which of the following is true?
A the median $<$ the mean $<$ the mode **B** the median = the mean = the mode
C the median $>$ the mean $>$ the mode **D** none of the above
- 7 The marks on a test are normally distributed with a mean of 63 and a standard deviation of 5.
a What mark is one standard deviation:
 i above the mean? **ii** below the mean?
b What percentage of scores lie between 58 and 68?
c What percentage of scores lie between 53 and 73?
d What percentage of scores lie between 48 and 78?
- 8 For a normal distribution, what percentage of scores have a z -score:
a between $z = 0$ and $z = 1$? **b** between $z = -1$ and $z = 0$?
c greater than 0? **d** greater than 1?
- 9 **a** Determine whether the frequency distribution given below is normal by first drawing a frequency graph.
b Then, if necessary, find the mean, mode and median to confirm your answer.

x	10	11	12	13	14	15	16	17	18
f	2	6	12	19	25	18	11	6	3

6B REVIEW SET

- 1 In an examination the mean mark was 70 and the standard deviation was 10. Write down the examination mark corresponding to a z -score of:
a 2 **b** -1 **c** 1.3 **d** -2.6
- 2 Matthew scored 62 in a Science test for which the mean was 50 and the standard deviation 12. He scored 58 in an English test for which the mean was 51 and the standard deviation 8.
a In which subject did he perform better, given that the classes are of equal ability?
b What mark would Matthew have to have scored in English for his performance to be equivalent to that in Science?
- 3 Rebecca scored 72 in Mathematics and 69 in Science. Serena scored 66 in Mathematics and 75 in Science. The mean in Mathematics was 60 and the standard deviation was 8. In Science the mean was 65 and the standard deviation was 10. By completing the table below, determine who had the better overall performance.

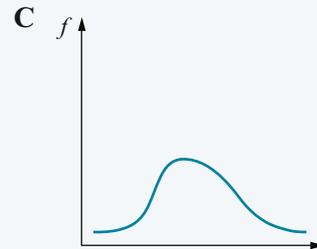
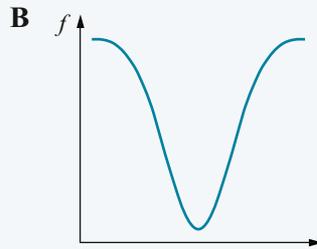
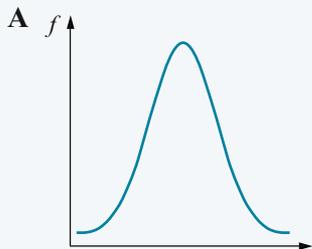
	Mathematics		Science		Total	
	Mark	z -score	Mark	z -score	Mark	z -score
Rebecca	72		69			
Serena	66		75			

- 4** Draw two normal curves with the same mean but different standard deviations.
- 5** Toothpicks are packaged in boxes with a mean of 98 and standard deviation of 4 toothpicks. What proportion of boxes will contain between:
- a** 94 and 102 toothpicks? **b** 90 and 106 toothpicks? **c** 86 and 110 toothpicks?
- 6** A machine produces bolts with a mean length of 25 mm and standard deviation 0.6 mm. Any bolts shorter than 23.2 mm or longer than 26.8 mm are rejected. What percentage of bolts made by this machine will be rejected?
- 7** The weights of a large sample of 18-year-old males is normally distributed with a mean of 74 kg and standard deviation of 6.5 kg. Between what limits would you expect the weights of:
- a** the middle 68% of these males to lie? **b** the middle 95% of these males to lie?
c almost all, 99.7%, of these males to lie?
- 8** For a normal distribution state the percentage of scores that have a z -score:
- a** between $z = -2$ and $z = 2$ **b** greater than $z = 2$ **c** less than $z = 2$
d less than $z = -2$ **e** greater than $z = -2$

6C REVIEW SET

- 1** In a Commerce test the mean was 63 and the standard deviation was 12. Calculate the standardised scores for test marks of:
- a** 81 **b** 39 **c** 66 **d** 58
- 2** The results for an examination had a mean score of 55 and a standard deviation of 10. Write the raw examination mark corresponding to a standardised score of:
- a** 0.6 **b** 1.3 **c** -0.9 **d** -2.1
- 3** The following are the scores in a Language test:
60 61 59 50 65 77 63 58 36 88 66 74, 61 43 58
- a** Calculate the mean and standard deviation of these marks.
b How many scores lie within 1 standard deviation of the mean?
c How many scores are more than 2 standard deviations above the mean?
- 4** Francis scored 52 in the half-yearly English test for which the mean was 68 and the standard deviation 11. In the yearly examination for which the mean was 60 and the standard deviation 6, she scored 56. Determine whether or not Francis improved her performance in English over the year. Justify your answer.

- 5** Which of the following is a normal curve?



- 6** The life of a brand of electric light globe is normally distributed with a mean of 1400 hours and a standard deviation of 120 hours. Within what limits would you expect:
- a** the middle 68% of globes to last? **b** the middle 95% of globes to last?
c nearly all, 99.7%, of globes to last?

- 7 a** Packets of sugar are labelled as having a weight of 500 g. The weight of these packets is normally distributed with a mean of 510 g and standard deviation of 10 g. Complete the following table.

z-score	-3	-2	-1	0	1	2	3
Weight (g)				510			

- b** What percentage of packets have a weight that lies between:
- i** 500 g and 520 g? **ii** 490 g and 530 g? **iii** 480 g and 540 g?
- c** A packet is chosen at random and its weight is 450 g. What conclusion could be drawn?
- 8 a** Determine whether the frequency distribution given below is normal by first drawing a frequency graph.
- b** Then, if necessary, find the mean, mode and median to confirm your answer.

x	7	8	9	10	11	12	13
f	21	16	8	5	9	15	19

6D REVIEW SET

- 1** In a test the mean was 64 and the standard deviation was 6. Calculate the standardised score for a mark of:
- a** 60 **b** 76 **c** 48 **d** 80
- 2** Melanie scored 67 in a Science test for which the mean was 56 and the standard deviation 11. In an English test, for which the mean was 54 and the standard deviation 8, she scored 60.
- a** In which subject did she perform better, given that the classes are of equal ability?
- b** What mark would Melanie have to have scored in English for her performance to be equivalent to that in Science?
- 3** The mean score for an examination was 50 and the standard deviation was 15. Write the raw examination mark corresponding to a standardised score of:
- a** 0.6 **b** 1.4 **c** -0.8 **d** -2.2
- 4** The marks scored by three students in tests in English, Mathematics, Science and Art are shown in the table below. List the students in order of merit on the basis of their total performance in all subjects.

Student	English	Mathematics	Science	Art
Michael	74	71	84	52
Janelle	67	70	75	76
Melissa	53	85	83	64
Mean	65	62	60	65
Standard deviation	12	15	14	12

- 5** Jelly babies are produced in monster packs with a mean of 50 and a standard deviation of 2. What proportion of packets will contain between:
- a** 48 and 52 jelly babies? **b** 46 and 54 jelly babies? **c** 44 and 56 jelly babies?
- 6** The life of a brand of torch battery is normally distributed with a mean of 42 hours and a standard deviation of 3.2 hours. Within what limits would you expect:
- a** the middle 68% of batteries to last? **b** the middle 95% of batteries to last?
- c** nearly all, 99.7%, of batteries to last?

