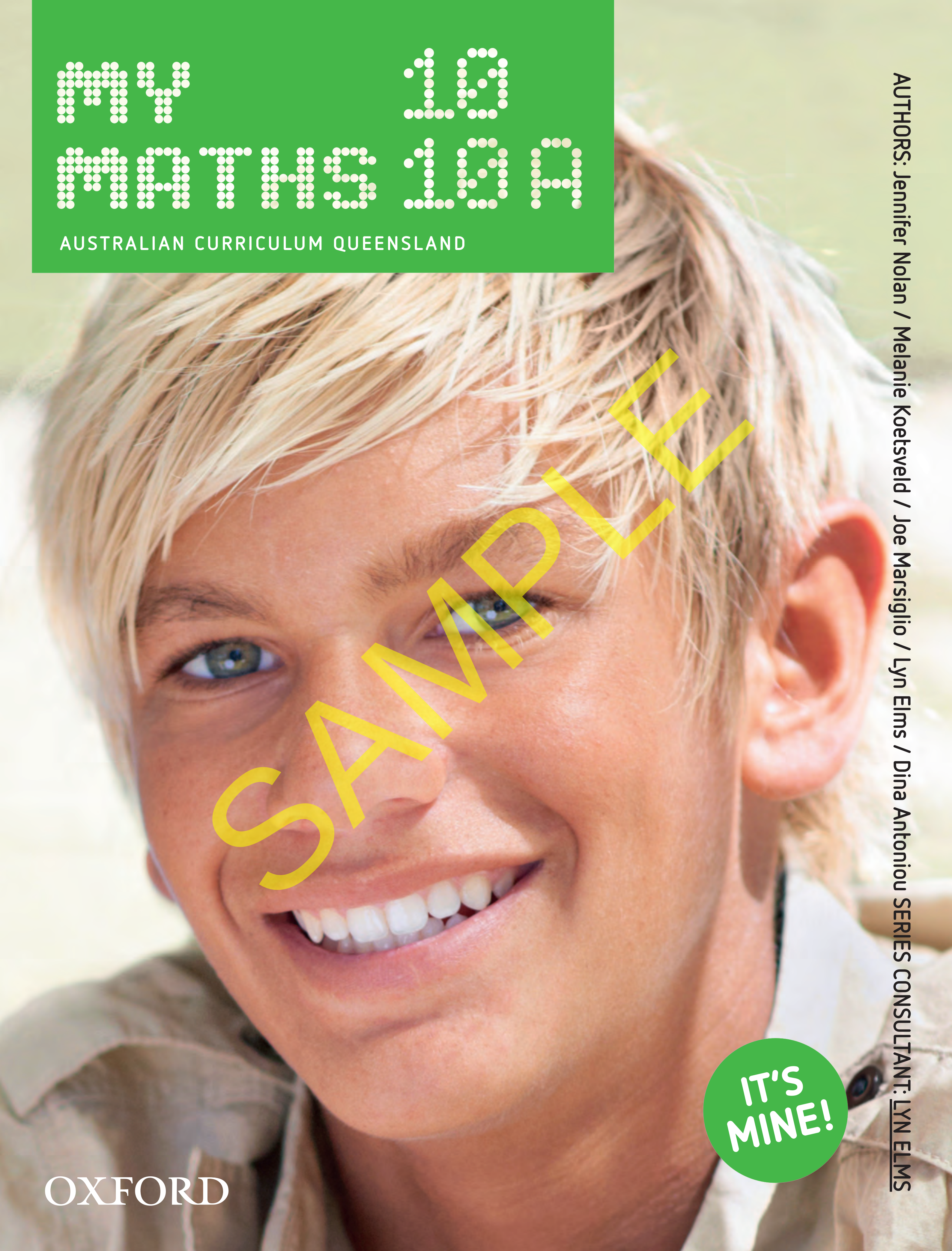
MY MATHS 10 10A

AUSTRALIAN CURRICULUM QUEENSLAND



AUTHORS: Jennifer Nolan / Melanie Koetsveld / Joe Marsiglio / Lyn Elms / Dina Antoniou SERIES CONSULTANT: LYN ELMS

IT'S MINE!

OXFORD

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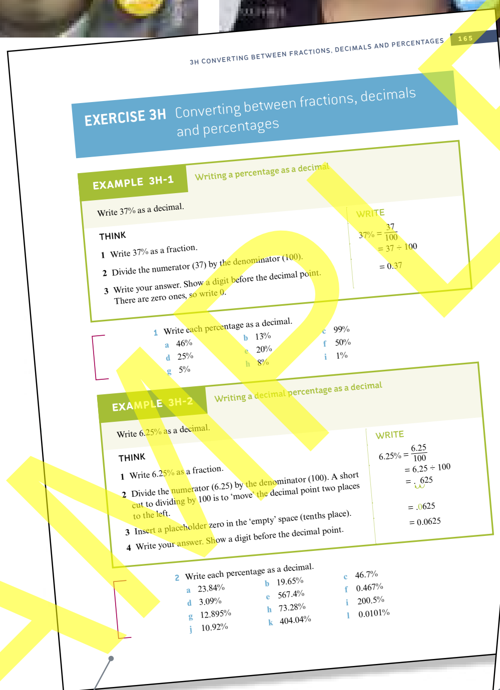
OXFORD MYMATHS FOR QUEENSLAND

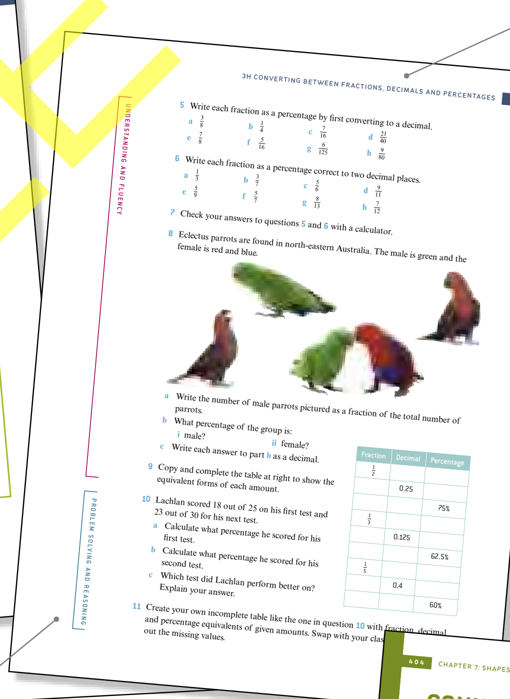
Oxford MyMaths for Queensland has been specifically developed to support students wherever and whenever learning happens: in class, at home, with teacher direction or in independent study.

STUDENT BOOK + OBOOK/ASSESS

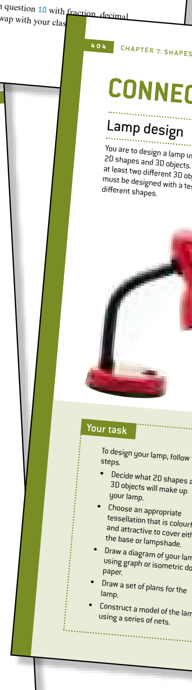
- Finely levelled exercises to ensure smooth progress
- Integrated worked examples – right where your students need them
- Learning organised around the 'big ideas' of mathematics
- Discovery, practice, thinking and problem-solving activities promote deep understanding
- A wealth of revision material to consolidate and prove learning
- Rich tasks to apply understanding
- Highly accessible and easy to navigate
- Comprehensive digital tutorials and guided examples to support independent progress



The student book accurately and carefully delivers the course.



Every question matched to the Australian Curriculum proficiencies.



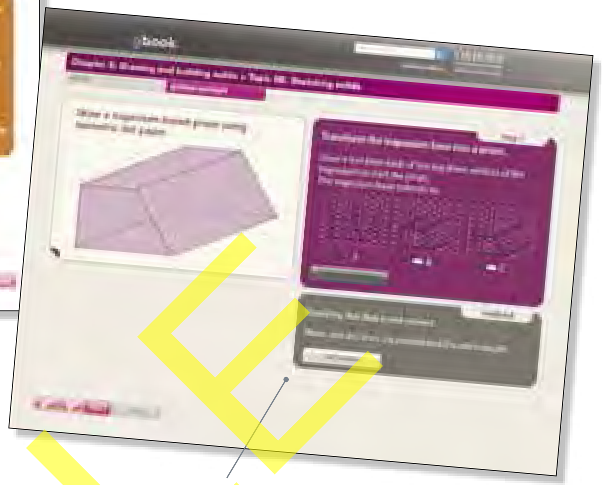
Worked examples are clearly laid out and located where students need them most.

24/7 LEARNING AND SUPPORT

E-tutors scaffold understanding of key concepts and build confidence.

Self-discovery opportunities for students through guided exploration.

Guided examples support practice and fluency



Finely levelled content enables students to progress with ease

Ample revision to consolidate understanding and prove that learning has happened

Students receive feedback for incorrect responses

Optimise understanding and performance.

Personalised learning: tailor the very best learning experiences for all.

Intervention and extension worksheets supplied for every topic.

Rich tasks where students can demonstrate understanding

TEACHER OBOOK/ASSESS

Practical classroom resources and tools:

- Manage student differentiation
- Correct common misconceptions
- Assign work
- Set tests
- Monitor results
- Any device, anytime, anywhere.

7 GEOMETRY

7A Geometry review

7B Congruence

7C Similarity

7D Understanding proofs

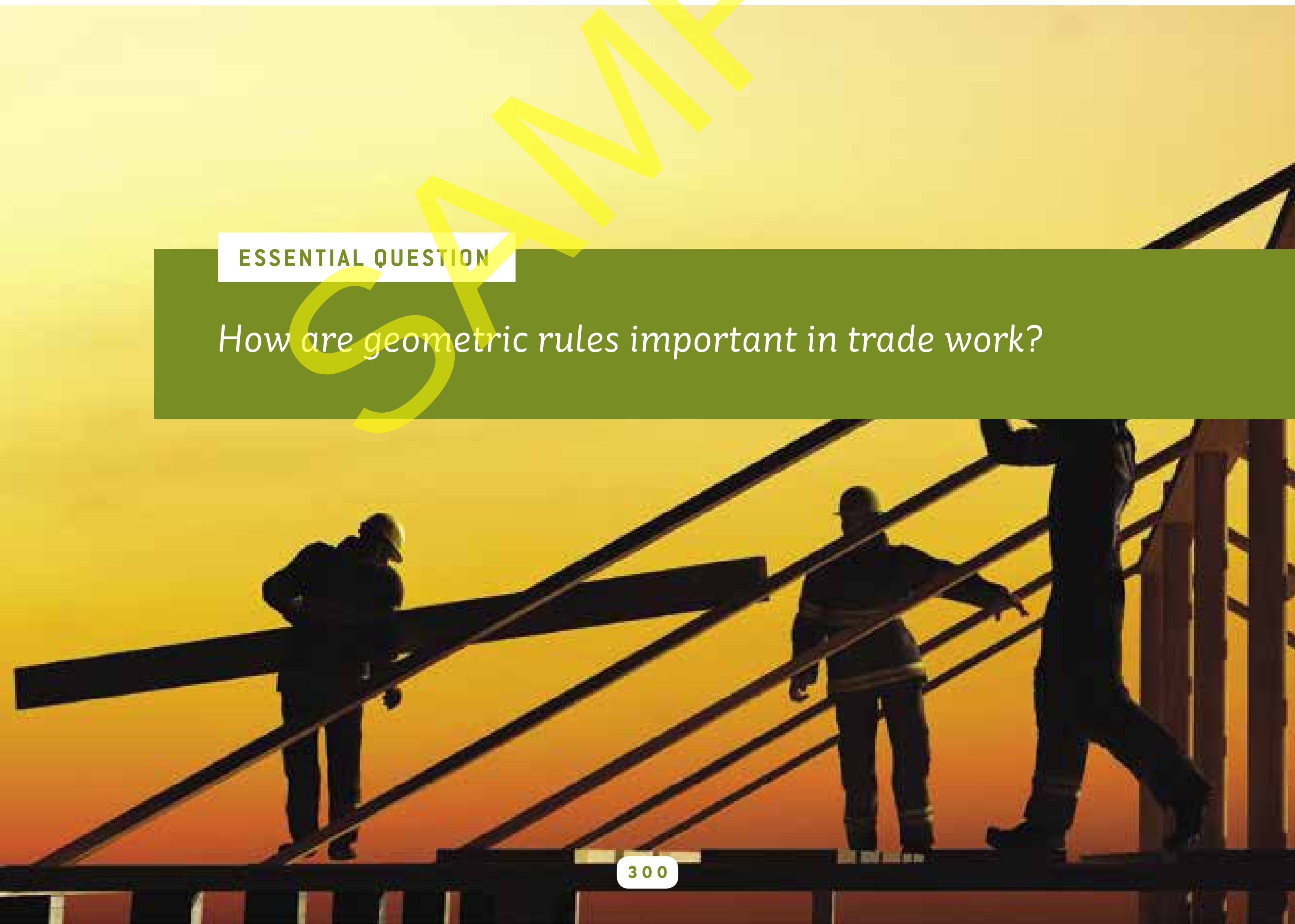
7E Proofs and triangles

7F Proofs and quadrilaterals

7G Circle geometry: circles and angles 10A

7H Circle geometry: chords 10A

7I Circle geometry: tangents and secants 10A



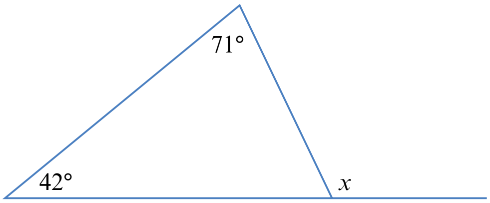
ESSENTIAL QUESTION

How are geometric rules important in trade work?

Are you ready?

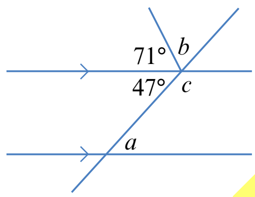
7A **1** What is the internal angle sum of a triangle?

7A **2** What is the value of x in this diagram?



A 71° B 29°
C 67° D 113°

7A **3** Consider this diagram.



a What is the value of a?

b What is the value of b?

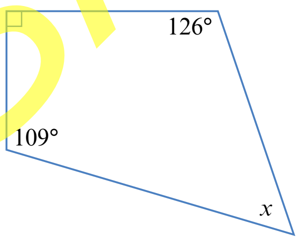
A 47° B 71°
C 62° D 118°

c What is the value of c?

7A **4** Which statement is false?

A A rhombus is a square.
B A rectangle is a parallelogram.
C A square is a rectangle.
D A rhombus is a parallelogram.

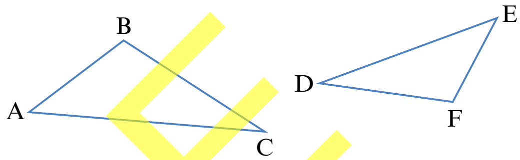
7A **5** What is the value of x in this diagram?



7B **6** Which of these is *not* a condition for congruence in triangles?

A SAS B AAA
C SSS D AAS

7C **7** Which option would allow you to calculate the scale factor for these similar figures?

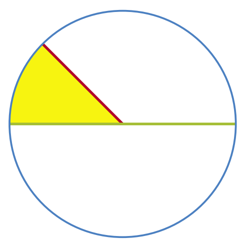


A $\frac{AB}{DE}$ B $\frac{AC}{ED}$
C $\frac{BC}{DE}$ D $\frac{AB}{DF}$

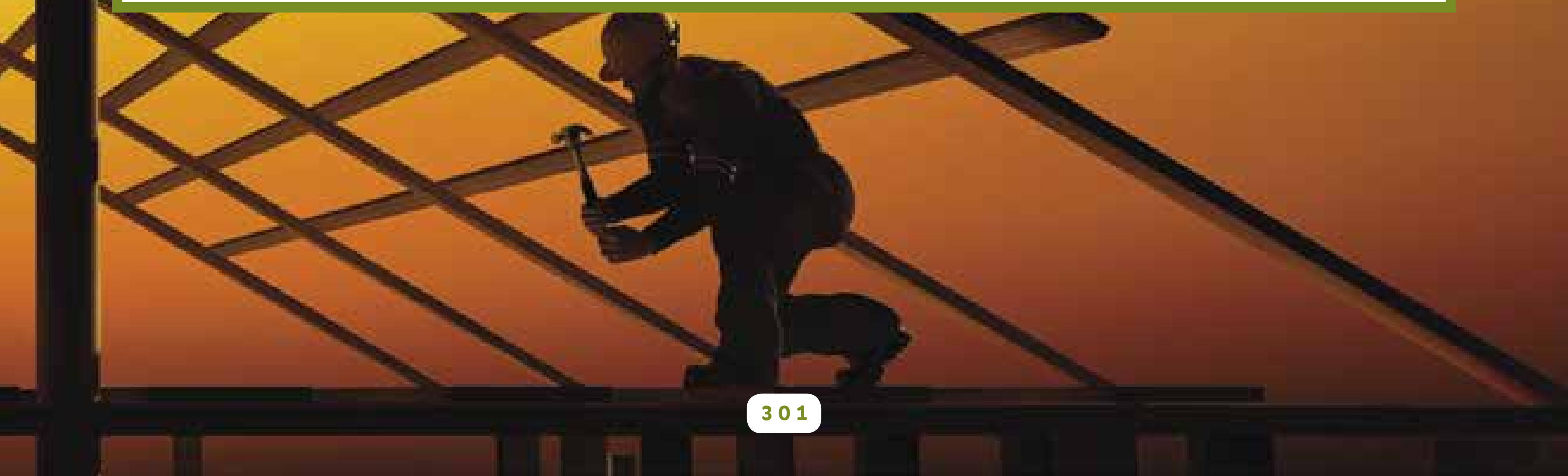
7G 10A **8** a If $4x + 10 = 24$, find the value of x.

b If $5x - 2 = 3x + 2$, find the value of x.

7G 10A **9** For this diagram of a circle, name the part shown:



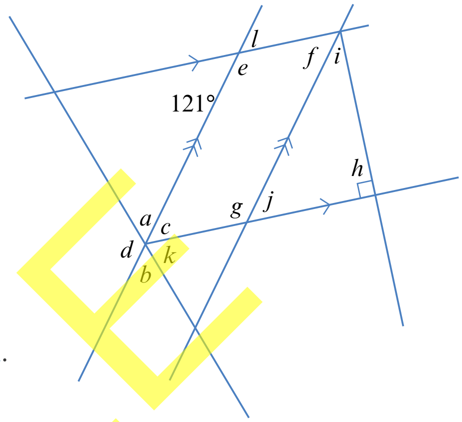
a in red
b in blue
c in green
d in yellow.



7A Geometry review

Start thinking!

Consider this diagram.



1 What is the relationship between angles:

a *a* and *b*? **b** *a* and *d*?

c *f* and *i*? **d** *c* and *g*?

e *f* and *j*? **f** *c* and *l*?

2 Make a glossary list of all angle relationships that you can see within the diagram. Be sure to include angles around a point, angles and parallel lines, angles within a triangle and a **quadrilateral**.

3 What type of angle is angle *e*? How is this different from angle *k*?

4 Make a glossary list of all the types of angles.

5 What is the angle sum of angles *h*, *i* and *j*? How do you know?

6 What type of triangle is this?

7 Make a list of the six different types of triangle and their attributes.

8 How is angle *g* related to the triangle from question 6?

9 What is the angle sum of angles *c*, *e*, *f* and *g*? How do you know?

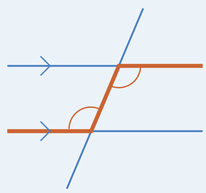
10 What type of quadrilateral is this?

11 Make a list of the six special types of quadrilateral and their attributes.

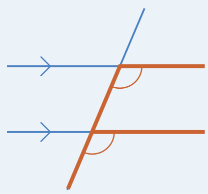
12 Explain why, if you knew the value of angles *a* and *e*, you could find the value of every other angle in the diagram, including those not labelled.

KEY IDEAS

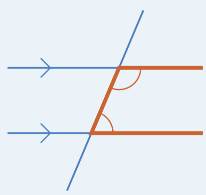
- **Complementary angles** add to 90°.
- **Supplementary angles** add to 180°.
- Angles around a point add to 360°.
- **Vertically opposite angles** are equal.
- When parallel lines are crossed by a **transversal**, a number of angles are formed, as shown at right.
- A **polygon** is a closed shape with straight sides.
- A **triangle** has an angle sum of 180°.
- A quadrilateral has an angle sum of 360°.



alternate angles (equal)



corresponding angles (equal)

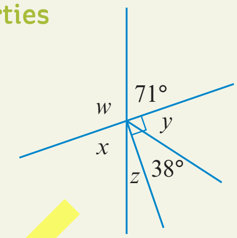


co-interior angles (supplementary)

EXERCISE 7A Geometry review

EXAMPLE 7A-1 Finding angle size using angle properties

Find the size of the labelled unknown angles in this diagram.



THINK

1 Angle w is supplementary to 71°.

2 Angle x is vertically opposite 71°.

3 Angle y is complementary to 38°.

4 Angle z is supplementary to angles 71°, y and 38°.

WRITE

$w = 180° - 71° = 109°$

$x = 71°$

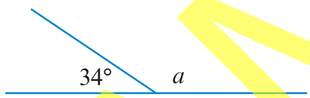
$y = 90° - 38° = 52°$

$z = 180° - 71° - 52° - 38° = 19°$

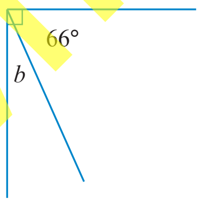
UNDERSTANDING AND FLUENCY

1 Find the size of the labelled unknown angles in these diagrams.

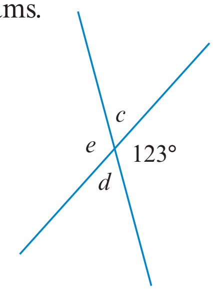
a



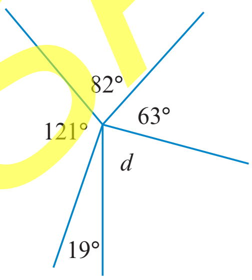
b



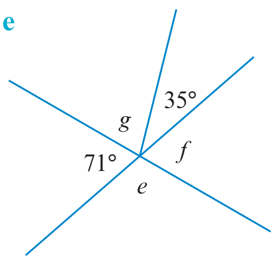
c



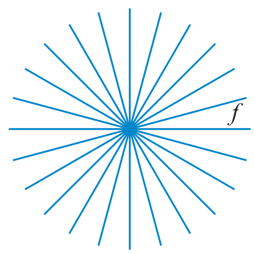
d



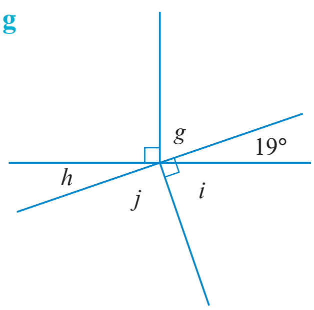
e



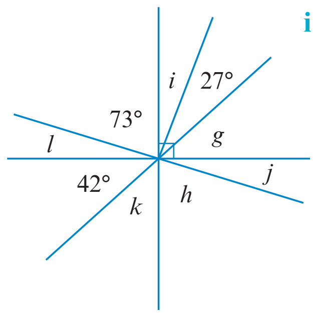
f



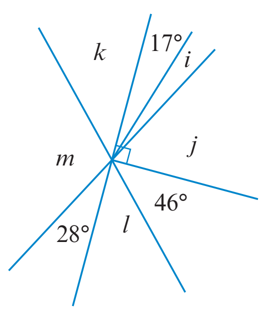
g



h

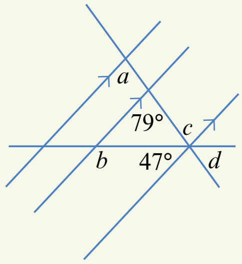


i



EXAMPLE 7A-2 Finding angle size using angle relationships with parallel lines

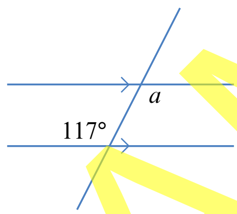
Find the value of the pronumerals in this diagram.

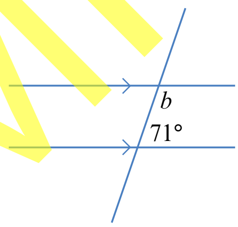


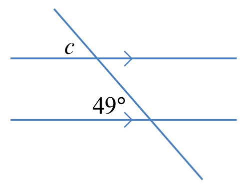
THINK	WRITE
1 Angle *a* is corresponding to 79°. Corresponding angles are equal.	$a = 79°$
2 Angle *b* is co-interior to angle 47°. Co-interior angles are supplementary.	$b = 180° - 47° = 133°$
3 Angle *c* is alternate to 79°. Alternate angles are equal.	$c = 79°$
4 Angle *d* is supplementary to angle *c* and the angle which is vertically opposite and hence equal to 47°.	$d = 180° - 79° - 47° = 54°$

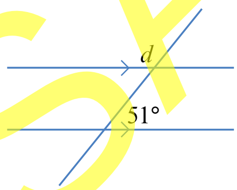
UNDERSTANDING AND FLUENCY

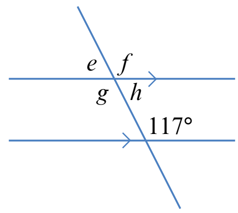
2 Find the value of each pronumeral in these diagrams.

a


b


c


d


e


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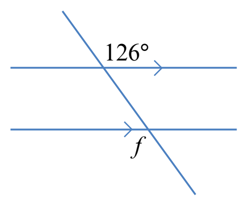
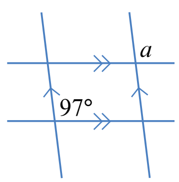


3 Find the value of each pronumeral in these diagrams.

a


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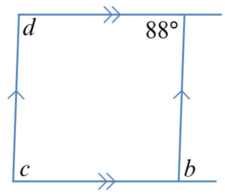

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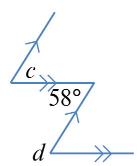

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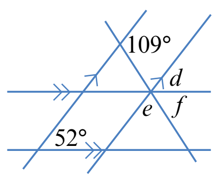

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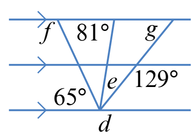

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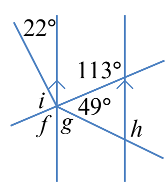
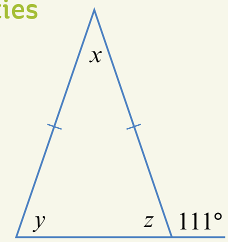


EXAMPLE 7A-3 Finding angle size using triangle properties

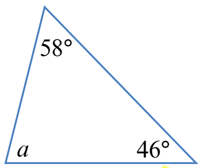
Find the value of each pronumeral in this diagram.



THINK	WRITE
1 Angle z is supplementary to 111°.	$z = 180° - 111° = 69°$
2 Angle y is equal to angle z because the triangle is an isosceles triangle.	$y = z = 69°$
3 Angle x is supplementary to angles y and z because there are 180° in a triangle.	$x = 180° - 69° - 69° = 42°$

4 Find the value of each pronumeral in these diagrams.

a



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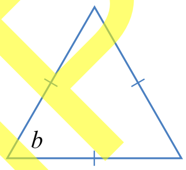

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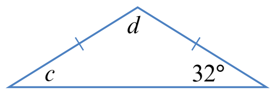

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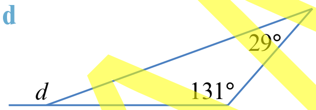

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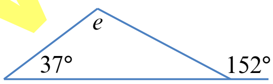

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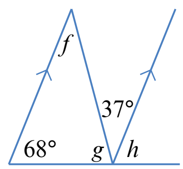
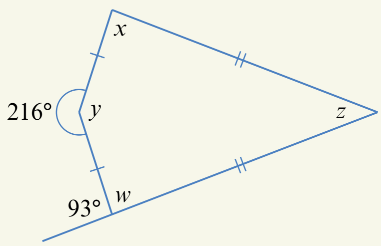


EXAMPLE 7A-4 Finding angle size using quadrilateral properties

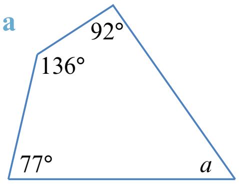
Find the value of each pronumeral in this diagram.

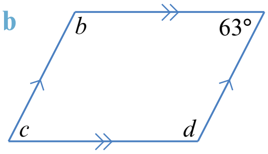


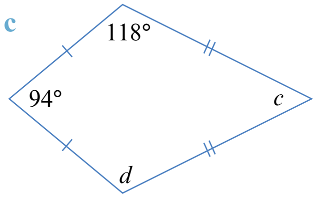
THINK	WRITE
1 Angle w is supplementary to 93°.	$w = 180° - 93° = 87°$
2 Angle x is equal to angle w because a **kite** has a pair of equal and opposite angles.	$x = w = 87°$
3 Angle y and 216° add to 360°.	$y = 360° - 216° = 144°$
4 Angles w, x, y and z add to 360° because the angle sum in a quadrilateral is 360°.	$z = 360° - 144° - 87° - 87° = 42°$

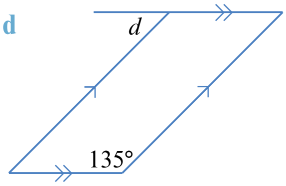
UNDERSTANDING AND FLUENCY

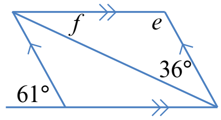
5 Find the value of each pronumeral in these diagrams.

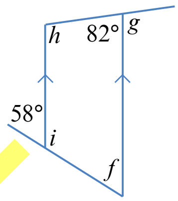
a 

b 

c 

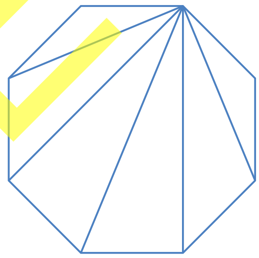
d 

e 

f 

PROBLEM SOLVING AND REASONING

6 You can use the internal angle sum of a triangle to find the internal angle sum for any polygon. Consider this octagon.



a How many sides does it have?
b How many triangles is it split into?
c What is the difference between these two numbers?
d Use the triangles to calculate the internal angle sum of an octagon.
e Repeat parts a–d to find the internal angle sum of a:
   i pentagon ii decagon iii heptagon iv dodecagon.
f Explain why the internal angle sum of any polygon can be found using the formula $(n - 2) \times 180°$, where n = number of sides.

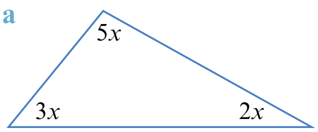
7 Use the formula obtained in question 6 to find the internal angle sum of a polygon with:
a 20 sides b 50 sides c 100 sides.

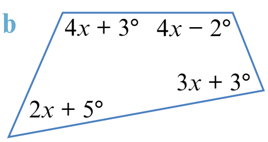
8 a Explain why you can use the formula $x = \dfrac{180°(n-2)}{n}$ to find the size of an individual internal angle in any **regular polygon**.
b Why can this formula only be used for regular polygons?

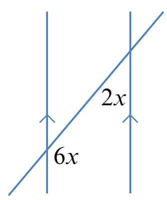
9 Find the size of an individual internal angle for a regular:
a heptagon b nonagon c hexagon.

10 Find the number of sides for a regular polygon that has each internal angle equal to:
a 160° b 144° c 170° d 179.64°.

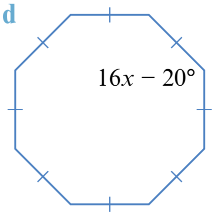
11 Find the value of x in each diagram.

a 

b 

c 

PROBLEM SOLVING AND REASONING

d


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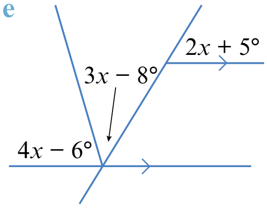

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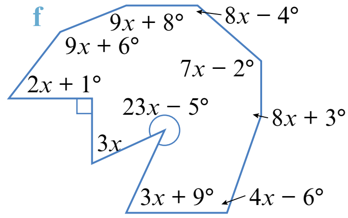
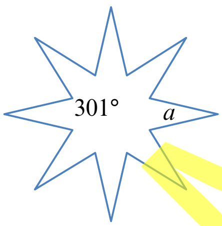


12 Decide if each statement is true or false. If false, write a correct version of the statement.

- **a** A kite has a pair of equal opposite angles.
- **b** Angles around a point add to 180°.
- **c** An **isosceles** triangles has two pairs of equal angles.
- **d** Corresponding angles in parallel lines are supplementary.
- **e** A square is a rhombus.
- **f** A parallelogram is a rectangle.
- **g** Complementary angles add to 180°.
- **h** A triangle that has one 60° angle must be **equilateral**.
- **i** A quadrilateral can have a maximum of one **concave** angle.

13 Find the value of each pronumeral, giving reasons.

a


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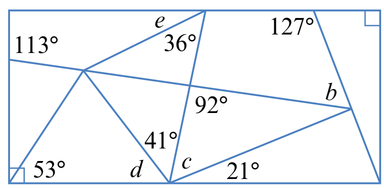

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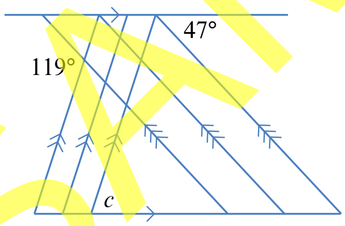

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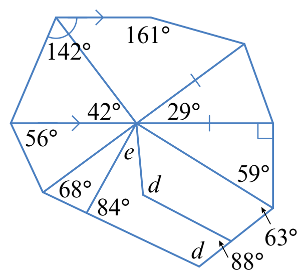
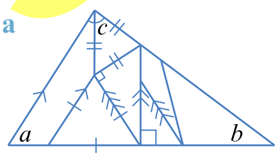


CHALLENGE

14 Find the value of each pronumeral, giving reasons.

a


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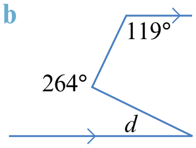

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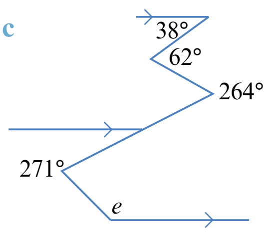

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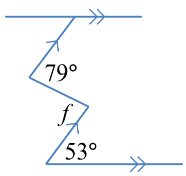

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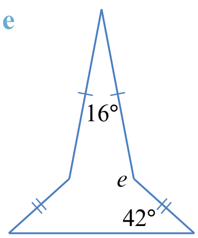

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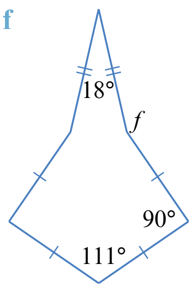
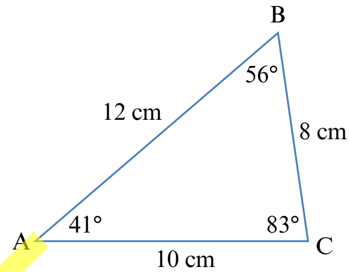


Reflect

What numbers are important when considering the relationships between angles, lines and polygons?

7B Congruence

Start thinking!



1 List the length of each side and the size of each angle in ΔABC.
For example, AB = 12 cm, ∠ABC = 56°, etc.

2 Draw ΔDEF, where DE = 8 cm, EF = 10 cm and DF = 12 cm.

For any two figures to be **congruent** (identical in shape and size), they must have all corresponding sides the same length and all corresponding angles the same size.

3 Is ΔDEF congruent to ΔABC? How do you know?

When two triangles have all their corresponding sides the same length (SSS), the corresponding angles are also the same size. This means that the triangles are congruent.

4 Measure the angles of ΔDEF to confirm that these triangles are congruent.

When two figures are congruent, you write this as a statement with the congruency symbol ≅. The same order of corresponding vertices is used in naming matching sides, angles and shapes.

5 List the corresponding vertices and hence write a congruency statement for the two triangles.

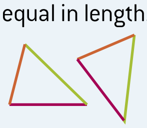
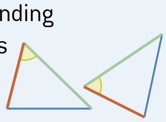
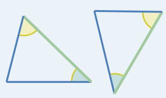
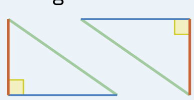
6 Draw ΔGHI, where ∠GHI = 41°, ∠HIG = 56° and ∠IGH = 83°.

7 Can you tell if ΔGHI is congruent to ΔABC? How do you know?

8 Explain why, if two triangles have all their corresponding angles the same size (AAA), this does not mean that they are congruent. You may like to include diagrams with your explanation.

KEY IDEAS

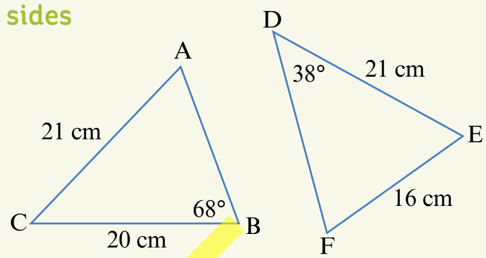
- Congruent figures are identical in shape and size but can be in any position or orientation.
- Two figures are congruent if their corresponding sides are all the same length and their corresponding angles all the same size. The symbol for congruence is ≅.
- Matching sides, angles and shapes in congruent figures are named using the same order of corresponding vertices.
- For triangles, there are four conditions for congruence (shown in the table).
- The specifications AAA and SSA do not necessarily mean congruence (more information is needed).

SSS	SAS	AAS	RHS
Three corresponding pairs of sides are equal in length. 	Two corresponding pairs of sides are equal in length and the corresponding pair of angles in between are equal. 	Two pairs of angles are equal and a corresponding pair of sides are equal in length. 	The hypotenuses and a corresponding pair of sides are equal in length in a right-angled triangle. 

EXERCISE 7B Congruence

EXAMPLE 7B-1 Matching corresponding sides in congruent triangles

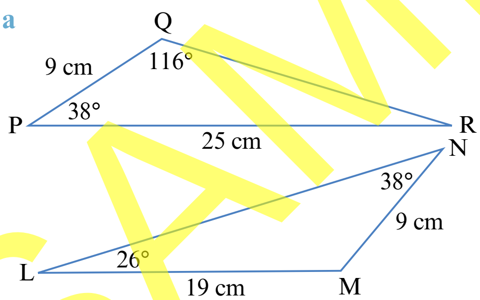
Assuming this pair of triangles is congruent, match the corresponding sides.



THINK	WRITE
1 Name the two sides that are 21 cm long according to the order of corresponding vertices; that is, A and E, C and D.	AC = ED
2 BC is 20 cm and EF is 16 cm, so if the triangles are congruent, BC must be equal to FD.	BC = FD
3 Write the last pair of corresponding sides.	AB = EF

UNDERSTANDING AND FLUENCY

1 Assuming each pair of triangles is congruent, match the corresponding sides.

a


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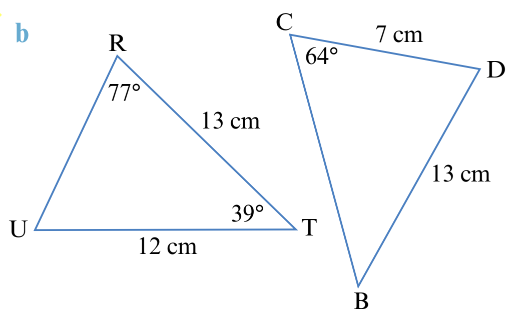

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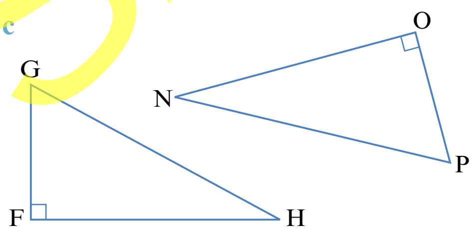

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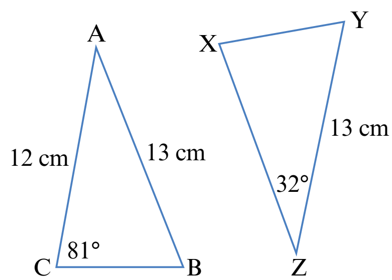

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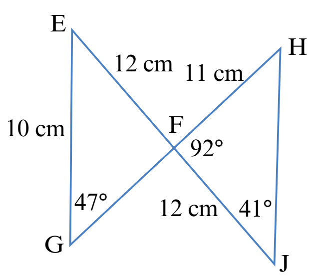

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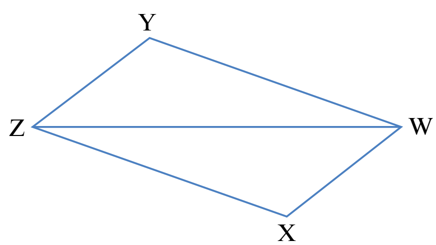
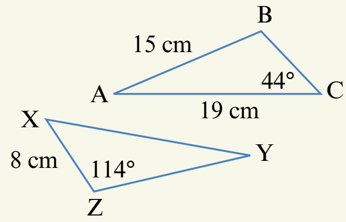


EXAMPLE 7B-2 Finding unknown side lengths and angles in congruent triangles

Find the unknown side lengths and angles in this pair of congruent triangles.



THINK

1 Match corresponding sides to identify the unknown side lengths.

2 Match corresponding angles to identify the unknown angles. Use the triangle sum of 180° if necessary.

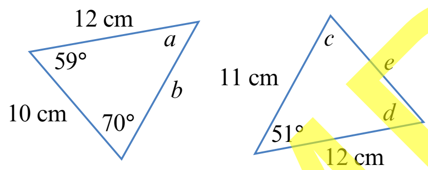
WRITE

AB = YZ = 15 cm
BC = ZX = 8 cm
AC = YX = 19 cm

∠ABC = ∠YZX = 114°
∠ACB = ∠YXZ = 44°
∠CAB = ∠XYZ = 180° − 114° − 44° = 22°

2 Find the unknown side lengths and angles in each pair of congruent triangles.

a



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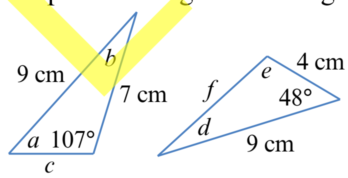

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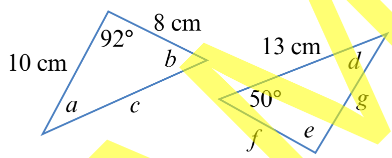

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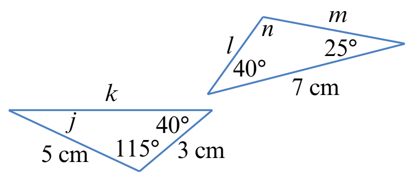
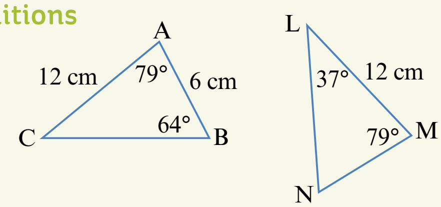


EXAMPLE 7B-3 Identifying congruence conditions

Decide which condition you would use to check if this pair of triangles is congruent.



THINK

1 Look at the triangles. ΔABC has two given side lengths and two angles and ΔLMN has two given angles and a side length.

2 Can any information be added to the triangles? Both triangles have two angles, so the third for each can be found.

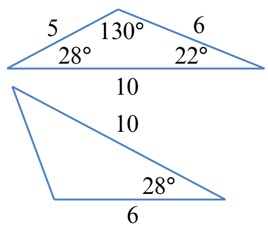
3 ΔABC now has information for AAS and SAS, but ΔLMN only has information for AAS.

WRITE

You would use the congruence condition AAS to check if these triangles are congruent.

3 Decide which condition you would use to check if each pair of triangles is congruent.

a



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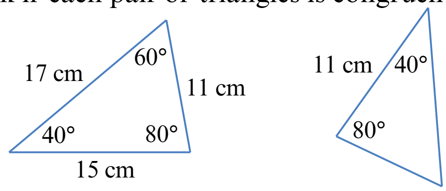

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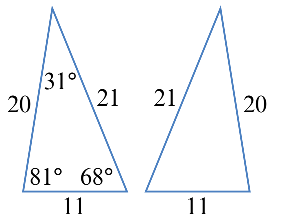

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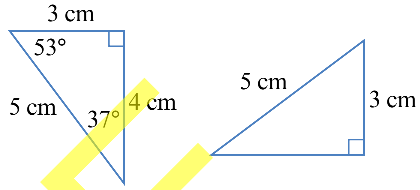

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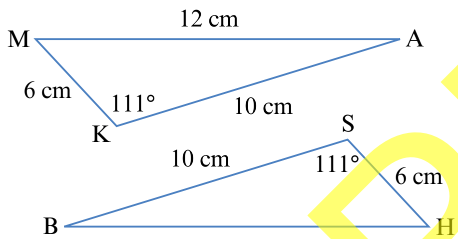

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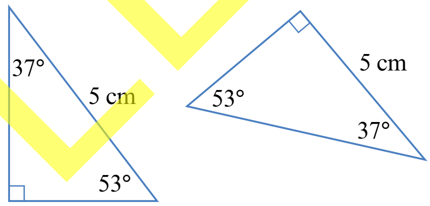
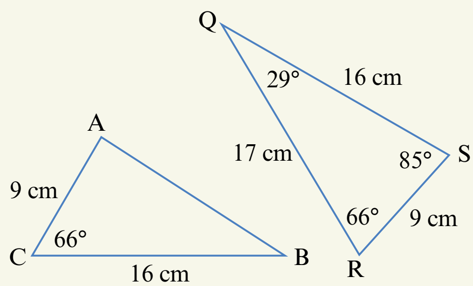


EXAMPLE 7B-4 Identifying congruence

Decide if this pair of triangles is congruent, giving a reason for your answer.



THINK

1 Two side lengths and an angle are given that are the same in both triangles (9 cm, 16 cm and 66°).

2 In ΔABC, the known angle is between the two sides, which fits the SAS condition for congruence.

3 The same sized angle in ΔQRS is not between the same two sides. A different sized angle (85°) is between the two sides, which means that these two triangles fail the SAS condition for congruence.

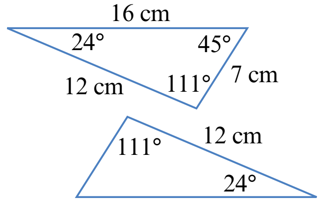
WRITE

The two triangles are not congruent as they fail the SAS condition for congruence.

UNDERSTANDING AND FLUENCY

4 Decide if each pair of triangles is congruent, giving a reason for your answer.

a



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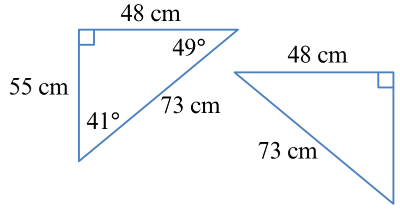

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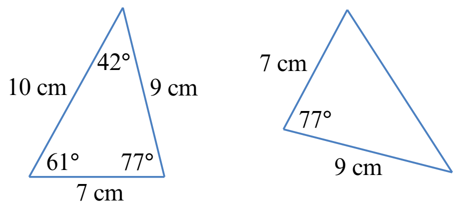

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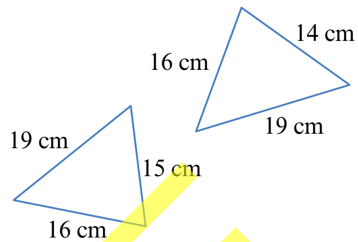

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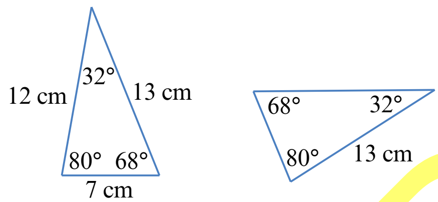

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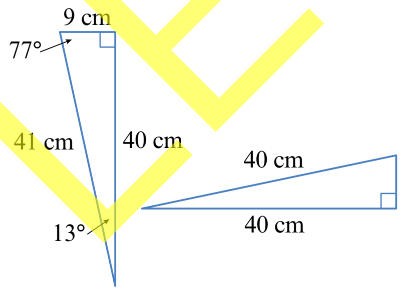
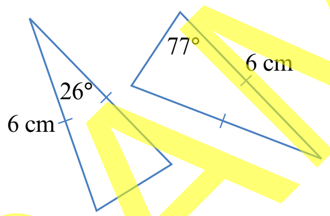


PROBLEM SOLVING AND REASONING

5 Use your understanding of triangle properties to decide whether each pair of triangles is congruent.

a



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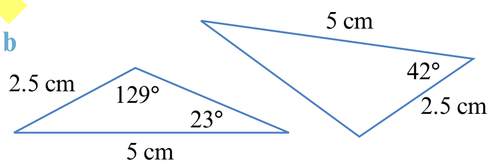

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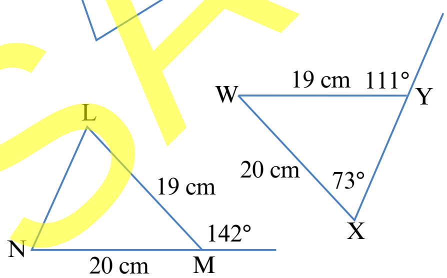

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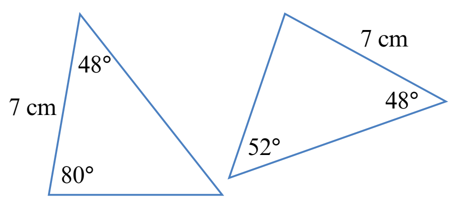


6 **a** Use a **square** and a **rhombus** to show that having all corresponding sides the same length is not enough to prove congruence for shapes other than triangles.

b Use a **parallelogram** and a rhombus to show that meeting SAS does not prove congruence for shapes other than triangles.

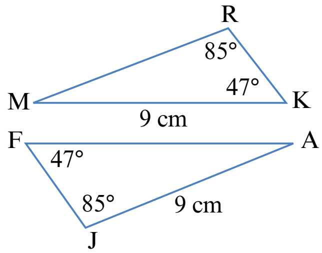
c Use a square and a **rectangle** to show that meeting AAS does not prove congruence for shapes other than triangles.

d Can you think of an alternative to SAS and AAS that could prove congruence in quadrilaterals? Investigate if this pattern continues for other polygons.

PROBLEM SOLVING AND REASONING

7 Lachlan said that each pair of triangles is congruent. Explain why he is wrong in each case.

a



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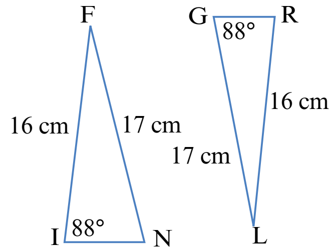

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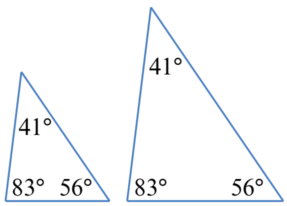

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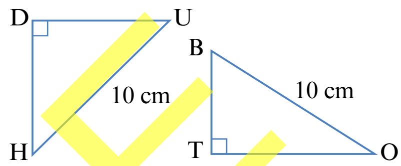


8 Explain why correspondence of side lengths and angles is important when deciding if a pair of triangles meets a congruency condition.

9 Decide if each statement is true or false.

a If two triangles meet AAA, they are not congruent.

b If two triangles each have two corresponding sides the same length and one corresponding angle of the same size, the triangles are congruent.

c A pair of triangles can meet one congruence condition but fail another.

d If ΔABC is congruent to ΔDEF, and ΔDEF is congruent to ΔGHI, then ΔGHI is congruent to ΔABC.

e If two quadrilaterals have all sides equal in length, then they are congruent.

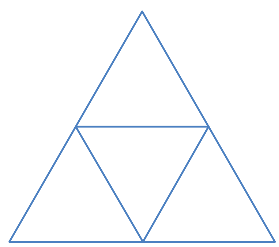
f All equilateral triangles are congruent.

CHALLENGE

10 Use your understanding of the relevant geometrical properties to draw each description and show that the triangles within them are congruent.

a A circle with centre O has a right-angled triangle drawn where the right angle is at the centre and the short sides are radii of the circle. A second right-angled triangle is drawn in the same circle under the same conditions.

b An equilateral triangle is split into four smaller triangles by drawing the three **vertices** of a single triangle at the midpoint of each side, as seen in the diagram at right.



c A kite is split into two triangles by a line drawn vertically down its centre.

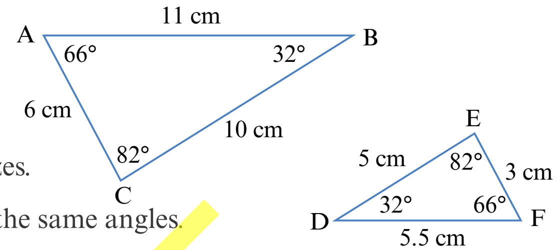
d A regular hexagon is split into six triangles by drawing 3 diagonal lines joining opposite vertices.

Reflect

What do you need to check to see if any pair of shapes is congruent?

7C Similarity

Start thinking!

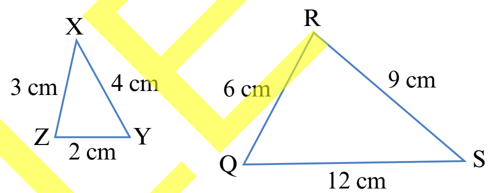


1 What is similar and what is different about the first pair of triangles?

Similar figures are exactly the same shape but can be different sizes.

2 Explain how you know two triangles are similar if they have the same angles.

3 How might you figure out if the second pair of triangles are similar?



4 How long is QS compared with YX?

It is important when checking that corresponding sides are in the same **ratio** that you check every pair of sides.

5 Check that the other two pairs of sides are in the same ratio.

This ratio is called **scale factor**, can be calculated using the formula: scale factor = $\frac{\text{image length}}{\text{original length}}$.

6 Explain why, if the **image** is bigger than the original, the scale factor will be bigger than 1, but if the image is smaller than the original, the scale factor will be a fraction less than 1.

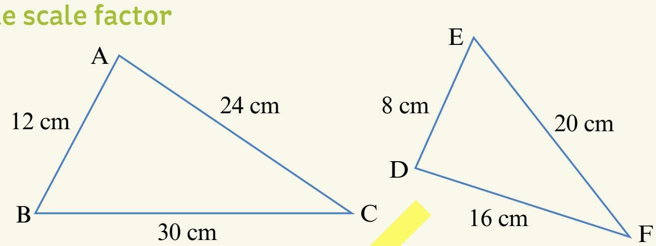
KEY IDEAS

- Similar figures are identical in shape but can be different in size.
- For two figures to be similar, all angles must be equal in size and all corresponding sides must be in the same ratio.
- If two figures are similar, an unknown side length or angle can be found if the scale factor is known.
- Scale factor is calculated using the formula: scale factor = $\frac{\text{image length}}{\text{original length}}$.
- For two triangles to be similar, they must meet one of four similarity conditions:
  - SSS: All three corresponding sides are in the same ratio.
  - SAS: Two corresponding sides are in the same ratio and the angles in between are equal.
  - AAA: All three corresponding angles are equal.
  - **RHS**: The hypotenuses are in the same ratio as another pair of sides in right-angled triangles.

EXERCISE 7C Similarity

EXAMPLE 7C-1 Finding the scale factor

Find the scale factor for this pair of similar triangles. Assume that the first triangle is the original.

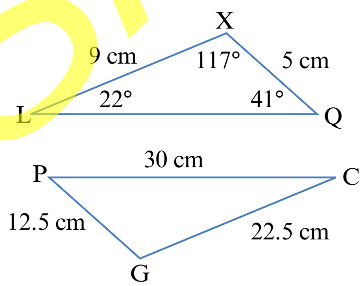


THINK	WRITE
1 To find the scale factor, compare corresponding sides. Choose the longest side of the original triangle (ΔABC) and the longest side of the image (ΔDEF).	BC = 30 cm, EF = 20 cm
2 Substitute these two side lengths into the formula for scale factor.	Scale factor $= \dfrac{\text{image length}}{\text{original length}}$
3 Check that the scale factor seems reasonable. (Number between 0 and 1 means a **reduction**.)	$= \dfrac{20\text{ cm}}{30\text{ cm}}$ $= \dfrac{2}{3}$
4 Match the remaining corresponding sides and check that each side is in the same ratio.	Scale factor $= \dfrac{16}{24} = \dfrac{8}{12} = \dfrac{2}{3}$
5 Write your answer.	Scale factor $= \dfrac{2}{3}$

UNDERSTANDING AND FLUENCY

1 Find the scale factor for each pair of similar triangles. Assume that the first triangle in the pair is the original.

a



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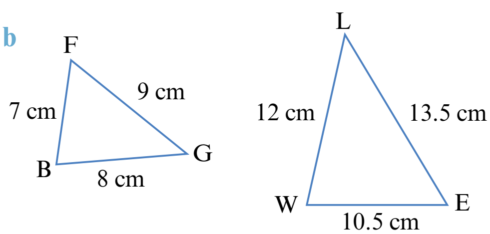
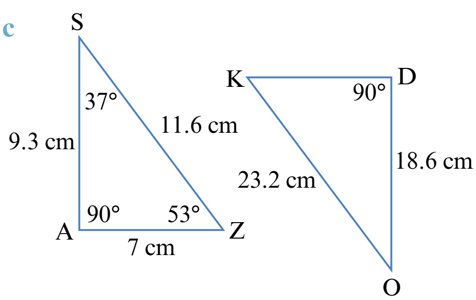
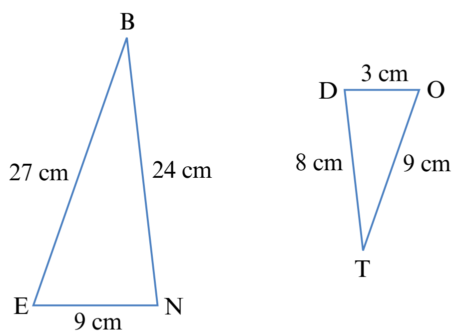


c

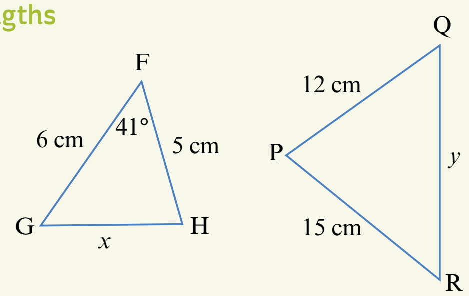


d



EXAMPLE 7C-2 Finding unknown side lengths in similar triangles

Find the unknown side lengths in this pair of similar triangles.



THINK

1 As the triangles are similar, all corresponding sides must be in the same ratio. Match corresponding sides.

WRITE

Corresponding pairs of sides are:
FG and RQ, GH and QP, FH and RP

THINK

2 Use the pair of corresponding sides that both have measurements (FH and RP) to find the scale factor from ΔFGH to ΔRQP.
Check that the scale factor seems reasonable. (Number larger than 1 means an **enlargement**.)

WRITE

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}} = \frac{RP}{FH} = \frac{15\text{ cm}}{5\text{ cm}} = 3$$

THINK

3 Since x is on the 'original' triangle, divide the length of the corresponding side in ΔRQP by the scale factor.

WRITE

$$x = \frac{QP}{\text{scale factor}} = \frac{12\text{ cm}}{3} = 4\text{ cm}$$

THINK

4 Since y is on the 'image' triangle, multiply the length of the corresponding side in ΔFGH by the scale factor.

WRITE

$$y = FG \times \text{scale factor} = 6\text{ cm} \times 3 = 18\text{ cm}$$

THINK

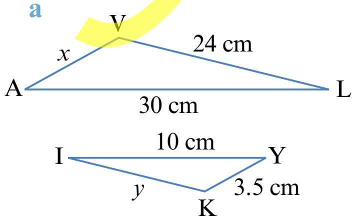
5 Write your final answer.

WRITE

$x = 4$ cm and $y = 18$ cm.

2 Find the unknown side lengths in each pair of similar triangles.

a



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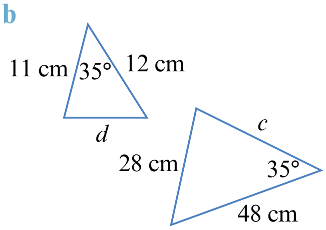

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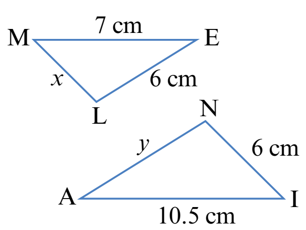

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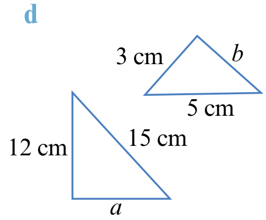

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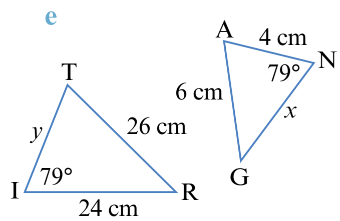

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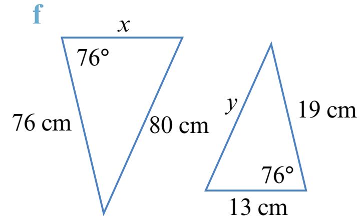
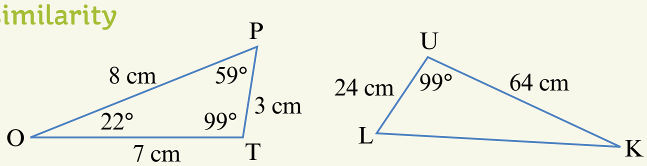


EXAMPLE 7C-3 Identifying similarity

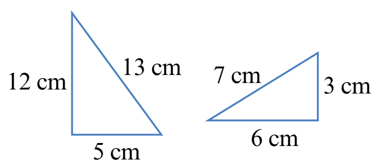
Decide if this pair of triangles is similar, giving a reason for your answer.



THINK	WRITE
1 ΔLUK has two sides and an angle in between, so use the similarity condition SAS to compare. Match corresponding sides and angles.	PT corresponds to LU ∠PTO corresponds to ∠LUK ∠TOP corresponds to ∠UKL ∠TPO corresponds to ∠ULK TO corresponds to UK OP corresponds to KL
2 Since ∠PTO corresponds to ∠LUK, check if the angles are equal.	∠PTO = ∠LUK = 99°
3 PT corresponds to LU. Find the scale factor from ΔPTO to ΔLUK.	$\text{Scale factor} = \frac{\text{image length}}{\text{original length}} = \frac{\text{LU}}{\text{PT}} = \frac{24\text{ cm}}{3\text{ cm}} = 8$
4 TO corresponds to UK. Find the scale factor from ΔPTO to ΔLUK.	$\text{Scale factor} = \frac{\text{image length}}{\text{original length}} = \frac{\text{UK}}{\text{TO}} = \frac{64\text{ cm}}{7\text{ cm}} \approx 9.14$
5 Look at your results and write your final answer.	The two triangles are not similar as the two scale factors are not the same, so they fail the similarity condition SAS.

3 Decide if each pair of triangles is similar, giving a reason for your answer.

a



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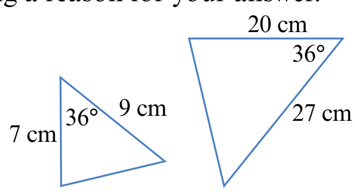

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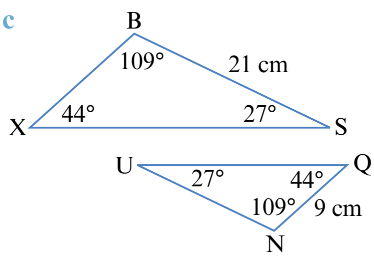

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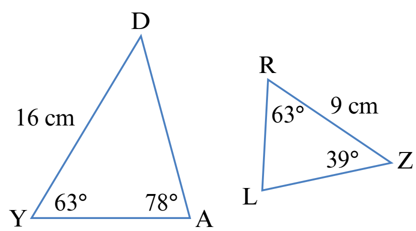
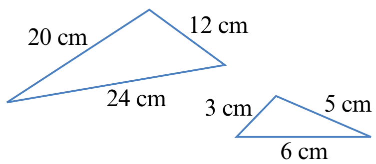


PROBLEM SOLVING AND REASONING

e


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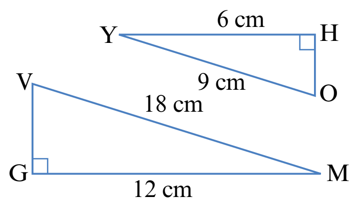
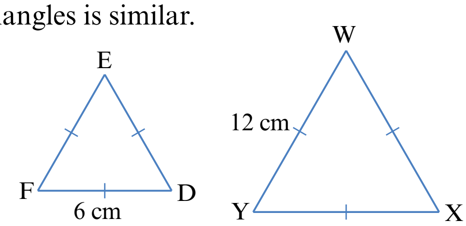


4 Use your understanding of triangle properties to decide whether each pair of triangles is similar.

a


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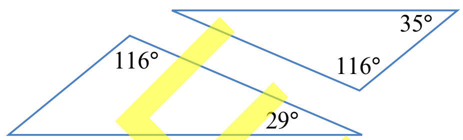

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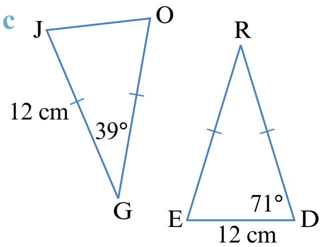

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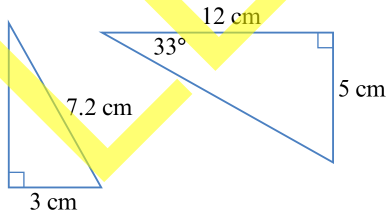
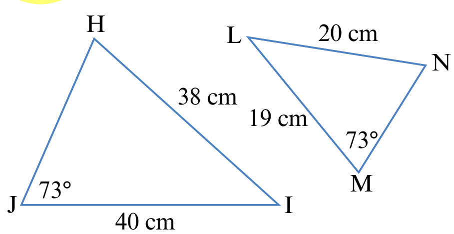


5 a Use a rectangle and a parallelogram to show that having all corresponding sides in the same ratio is not enough to prove similarity for shapes other than triangles.
b Use a parallelogram and a rhombus to show that having all corresponding angles the same size is not enough to prove similarity for shapes other than triangles.
c Use a rectangle and a square to show that meeting SAS does not prove similarity for shapes other than triangles.
d Use a parallelogram and a rhombus to show that meeting AAS does not prove similarity for shapes other than triangles.
e Can you think of an alternative to SAS and AAS that could prove similarity in quadrilaterals? Investigate if this pattern continues for other polygons.

6 Jessica said that each pair of triangles is similar. Explain why she is wrong in each case.

a


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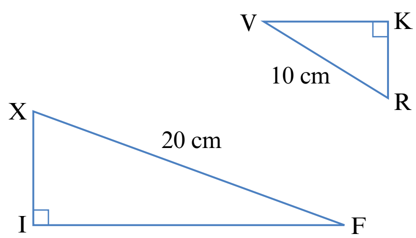
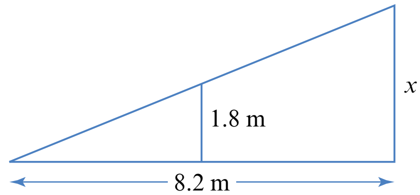


7 Explain why correspondence of sides and angle is important when deciding if a pair of triangles meets a similarity condition.

8 A 1-m ruler is placed upright next to a tree. If the ruler casts a 2.3 m shadow and the tree casts an 11.2 m shadow, how tall is the tree?

PROBLEM SOLVING AND REASONING

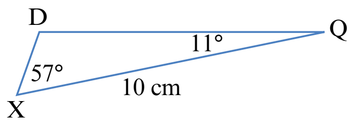
9 A ski jump is 8.2 m long. It has vertical supports at the half-way mark that are 1.8 m tall. How tall is the ski jump at its end?



10 Use similar triangles to calculate the unknown height or length of something in your classroom. Include a diagram in your answer.

11 Select two pairs of similar triangles from these options. Provide reasons for your selection.

a



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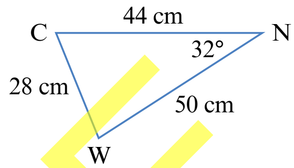

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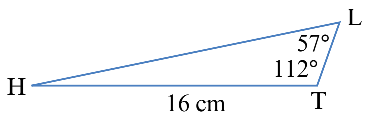

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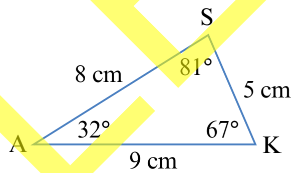

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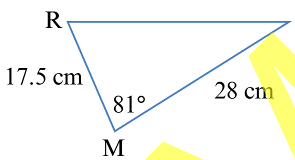

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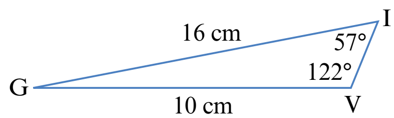


12 Decide if each statement is true or false.

- **a** If two triangles meet AAA, they are not necessarily similar.
- **b** If ΔABC is similar to ΔDEF, and ΔDEF is similar to ΔGHI, then ΔGHI is similar to ΔABC.
- **c** All isosceles triangles are similar.
- **d** All squares are similar.
- **e** If two triangles fail SAS, they are not necessarily similar.
- **f** If two quadrilaterals have all sides in the same ratio, they are not necessarily similar.

13 Dilating ΔSIJ by $\frac{4}{3}$ gives ΔPWX. Is this a reduction or enlargement? Explain.

CHALLENGE

14 A flagpole has a guide wire attached at its midpoint, tethered to the ground 2.7 m away. A metre ruler standing vertically touches the guide wire when placed 75 cm away from where it is tethered to the ground. How tall is the flagpole?

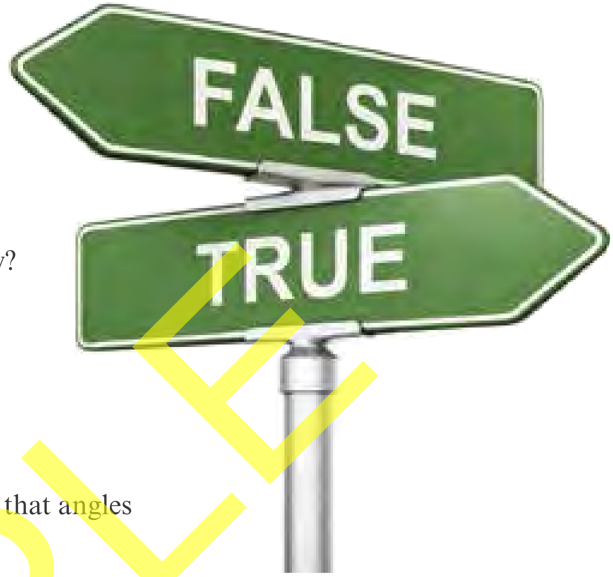
15 Explain how you can use similar triangles to find the width of a river. (Hint: you will need to use four markers on one side of the river, one of which lines up with a landmark on the opposite side.) Draw a diagram to support your answer.

Reflect

What is the difference between the congruence and similarity conditions?

7D Understanding proofs

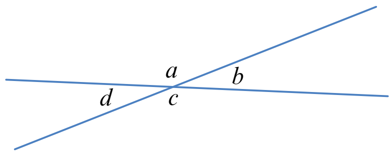
Start thinking!



1 How do you know when something is true?

It is important in mathematics not to just demonstrate that something is true or that it works, but to *prove* that it is true.

2 Which angles are equal in this diagram? How do you know?



3 Copy the diagram and cut out the angles and demonstrate that angles a and c are equal in size.

This is a practical demonstration that vertically opposite angles are equal, but it is not a proof.

A mathematical **proof** is a series of statements that show that a theory is true in all cases. Usually the proof uses some self-evident or assumed statements, known as **axioms**, in showing that something is true. One of the most famous mathematicians, Euclid (who lived around 300 BC), was able to prove much of the mathematics you use today, using only five axioms.

You can use the fact that there is 180° around a straight line as an axiom to prove that vertically opposite angles are equal.

4 What is $a + b$ equal to?

5 What is $b + c$ equal to?

6 How do your answers to questions **4** and **5** show that $a + b$ is equal to $c + b$?

7 How does this show that a is equal to c?

8 Write these steps as a mathematical proof showing that angle a is equal to angle c.

9 Why do you think proofs are important in mathematics?

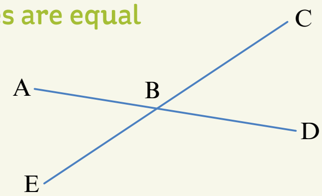
KEY IDEAS

▶ A mathematical proof is a series of statements that show that a theory is true in all cases.

EXERCISE 7D Understanding proofs

EXAMPLE 7D-1 Proving that vertically opposite angles are equal

Prove that ∠ABC = ∠DBE.



THINK	WRITE
1 Use angles around a straight line to write a statement about ∠ABC and a statement about ∠DBE that share a common angle.	∠ABC + ∠CBD = 180° ∠DBE + ∠CBD = 180°
2 Since the right side of each equation is the same, the left sides must be equal.	∠ABC + ∠CBD = ∠DBE + ∠CBD
3 Simplify by subtracting ∠CBD from both sides of the equation.	∠ABC = ∠DBE

UNDERSTANDING AND FLUENCY

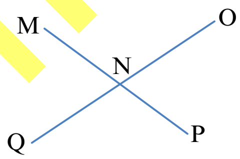
1 Fill in the gaps to prove that ∠MNQ = ∠ONP.

∠MNQ + ∠QNP = _____°

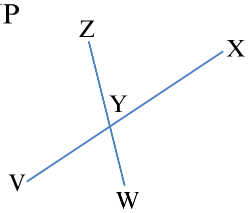
∠ONP + ∠_____ = 180°

∠_____ + ∠QNP = ∠_____ + ∠QNP

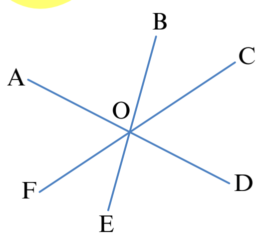
∠_____ = ∠_____



2 Prove that ∠XYW = ∠ZYV.



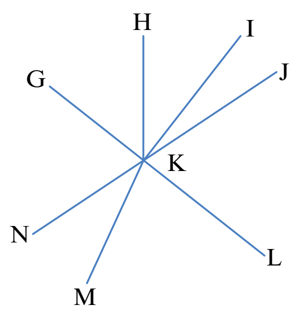
3 Show that ∠EOF = ∠BOC, starting with the opening statement:



a ∠EOF + ∠FOB = 180°

b ∠EOF + ∠AOF + ∠AOB = 180°.

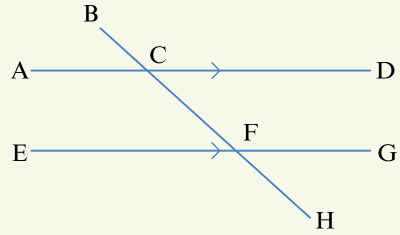
4 Show that ∠GKJ = ∠NKL.



NOTE In some books you may notice the letters 'QED' written at the end of a proof. This is an acronym for the Latin phrase *quod erat demonstrandum*, which means 'which had to be demonstrated'.

EXAMPLE 7D-2 Proving that alternate angles are equal

Prove that ∠ACF = ∠CFG,
given that ∠CFG = ∠BCD.



THINK

1 Write the given information.

2 Write statements linking ∠ACF and ∠BCD.

3 Use the given information to complete the proof.

WRITE

∠CFG = ∠BCD

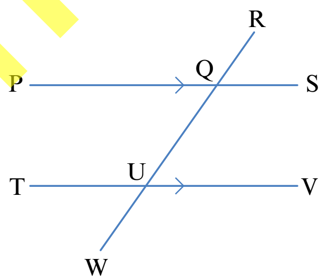
∠ACF + ∠DCF = 180°
∠BCD + ∠DCF = 180°
∠ACF + ∠DCF = ∠BCD + ∠DCF
∠ACF = ∠BCD

but ∠BCD = ∠CFG
so ∠ACF = ∠CFG

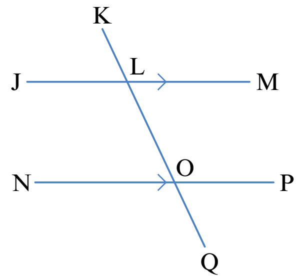
UNDERSTANDING AND FLUENCY

5 Fill in the gaps to prove that ∠TUQ = ∠SQU, given that ∠TUQ = ∠PQR.

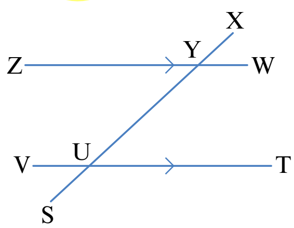
∠TUQ = ∠PQR
∠PQR + ∠PQU = ______
∠SQU + ∠______ = 180°
∠PQR + ∠______ = ∠SQU + ∠______
∠PQR = ∠______
but ∠PQR = ∠TUQ
so ∠TUQ = ∠______



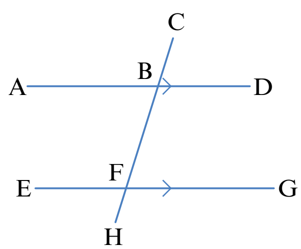
6 Given that ∠JLK = ∠LON, show that ∠MLO = ∠LON.



7 Given that ∠XYW = ∠YUT, show that ∠WYU = ∠VUY.



8 Given that ∠ABC = ∠EFB, show that ∠DBF is supplementary to ∠BFG.



9 Is the sum of two even integers always even? To investigate, consider any two even numbers; call them x and y.

- **a** How do you define an even number?
- **b** Explain why it follows that you can write x as $2a$.
- **c** Write y in the same format using the pronumeral b.
- **d** Write $x + y$ using the format from parts **b** and **c**.
- **e** Factorise your answer to part **d**.
- **f** Explain why your answer to part **e** proves that the sum of two even integers is always even.
- **g** Write the proof in full.

10 Use a similar method to that shown in question **9** to prove that the square of any even number is always even.

11 There are different types of mathematical proof – so far you have seen a type called **direct proof**. Another method of proof is called **proof by contradiction**, where for a statement to be true a contradiction would have to occur, so that the statement is shown to be false.

One such proof involves the square root of 2. To prove that it is an irrational number, you instead try to prove that it is a rational number.

- **a** What is a rational number?

A rational number can be written in the form $\frac{a}{b}$, where a and b are whole numbers, and b is not equal to zero.

- **b** So, if the square root of 2 is rational, $\sqrt{2} =$ ______.

You assume that this fraction is in simplest form; that is, a and b have no common factors.

- **c** Explain why a or b may be even, but a and b cannot both be even.
- **d** Square both sides of the equation you found in part **b**.
- **e** Explain why you can write this in the form $a^2 = 2b^2$.
- **f** How do you know that a^2 is even?
- **g** How do you know that a is an even number? (Hint: look at question **10**.)

So you know that a is an even number, which means that b cannot also be an even number.

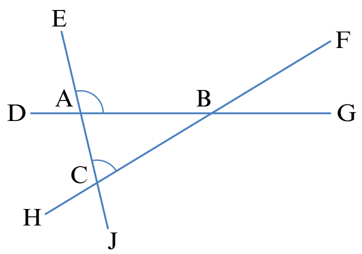
- **h** Explain why you can now write a as $2k$.
- **i** Substitute this into the equation $a^2 = 2b^2$.
- **j** Expand this and explain why it shows that b is also an even number.
- **k** Explain why this means that $\sqrt{2}$ is not rational.
- **l** Write the proof in full.

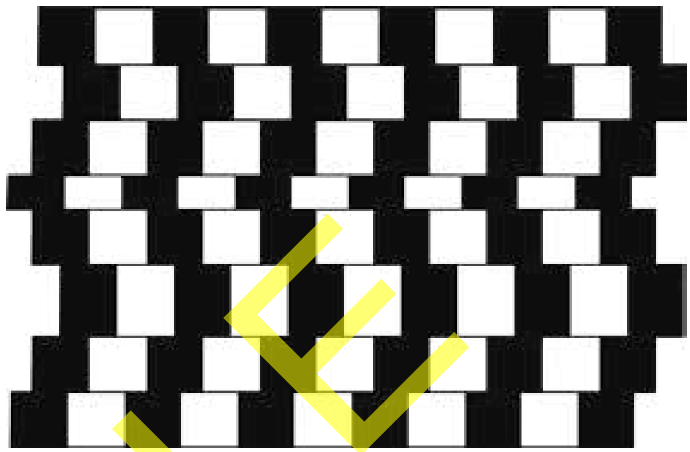
12 Follow a process similar to that in question **11** to show that $\sqrt[3]{2}$ is an irrational number.

13 Another example of proof by contradiction is used in proving that if corresponding angles are equal, the lines must be parallel. Start with the (false) assumption that if two corresponding angles are equal, then the lines are *not* parallel.

a Why does this mean that the lines would intersect?

Consider this diagram, showing intersecting lines DG and HF, cut by the transversal EJ.





b Name the two angles in the diagram that you are assuming are equal.

The other assumption that you must make is that the length AC > 0.

c Why do you need to make this assumption?

Say that ∠EAB = ∠ACB = x.

d Show that ∠BAC = 180° − x

e If ∠ABC can be represented by y, write a statement adding all the angles in ΔABC.

f Show that simplifying this statement leaves you with y = 0°.

g How does this imply that AC does in fact equal 0?

h Explain how this is proof by contradiction that if corresponding angles are equal, then the lines must be parallel.

i Write the proof in full.

14 Another method of proof is called **proof by contraposition**. In its simplest form, you can prove that 'if a then b' by first proving 'if not b then not a'. One example of proof by contraposition is to show if p^2 is odd, then p must be odd.

a What is the contrapositive statement to 'if p^2 is odd, then p must be odd'?

b Prove this contrapositive statement. Hint: this is what you are asked to do for question 10.

c Write a final statement to complete the proof by contraposition.

15 Use proof by contraposition to prove this statement.

If the product of a and b is odd, then both a and b must be odd.

Hint: write the contrapositive statement and prove this first.

16 Use proof by contraposition to prove that if $a^2 - 2a + 3$ is even, then a must be odd.

CHALLENGE

17 Another method of proof is **proof by mathematical induction**. This is where you initially prove the statement for a single case, for example $n = 1$, and then prove it to be true for a rule, such as $n = k + 1$, where k is an integer.

a Explain why, if something is true for $n = 1$ and $n = k + 1$, then it must be true for all integers.

An example is to prove that all integers that can be written as $2n + 1$ must be odd.

b Prove this statement for:

i $n = 1$ **ii** $n = 2$

iii $n = 99$ **iv** $n = 100$.

c Substitute $n = k + 1$ into $2n + 1$ and prove that this is also an odd number.

d Write a closing statement to finish the proof.

18 Use proof by induction to prove that $3^n - 1$, where n is a positive integer, is a multiple of 2.

19 It is important when you complete proofs that you do not make errors because, if you are not careful, you may think you have proven something that is actually false. The following example 'proves' that 2 = 1. Can you find the error?

$a = b$, where a and b are not equal to zero.

Then $a^2 = ab$

and $a^2 - b^2 = ab - b^2$

Factorising gives $(a + b)(a - b) = b(a - b)$

Dividing by $(a - b)$ gives $a + b = b$

Remember that $a = b$

So $b + b = b$

So $2b = b$

Dividing by b gives $2 = 1$

20 Follow a process similar to that in question **11** to show that $\sqrt{5}$ is an irrational number.

21 Create your own proofs using:

a direct proof

b proof by contradiction

c proof by contraposition

d proof by mathematical induction.

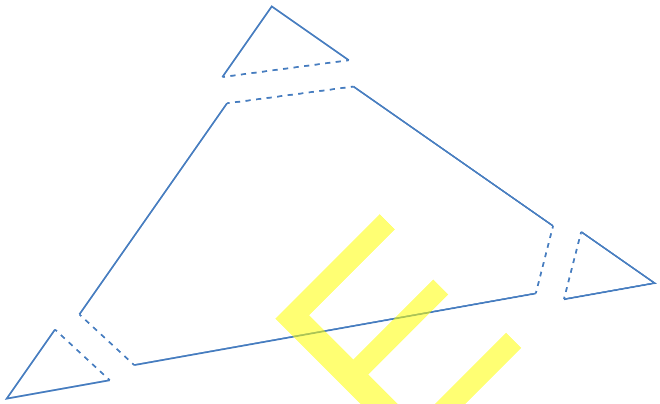
You may wish to use the Internet to help you.

Reflect

How are the different methods of proof similar and how are they different?

7E Proofs and triangles

Start thinking!

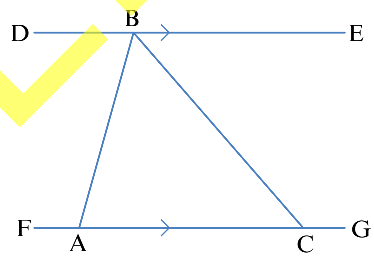


1 What is the internal angle sum of a triangle?

2 Draw a triangle and by either measuring its angles or by cutting out its angles and placing them in a straight line, show that it has an internal angle sum of 180º.

3 Why is drawing 10 different triangles and showing each has an angle sum of 180º not a proof?

You can use your knowledge of angles and parallel lines to prove that the internal angle sum of a triangle is 180°.



4 Name the internal angles of the triangle.

5 Show that ∠CBE = ∠BCA.

6 Which angle is ∠BAC equal to?

7 Write an equation for the angles around the straight line DE. (Hint: the three angles add to 180º).

8 Substitute the values you found from questions **5** and **6** into this equation.

9 Explain why this is the end of the proof.

10 Write the proof in full.

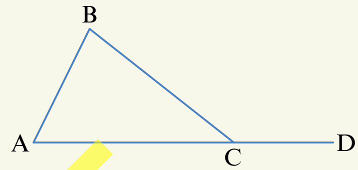
KEY IDEAS

- Many properties of triangles can be proved using knowledge of angles around a straight line and in parallel lines.
- This information can also be used to prove congruence in triangles.
- The symbol for congruence is ≅.

EXERCISE 7E Proofs and triangles

EXAMPLE 7E-1 Proving the exterior angle of a triangle is the sum of the two opposite internal angles

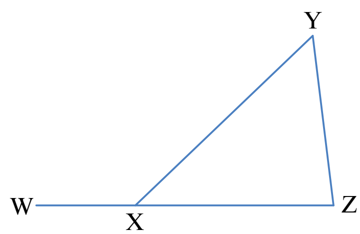
Prove that ∠BCD = ∠ABC + ∠CAB.



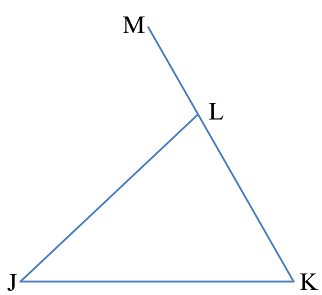
THINK	WRITE
1 Write an equation linking the exterior angle of the triangle and the straight line it lies on.	∠BCD + ∠BCA = 180°
2 Write an equation linking the internal angles of the triangle.	∠ABC + ∠CAB + ∠BCA = 180°
3 Equate the left sides of the two equations.	∠BCD + ∠BCA = ∠ABC + ∠CAB + ∠BCA
4 Simplify by subtracting ∠BCA from both sides.	∠BCD = ∠ABC + ∠CAB

UNDERSTANDING AND FLUENCY

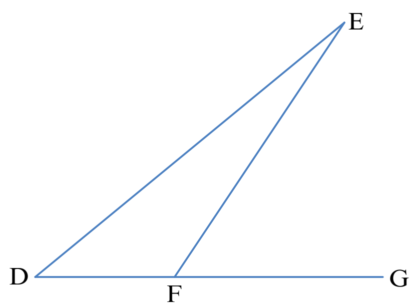
1 Prove that ∠WXY = ∠XZY + ∠ZYX by following these steps.
   a Write an equation linking the exterior angle of the triangle and the straight line it lies on.
   b Write an equation linking the internal angles of a triangle.
   c Equate the left sides of the two equations.
   d Simplify the resulting equation.



2 Prove that ∠JLM = ∠JKL + ∠LJK.



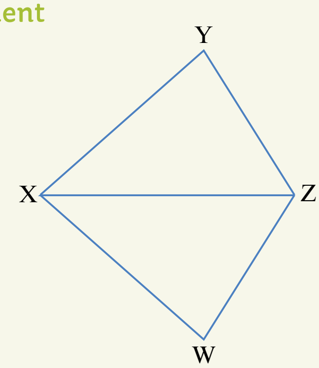
3 Prove that ∠EFG = ∠FDE + ∠DEF.



EXAMPLE 7E-2

Proving that two triangles are congruent using equal sides

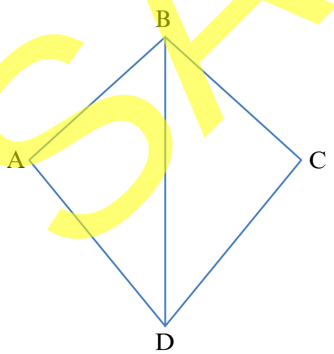
If XY = XW and ZY = ZW, prove that $\Delta XYZ \cong \Delta XWZ$.



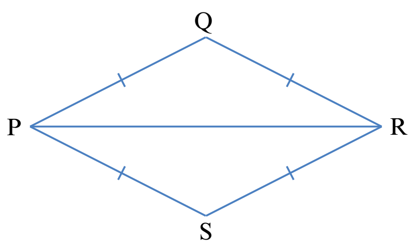
THINK	WRITE
1 If ΔXYZ and ΔXWZ are congruent, they have corresponding vertices. Match them for the two triangles.	X is common to both triangles. Z is common to both triangles. Y corresponds to W.
2 Decide if the given information is enough to satisfy a congruence condition.	Two pairs of corresponding sides are equal in length. Need another piece of information to check for congruence.
3 List a third piece of information.	XZ is common to both triangles.
4 Check for congruence and write the proof.	XY = XW (given) ZY = ZW (given) XZ is common to both triangles. $\Delta XYZ \approx \Delta XWZ$ (using SSS)

UNDERSTANDING AND FLUENCY

4 If AB = CB and AD = CD, prove that $\Delta ABD \cong \Delta CBD$.

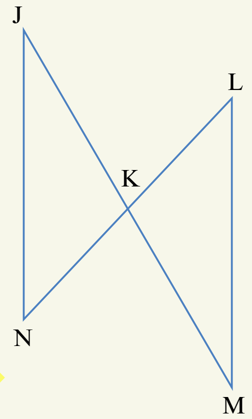


5 Prove that $\Delta PQR \cong \Delta PSR$.



EXAMPLE 7E-3 Proving that two triangles are congruent using AAS

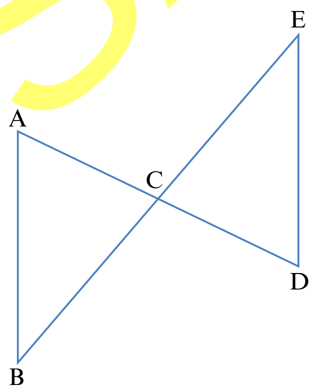
If KL = KN, ∠LKM = ∠NKJ and ∠LMK = ∠NJK, prove that ΔJKN ≅ ΔMKL using a congruence condition.



THINK	WRITE
1 If ΔJKN and ΔMKL are congruent, they have corresponding vertices. Match them for the two triangles.	J corresponds to M N corresponds to L K is common to both triangles
2 Consider the given information.	Sides KL and KN are corresponding and equal in length. Angles LKM and NKJ are corresponding and are equal. Angles LMK and NJK are corresponding and are equal.
3 Decide if you have enough information to meet a congruence condition (two pairs of equal angles and a corresponding pair of sides that are equal in length). Write the proof.	KL = KN (given) ∠LKM = ∠NKJ (given) ∠LMK = ∠NJK (given) ΔJKN ≈ ΔMKL (using AAS)

UNDERSTANDING AND FLUENCY

6 If AC = DC, ∠ACB = ∠DCE and ∠CBA = ∠CED, prove that ΔABC ≅ ΔDEC by following these steps.



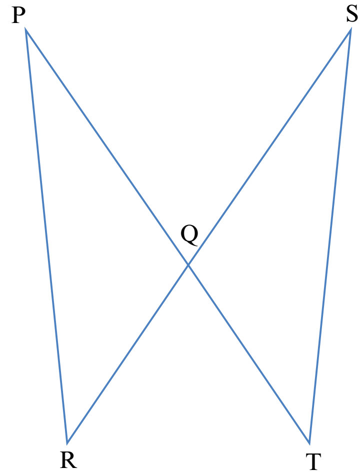
a Write the given information.

b Write a statement about the third angle of each triangle.

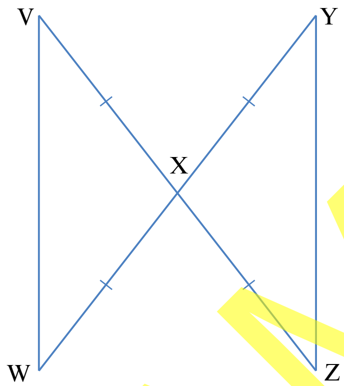
c Prove that the triangles are congruent by showing that they meet a congruence condition.

UNDERSTANDING AND FLUENCY

7 If QR = QT, ∠PQR = ∠SQT and ∠RPQ = ∠TSQ, prove that ΔPQR ≅ ΔSQT.

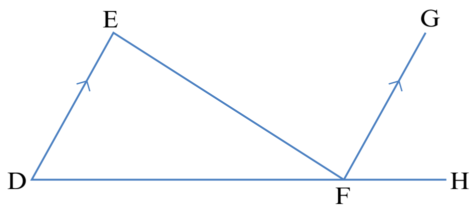


8 Prove that ΔVWX ≅ ΔYZX.

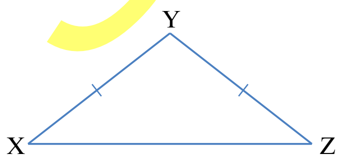


PROBLEM SOLVING AND REASONING

9 Use the fact that DE and FG are parallel to prove that ∠EFH = ∠DEF + ∠FDE.



10 Question 8 used the fact that if a triangle has two sides that are equal in length, then the angles opposite those sides are equal. You can prove this fact using congruent triangles. Consider ΔXYZ.

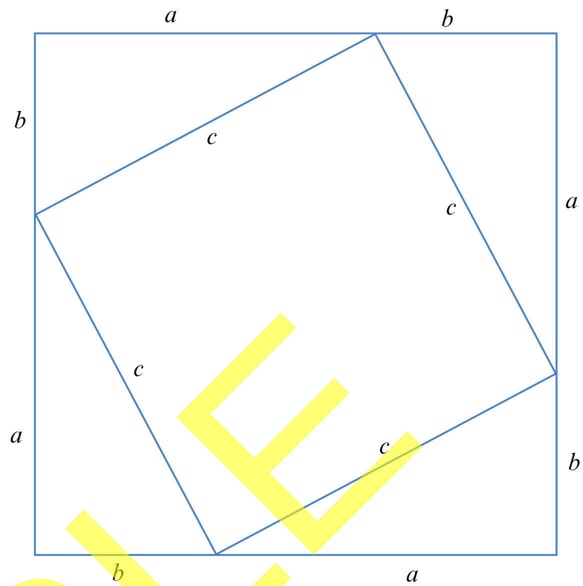


a Draw a line from Y to the midpoint of XZ, and name this point W.
b Name the two triangles now formed.
c Which two sides do you know are equal in these two triangles?
d Name the side common to both triangles.
e Explain why ∠XYW = ∠ZYW.
f Use a congruence condition to prove that ΔWXY ≅ ΔWZY.
g Explain why you now know that ∠WXY = ∠WZY.
h Write the proof in full.

PROBLEM SOLVING AND REASONING

11 Use what you have proven in question 10 to prove that the angles of an equilateral triangle are 60°.

12 Pythagoras' Theorem is another theorem that is reasonably simple to prove. Consider this diagram, which shows one square drawn inside another so that four congruent right-angled triangles are formed.

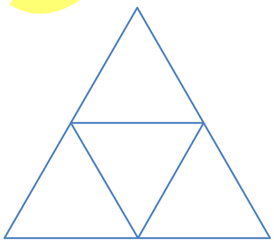


a How do you know that the hypotenuse of each triangle is equal in length?
b Explain why the area of the larger square can be represented by the equation $A = (a + b)^2$.
c Write an expression for the area of the smaller square.
d Write an expression for the area of one of the triangles.
e Explain why the area of the smaller square plus the four triangles can be represented by the equation $A = c^2 + 2ab$.
f Why can you now write $(a + b)^2 = c^2 + 2ab$?
g Expand and simplify the expression to prove Pythagoras' Theorem.
h Write the proof in full.

CHALLENGE

13 Prove that the four right-angled triangles in question 12 are congruent and hence show that the corresponding angles in the triangles are equal.

14 Prove that drawing a triangle along the midpoints of an equilateral triangle splits that equilateral triangle into four congruent, equilateral triangles. (Hint: you will need to make use of your knowledge of angles around a straight line and the isosceles triangle theorem.)



15 Use the Internet to research other proofs involving triangles.

Reflect

How are angles in parallel lines useful in proofs involving triangles?

7F Proofs and quadrilaterals

Start thinking!

Different quadrilaterals have different properties, and you can use your understanding of angles in parallel lines and triangles to prove these properties.

Shape	Properties
Parallelogram	Opposite angles are equal Opposite sides are parallel and equal Diagonals **bisect** each other
Rhombus	All sides are equal and opposite sides are parallel Opposite angles are equal Diagonals bisect each other at right angles Diagonals bisect the interior angles
Square	All sides are equal and opposite sides are parallel All angles are right angles Diagonals are equal in length and bisect each other at right angles Diagonals bisect the interior angles
Rectangle	Opposite angles are equal Opposite sides are parallel and equal Diagonals are equal and bisect each other
Trapezium	One pair of opposite sides is parallel

1 For each quadrilateral:
   - **a** draw an example
   - **b** demonstrate that each property is true.

2 Use the table of properties to decide if each statement is true or false. If false, give a reason.
   - **a** A square is a rhombus.
   - **b** A **trapezium** is a parallelogram.
   - **c** A parallelogram is a rectangle.
   - **d** A rectangle is a parallelogram.
   - **e** A square is a parallelogram.
   - **f** A rhombus is a square.

3 How might you prove properties involving:
   - **a** angles?
   - **b** side lengths?
   - **c** diagonals?

4 Why do you think that congruence might be important in these proofs?

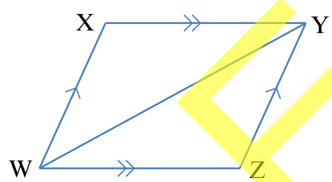
KEY IDEAS

- Quadrilaterals have many properties that can be shown using mathematical proofs.
- Understanding of angle and triangle properties is essential to these proofs.
- Bisect means to cut in half.
- The symbol for 'is parallel to' is $\parallel$.
- The symbol for 'is perpendicular to' is $\perp$.

EXERCISE 7F Proofs and quadrilaterals

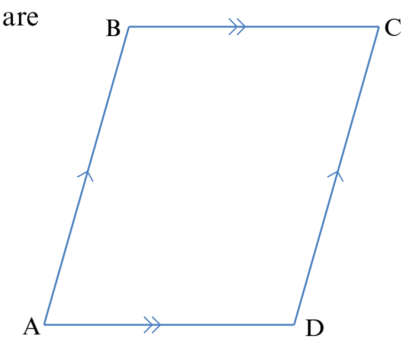
EXAMPLE 7F-1 Proving opposite sides of a parallelogram are equal

Prove that the opposite sides of a parallelogram are equal in length.

THINK	WRITE
1 Draw and label a parallelogram. Mark in a diagonal.	
2 Name the two triangles.	The two triangles are ΔWXY and ΔYZW.
3 Name the common side (the diagonal of parallelogram).	The common side is WY.
4 Use knowledge of angles and parallel lines to show angle equivalence.	∠XYW = ∠ZWY (alternate angles) ∠YWX = ∠WYZ (alternate angles)
5 Use a congruence condition to show the triangles are congruent.	ΔWXY ≅ ΔYZW (AAS)
6 Show that the opposite sides of the parallelogram are equal in length by matching corresponding sides of the congruent triangles.	WX = YZ because they are corresponding sides of congruent triangles. WZ = YX because they are corresponding sides of congruent triangles.
7 Complete the proof with a closing statement.	Therefore, the opposite sides of a parallelogram are equal in length.

UNDERSTANDING AND FLUENCY

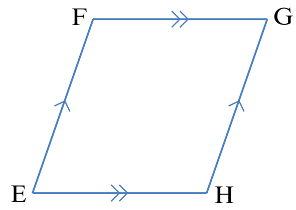
1 Prove that the opposite sides of this parallelogram are equal in length by following these steps.



a Copy the parallelogram and draw a diagonal between vertices A and C.
b Name the two triangles formed by the diagonal AC.
c Name the side common to both triangles.
d Use an understanding of angles and parallel lines to show angle equivalence.
e Use a congruence condition to show the triangles are congruent.
f Show that the opposite sides of the parallelogram are equal in length by matching corresponding sides of the congruent triangles.

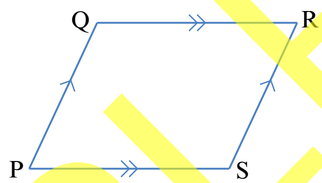
2 Draw the parallelogram LMNO and prove that its opposite sides are equal in length.

3 If FG = EF, prove that this shape is a rhombus.



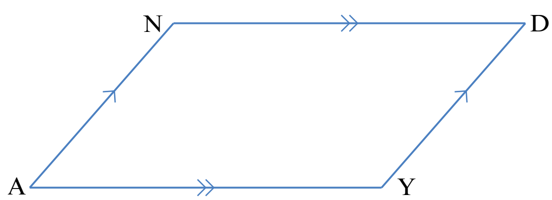
EXAMPLE 7F-2 Proving opposite angles in a parallelogram are equal

Prove that the opposite angles of a parallelogram are equal in size.

THINK	WRITE
1 Draw and label a parallelogram.	
2 Use an understanding of angles and parallel lines to write statements about co-interior angles.	∠QPS + ∠PSR = 180° (co-interior angles) ∠QPS + ∠PQR = 180° (co-interior angles)
3 Equate the left sides of the equations.	∠QPS + ∠PSR = ∠QPS + ∠PQR
4 Simplify by subtracting the same angle from both sides.	∠PSR = ∠PQR
5 Repeat steps **2–4** for the other pair of opposite angles.	Similarly, ∠PSR + ∠QRS = 180° (co-interior angles) ∠PSR + ∠QPS = 180° (co-interior angles) ∠PSR + ∠QRS = ∠PSR + ∠QPS ∠QRS = ∠QPS
6 Complete the proof with a closing statement.	Therefore, the opposite angles of a parallelogram are equal in size.

UNDERSTANDING AND FLUENCY

4 Prove that the opposite angles of this parallelogram are equal by following these steps.



a Use an understanding of angles and parallel lines to write statements about co-interior angles.

b Equate the left sides of the equations and simplify the resulting equation by subtracting the same angle from both sides.

c Repeat steps a and b to show equivalence for the other angle pair.

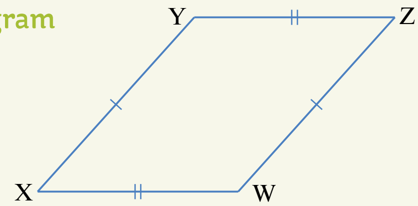
d Complete the proof with a closing statement.

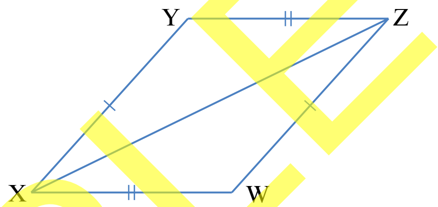
5 Draw the parallelogram MARK and prove that its opposite angles are equal.

6 Draw the rhombus ISAC and prove that its opposite angles are equal.

EXAMPLE 7F-3 Proving a shape is a parallelogram

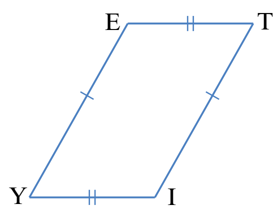
Prove that XYZW is a parallelogram.



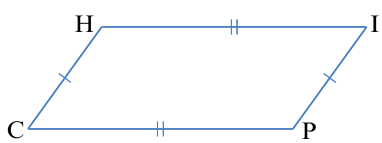
THINK	WRITE
1 Copy the figure and include a diagonal.	
2 Show that the two triangles formed are congruent.	XY = WZ YZ = WX XZ is common side ΔXWZ ≅ ΔZYX (using SSS)
3 Match the corresponding angles in these congruent triangles.	∠WXZ = ∠YZX ∠XZW = ∠ZXY
4 If alternate angles are equal, then the lines must be parallel. Note: ‖ means 'is parallel to'.	YZ ‖ WX because the alternate angles ∠WXZ and ∠YZX are equal. XY ‖ ZW because the alternate angles ∠XZW and ∠ZXY are equal.
5 Use the definition of a parallelogram to prove that WXYZ is a parallelogram.	WXYZ is a parallelogram because opposite sides are parallel and equal in length.

UNDERSTANDING AND FLUENCY

7 Prove that YETI is a parallelogram by following these steps.
   a Copy the figure and include a diagonal.
   b Show that the two triangles formed are congruent.
   c Match the corresponding angles in these congruent triangles.
   d If alternate angles are equal, then the lines must be parallel.
   e Use the definition of a parallelogram to prove that YETI is a parallelogram.

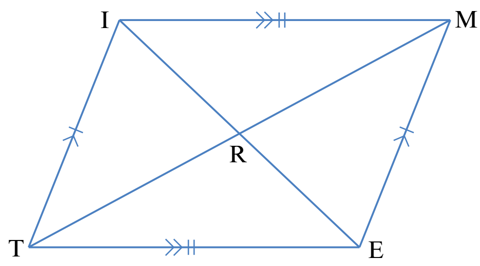


8 Prove that CHIP is a parallelogram.



9 Prove that a quadrilateral ABCD with four equal side lengths is a rhombus.

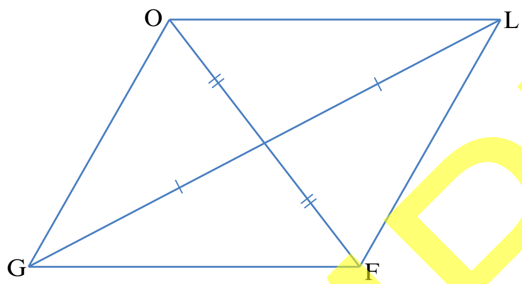
10 Consider this parallelogram.



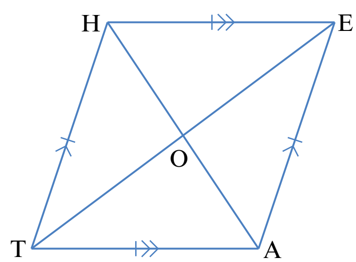
- a What angle is equal to ∠IMR?
- b What angle is equal to ∠MIR?
- c Show that ΔMIR ≅ ΔTER.
- d Copy the figure and mark corresponding sides for the congruent triangles.
- e Explain how this shows that the diagonals of the parallelogram TIME bisect each other.
- f Write the proof in full.

11 Draw a parallelogram and prove that its diagonals bisect each other.

12 Prove that this shape is a parallelogram.



13 Consider this rhombus.



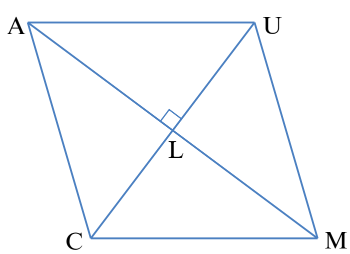
- a Follow the steps from question 10 to prove that the diagonals bisect each other.
- b Hence show that the four triangles are congruent to one another.
- c Redraw the rhombus with markings showing the corresponding sides.
- d Show that ∠HOE = ∠AOE.
- e Use your understanding of angles around a straight line to prove that the diagonals bisect each other at right angles.
- f Write the proof in full.

> **NOTE** Remember you can use the symbol ⊥ for 'is perpendicular to' or 'at right angles to'.

14 Draw a rhombus and prove that its diagonals bisect each other at right angles.

15 Prove that this is a rhombus if:

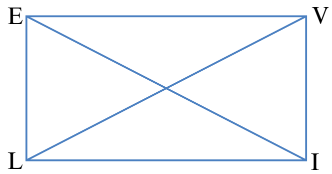
- a AL = ML and CL = UL
- b AU || MC and AC || MU.



16 Prove that parallelogram REMY is a rectangle if ∠REM = 90°.

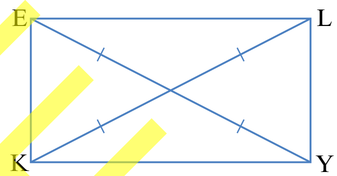
PROBLEM SOLVING AND REASONING

17 Consider figure LEVI, where EV = IL, EV || IL and ∠EVI = 90°.



a Prove that the remaining angles are 90°.
b Prove that EL = IV and EL || IV.
c Use Pythagoras' Theorem to show that LV = EI.
d Hence prove that ΔLEV ≅ ΔELI.

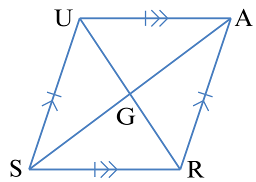
18 Prove that this shape is a rectangle by following these steps.



a Use the isosceles triangle theorem and what you know about equal alternate angles to first prove that it is a parallelogram.
b Use ∠LYE + ∠YEL + ∠ELK + ∠KLY to prove that ∠ELY = 90°.
c Prove that all interior angles are 90°.

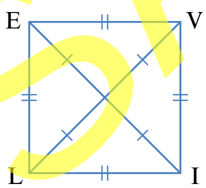
19 Draw the quadrilateral DANE where the diagonals DN and AE are equal and bisect each other. Prove that it is a rectangle.

20 Consider this rhombus.



a Prove that UG = RG and AG = SG (that the diagonals bisect each other).
b Prove that ΔUAG ≅ ΔRAG.
c Hence show that ∠UAS = ∠RAS.
d Explain why this shows that the diagonals of a rhombus bisect the interior angles.

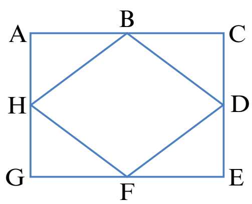
21 Prove that ∠EVL = 45°.



22 What property of rhombuses (including squares) means that the diagonals bisect the internal angles?

CHALLENGE

23 ACEG is a rectangle. If the vertices of HBDF touch the midpoints of each of the rectangle's sides, prove that HBDF is a rhombus.



Reflect

How are congruent triangles useful in proofs involving quadrilaterals?

7G Circle geometry: circles and angles

Start thinking!

There are many different theorems involving circles that allow you to solve problems. Before working with these theorems, it is important you understand the special terms used in circle geometry.

1 Consider each term in this list:
arc, centre, **chord**, **circumference**, **diameter**, **radius**, **sector**, **segment**
  a Look them up in the glossary and write a definition.
  b Match each term with one of the pronumerals, *a*–*h*, in figures 1 and 2.

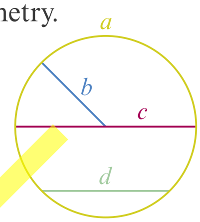


Figure 1

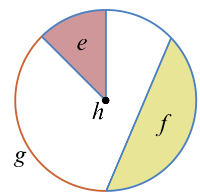


Figure 2

2 Consider figure 3.
  a Name the two chords.
  b Name the minor arc formed by the two chords.
  c Name the angle formed by the two chords.
  In circle geometry, you say that the angle is subtended by the arc, rather than saying that an angle is formed by two chords.
  d Copy and complete this sentence.
  The arc _____ subtends the angle _______.
  e Imagine that there is a chord AC. Copy and complete this sentence about it:
  The chord AC subtends the angle _______.

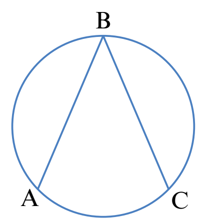


Figure 3

3 Explain what 'subtend' means.

4 Why is it important to be familiar with the terms used in circle geometry before carrying out any proofs?

KEY IDEAS

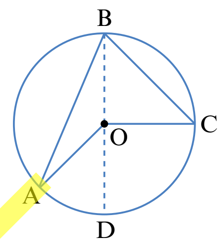
- Understanding the terms arc, centre, chord, circumference, diameter, radius, sector and segment is important in circle geometry. If necessary, refer to the glossary.
- An angle is subtended by the chord or arc that connects the endpoints of the two chords that form the angle.
- Theorem 1: the angle subtended at the centre of the circle is twice the size of the angle on the circumference subtended by the same arc.
- Theorem 2: any angle subtended by the diameter is a right angle.
- Theorem 3: all angles at the circumference of a circle that are subtended by the same arc are equal in size.

EXERCISE 7G Circle geometry: circles and angles

UNDERSTANDING AND FLUENCY

1 Draw an example of a circle with each of these.
  a a chord XY
  b an arc between points S and T
  c the major segment formed by the chord UP
  d the minor sector formed by the radii OM and ON
  e the arc that subtends ∠XYZ
  f a chord that subtends ∠DEF

2 Consider this diagram.

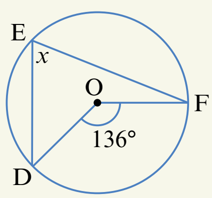


  a How do you know that AO = BO = CO?
  b How do you know that ∠ABO = ∠OAB?
  c Use the theorem for the exterior angle of a triangle to show that ∠AOD = 2∠ABO.
  d Use your answers to parts b and c to show that ∠DOC = 2∠OBC.
  e Explain how this shows that ∠AOC = 2∠ABC.

This question demonstrates Circle Theorem 1:
the angle subtended at the centre of the circle is twice the size of the angle on the circumference subtended by the same arc.

EXAMPLE 7G-1 Using Circle Theorem 1

Find the value of x in this circle.



THINK

1 Using Theorem 1 means that ∠DOF = 2∠DEF.

2 Write your answer.

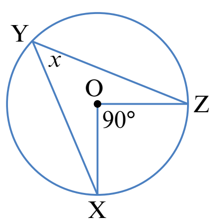
WRITE

$136° = 2x$

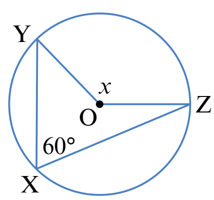
$x = 68°$

3 Find the value of x in each circle.

a



b



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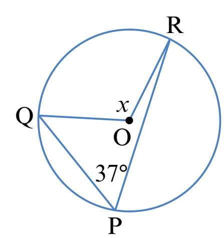
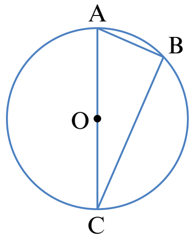


4 Consider this circle.



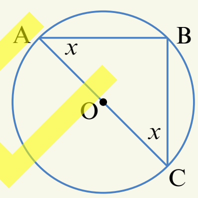
This question demonstrates Circle Theorem 2:
any angle subtended by the diameter is a right angle.

a What is the angle at the centre of the circle?

b Use Theorem 1 to show that ∠ABC = 90°.

EXAMPLE 7G-2 Using Circle Theorem 2

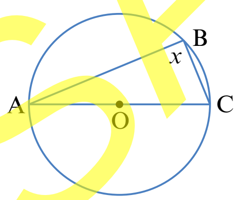
Find the value of x in this circle.



THINK	WRITE
1 Using Theorem 2, ∠ABC is a right angle.	∠ABC = 90°
2 Use the angle sum of a triangle to write an equation to find the value of x.	$2x + 90° = 180°$
3 Solve the equation and write your answer.	$2x = 90°$ $x = 45°$

5 Find the value of x in each circle.

a



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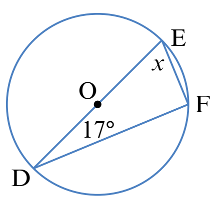

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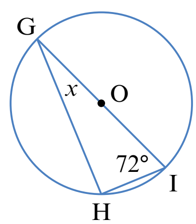
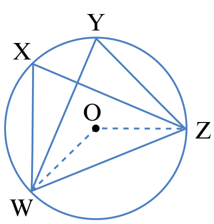


6 Consider this circle.



This question demonstrates Circle Theorem 3:
All angles at the circumference of a circle that are subtended by the same arc are equal in size.

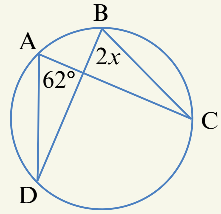
a Use Theorem 1 to explain why ∠WOZ = 2∠WXZ.

b Similarly, show that ∠WOZ = 2∠WYZ.

c What can you say about ∠WYZ and ∠WXZ?

EXAMPLE 7G-3 Using Circle Theorem 3

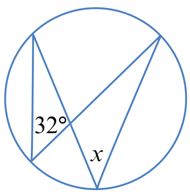
Find the value of x in this circle.



THINK	WRITE
1 Using Theorem 3 means that ∠DBC and ∠DAC are equal in size.	∠DBC = ∠DAC
2 Substitute values and solve for x.	$2x = 62°$ $x = 31°$

7 Find the value of each pronumeral.

a



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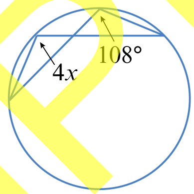

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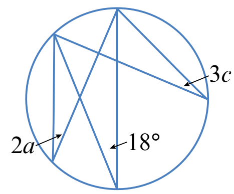
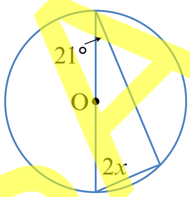


PROBLEM SOLVING AND REASONING

8 Find the value of each pronumeral, giving a reason.

a



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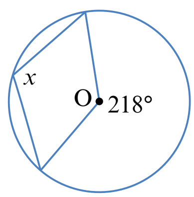

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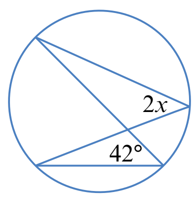

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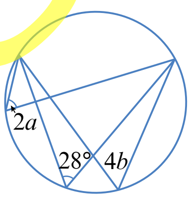

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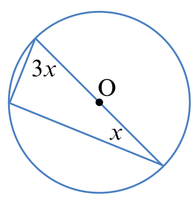

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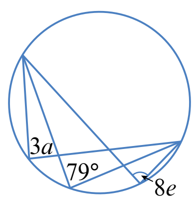

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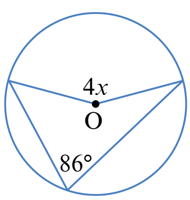

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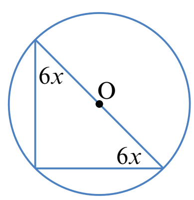

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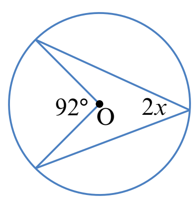
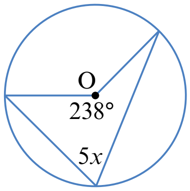


9 Use your understanding of triangle and circle properties and theorems to find the value of x.

a



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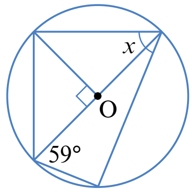

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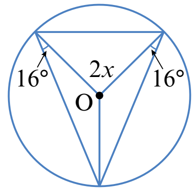

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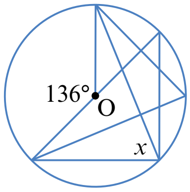

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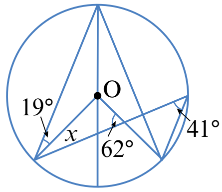

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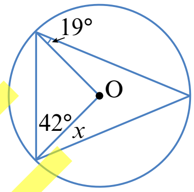
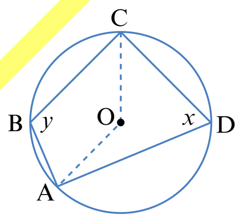


10 Consider this diagram, showing a cyclic quadrilateral (a quadrilateral that has each of its vertices on the circumference of a circle).



a Explain why the obtuse angle $\angle AOC = 2x$ and the reflex angle $\angle AOC = 2y$.

b Use angles around a point to explain why the reflex $\angle AOC = 360° - 2x$.

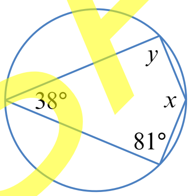
c Use your answers from parts **a** and **b** to show that $y = 180° - x$.

d Copy the figure, replacing radii OC and OA with OB and OD, and use this to show that $\angle BAD = 180° - \angle BCD$.

e Find the value of each pronumeral.

> This question demonstrates Circle Theorem 4:
> the opposite angles of a cyclic quadrilateral are supplementary.

i



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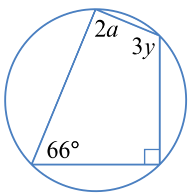
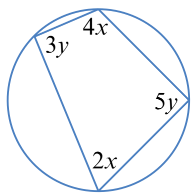
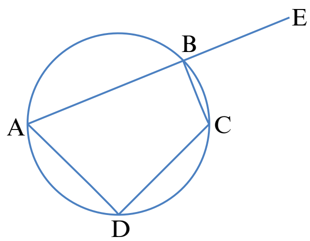


iii



11 Consider this diagram, showing another cyclic quadrilateral.



a Use Theorem 4 to explain why $\angle ADC = 180° - \angle ABC$.

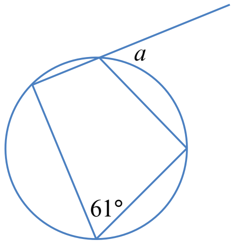
b Why does $\angle EBC = 180° - \angle ABC$?

c Explain why this shows that $\angle ADC = \angle EBC$.

> This question demonstrates Circle Theorem 5:
> the exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

d Find the value of each pronumeral.

i



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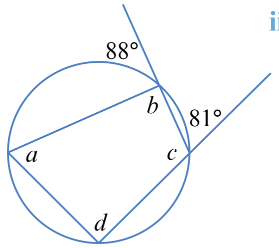

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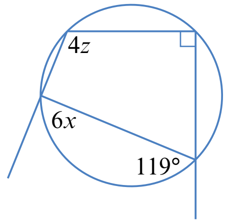
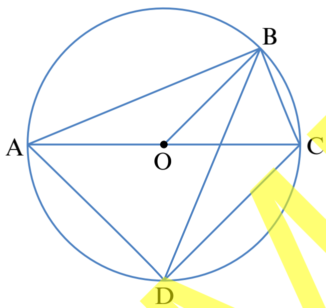


CHALLENGE

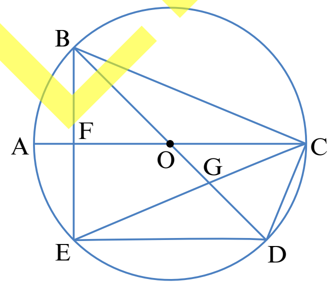
12 Write a complete proof for each of the Circle Theorems, 1 to 5.

13 Use the given information to answer these.

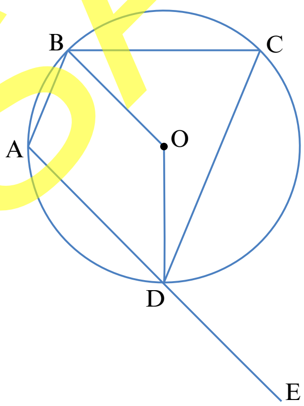
a Find ∠DBC, given that ∠ADB = 68°, OA = OB = OC and ∠BCD = 92°.



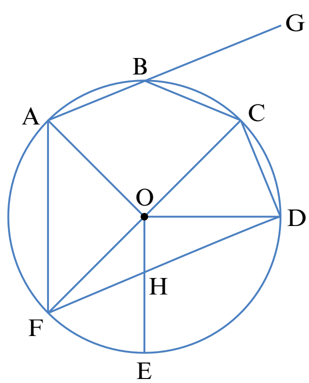
b Find ∠DEG, given that BE ⊥ AC, ∠OCG = 19° and ∠FOG = 142°.



c Find ∠ODC, given that ∠BAD = 96° and ∠CBO = 46°.



d Find ∠OAB, given that ∠GBC = 46°, ∠BCD = 138° and ∠AFD = 70°.



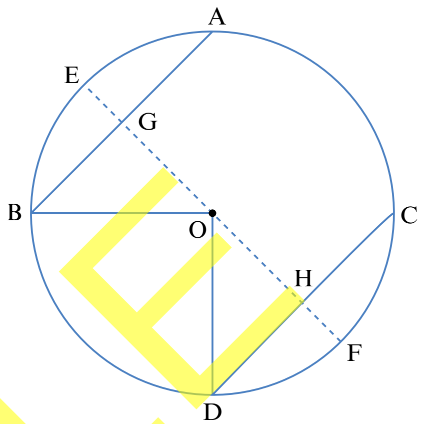
14 Use dynamic geometry software or other digital technology to demonstrate each theorem.

Reflect

How are all the proofs in this section reliant on triangle properties?

7H Circle geometry: chords

Start thinking!



Consider this diagram, ignoring the dashed line to begin with.

1 Name the two chords shown.

2 Name the two radii shown.

3 The two radii are perpendicular to one another. What does this mean?

4 Name the arc that subtends ∠BOD.

Now consider the dashed line.

5 Name the dashed line and describe it using circle terminology.

6 EF bisects AB and DC. What does this mean?

GO = OH. This means that CD and AB are **equidistant** from the centre of the circle.

7 What does equidistant mean?

8 Explain how you know CD and AB are equidistant from the centre of the circle.

9 What else can you describe from this diagram?

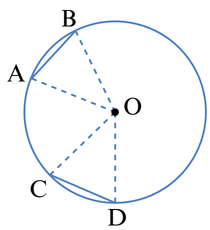
KEY IDEAS

- A chord is a straight line from one point on the circumference of a circle to another.
- Theorem 6: chords that are equal in length subtend equal angles at the centre of the circle
- Theorem 7: if a radius and a chord intersect perpendicularly, then the radius bisects the chord.
- Theorem 8: chords that are equal in length are equidistant from the circle centre.
- Theorem 9: when two chords intersect inside a circle, this divides each chord into two line segments in which the product of the lengths of the line segments for both chords is the same.

EXERCISE 7H Circle geometry: chords

UNDERSTANDING AND FLUENCY

1 Consider this diagram, where AB = CD.



> This question demonstrates Circle Theorem 6: chords that are equal in length subtend equal angles at the centre of the circle.

a Given that OA, OB, OC and OD are radii, what can you say about them?

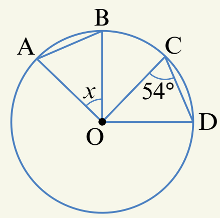
b What type of triangle does this mean ΔOAB and ΔOCD are?

c Explain why ΔOAB ≅ ΔOCD.

d Explain why ∠COD = ∠AOB.

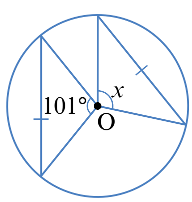
EXAMPLE 7H-1 Using Circle Theorem 6

Find the value of x in this circle if AB = CD.



THINK	WRITE
1 Since OC and OD are radii, they have equal length.	OC = OD
2 This means that ΔCOD is an isosceles triangle so you can find the other two angles within the triangle.	ΔCOD is isosceles, so ∠OCD = ∠ODC = 54° ∠COD = 180° − 54° − 54° = 72°
3 Use Theorem 6 to identify the equal angles at the centre of the circle.	∠AOB = ∠COD = 72°
4 Write your answer.	x = 72°

2 Find the value of x in each circle.

a


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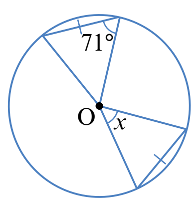
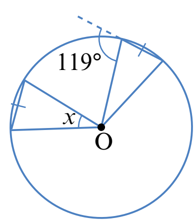
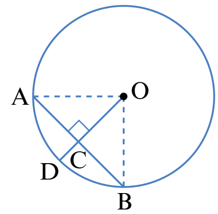


c


3 Consider this diagram.

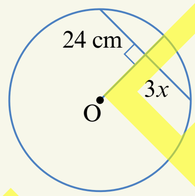
- **a** Explain why OA = OB.
- **b** Explain why ∠OCA = ∠OCB.
- **c** Use a triangle congruency condition to explain why ΔACO ≅ ΔBCO.
- **d** Explain why AC = CB.



This question demonstrates Circle Theorem 7:
if a radius and a chord intersect perpendicularly, then the radius bisects the chord.

EXAMPLE 7H-2 Using Circle Theorem 7

Find the value of x in this circle.

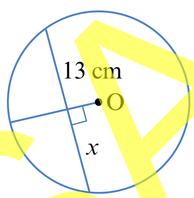


THINK	WRITE
1 The radius and chord intersect at right angles. Using Theorem 7, the length $3x$ is the same as 24 cm.	$3x = 24$ cm
2 Solve for x.	$x = 8$ cm

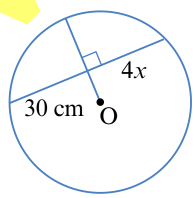
UNDERSTANDING AND FLUENCY

4 Find the value of x in each circle.

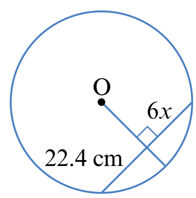
a



b

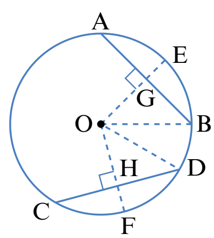


c



5 Consider this diagram, where AB = CD.

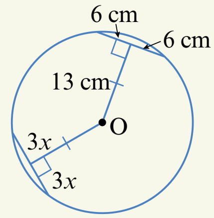
- **a** Explain why OE = OB = OD = OF.
- **b** Use Theorem 7 to explain why AG = GB and CH = HD.
- **c** Likewise, use Theorem 7 and the fact that AB = CD to explain why GB = HD.
- **d** Use a triangle congruency condition to explain why ΔOHD ≅ ΔOGB.
- **e** Explain why OH = OG.
- **f** Use your understanding of Pythagoras' Theorem to explain why the shortest distance from the centre of a circle to a chord is perpendicular to that chord.



This question demonstrates Circle Theorem 8:
chords that are equal in length are equidistant from the circle centre.

EXAMPLE 7H-3 Using Circle Theorem 8

Find the value of x in this circle.



THINK

1 The chords are equidistant from the centre circle. Using Theorem 8, the chords must be equal in length.

2 Solve for x and include the length unit.

WRITE

$3x + 3x = 6 + 6$

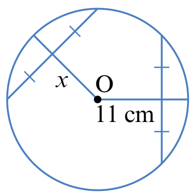
$6x = 12$

$x = 2$ cm

UNDERSTANDING AND FLUENCY

6 Find the value of each pronumeral.

a



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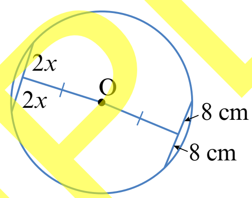

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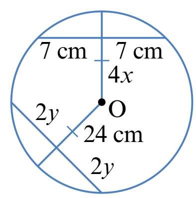
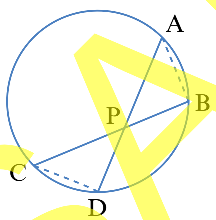


7 Consider this diagram.



a Explain why ∠CPD = ∠APB.

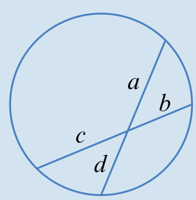
b Use Theorem 3 to explain why ∠DCB = ∠DAB.

c Explain why ∠PDC = ∠PBA.

d Explain how you can tell that ΔCPD is similar to ΔAPB.

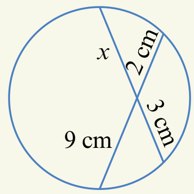
e Explain why you can state that $\frac{AP}{BP} = \frac{CP}{DP}$ and hence that $AP \times DP = CP \times BP$.

This question demonstrates Circle Theorem 9: when two chords intersect inside a circle, this divides each chord into two line segments in which the product of the lengths of the line segments for both chords is the same. In the figure below, $a \times d = c \times b$.



EXAMPLE 7H-4 Using Circle Theorem 9

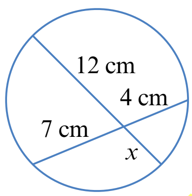
Find the value of x in this circle.



THINK	WRITE
1 Write the product of the lengths of the two line segments for each chord.	chord 1: $x \times 3$ cm chord 2: 2 cm × 9 cm
2 Use Theorem 9 to write an equation.	$x \times 3 = 2 \times 9$
3 Simplify the equation.	$3x = 18$
4 Solve for x and include the length unit.	$x = 6$ cm

8 Find the value of x in each circle, correct to one decimal place.

a



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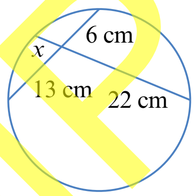

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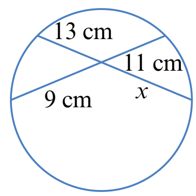
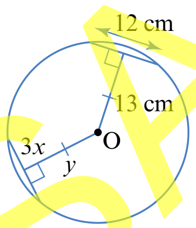


PROBLEM SOLVING AND REASONING

9 Find the value of each pronumeral, giving a reason.

a



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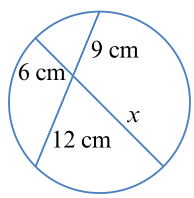

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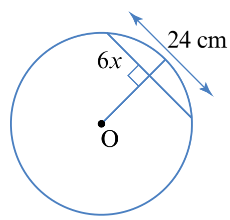

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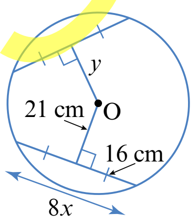

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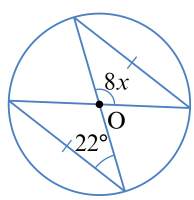

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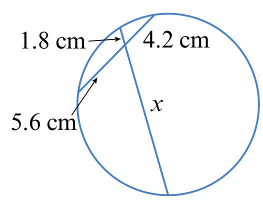

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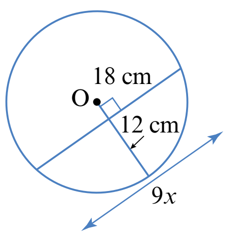

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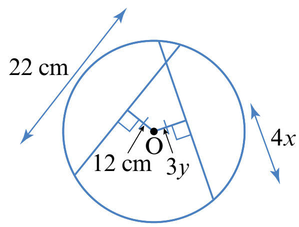

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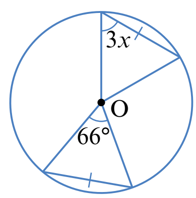
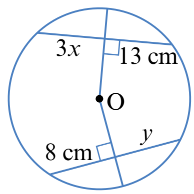


PROBLEM SOLVING AND REASONING

10 Use your understanding of circle properties and theorems, and any other necessary geometric properties, to find the value of each pronumeral.

a



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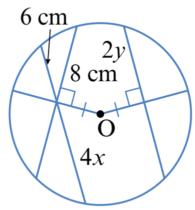

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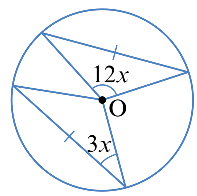

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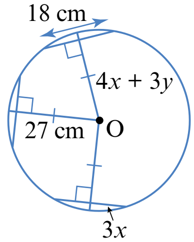

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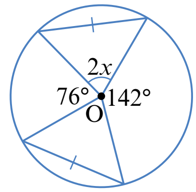

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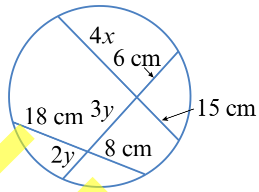
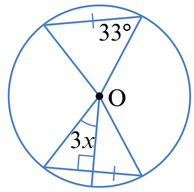


11 Find the value of each pronumeral.

a



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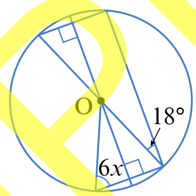

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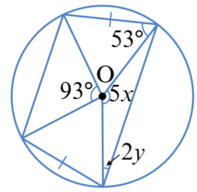

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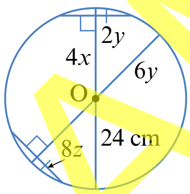

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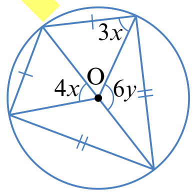

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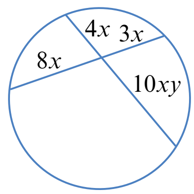


12 Draw a circle with two chords. Show that constructing perpendicular bisectors of these two chords locates the centre of the circle.

CHALLENGE

13 Write a complete proof for each theorem in this exercise.

14 Use dynamic geometry software or other digital technology to demonstrate each theorem.

15 For each theorem in this exercise, create a problem to solve that involves the use of:

a simultaneous equations

b Pythagoras' Theorem (where possible).

16 Use the Internet to investigate the alternate segment theorem. Write out a proof of what you find. Include an example of a problem and solution in your findings.

Reflect

How is the centre of the circle important in many circle theorems?

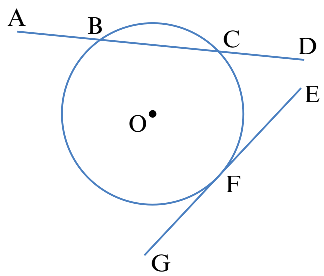
7I Circle geometry: tangents and secants

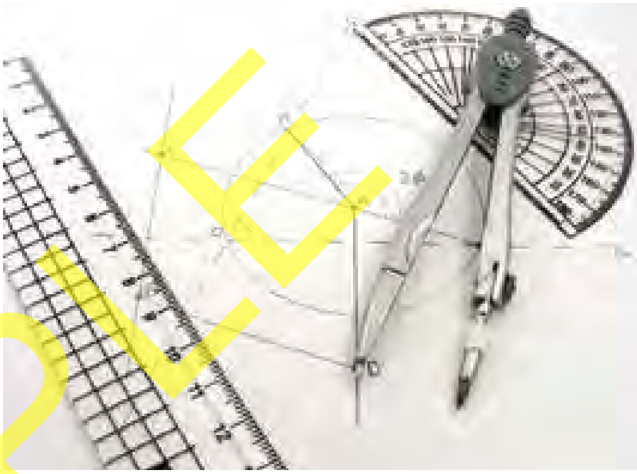
Start thinking!

Two important concepts that have not yet been covered are the **tangent** and the **secant**.

1 Use your glossary or other means to write the definitions of these terms.

2 Name the tangent and the secant on this diagram.





3 At what point does the tangent touch the circumference of the circle?

4 Name the points where the secant cuts the circle.

5 Explain the difference between a chord, a secant, a tangent and an arc.

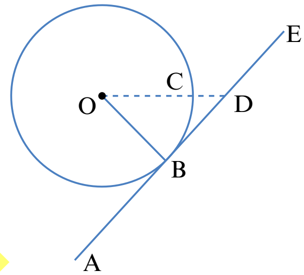
KEY IDEAS

- A secant is a line that cuts a circle twice.
- A tangent is a line that touches the circumference of a circle at one point only.
- Theorem 10: a tangent drawn at the same point to a radius will be perpendicular to that radius.
- Theorem 11: if two tangents intersect outside a circle, the distances along the tangent from the intersection to the circumference of the circle are equal.
- Theorem 12: if two secants intersect outside a circle, the products of the entire secant length by the external secant length will be the same.
- Theorem 13: if a tangent and a secant intersect outside a circle, the product of the entire secant length by the external secant length will be equal to the square of the tangent length.

EXERCISE 7I Circle geometry: tangents and secants

UNDERSTANDING AND FLUENCY

1 Consider this diagram, showing the intersection of a radius and a tangent.



a Name the two radii and explain why they are equal.

b Name the tangent at point B.

You may guess that ∠OBD = 90° and, to prove this, you instead imagine that another angle, in this case ∠ODB, is 90°.

c Which side would be the hypotenuse of ΔODB?

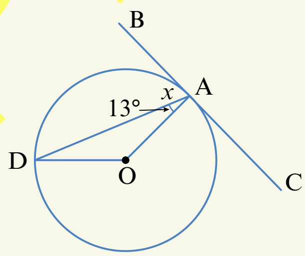
d This would mean that OB > OD. Why does this not make sense?

e How does this prove that ∠OBD = 90°?

This question demonstrates Circle Theorem 10: a tangent drawn at the same point to a radius will be perpendicular to that radius.

EXAMPLE 7I-1 Using Circle Theorem 10

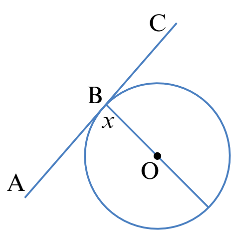
Find the value of x in this diagram.



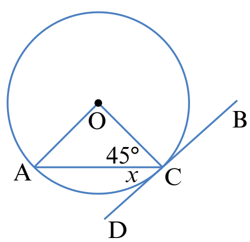
THINK	WRITE
1 Using Theorem 10 means that the angle between the tangent BC and the radius OA is 90°.	∠OAB = 90°
2 Use your understanding of complementary angles to find x.	$x = 90° - 13°$ $= 77°$

2 Find the value of x in each circle.

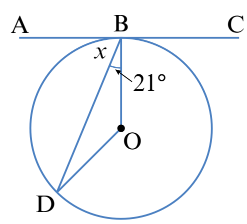
a



b



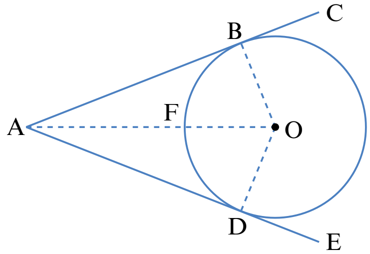
c



10A

UNDERSTANDING AND FLUENCY

3 Consider this diagram, showing two tangents intersecting outside a circle.

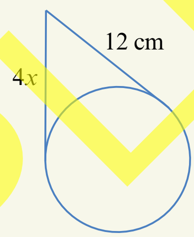


> This question demonstrates Circle Theorem 11:
> if two tangents intersect outside a circle, the distances from the intersection to the circumference of the circle are equal.

- **a** How do you know that ∠ABO = ∠ADO = 90°?
- **b** Explain why OB = OD.
- **c** Explain why ΔABO ≅ ΔADO.
- **d** Explain why this means that AB = AD.

EXAMPLE 7I-2 Using Circle Theorem 11

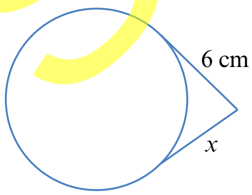
Find the value of x in this diagram.



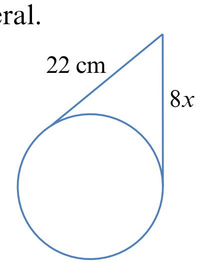
THINK	WRITE
1 Using Theorem 11 means that the distances from the intersection of the two tangents to the circumference of the circle are equal. Write this as an equation.	$4x = 12$
2 Solve for x and include the length unit.	$x = 3$ cm

4 Find the value of each pronumeral.

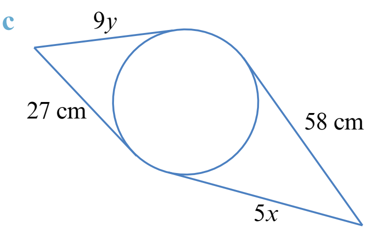
a



b

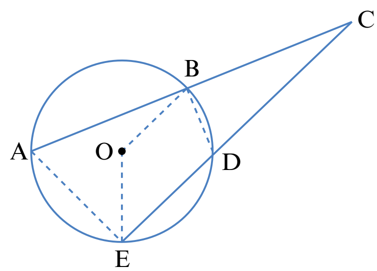


c



5 Consider this diagram, showing two secants intersecting outside a circle.

- **a** Explain why ∠BOE = 2∠BAE.
- **b** Explain why the reflex angle ∠BOE = 360° − 2∠BAE.
- **c** Explain how you know ∠BDE = 180° − ∠BAE.



d Hence explain why ∠BDC = ∠BAE.

Now consider ΔDCB and ΔACE.

e What angle do they have in common?

f What pair of angles have you already shown are equal?

g Explain why these triangles must therefore be similar.

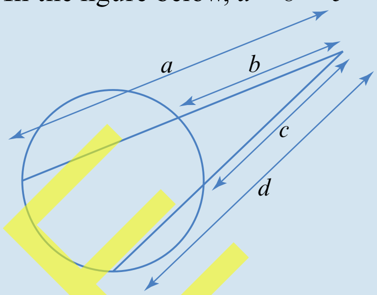
h Show that $\frac{CA}{CD} = \frac{EC}{BC}$.

i Explain why this can also be written as AC × BC = CE × CD.

This question demonstrates Circle Theorem 12:

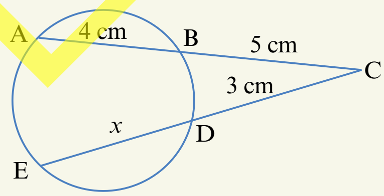
if two secants intersect outside a circle, the products of the entire secant length by the external secant length will be the same.

In the figure below, $a \times b = c \times d$.



EXAMPLE 7I-3 Using Circle Theorem 12

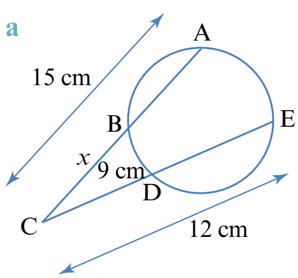
Find the value of x in this diagram.



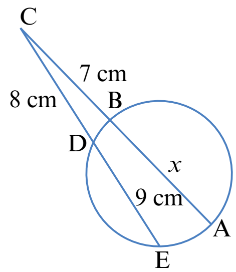
THINK	WRITE
1 Using Theorem 12, write the relationship between the secant lengths.	AC × BC = CE × CD
2 Write the length of each line segment.	AC = 4 + 5 = 9 cm BC = 5 cm CE = $(3 + x)$ cm CD = 3 cm
3 Substitute these lengths into the relationship and simplify.	$9 \times 5 = (3 + x) \times 3$ $45 = 3(3 + x)$
4 Solve for x and include the length unit.	$15 = 3 + x$ $x = 12$ cm

6 Find the value of x in each diagram.

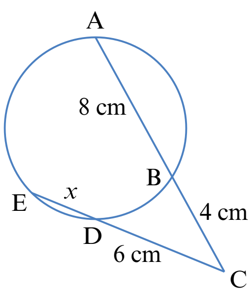
a



b

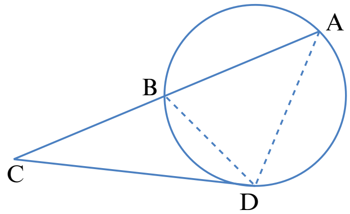


c



UNDERSTANDING AND FLUENCY

7 Consider this diagram, showing a tangent and a secant intersecting.



a Which angle do ΔBCD and ΔDAC have in common?

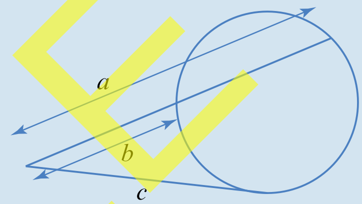
b Construct OD and OB, where O is the centre of the circle, and explain why ∠CAD = ∠CDB.

c Explain why ΔBCD and ΔDCA must be similar.

d Explain how you know that $\frac{CB}{CD} = \frac{CD}{CA}$.

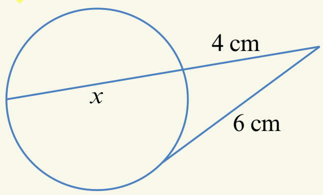
e Explain why this can be written as $CA \times CB = (CD)^2$.

This question demonstrates Circle Theorem 13: if a tangent and a secant intersect outside a circle, the product of the entire secant length by the external secant length will be equal to the square of the tangent length. In the figure below, $a \times b = c^2$.



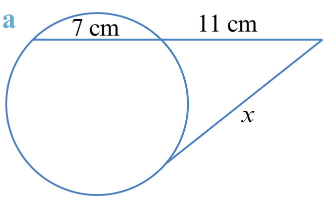
EXAMPLE 7I-4 Using Circle Theorem 13

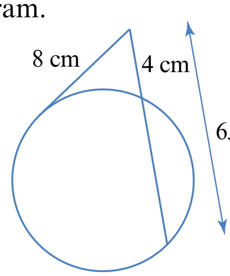
Find the value of x in this diagram.

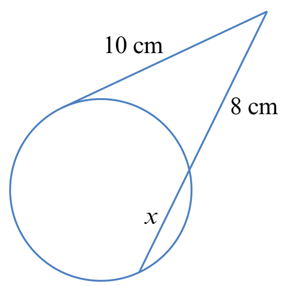


THINK	WRITE
1 Using Theorem 13, write the relationship for an intersecting secant and tangent.	$a \times b = c^2$
2 Write the length for each line segment.	$a = (x + 4)$ cm $b = 4$ cm $c = 6$ cm
3 Substitute into the formula and simplify.	$(x + 4) \times 4 = 6^2$ $4(x + 4) = 36$
4 Solve for x and include the length unit.	$x + 4 = 9$ $x = 5$ cm

8 Find the value of x in each diagram.

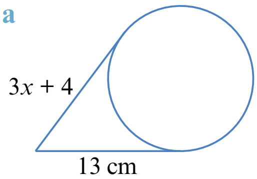
a


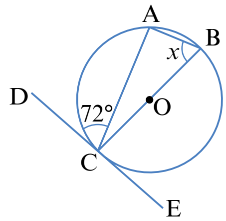
b


c


PROBLEM SOLVING AND REASONING

9 Find the value of each pronumeral, giving reasons.

a


b


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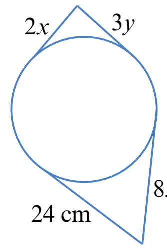
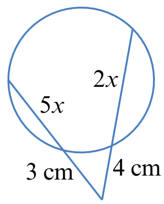
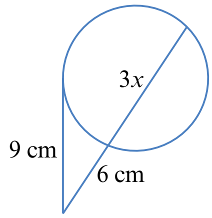
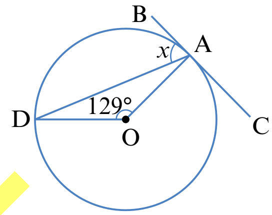
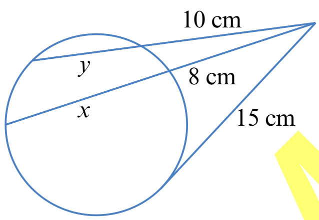


d


e


f


10 Use your understanding of any relevant circle theorems and geometric properties to find the value of each pronumeral, giving reasons.

a


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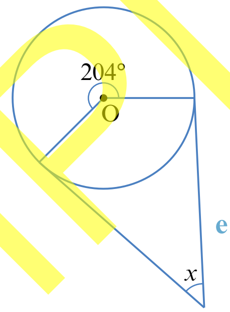
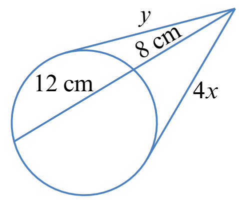


c


CHALLENGE

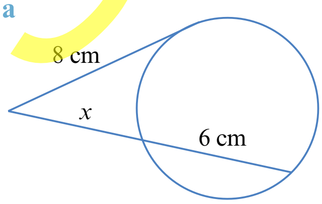
11 Write a complete proof for each theorem in this Exercise.

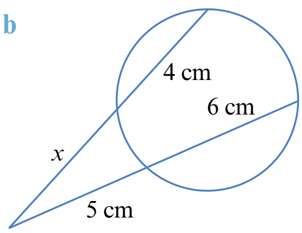
12 For each theorem in this Exercise, create a problem to solve that involves the use of:

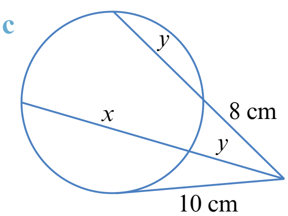
a simultaneous equations

b Pythagoras' Theorem (where possible).

13 Find the value of each pronumeral in these diagrams.

a


b


c


14 The diagram in question 3 shows two tangents that intersect outside a circle. Prove that a straight line drawn from the centre of the circle to the intersection point bisects the angle formed by the two tangents.

15 Use dynamic geometry software or other digital technology to demonstrate each theorem.

Reflect

What geometric properties do the circle theorems rely on?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

corresponding angles	regular polygon	trapezium	circle	diameter
alternate angles	quadrilateral	congruent figures	arc	subtended
co-interior angles	parallelogram	bisect	chord	cyclic quadrilateral
complementary angles	rectangle	similar figures	segment	equidistant
supplementary angles	square	scale factor	sector	tangent
vertically opposite angles	rhombus	axiom	circumference	secant
transversal	kite	proof	radius	line segment

MULTIPLE-CHOICE

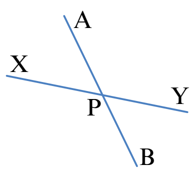
7A **1** What angle is supplementary to 25°?
A 75° B 155° C 65° D 335°

7B **2** ΔANP ≅ ΔGWK, where the order of the letters represents corresponding vertices of the two triangles. Which statement is correct?
A ∠APN = ∠KGW
B ∠PAN = ∠KWG
C ∠NPA = ∠GKW
D ∠NAP = ∠WGK

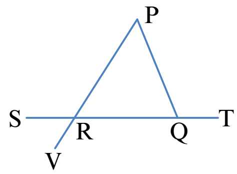
7B **3** Which of these is *not* a test for congruent triangles?
A AAA B SSS C SAS D AAS

7C **4** Which of these is *not* a test for similar triangles?
A AAA B SSS C SAS D AAS

7D **5** Straight lines AB and XY intersect at P. Given that ∠APY = 2∠APX, which statement is true?
A ∠BPX = 2∠APY
B ∠XPB = 2∠APX
C ∠YPB = 2∠APX
D ∠APX = 2∠BPY



7E **6** Which statement is true?



A ∠RPQ = ∠RQP
B ∠PRS = ∠PQT
C ∠TQP = ∠PRQ + ∠RQP
D ∠RPQ + ∠PQR = ∠SRP

7F **7** Which one is *not* a parallelogram?
A square B kite
C rhombus D rectangle

7G 10A **8** The angle in the centre of a circle and the angle at the circumference subtended by the same arc are complementary angles. Which values for the two angles are possible?
A 60° and 30° B 45° and 45°
C 90° and 45° D 120° and 60°

7H 10A **9** A circle contains two chords of the same length. Which of these is *not* true?
A The arcs on which they stand are the same length.
B The chords are the same perpendicular distance from the centre.
C The chords can not intersect.
D The chords subtend equal angles at the centre of the circle.

SHORT ANSWER

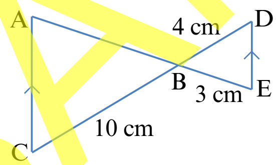
7A **1** Decide whether each statement is true or false. For any false statement, provide a reason.

- **a** The diagonals of a kite intersect at 90°.
- **b** It is not possible for a triangle to have a concave angle.
- **c** If a kite has four equal sides, it can be classified as a rhombus.
- **d** It would be possible to construct a triangle with side lengths of 3 cm, 7 cm and 4 cm.

7B **2** A rectangle ABCD has side lengths of 4 cm and 3 cm. Its diagonals intersect at the point P.

- **a** Draw a diagram to represent this rectangle.
- **b** Explain why ∠ABP is equal to ∠CDP, and ∠PAB is equal to ∠PCD.
- **c** Explain why ΔPAB ≅ ΔPCD.
- **d** If ∠BDC = 37°, find the size of ∠CBD.

7C **3** **a** Provide a reason why ΔABC is similar to ΔEBD.



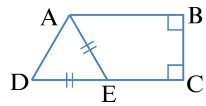
- **b** Consider ΔEBD to be the image of ΔABC. What is the scale factor?
- **c** What is the length of AB?

7D **4** ΔABC has an internal line drawn from the vertex B, meeting the opposite side AC at the point D. These are the facts known about this triangle.

∠ABD + ∠DBC = 90°
∠BAD + ∠BCD = 90°
∠BAD + ∠ADB + ∠ABD = 180°

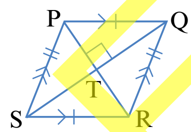
- **a** Draw a sketch of this triangle.
- **b** Use only the given information to prove that ∠ADB = ∠DBC + ∠BCD.

7E **5** Refer to this diagram and write a proof to show that:

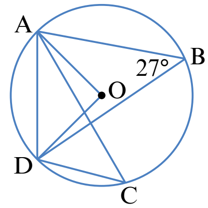


- **a** ∠AEC = 2∠ADE
- **b** 2∠ADE + ∠BAE = 180°

7F **6** Use the information in the diagram below to prove that figure PQRS is a rhombus.

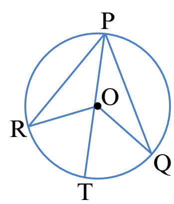


7G 10A **7** Determine the size of these angles.

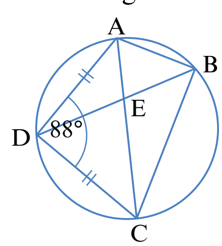


- **a** ∠AOD
- **b** ∠ACD
- **c** ∠OAD

7G 10A **8** Prove that 2∠RPO + 2∠QPO + ∠POR + ∠POQ = 360°



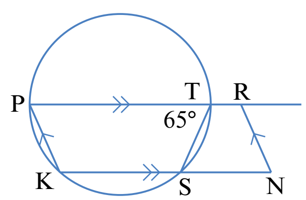
Questions **9** and **10** refer to this diagram.



7H 10A **9** Determine the size of ∠DBC.

7H 10A **10** Prove that ∠DBC = ∠ABD.

Questions **11** and **12** refer to this diagram.



7I 10A **11** Determine the size of ∠KPT.

7I 10A **12** Prove that ∠TSN = ∠RNS

MIXED PRACTICE

1 If ∠ABC + ∠CBD = 180° and ∠EBD + ∠DBC = 180°, write a relationship between ∠ABC and ∠EBD.

2 ΔADM ≅ ΔRWZ, where the order of the letters represents corresponding vertices of the two triangles. These details are known about ΔADM: ∠DAM = 90°, ∠AMD = 67°, AM = 5 cm, AD = 12 cm, DM = 13 cm

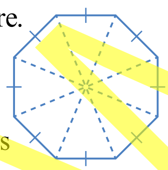
Use congruency to find these.

a RZ **b** ∠WRZ
c WZ **d** ∠RWZ.

3 ΔXGR is similar to ΔKFY, where the order of the letters represents corresponding vertices of the two triangles. Which of the following statements about the two triangles is true?

A ∠GRX = ∠FKY **B** $\frac{XG}{KF} = \frac{RX}{YF}$
C ∠RXG = ∠KFY **D** $\frac{FY}{GR} = \frac{KY}{XR}$

4 **a** Name the shape of this figure.
b Give the size of each internal angle.



There are eight smaller triangles within the figure.

c What is the size of the central angle of each of these triangles?
d Explain whether these triangles are similar.
e Are the triangles all congruent to each other? Give a reason for your answer.

5 Explain these statements.

a Two similar figures may also be congruent.
b Two congruent figures must also be similar.

6 If $a + b = 180°$, $b + c = 180°$ and $c + d = 180°$, which of these is true?

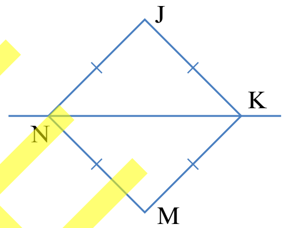
A $a + d = 180°$ **B** $a = b$
C $b = c$ **D** $a + c = 180°$

7 A rational number n can be represented in the form $\frac{x}{y}$, where $y \neq 0$, and x and y have no common factors. Explain whether the reciprocal of n is also a rational number.

8 A regular polygon has internal angles of 150°.

a Write a formula you could use to determine the number of sides in the polygon.
b Substitute into this formula to calculate the number of sides in the polygon.
c What is the name of this polygon?

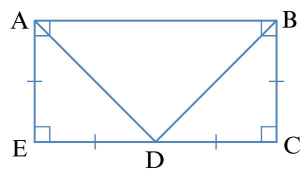
9 What would be the simplest congruency condition you could use to prove ΔNJK ≅ ΔNMK?



10 Which statement is *not* correct for two congruent triangles?

A the ratio of corresponding sides is 1
B corresponding sides are equal in length
C corresponding angles are equal in size
D they are not similar

Questions **11** and **12** refer to this diagram. This represents the back view of a rectangular envelope with the flap folded down. The envelope is twice as long as it is wide.

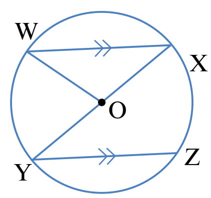


11 Using only the information provided in the diagram, which congruency condition could you use to prove ΔADE ≅ ΔBDC?

A RHS **B** SSS **C** SAS **D** AAS

12 Prove that ∠ADB is a right angle.

Questions **13** and **14** refer to this diagram.



10A **13** Find a relationship between ∠WOY and ∠OYZ.

10A **14** Which statement is incorrect?

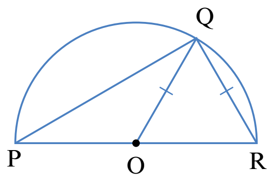
A OW = OX

B ∠XWO = ∠WXO

C ∠WOY = ∠OWX

D WX = YZ

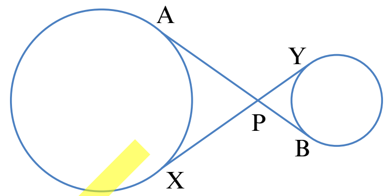
Questions **15** and **16** refer to this diagram.



10A **15** Determine the size of ∠PQO.

10A **16** State the relationship between ∠POQ and ∠OQR.

10A **17** AB and XY are tangents to the two circles. Prove that AB = XY.



10A **18** Prove that, if a cyclic quadrilateral is a parallelogram, then it must be a rectangle.

ANALYSIS

Congruent and similar figures are everywhere in everyday life. These figures can create quite pleasing, and often unusual, images.

1 Car logos

Here is a stylised image of the logo displayed on Volkswagen cars.

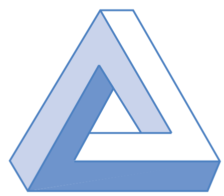
a How many sets of parallel lines are in the drawing of the logo?

b The angles within the V and the W are all equal in size. Explain how you know this is so.

c The logo displays vertical symmetry. Does this mean that there are congruent shapes within the letters? Explain.

d Are there any similar shapes within the letters?

2 Art



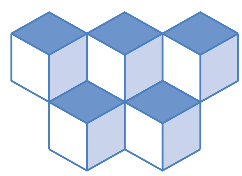
This Penrose triangle is said to be 'impossible' because it cannot be constructed as a 3D object. Appropriate shading creates the perception of the 3D image. (Use an Internet search to see a 3D sculpture of this triangle in Perth, Western Australia. There is a trick, however!)

a Trace the figure, and cut around the edges.

b Cut out the three shaded shapes.

c Describe the congruence of these shapes, and explain how they fit together.

3 Quilting



The patterns in quilting are structured around congruent figures.

a How many different congruent shapes are shown in this pattern?

b Describe the shape/s.

c Copy the pattern, and extend it in all directions.

CONNECT

Euclid's elements

Euclid was a Greek mathematician who lived around 300 BC. He is sometimes called the 'Father of Geometry'. His most famous and influential work was his series of 13 books, Elements, in which he set out definitions, axioms, theorems and proofs of the theorems.

Your task

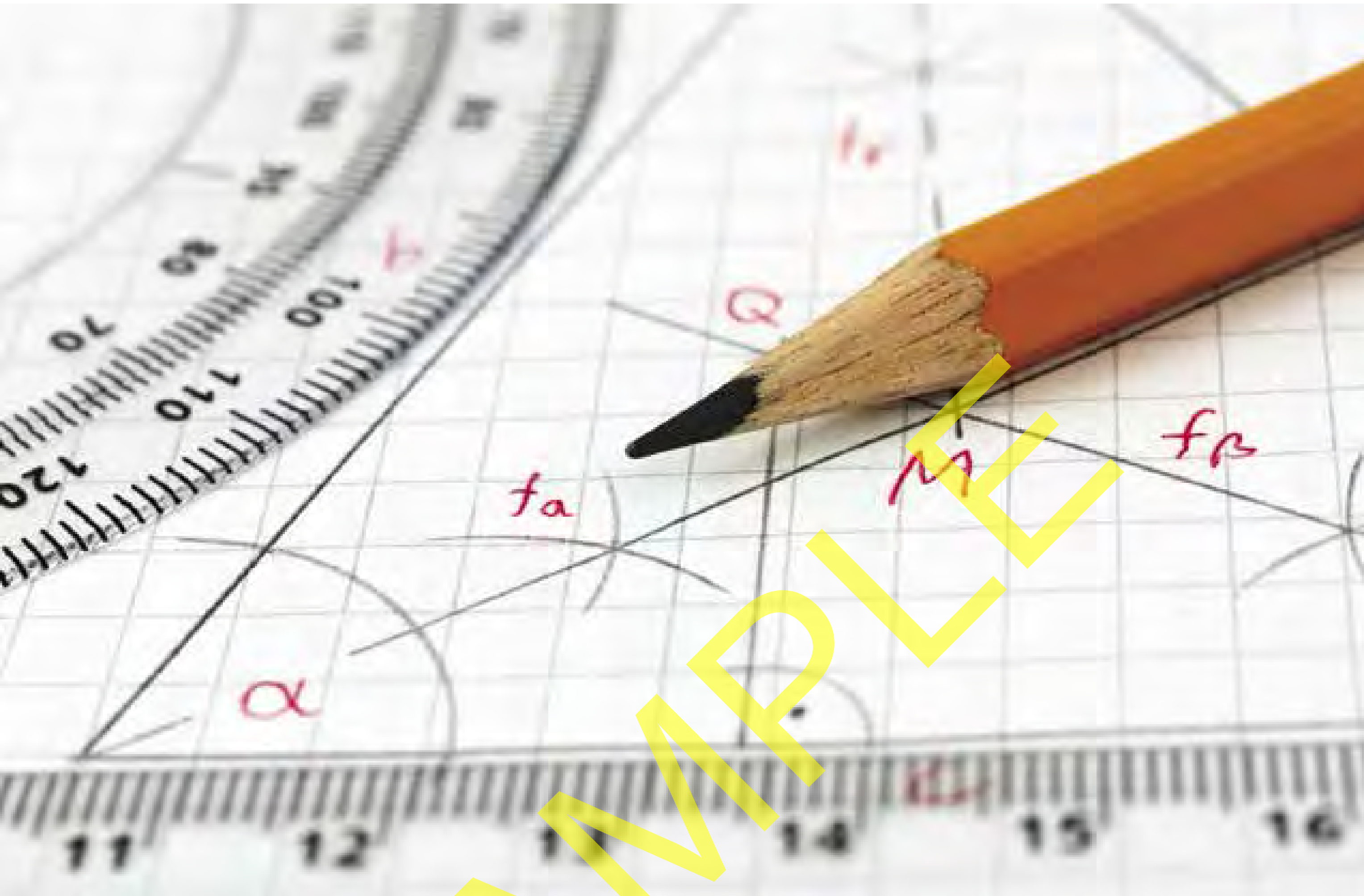
Select five different proofs from Euclid's elements and use your own examples to demonstrate them. To complete this task you will need to follow these steps.

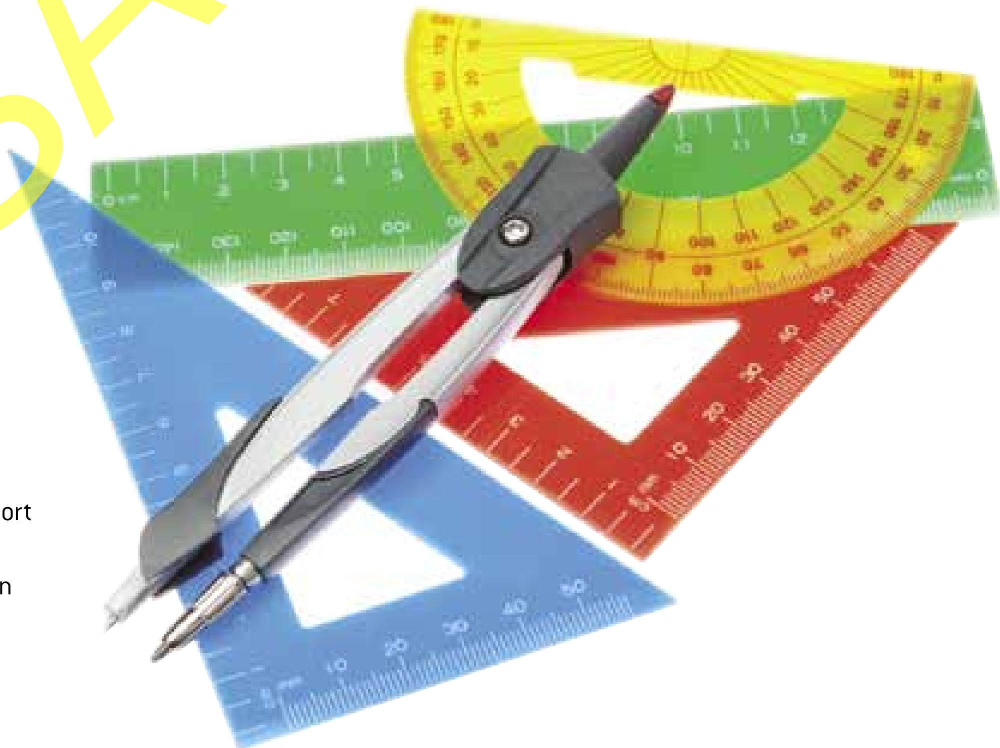
Select five different proofs from Euclid's Elements.

For each proof:

- draw a relevant diagram complete with labels on each vertex
- use your understanding of parallel lines and angles and congruence in order to demonstrate the proof using your diagram
- set out the proof in a logical sequence; you may wish to redraw the diagram several times to help you.

Ensure that you use proper mathematical terminology.





You will need:

- access to the Internet
- ruler
- protractor
- pair of compasses.

You may like to present your findings as a report. Your report could include:

- a PowerPoint presentation
- a booklet
- a poster series
- other (check with your teacher).