12

Probability and Venn diagrams

This chapter deals with further ideas in chance.

At the end of this chapter you should be able to:

- identify complementary events and use the sum of probabilities to solve problems
- describe events using language of 'at least', exclusive 'or' (*A* or *B* but not both), inclusive 'or' (*A* or *B* or both) and 'and'
- represent such events in two-way tables and Venn diagrams and solve related problems.

NSW Syllabus references: S4 S&P Probability 2
Outcomes: MA4-1WM, MA4-2WM, MA4-3WM, MA4-21SP
STATISTICS & PROBABILITY – ACMSP204, ACMSP205, ACMSP292

Diagnostic test

1 A normal six-sided die is thrown once. The number of outcomes in the sample space is:
A 1 **B** 3 **C** 5 **D** 6

2 A ticket is selected at random from a hat containing 1 blue, 2 red and 1 green ticket. The sample space is:
A {blue, red, green}
B {red}
C {blue, red, red, green}
D {not red}

3 A bag contains 1 pink, 1 purple and 1 orange ball. The probability of randomly selecting a purple ball is:
A 1 **B** 3 **C** $\frac{1}{3}$ **D** $\frac{2}{3}$

4 The probability of an event is $\frac{2}{5}$. As a percentage this is:
A 20% **B** 40%
C 50% **D** none of these

5 A hat contains 3 blue and 5 black tickets. If one ticket is chosen at random from the hat, the probability that it is blue is:
A $\frac{1}{2}$ **B** $\frac{3}{5}$ **C** 3 **D** $\frac{3}{8}$

6 A letter is chosen randomly from the word PROBABILITY. The probability that the letter chosen is B is:
A $\frac{1}{11}$ **B** $\frac{2}{11}$ **C** 1 **D** $\frac{2}{9}$

7 A term describing a probability of about 80% is:
A certain **B** highly probable
C evens **D** low probability

8 A coin is tossed once. An impossible event would be getting:
A heads **B** tails
C heads or tails **D** a six

9 When a normal six-sided die is rolled, the probability of getting a 7 is:
A $\frac{7}{6}$ **B** $\frac{6}{7}$ **C** $\frac{1}{6}$ **D** 0

10 If an event is certain to happen, its probability is:
A 0 **B** $\frac{1}{2}$ **C** 1 **D** 2

The Diagnostic test questions refer to the Year 7 outcomes from ACMSP167 and ACMSP168.

A Probability review

The sample space (S) of an experiment is the set of all possible outcomes. For example, if a normal six-sided die is rolled, S = {1, 2, 3, 4, 5, 6}.

$$\text{Probability (event)} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

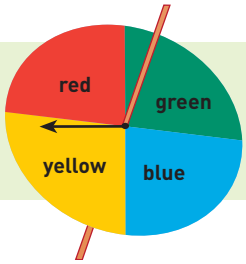
For convenience this may be written as $P(E) = \frac{n(E)}{n(\text{S})}$

where $P(E)$ = probability of the event E
$n(E)$ = number of outcomes favourable to event E
$n(\text{S})$ = number of possible outcomes.

EXAMPLE 1

A spinner is divided into 4 equal portions coloured red, blue, green and yellow. If the spinner is spun, find the probability that it lands on:

a red **b** green **c** blue or yellow



Sample space, S = {R, B, G, Y} Number of possible outcomes, $n(\text{S}) = 4$

a Favourable outcomes = {R} Number of favourable outcomes = 1
$P(\text{R}) = \frac{1}{4}$

b Favourable outcomes = {G} Number of favourable outcomes = 1
$P(\text{R}) = \frac{1}{4}$

c Favourable outcomes = {B, Y} Number of favourable outcomes = 2
$P(\text{B or Y}) = \frac{2}{4} = \frac{1}{2}$

Exercise 12A

1 A bag contains 1 red (R), 1 blue (B), 1 green (G), 1 yellow (Y) and 1 white (W) marble. One marble is chosen at random from the bag. Complete to find the following probabilities.

S = {_____} Number of possible equally likely outcomes = _____

a Probability that the marble is red:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{R}) =$ _____

b Probability that the marble is green:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{G}) =$ _____

c Probability that the marble is red or green:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{R or G}) =$ _____

d Probability that the marble is blue or white:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{B or W}) =$ _____

e Probability that the marble is blue or yellow or white:
Favourable outcomes = {_____} Number of favourable outcomes = _____
$P(\text{B or Y or W}) =$ _____

2 A normal six-sided die is rolled. Find the probability of getting:

a 6 **b** 3 **c** 3 or 6

d an even number **e** an odd number **f** a number > 3

3 The numbers 0, 1, 2, 3, … 9 are written on 10 cards. The cards are shuffled and one is selected at random. Find the probability that it shows:

a 3 **b** 9 **c** 3 or 9

d an even number **e** an odd number **f** a number > 6

g a multiple of 4

4 The numbers 1, 2, 3, …7 are written on cards. The cards are shuffled and one is selected at random. Find the probability that it is:

a 4 **b** 5 **c** 4 or 5

d an even number **e** an odd number **f** a number > 4

5 The letters of the alphabet are written on cards. The cards are shuffled and one is selected at random. Find the probability that the letter shown is:

a K **b** Z **c** K or Z

d a vowel **e** a consonant **f** a letter of the word RANDOM

EXAMPLE 2

A hat contains 3 red, 4 blue and 5 green tickets. If one ticket is chosen at random, what is the probability that it is:

a red? **b** blue? **c** red or blue? **d** green? **e** blue or green?

S = {R, R, R, B, B, B, B, G, G, G, G, G}

Number of possible equally likely outcomes = 12

a Favourable outcomes = {R, R, R} Number of favourable outcomes = 3

$P(\text{R}) = \frac{3}{12} = \frac{1}{4}$

b Favourable outcomes = {B, B, B, B} Number of favourable outcomes = 4

$P(\text{B}) = \frac{4}{12} = \frac{1}{3}$

c Favourable outcomes = {R, R, R, B, B, B, B} Number of favourable outcomes = 7

$P(\text{R or B}) = \frac{7}{12}$

d Favourable outcomes = {G, G, G, G, G} Number of favourable outcomes = 5

$P(\text{G}) = \frac{5}{12}$

e Favourable outcomes = {B, B, B, B, G, G, G, G, G} Number of favourable outcomes = 9

$P(\text{B or G}) = \frac{9}{12} = \frac{3}{4}$

6 A bag contains 5 blue and 2 red marbles. One marble is drawn at random. Complete to find the following probabilities.

S = {_____}

Number of possible equally likely outcomes = ___

a Probability that the marble is blue:

Favourable outcomes = {_____} Number of favourable outcomes = ___

$P(\text{B}) =$ ___

b Probability that the marble is red:
Favourable outcomes = {_____} Number of favourable outcomes = ___
$P(\text{R}) =$ ___

c Probability that the marble is blue or red:
Favourable outcomes = {_____} Number of favourable outcomes = ___
$P(\text{B or R}) =$ ___

7 A purse contains two 5-cent coins, three 10-cent coins, four 20-coins and one 50-cent coin. A coin is chosen at random from this purse. Find the probability that the value of the coin is:

a 10 cents **b** 20 cents **c** 10 cents or 20 cents
d 5 cents **e** 5 cents or 10 cents **f** more than 10 cents.

8 One hundred tickets are sold in a raffle. What is the probability of winning the raffle if you buy:

a 1 ticket? **b** 2 tickets? **c** 5 tickets? **d** 10 tickets?

9 A card is selected at random from a normal playing pack of 52 cards. Find the probability that it is:

a the king of clubs **b** a king **c** a spade
d black **e** a red 7 **f** a jack or a queen.

10 A letter is chosen at random from the letters of the word HIPPOPOTAMUS. What is the probability that the letter chosen is:

a H? **b** O? **c** P?

12040_Photo of a hippopotamus

11 A set of traffic lights shows green for 45 seconds, amber for 5 seconds and red for 30 seconds. When approaching this set of lights, what is the probability that it will be showing:

a green? **b** amber? **c** red?

12 **a** A normal six-sided die is rolled. Find the probability of getting a:

i 7 **ii** 9

b What kind of events are those in part **a**?

c Find the probability of getting:

i a number < 9 **ii** an odd or even number.

d What kind of events are those in part **c**?

From question **12**, we can see that:

- If an event is impossible then its probability is 0.
- If an event is certain then its probability is 1.

Hence, the probability of any event, E, lies in the range 0 to 1, inclusive.
That is: $0 \leqslant P(E) \leqslant 1$

13 A spinner is divided into four equal parts coloured red, yellow, green and blue. When the spinner is spun, what is the probability that it lands on:

a purple? **b** red or yellow or green or blue?

14 The numbers 1 to 7 are written on 7 cards. The cards are shuffled and one card is selected at random. Find the probability that it shows:

a an odd or even number
b a number < 10
c 9
d 0

15 A bag contains 1 red, 1 blue and 1 green marble. A marble is selected at random from this bag. Write down an event E for which:

a $P(E) = 0$
b $P(E) = 1$

16 Jack calculated the answer to a probability question in a test to be $\frac{4}{3}$. Explain why Jack's answer was wrong.

12041_Photo of tossing a coin. Have used 10026_LEY_IM7-rf

Investigation 1 Probability certainties

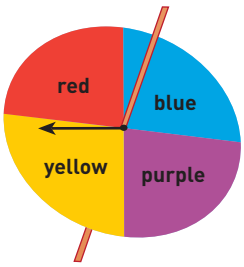
1 A coin is tossed once.

a List the sample space.
b What is the probability of heads?
c What is the probability of tails?
d Add the probability of heads and the probability of tails.
e Describe an event that results in a certainty.
f What is the probability of the event in part **e**?

2 A normal six-sided die is thrown once.

a List the sample space.
b Find these probabilities.
i $P(6)$ **ii** $P(5)$
iii $P(4)$ **iv** $P(3)$
v $P(2)$ **vi** $P(1)$
c Add the probabilities of getting 6, 5, 4, 3, 2 and 1.
d What is the probability of getting 6 or 5 or 4 or 3 or 2 or 1 when throwing a die once?
e Why is this a certainty?

3 A spinner has four equal-sized segments coloured blue, purple, red and yellow. The spinner is spun once.



a List the sample space.
b What is the probability of the outcome blue, purple, red or yellow?
c What is:
i P(blue)? **ii** P(purple)?
iii P(red)? **iv** P(yellow)?
d Add the probabilities from part **c**. What do you notice? Explain.

4 Complete:
The sum of the probabilities of all possible outcomes of a single experiment is _____.

5 A normal six-sided die is thrown once.

a Find $P(6)$.
b Find $P(1, 2, 3, 4 \text{ or } 5)$.
c Find $P(\text{not } 6)$. (*Hint:* See part **b**.)
d Add the probability of getting 6 to the probability of *not* getting 6. What do you notice? Explain your answer.

B Sum of probabilities

In Investigation 1 you found that the sum of the probabilities of all possible outcomes of an experiment is 1.

EXAMPLE 1

A bag contains red, blue and green marbles. When one marble is chosen at random from the bag, the probability that it is red is $\frac{7}{19}$ and the probability that it is blue is $\frac{4}{19}$. What is the probability that it is green?

$$\begin{aligned} P(\text{red}) + P(\text{blue}) + P(\text{green}) &= 1 \\ \frac{7}{19} + \frac{4}{19} + P(\text{green}) &= 1 \\ \frac{11}{19} + P(\text{green}) &= 1 \\ P(\text{green}) &= 1 - \frac{11}{19} \\ &= \frac{8}{19} \end{aligned}$$

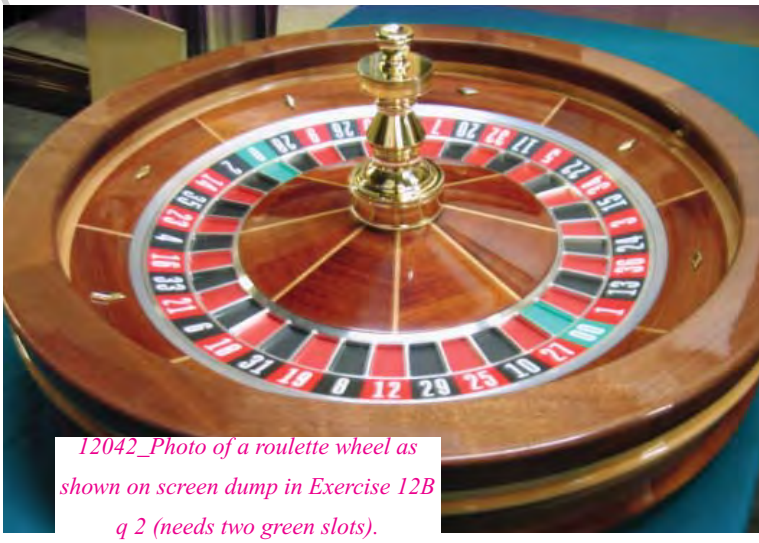
The sum of the probabilities of all the possible outcomes of an experiment is 1.

Exercise 12B

1 A bag contains black, white and orange counters. If a counter is chosen at random from this bag the probability that it is black is $\frac{6}{13}$ and the probability that it is white is $\frac{2}{13}$. Complete the following to calculate the probability that it is orange.

$$\begin{aligned} P(\text{black}) + P(\text{white}) + P(\text{orange}) &= \_\_\_\_ \\ \frac{6}{13} + \frac{2}{13} + P(\text{orange}) &= \_\_\_\_ \\ \_\_\_\_ + P(\text{orange}) &= \_\_\_\_ \\ P(\text{orange}) &= \_\_\_\_ - \_\_\_\_ \\ &= \_\_\_\_ \end{aligned}$$

2 A roulette wheel has red, black and green slots. When spun, the probability that the ball will stop in a red slot is $\frac{9}{19}$ and the probability that it will stop in green slot is $\frac{1}{19}$. What is the probability that it will stop in a black slot?



12042_Photo of a roulette wheel as shown on screen dump in Exercise 12B q 2 (needs two green slots).

3 A spinner consists of pink, blue and green sections. If the spinner is spun the probability that it will stop on blue is $\frac{1}{2}$ and the probability that it will stop on green is $\frac{1}{3}$. Find the probability that it will stop on pink.

4 When three coins are tossed, the possible outcomes are 3 heads, exactly 2 heads, exactly 1 head or no heads. If the probability of 3 heads is $\frac{1}{8}$, the probability of exactly 1 head is $\frac{3}{8}$ and the probability of getting no heads is $\frac{1}{8}$. Calculate the probability of getting exactly 2 heads.

5 A purse contains 5-cent, 10-cent, 20-cent and 50-cent coins. When a coin is chosen at random from this purse, the probability of getting a 5-cent coin is $\frac{1}{5}$, the probability of getting a 10-cent coin is $\frac{1}{3}$ and the probability of getting a 50-cent coin is $\frac{1}{10}$. Calculate the probability of getting a 20-cent coin.

C Identifying complements of events

In an experiment with sample space S, the **complement** of an event A is the set of outcomes that are in the sample space S but are not in A.

12043_Photo of 6-sided die Have used 10001_LEY_IM7-rf but another pic would be good.

EXAMPLE 1

When a normal six-sided die is thrown, what is the complement of the event:

a throwing 2?

b throwing 1 or 2?

c throwing an even number?

a S = {1, 2, 3, 4, 5, 6}. Let A be the event 'throwing a 2'; then A = {2}.
The complement of A is the set of outcomes that are in S but are not in A.
Complement of A = {1, 3, 4, 5, 6}
The complement of A is the event 'throwing a 1, 3, 4, 5 or 6' or 'not throwing a 2'.

Write the event as a set of outcomes, even if there is only one outcome in the set.

b S = {1, 2, 3, 4, 5, 6}. Let B be the event 'throwing a 1 or 2'; then B = {1, 2}.
The complement of B is the set of outcomes that are in S but are not in B.
Complement of B = {3, 4, 5, 6}
The complement of B is the event 'throwing a 3 or 4 or 5 or 6' or 'not throwing a 1 or 2' or 'throwing a number greater than 2'.

c S = {1, 2, 3, 4, 5, 6}. Let C be the event 'throwing an even number'; then C = {2, 4, 6}.
The complement of C is the set of outcomes that are in S but are not in C.
Complement of C = {1, 3, 5}
The complement of C is the event 'throwing a 1, 3 or 5' or 'not throwing an even number' or 'throwing an odd number'.

Exercise 12C

1 A normal die is thrown. Complete the following to find the complement of each event given.

a Throwing a 6
S = {_____}. Let A be the event 'throwing a 6'; then A = {_____}.
The complement of A is the set of outcomes that are in S but are not in A.
Complement of A = {_____}
The complement of A is the event '_____' or '_____' or '_____'.

b Throwing a 5 or 6
S = {_____}. Let B be the event 'throwing a 5 or 6'; then B = {_____}.
The complement of B is the set of outcomes that are in S but are not in B.
Complement of B = {_____}
The complement of B is the event '_____' or '_____' or '_____'.

c Throwing an odd number
S = {_____}. Let C be the event 'throwing an odd number'; then C = {_____}.
The complement of C is the set of outcomesthat are in S but are not in C.
Complement of C = {_____}
The complement of C is the event '_____' or '_____' or '_____'.

2 A normal die is thrown. Find the complement of each event.

a throwing 3 **b** throwing 3 or 5 **c** throwing a number greater than 2

3 The numbers 1 to 9 are written on cards and one card is chosen at random. Find the complement of these event.

a choosing 7 **b** choosing 1 or 7

c choosing an even number **d** choosing a number greater than 4

EXAMPLE 2

a A student is chosen from a class consisting of boys and girls. What is the complement of the event 'choosing a boy'?

b A marble is chosen from a bag containing black and yellow marbles .What is the complement of the event 'selecting a black marble'?

a The complement of the event 'choosing a boy' is 'not choosing a boy' or 'choosing a girl'.

b The complement of the event 'selecting a black marble' is 'not selecting a black marble' or 'selecting a yellow marble'.

4 Complete the following.

a A student is chosen from a class consisting of boys and girls. The complement of the event 'choosing a girl' is '_____ choosing a girl' or 'choosing a _____'.

b A marble is chosen from a bag containing red and blue marbles. The complement of the event 'choosing a red marble' is '_____ choosing a red marble' or 'choosing a _____ marble'.

c A marble is chosen from a bag containing red, blue and green marbles. The complement of the event 'choosing a red marble' is '_____ choosing a red marble' or 'choosing a _____ marble'.

d A marble is chosen from a bag containing red, blue and green marbles. The complement of the event 'choosing a red or blue marble' is '_____ choosing a red or blue marble' or 'choosing a _____ marble'.

5 A coin is tossed. What is the complement of the event 'getting a head'?

6 A spinner has four equal-sized segments coloured red, blue, green and yellow. The spinner is spun and the colour on which it stops is noted. What is the complement of the event:

a stops on red? **b** stops on red or blue?

c stops on blue or green? **d** stops on red or blue or green?

7 A card is selected from a normal playing pack. What is the complement of selecting:

a a red card?

b a spade?

c a spade or a club?

d a picture card?

8 Sydney plays Melbourne in a soccer final. What is the complement of the event:

a Sydney wins or draws?

b Sydney wins?

c a draw?

d Melbourne loses or draws?

12044_Photo of students selecting a card. Have used 10030_LEY_IM7_A but maybe better to get a new pic where student is not looking at camera

Investigation 2 Sum of probabilities of complementary events

1 A coin is tossed once. Find these probabilities.

a $P(\text{head})$ **b** $P(\text{not head})$ **c** $P(\text{head}) + P(\text{not head})$

2 A normal six-sided die is rolled. Find:

a **i** $P(2)$ **ii** $P(\text{not } 2)$ **iii** $P(2) + P(\text{not } 2)$

b **i** $P(6)$ **ii** $P(\text{not } 6)$ **iii** $P(6) + P(\text{not } 6)$

c **i** $P(\text{even number})$ **ii** $P(\text{not even number})$ **iii** $P(\text{even number}) + P(\text{not even number})$

d **i** $P(\text{number} < 5)$ **ii** $P(\text{number not} < 5)$ **iii** $P(\text{number} < 5) + P(\text{number not} < 5)$

3 A bag contains 2 red, 5 blue and 4 white counters. A counter is chosen at random from the bag. Find:

a **i** $P(\text{red})$ **ii** $P(\text{not red})$ **iii** $P(\text{red}) + P(\text{not red})$

b **i** $P(\text{blue})$ **ii** $P(\text{not blue})$ **iii** $P(\text{blue}) + P(\text{not blue})$

c **i** $P(\text{white})$ **ii** $P(\text{not white})$ **iii** $P(\text{white}) + P(\text{not white})$

12045_Photo of a student choosing a red, blue or white counter from a bag.

D Probabilities of complements of events

Pairs of events in Investigation 2 are:

- getting a head, not getting a head
- getting 2, not getting 2
- getting a white counter, not getting a white counter.

These are **complementary events**. Each event is the complement of the other. The complement of the set E, called **not** E, can be written $\tilde{E}$ **or** E'.

From Investigation 2 we see that: $P(E) + P(\text{not } E) = 1$ or $P(E) + P(\tilde{E}) = 1$.

EXAMPLE 1

When two coins are tossed, the probability of getting 2 heads is $\frac{1}{4}$. What is the probability of *not* getting 2 heads when two coins are tossed?

$P(2\text{H}) + P(\text{not } 2\text{H}) = 1$

$\frac{1}{4} + P(\text{not } 2\text{H}) = 1$

$\therefore P(\text{not } 2\text{H}) = 1 - \frac{1}{4} = \frac{3}{4}$

From Example 1 we can see that $P(E) + P(\text{not } E) = 1$ is equivalent to $P(\text{not } E) = 1 - P(E)$.

Exercise 12D

1 On any weekday, the probability of a particular bus arriving at its destination on time is $\frac{9}{11}$. Complete the following to find the probability that the bus does *not* arrive on time.

$P(\text{on time}) + P(\text{not on time}) = \_\_\_$

$\frac{9}{11} + P(\text{not on time}) = \_\_\_$

$\therefore P(\text{not on time}) = \_\_\_ - \frac{\square}{\square} = \frac{\square}{\square}$

2 The probability of choosing a 20-cent coin from a particular purse is $\frac{3}{10}$. When a coin is chosen at random from this purse, what is the probability that it is not a 20-cent coin?

3 When a lolly is selected at random from a bag, the probability that it is red is $\frac{5}{13}$. What is the probability that the lolly is not red?

4 The letters of a word are written on cards and placed in a hat. When a card is chosen at random from this hat, the probability that the letter is M is $\frac{2}{7}$. What is the probability that the letter is not M?

5 On approaching a set of traffic lights, the probability that the light will be red is $\frac{30}{67}$. What is the probability that the light will not be red?

6 On a European roulette wheel, the probability that the ball will stop on a red number is $\frac{18}{37}$. What is the probability that it will *not* stop on a red number?

7 There are 1000 tickets sold in a raffle. If Angela buys 1 ticket, what is the probability that:

a she will win the raffle? **b** she will not win the raffle?

8 The numbers 1 to 9 are written on cards. The cards are shuffled and one is selected at random. Find these probabilities.

a $P(9)$ **b** $P(\text{not } 9)$ **c** $P(\text{even number})$

d $P(\text{not even number})$ **e** $P(\text{number} < 3)$ **f** $P(\text{number not} < 3)$

9 The numbers 0 to 9 are written on cards. The cards are shuffled and one is selected at random. Find these probabilities.

a $P(6)$ **b** $P(\text{not } 6)$ **c** $P(\text{odd number})$

d $P(\text{not odd number})$ **e** $P(\text{number} > 7)$ **f** $P(\text{number not} > 7)$

10 A bag contains 6 red, 9 blue and 8 white marbles. One marble is chosen at random from the bag. Find these probabilities.

a $P(\text{red})$ **b** $P(\text{not red})$ **c** $P(\text{not blue})$ **d** $P(\text{not white})$

12046_Photo of traffic lights or selling raffle tickets

E Mutually exclusive events

Two events are mutually exclusive if they cannot both happen at the same time. For example, when a die is rolled once, the events 'getting a 1' and 'getting a 4' are mutually exclusive. Only one number can occur on the uppermost face and hence both events cannot happen at the same time.

The event 'getting a number bigger than 3' and the event 'getting an even number' are not mutually exclusive, as the outcomes a 4 or a 6 are bigger than 3 and even; that is, both events can happen at the same time. (These are sometimes called inclusive events.)

EXAMPLE 1

A die is rolled. State whether events A and B are mutually exclusive or non-mutually exclusive.

a A is the event a 2 and B is the event a 3.

b A is the event a 2 and B is the event an even number.

c A is the event a 2 and B is the event a number bigger than 4.

d A is the event a number less than 5 and B is the event a number bigger than 2.

a $A = \{2\}$, $B = \{3\}$. Both A and B cannot happen at the same time. A and B are mutually exclusive.

b $A = \{2\}$, $B = \{2, 4, 6\}$. Both A and B can happen at the same time (the outcome rolling a 2). A and B are non-mutually exclusive.

c $A = \{2\}$, $B = \{5, 6\}$. Both A and B cannot happen at the same time. A and B are mutually exclusive.

d $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$. Both A and B can happen at the same time (the outcomes 3 or 4). A and B are non-mutually exclusive.

Exercise 12E

1 A die is rolled. Complete the following to determine whether the events A and B are mutually exclusive or non-mutually exclusive.

a A is the event a 5: $A = \{\_\_\_\_\}$. B is the event a 6: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

b A is the event a 5: $A = \{\_\_\_\_\}$. B is the event an odd number: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

c A is the event an even number: $A = \{\_\_\_\_\}$. B is the event an odd number: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

d A is the event a multiple of 3: $A = \{\_\_\_\_\}$. B is the event an even number: $B = \{\_\_\_\_\}$.
Hence the events A and B are mutually exclusive/non-mutually exclusive.

2 A card is selected at random from a normal playing pack. State whether the following events A and B are mutually exclusive or non-mutually exclusive.

a A is the event a king. B is the event an ace.

b A is the event a king. B is the event a red card.

c A is the event a 10, B is the event a queen.

d A is the event a jack, B is the event a black card.

e A is the event a 7, B is the event a diamond.

f A is the event a club, B is the event a red card.

g A is the event a black card, B is the event an ace.

3 The numbers 0 to 9 are written on 10 cards. The cards are shuffled and one card is selected at random. State whether the following events are mutually exclusive or non-mutually exclusive.

a getting a 5, getting an even number
b getting an even number, getting a number less than 5
c getting an odd number, getting a number bigger than 5
d getting a number less than 4, getting a number bigger than 5
e getting a number less than 4, getting a number less than 7

4 A pair of dice are rolled at the same time. State whether the following events are mutually exclusive or non-mutually exclusive.

a getting a total of 6, getting a double
b getting a total of 9, getting a double

F Compound events

A compound event is an event that can be expressed as a combination of simple events.

EXAMPLE 1

A normal six-sided die is thrown. Match each statement in the left-hand column with its equivalent in the right-hand column.

a throwing a 4, 5 or 6	**A** throwing at least 3
b throwing a 1, 2, 3 or 4	**B** throwing at most 5
c throwing a 3, 4, 5 or 6	**C** throwing at most 4
d throwing a 1, 2, 3, 4, or 5	**D** throwing at least 4

a throwing a 4, 5 or 6	is equivalent to	**D** throwing at least 4
b throwing a 1, 2, 3 or 4	is equivalent to	**C** throwing at most 4
c throwing a 3, 4, 5 or 6	is equivalent to	**A** throwing at least 3
d throwing a 1, 2, 3, 4, or 5	is equivalent to	**B** throwing at most 5

Exercise 12F

1 A normal six-sided die is thrown. Complete the following.

a Throwing 4, 5 or 6 is equivalent to throwing at least ___.
b Throwing a 1, 2 or 3 is equivalent to throwing at most ___.
c Throwing a 2, 3, 4, 5 or 6 is equivalent to throwing at least ___.
d Throwing a 1 or 2 is equivalent to throwing at most ___.

2 A regular eight-sided die is rolled. Complete the following.

a Throwing 1, 2, 3 or 4 is equivalent to throwing at most ___.
b Throwing a 6, 7 or 8 is equivalent to throwing at least ___.
c Throwing a 4, 5, 6, 7 or 8 is equivalent to throwing at least ___.
d Throwing a 1, 2, 3, 4 or 5 is equivalent to throwing at most ___.

3 The numbers 0 to 9 are written on cards. A card is selected at random. List the outcomes for the event that the number on the card is:

a at most 4 **b** at least 7 **c** at least 5 **d** at most 2.

4 When two children are born, the sample space for the order of birth is S = {bb (boy followed by boy), bg (boy followed by a girl), gb (girl followed by a boy), gg (girl followed by a girl)}.
List the outcomes for each event.

a at most 1 boy **b** at least 1 boy **c** at least 1 girl **d** at most 1 girl

5 When two coins are tossed, the sample space is S = {HH, HT, TH, TT}. List the outcomes for each event.

a at least 1 tail **b** at most 1 tail **c** at least 1 head **d** at most 1 head

6 A spinner has three equal parts numbered 1, 2 and 3. When this spinner is spun twice the sample space is: S = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}. List the outcomes for these events.

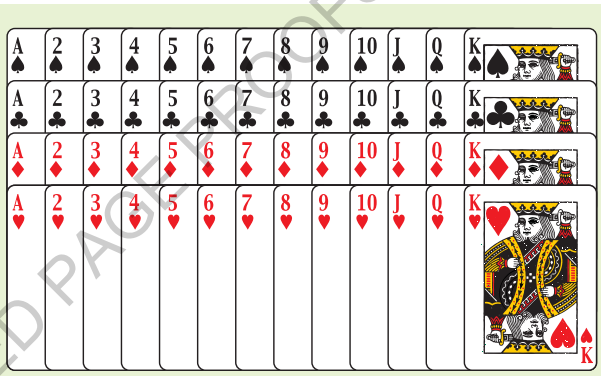
a at least one 3 **b** at most one 3

EXAMPLE 2

A card is chosen at random from a normal 52-card playing pack. List the outcomes in these events.

a choosing a jack and a red card

b choosing a club and a picture card.



a Let A be the event 'a jack' and let B be the event 'a red card'.
A = {J of hearts, J of diamonds, J of spades, J of clubs}
B = {2, 3, 4…, J, Q, K, A of hearts, 2, 3, 4…, J, Q, K, A of diamonds}
The event 'A and B' is the set of outcomes that have the attributes of both A and B; that is, the outcomes that are 'a jack and a red card'.
Hence event 'A and B' = {J of hearts, J of diamonds}

b Let X be the event 'a club' and let Y be the event 'a picture card'.
X = {2, 3, 4…, J, Q, K, A of clubs}
Y = {J, Q, K of hearts, J, Q, K of diamonds, J, Q, K of spades, J, Q, K of clubs}
The event 'X and Y' is the set of outcomes that are 'a club and a picture card'.
Hence event 'X and Y' = {J, Q, K of clubs}.

7 A card is selected at random from a normal playing pack. List the outcomes in these events.

a selecting a card that is a 10 and a heart

b selecting a card that is a 10 and a red card

c selecting a card that is black and a king

d selecting a card that is a club and even numbered

e selecting a card that is red and odd numbered

f selecting a card that is black and a picture card

EXAMPLE 3

A die is rolled. A is the event 'an odd number', B is the event 'an even number' and C is the event 'a number < 3'. List the outcomes for these events.

a A and B **b** A or B **c** B and C **d** B or C

a The event 'A and B' $= \{\ \}$ as a number cannot be both odd and even. These are mutually exclusive events.

The set { } has no outcomes and is known as an empty set.

b The event 'A or B' $= \{1, 2, 3, 4, 5, 6\}$
If the events A and B are mutually exclusive then the event 'A or B' = set of outcomes for A + set of outcomes for B.

c The event 'B and C' is the set of outcomes that belong to both B and C; that is, the outcomes that are even and less than 3. Event 'B and C' $= \{2\}$.

d The event 'B or C' is the set of outcomes that belong to B or C.
The problem raised is whether to include the outcome 2, which is in both events. If two events are not mutually exclusive, an 'or' statement needs a qualifier to determine inclusivity (the 2 is included) or exclusivity (the 2 is excluded).
The event 'B or C or both' $= \{1, 2, 4, 6\}$ is known as an 'inclusive or' compound event.
The event 'B or C but not both' $= \{1, 4, 6\}$ is known as an 'exclusive or' compound event.

EXAMPLE 4

a A card is selected at random from a pack. A is the event 'a red card', B is the event 'a black card'.
i List the outcomes for the event 'A and B'.
ii Does the event 'A or B' need a qualifier?

b A card is selected at random from a pack. A is the event 'a red card', B is the event 'an ace'.
i List the outcomes for 'A and B'.
ii Does the event 'A or B' need a qualifier?

c A person is chosen at random from a group. A is the event 'male' and B is the event 'left-handed'. Does the event 'A or B' need a qualifier?

a **i** A and $B = \{\ \}$
ii No. The events are mutually exclusive so the event 'A or B' = set of outcomes in A + set of outcomes for B.

b **i** A and $B = \{$ace of hearts, ace of diamonds$\}$
ii The events A and B are non-mutually exclusive, so the event 'A or B' needs a qualifier to indicate whether the outcomes for both events (ace of hearts, ace of diamonds) are included or excluded.

c If there are no left-handed males in the group then a qualifier is not necessary as the events are mutually exclusive. If there are left-handed males in the group then a qualifier is needed to indicate whether the left-handed males are to be included or excluded.

8 State whether a qualifier is needed for the event 'A or B'.
a A die is rolled. A is the event 'a number < 3'. B is the event ' a number > 4'.
b A die is rolled. A is the event 'an odd number'. B is the event 'a number > 4'.
c A student is chosen from a class of boys and girls. A is the event 'female'. B is the event 'wears glasses'.
d A student is chosen from a class. A is the event 'has blond hair'. B is the event 'has brown eyes'.

9 A die is rolled. A is the event 'a number > 3' and B is the event 'an odd number'. List the outcomes for these events.

a A **b** not A **c** B **d** not B

e A and B **f** A or B or both **g** A or B but not both

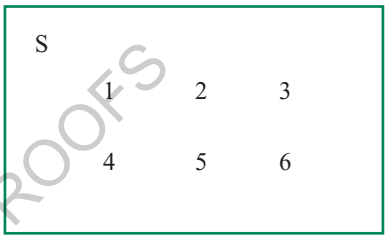
10 A card is selected at random from a normal playing pack. X is the event 'a diamond' and Y is the event 'a king'. List the outcomes for these events.

a X **b** not X **c** Y

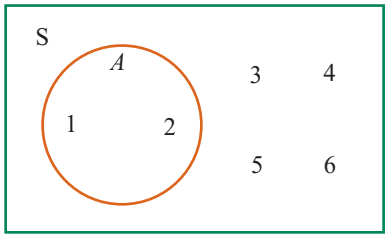
d X and Y **e** X or Y or both **f** X or Y but not both

G Venn diagrams

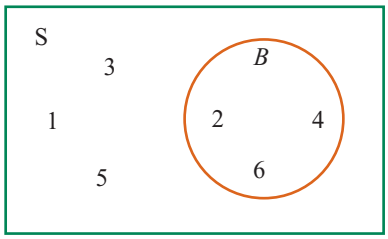
The outcomes of an event can be represented using a Venn diagram.
The sample space is represented by the region or points inside a rectangle.
For example, if a die is rolled, S $= \{1, 2, 3, 4, 5, 6\}$.
These outcomes are written inside a rectangle, labelled S, as shown.



The outcomes favourable to an event are represented by points or the region inside a circle. For example, if a six-sided die is rolled and A is the event 'a number < 3', then $A = \{1, 2\}$. These outcomes are written inside a circle labelled A, as shown.
Note: All the outcomes that are not favourable to event A are outside this circle: the complement of A is the set of outcomes outside circle A.
From the diagram, the event 'not A' $= \{3, 4, 5, 6\}$.



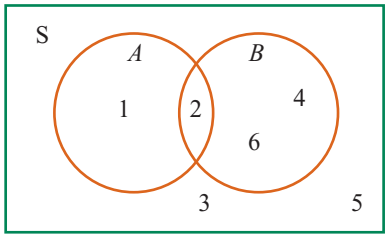
If B is the event 'an even number', then $B = \{2, 4, 6\}$. These outcomes are written inside another circle labelled B, as shown.
Note: All the outcomes that are not favourable to event B are outside this circle: the complement of B is the set of outcomes outside circle B.
From the diagram, the event 'not B' $= \{1, 3, 5\}$.



Both events A and B can be represented in a Venn diagram.

The outcome $\{2\}$ is common (belongs) to both events and is placed in the area that is common to both circles. It is placed in the area where the two circles intersect. The outcome $\{2\}$ is 'a number < 3' and 'an even number'.

The outcomes $\{1, 2, 4, 6\}$, which are within the combined areas of both circles, including the intersection of the circles, are numbers that are 'a number < 3' or 'an even number' or both (less than 3 and even).

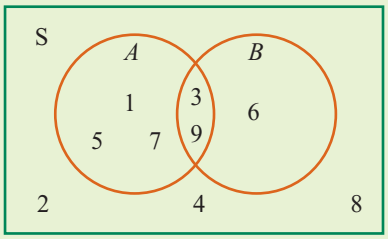


The event 'A or B or both' is an 'inclusive or' event because it includes 'both A and B'.

The outcomes $\{3, 5\}$ do not lie within either circle. These are the numbers that are neither 'less than 3' nor 'even'.

EXAMPLE 1

Numbers are written on cards, the cards shuffled and one is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram.

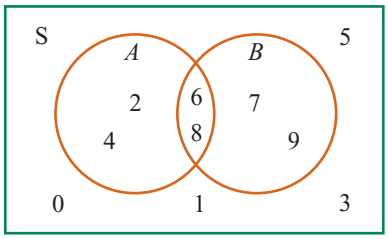


a List the sample space.
b List the set of outcomes favourable to event A.
c List the set of outcomes not favourable to the event A.
d List the set of outcomes favourable to event B.
e List the set of outcomes not favourable to the event B.
f List the set of outcomes favourable to the events 'A and B'.
g **i** List the set of outcomes favourable to the event 'A or B or both'.
ii Is this an 'inclusive or' event or an 'exclusive or' event?
h **i** List the set of outcomes favourable to the event 'A or B but not both'.
ii Is this an 'inclusive or' event or an 'exclusive or' event?
i List the outcomes that belong to neither A nor B.

a These are all the outcomes inside the rectangle. Hence S = {1, 2, 3, 4, 5, 6, 7, 8, 9}.
b These are all the outcomes inside circle A. Hence A = {1, 3, 5, 7, 9}.
c These are all the outcomes outside circle A (complement of the event A). Hence 'not A' = {2, 4, 6, 8}.
d These are all the outcomes inside circle B. Hence B = {3, 6, 9}.
e These are all the outcomes outside circle B (complement of the event B). Hence 'not B' = {1, 2, 4, 5, 7, 8}.
f These are all the outcomes in the intersection of the two circles. Hence 'A and B' = {3, 9}.
g **i** These are all the outcomes within both circles, including the intersection.
Hence 'A or B or both' = {1, 3, 5, 6, 7, 9}
ii It is an 'inclusive or' event.
h **i** These are all the outcomes within both circles, but excluding the intersection.
Hence 'A or B but not both' = {1, 5, 6, 7}.
ii It is an 'exclusive or' event.
i These are all the outcomes outside the two circles. Hence 'neither A nor B' = {2, 4, 8}.

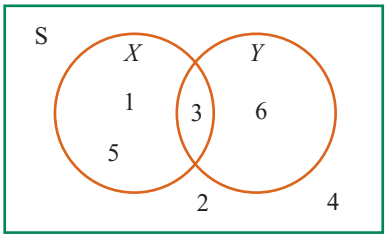
Exercise 12G

1 Numbers are written on cards, the cards are shuffled and one is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. Complete the following.



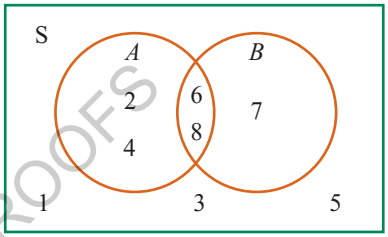
a The sample space S = {_____} (outcomes inside the rectangle)
b Event A = {_____} (outcomes inside the circle A)
c Event 'not A' = {_____} (outcomes outside the circle A)
d Event B = {_____} (outcomes inside circle B)
e Event 'not B' = {_____} (outcomes outside circle B)
f Event 'A and B' = {_____} (outcomes in the intersection of the circles)
g **i** Event 'A or B or both' = {_____} (outcomes within both circles, including the intersection)
ii Is this an 'inclusive or' event or an 'exclusive or' event?
h **i** Event 'A or B but not both' = {_____} (outcomes within both circles, excluding the intersection)
ii Is this an 'inclusive or' event or an 'exclusive or' event?
i Event 'neither A nor B' = {_____} (outcomes outside both circles)

2 A six-sided die is rolled. The outcomes for this experiment and the events X and Y are shown in the Venn diagram.



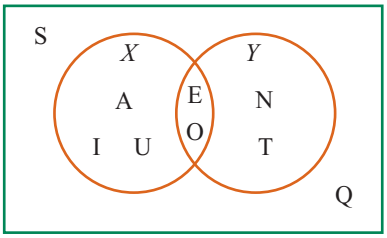
- **a** List the sample space.
- **b** List the set of outcomes for event X.
- **c** List the set of outcomes for the event 'not X'.
- **d** List the set of outcomes for event Y.
- **e** List the set of outcomes for the event 'not Y'.
- **f** List the set of outcomes for event 'X and Y'.
- **g** **i** List the set of outcomes for the event 'X or Y or both'.
  - **ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- **h** **i** List the set of outcomes for the event 'X or Y but not both'.
  - **ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- **i** List the outcomes that belong to neither X nor Y.

3 A spinner with eight equal parts numbered 1 to 8 is spun. A is the event 'the spinner lands on an even number' and B is the event 'the spinner lands on a number $>$ 5'. The outcomes for this experiment and events A and B are shown in the Venn diagram. Use the diagram to list the outcomes for the following events.



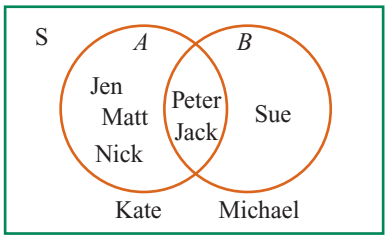
- **a** an even number
- **b** not an even number
- **c** a number $>$ 5
- **d** a number not $>$ 5
- **e** a number that is even and $>$ 5
- **f** a number that is even or $>$ 5 or both
- **g** a number that is even or $>$ 5 but not both
- **h** a number that is neither even nor $>$ 5
- **i** a number that is even but not $>$ 5
- **j** a number that is $>$ 5 but not even

4 The letters of the word EQUATION are written on cards and placed in a hat. One card is drawn at random. X is the event 'a vowel' and Y is the event 'a letter of the word NOTE'. The outcomes for this experiment and events X and Y are shown in the Venn diagram. Use the diagram to list the outcomes for the following events.



- **a** a vowel
- **b** not a vowel
- **c** a letter of the word NOTE
- **d** not a letter of the word NOTE
- **e** a vowel and a letter of the word NOTE
- **f** a vowel or a letter of the word NOTE or both
- **g** a vowel or a letter of the word NOTE but not both
- **h** neither a vowel nor a letter of the word NOTE
- **i** a vowel but not a letter of the word NOTE
- **j** a letter of the word NOTE but not a vowel

5 A person is chosen at random from a group of friends. A is the event 'has blond hair' and B is the event 'has blue eyes'. The outcomes for the experiment and events A and B are shown in the Venn diagram.

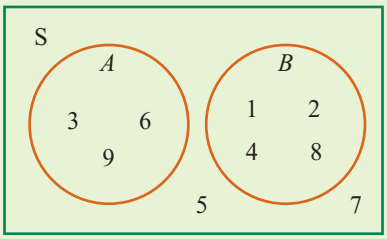


- **a** List the sample space.
- **b** List the set of outcomes for the following events.
  - **i** has blond hair
  - **ii** does not have blond hair
  - **iii** has blue eyes
  - **iv** does not have blue eyes
  - **v** has blond hair and blue eyes
  - **vi** has blond hair or blue eyes or both
  - **vi** has blond hair or blue eyes but not both
  - **vii** has neither blond hair nor blue eyes
  - **ix** has blond hair but not blue eyes
  - **x** has blue eyes but not blond hair

EXAMPLE 2

The numbers 1 to 9 are written on cards, the cards shuffled and one card is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. List the set of outcomes for the following events.

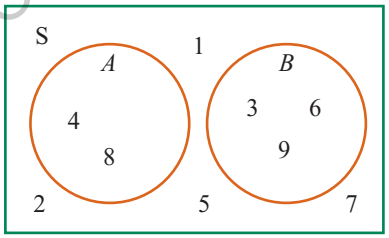
a A **b** not A **c** B **d** not B
e A and B **f** A or B **g** neither A nor B



a $A = \{3, 6, 9\}$
b not $A = \{1, 2, 4, 5, 7, 8\}$
c $B = \{1, 2, 4, 8\}$
d not $B = \{3, 5, 6, 7, 9\}$
e The events A and B have no intersection. They are mutually exclusive events: there are no outcomes of this experiment that have the properties of both A and B so 'A and B' = { }
f The event 'A or B' does not have to be qualified by 'or both' or 'but not both', as in this case, A and B are mutually exclusive events. A or $B = \{1, 2, 3, 4, 6, 8, 9\}$
g Neither A nor $B = \{5, 7\}$

6 The numbers 1 to 9 are written on cards, the cards shuffled and one card is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. A is the event 'a multiple of 4' and B is the event 'a multiple of 3'. List the set of outcomes for the following events.

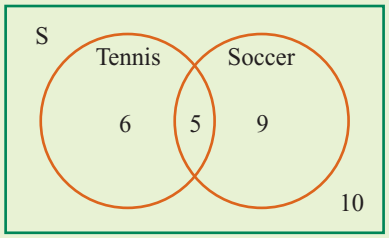
a A **b** not A **c** B **d** not B
e A and B **f** A or B **g** neither A nor B



EXAMPLE 3

This Venn diagram shows the sports played by a class of students. How many students:

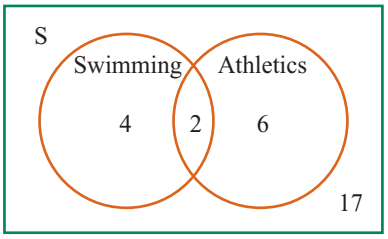
a are in the class?
b play tennis and soccer?
c play tennis?
d play soccer?
e play tennis or soccer or both?
f play tennis or soccer but not both?
g play neither tennis nor soccer?
h play tennis but not soccer?
i play soccer but not tennis?



a Number in class = 6 + 5 + 9 + 10 = 30
b Tennis and soccer = 5
c Tennis = 6 + 5 = 11
d Soccer = 9 + 5 = 14
e Tennis or soccer or both = 6 + 5 + 9 = 20
f Tennis or soccer but not both = 6 + 9 = 15
g Neither tennis nor soccer = 10
h Tennis but not soccer = 6
i Soccer but not tennis = 9

Note that the number of students, not the individual students, is shown.

7 This Venn diagram represents the number of students in a class who made the school swimming and athletics teams.



12047_Photo of swimming or athletics

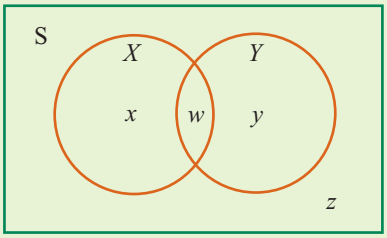
a How many students are in the class?

b How many students made:

- **i** both the swimming and athletics teams?
- **ii** the swimming team?
- **iii** the athletics team?
- **iv** the swimming team or the athletics team or both?
- **v** the swimming team or the athletics team but not both?
- **vi** neither the swimming team nor the athletics team?
- **vii** the swimming team but not the athletics team?
- **viii** the athletics team but not the swimming team?

EXAMPLE 4

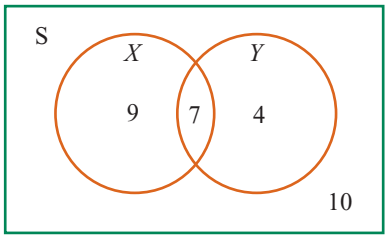
In a class of 30 students, 16 have black hair, 11 have brown eyes and 7 have both black hair and brown eyes.



a Complete the Venn diagram representing this information, where X is the event 'black hair' and Y is the event 'brown eyes'. Note that the number of students, not the individual students, is shown in each region of the diagram.

b How many students:

- **i** do not have black hair?
- **ii** do not have brown eyes?
- **iii** have black hair or brown eyes or both?
- **iv** have black hair or brown eyes but not both?
- **v** have neither black hair nor brown eyes?
- **vi** have black hair but not brown eyes?
- **vii** have brown eyes but not black hair?

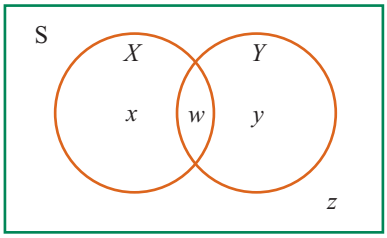


a There are 7 students with both black hair and brown eyes so $w = 7$.
This number is placed in the intersection of the circles.
There is a total of 16 students with black hair so $x + 7 = 16$.
$\therefore x = 9$
There is a total of 11 students with brown eyes so $y + 7 = 11$.
$\therefore y = 4$
There is a total of 30 students in the class so $x + w + y + z = 30$.
$9 + 7 + 4 + z = 30 \ \therefore z = 10$

b Number of students who:

- **i** do not have black hair $= 4 + 10 = 14$
- **ii** do not have brown eyes $= 9 + 10 = 19$
- **iii** have black hair or brown eyes or both $= 9 + 7 + 4 = 20$
- **iv** have black hair or brown eyes but not both $= 9 + 4 = 13$
- **v** have neither black hair nor brown eyes $= 10$
- **vi** have black hair but not brown eyes $= 9$
- **vii** have brown eyes but not black hair $= 4$

8 In a class of 29 students, 13 have black hair, 10 have brown eyes and 5 have both black hair and brown eyes.



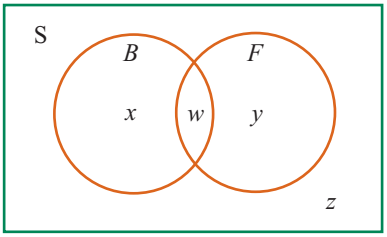
a Complete the Venn diagram representing this information, where X is the event 'black hair' and Y is the event 'brown eyes'.
Note that the number of students, not the individual students, is shown in each region of the diagram.
There are 5 students with both black hair and brown eyes so $w =$ ___. This number is placed in the intersection of the circles.
There is a total of 13 students with black hair so $x + 5 =$ ___.
$\therefore x =$ ___
There is a total of 10 students with brown eyes so $y + 5 =$ ___.
$\therefore y =$ ___
There are 29 students in the class so $x + 5 + y + z =$ ___.
___ + ___ + ___ + $z =$ ___
$\therefore z =$ ___

b The number of students who:

i do not have black hair = ___
ii do not have brown eyes = ___
iii have black hair or brown eyes or both = ___
iv have black hair or brown eyes but not both = ___
v have neither black hair nor brown eyes = ___
vi have black hair but not brown eyes = ___
vii have brown eyes but not black hair = ___

12048_Photo of students with black hair and brown eyes (plus one who do not)

9 In a group of 25 males, 13 play basketball, 11 play football and 6 play both basketball and football.

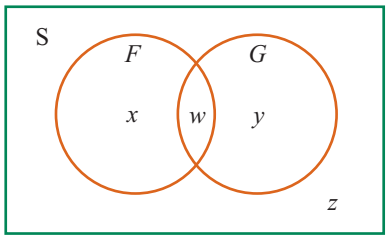


a Complete the Venn diagram representing this information, where B is the event 'plays basketball' and F is the event 'plays football', by finding the numbers w, x, y and z.

b How many males:

i do not play basketball?
ii do not play football?
iii play basketball or football or both?
iv play basketball or football but not both?
v play neither basketball nor football?
vi play basketball but not football?
vii play football but not basketball?

10 In a class of 30 students, 12 study French, 15 study German and 5 study both French and German.



a Complete the Venn diagram representing this information, where F is the event 'studies French' and G is the event 'studies German' by finding the numbers w, x, y and z.

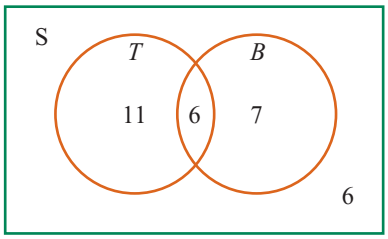
b How many students:

i do not study French?
ii do not study German?
iii study French or German or both?
iv study French or German but not both?
v study French but not German?
vi study German but not French?
vii do not study a language?
viii study at least one language?
ix study at most one language?

EXAMPLE 5

In a survey of her class Sally found that 17 students travelled to school by train, 13 travelled by bus and 6 travelled by both train and bus. There were 30 students in Sally's class. Draw a Venn diagram to represent this information, using T is the event 'travels by train' and B is the event 'travels by bus'.

The number of students who travel by both train and bus is 6, so 6 goes in the intersection of the circles. A total of 17 students travel by train, so $17 - 6 = 11$ students travel by train only, so 11 goes in the T circle. A total of 13 students travel by bus, so $13 - 6 = 7$ students travel by bus only, so 7 goes in the B circle. There are 30 students in the class so the number who do not travel by either train or bus is $30 - (11 + 6 + 7) = 6$ so 6 goes outside the circles.



11 In a group of 19 people waiting at a bus stop one morning, 13 people had an umbrella, 4 people had a raincoat and 3 people had both an umbrella and a raincoat.

a Draw a Venn diagram to represent this information, using U for the event 'has an umbrella' and R for the event 'has a raincoat'.

b How many people in this group:

i do not have an umbrella?
ii do not have a raincoat?
iii have an umbrella or a raincoat or both?
iv have an umbrella or a raincoat but not both?
v have neither an umbrella nor a raincoat?
vi have an umbrella but not a raincoat?
vii have a raincoat but not an umbrella?

12 In a survey of a music class it was found that 9 students played the piano, 4 played the guitar and no students played both piano and guitar. There were 18 students in the class.

a Draw a Venn diagram to represent this information, using P for the event 'plays the piano' and G for the event 'plays the guitar'.

b How many students:

i do not play the piano?
ii do not play the guitar?
iii play the piano or the guitar?
iv play neither the piano nor the guitar?
v play at least one instrument?
vi play at most one instrument

12049_Photo of students playing a piano or guitar.

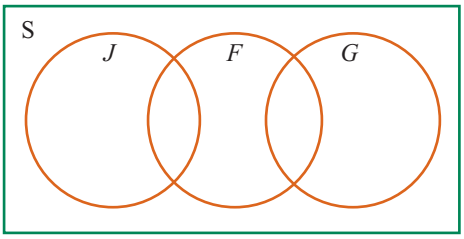
13 In a class of 30 students 25 passed the term mathematics test, 24 passed the science test and 23 students passed both tests.

a Draw a Venn diagram to represent this information using M for the event 'passed mathematics' and S for the event 'passed science'.

b How many student from this class:

i did not pass mathematics?
ii did not pass science?
iii passed neither mathematics nor science?
iv passed mathematics or science or both?
v passed mathematics or science but not both?
vi passed mathematics but not science?
vii passed science but not mathematics?
viii passed at least one subject?
ix passed at most one subject?
x failed at least one subject?
xi failed at most one subject?

14 In a class of 30 students 5 study Japanese, 13 study French, 9 study German, 2 study both Japanese and French and 5 study both French and German.

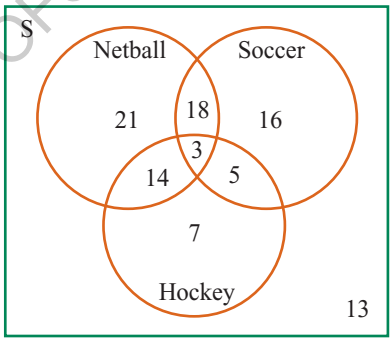


a Complete the Venn diagram where J is the event 'studies Japanese', F is the event 'studies French' and G is the event 'studies German'.

b How many students in this class:

i do not study a language?
ii do not study Japanese?
iii do not study French?
iv do not study German?
v study Japanese or French or both?
vi study Japanese or French but not both?
vii study French or German or both?
viii study French or German but not both?
ix study Japanese or German?
x study Japanese and French and German?
xi study Japanese only?
xii study French only?
xiii study German only?
xiv study exactly one language?
xv study exactly two languages?
xvi study at least one language?
xvii study at most one language

15 The Venn diagram represents the number of girls in Year 8 who play netball, soccer or hockey. How many girls:



a play netball?
b play soccer?
c play hockey?
d play netball and soccer?
e play netball and hockey?
f play soccer and hockey?
g play netball and soccer and hockey?
h netball or soccer or both?
i play netball or soccer but not both?
j play netball or hockey or both?
k play netball or hockey but not both?
l play soccer or hockey or both?
m play soccer or hockey but not both?
n do not play netball?
o do not play soccer?
p do not play hockey?
q do not play any of these sports?
r play netball but not soccer?
s play soccer but not netball?
t play netball but not hockey?
u play hockey but not netball?
v play soccer but not hockey?
w play hockey but not soccer?
x play exactly one of these sports?
y play at least one sport?
z play at most one sport?

12050 Photo of girls playing soccer or whatever from question 15

H Two-way tables

The information collected from data and surveys can also be represented in two-way tables.

EXAMPLE 1

The table shows the data collected from a survey of a Year 8 class.

	Left-handed	Right-handed	
Male	2	14	16
Female	1	13	14
	3	27	30

Use the table to find the number of students who are:

a in the class
b male
c female
d left-handed
e right-handed
f male and left-handed
g female and right-handed
h male or right-handed or both
i male or right-handed but not both
j neither male nor right-handed
k right-handed but not male
l left-handed but not female.

a 30 students in the class
b 16 male students
c 14 female students
d 3 left-handed students
e 27 right-handed students
f 2 male and left-handed students
g 13 female and right-handed students
h 29 male or right-handed students or both
i 15 male or right-handed students but not both
j 1 neither male nor right-handed student
k 13 right-handed but not male students
l 2 left-handed but not female students

Exercise 12H

1 The information in the table was collected from a group of athletes.

	Heavy	Light	
Tall	8	9	17
Short	3	10	13
	11	19	30

Use the table to find the number of people who are:

a in the group of athletes
b tall
c short
d heavy
e light
f short and light
g short and heavy
h tall and light
i tall and heavy
j tall or light or both
k tall or light but not both
l short or heavy or both
m short or heavy but not both
n neither short nor heavy
o neither tall nor light
p heavy but not short
q light but not tall.

2 The table shows the results of a survey of a group of students.

	Born in Australia	Born overseas	
Male	87	29	116
Female	98	16	114
	185	45	230

Use the table to find how many students were:

a surveyed
b male
c female
d born in Australia
e born overseas
f male and born overseas
g female and born in Australia
h male or born overseas or both
i male or born overseas but not both
j neither male nor born overseas
k neither female nor born in Australia
l born in Australia but not male
m female but not born in Australia.

3 The table shows data collected from a group of students. How many students have:

	Black hair	Not black hair	
Brown eyes	11	6	17
Not brown eyes	5	8	13
	16	14	30

a black hair?
b brown eyes?
c black hair and brown eyes?
d black hair or brown eyes or both?
e black hair or brown eyes but not both?
f neither black hair nor brown eyes?
g black hair but not brown eyes?
h brown eyes but not black hair?

4 The table shows the results of a survey of the way a group of employees get to work each day.

	Bus	Not bus	
Train	8	v	11
Not train	y	5	z
	x	w	23

a Complete the table by finding the missing numbers.

b How many employees travel by:
i bus?
ii train?
iii bus and train?
iv bus or train or both?
v bus or train but not both?
vi neither bus nor train?
vii bus but not train?
viii train but not bus?

5 The table shows data collected from a group of students.

	Passed Mathematics	Did not pass Mathematics	
Passed English	k	3	m
Did not pass English	l	n	6
	31	p	36

a Complete the table by finding the missing numbers.
b How many students:
i passed Mathematics?
ii passed English?
iii passed Mathematics and English?
iv passed Mathematics or English or both?
v passed Mathematics or English but not both?
vi failed both subjects?
vii passed Mathematics but not English?
viii passed English but not Mathematics?

6 A group of students were surveyed about whether or not they played soccer or netball. It was found that 15 played soccer, 16 played netball, 10 played both soccer and netball and 7 played neither soccer nor netball.

	Soccer	Not soccer	
Netball			
Not netball			

a Put this information in a two-way table like the one shown and find the missing numbers.

b How many students:

i do not play soccer?

ii do not play netball?

iii play soccer or netball or both?

iv play soccer or netball but not both?

v play soccer but not netball?

vi play netball but not soccer?

7 100 people were surveyed about their smoking and drinking habits. It was found that 19 people smoked cigarettes, 13 both smoked and drank alcohol and 15 people neither smoked nor drank alcohol.

	Drink	Do not drink	
Smoke			
Do not smoke			

a Put this information in a table like the one shown and find the missing numbers.

b How many people:

i drink alcohol?

ii do not drink alcohol?

iii do not smoke?

iv smoke or drink or both?

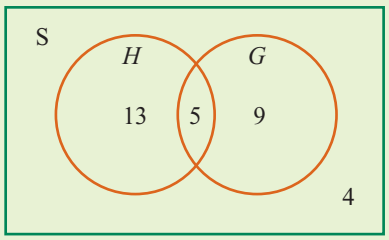
v smoke or drink but not both?

vi drink but do not smoke?

vii smoke but do not drink?

EXAMPLE 2

Information was collected from a group of students about the subjects that they studied. The number of students studying history (H) or geography (G) is shown in the Venn diagram. Represent the information in the Venn diagram in a two-way table like the one shown.



	History	Not history	
Geography			
Not geography			

There are $13 + 5 + 9 + 4 = 31$ in the group.

$13 + 5 = 18$ students study history

$9 + 5 = 14$ students study geography

5 study both history and geography

4 study neither history nor geography

9 study geography but not history

13 study history but not geography

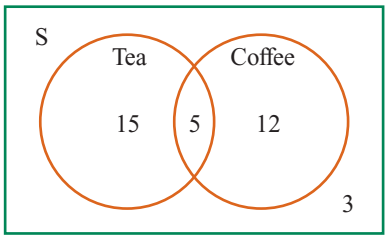
$13 + 4 = 17$ do not study geography

$9 + 4 = 13$ students do not study history

	History	Not history	
Geography	5	9	14
Not geography	13	4	17
	18	13	31

Note: Once you have transferred some information from the Venn diagram it is sometimes possible to use your knowledge of tables to complete it.

8 Data was collected from a group of people about whether they drink tea or coffee. The information is shown in the Venn diagram.



a Represent this information in a two-way table like the one below.

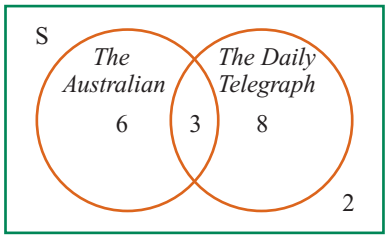
	Drink tea	Do not drink tea	
Drink coffee			
Do not drink coffee			

b Use the Venn diagram or the table to find how many people:

- **i** drink tea
- **ii** do not drink tea
- **iii** drink both tea and coffee
- **iv** drink tea but not coffee
- **v** drink tea or coffee but not both
- **vi** drink neither tea nor coffee
- **vii** at least one beverage
- **viii** exactly one beverage
- **ix** at most one of these beverages.

12051_Photo of people, maybe in a cafe, drinking tea and coffee

9 The people sitting in a café were surveyed about which newspaper they read. The number who read *The Australian* and *The Daily Telegraph* is shown in the Venn diagram.



a Represent this information in a two-way table like the one shown below.

	Read *The Australian*	Do not read *The Australian*	
Read *The Daily Telegraph*			
Do not read *The Daily Telegraph*			

b How many people read:

- **i** *The Australian*?
- **ii** *The Daily Telegraph*?
- **iii** both *The Australian* and *The Daily Telegraph*?
- **iv** neither *The Australian* nor *The Daily Telegraph*?
- **v** *The Australian* but not *The Daily Telegraph*?
- **vi** *The Daily Telegraph* but not *The Australian*?
- **vii** *The Australian* or *The Daily Telegraph* or both?
- **viii** *The Australian* or *The Daily Telegraph* but not both?
- **ix** at least one newspaper?
- **x** exactly one newspaper?
- **xi** at most one newspaper?

12052_Photo of someone reading a Sydney newspaper

Language in mathematics

1 Complete the following statements.

a Probability of an event $= \dfrac{\text{number of \_\_\_\_\_ outcomes}}{\text{number of \_\_\_\_\_ outcomes}}$

b If an event is impossible then its probability is _____.

c If an event is certain then its probability is _____.

d The probability of any event lies in the range _____ to _____ inclusive.

e The sum of the probabilities of all possible outcomes for an experiment is _____.

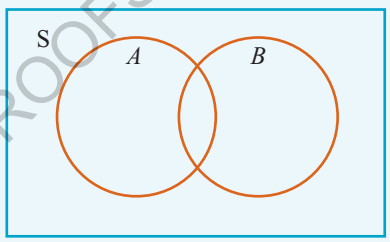
f The events 'getting a 2' and 'not getting a 2' are called _____ events.

g In a Venn diagram the sample space is represented by points, or the region, inside a _____ and the outcomes favourable to an event are represented by points, or the region, inside a _____.

2 What is the complement of the event:

a selecting a blue marble? b tossing a head?

3 The Venn diagram shows the events A and B in the sample space S. Describe in words the region where you would find the following set of outcomes.



a favourable to event A

b the complement of A

c favourable to events A and B

d favourable to events A or B or both

e favourable to events A or B but not both

Terms

certain	common	complement	complementary events		equally likely
excluding	exclusive or	divisible	favourable	impossible	including
inclusive or	intersection	outcome	possible	probability	random
sample space	shuffled	theoretical	Venn diagram		

Check your skills

1 The numbers 1 to 7 are written on cards, the cards are shuffled and one is selected at random. The probability that it is a number less than 4 is:

A $\frac{3}{7}$ **B** $\frac{4}{7}$ **C** $\frac{3}{4}$ **D** $\frac{7}{3}$

2 The letters of the word DISTRIBUTION are written on cards and one card is selected at random. The probability that the letter chosen is I is:

A $\frac{1}{10}$ **B** $\frac{1}{4}$ **C** $\frac{3}{10}$ **D** $\frac{1}{3}$

3 If the probability of an event is 1 then the event is:

A certain **B** quite likely **C** unlikely **D** impossible

4 When calculating the probability of choosing at random an apple from a bag of fruit, which of the following answers could not be correct?

A 0 **B** $\frac{3}{11}$ **C** $\frac{11}{3}$ **D** 0.3

5 A roulette wheel has red, green and black slots. When spun the probability that the ball will stop in a red slot is $\frac{18}{37}$ and the probability that it will stop in black slot is also $\frac{18}{37}$. The probability that it will stop in a green slot is:

A $\frac{18}{37}$ **B** $\frac{1}{37}$ **C** $\frac{1}{18}$ **D** $\frac{36}{37}$

6 The probability that a particular set of traffic lights is green is $\frac{43}{73}$. The probability that the light will not be green is:

A $\frac{30}{43}$ **B** $\frac{30}{73}$ **C** $\frac{43}{30}$ **D** $\frac{1}{2}$

7 If a marble is chosen from a bag containing red, blue, green and white marbles, the complement of the event 'choosing a red marble' is:

A choosing a blue marble
B choosing a blue or green marble
C choosing a blue or white marble
D choosing a blue or green or white marble.

8 The probability of choosing a red disc from a box is $\frac{4}{23}$. The probability of not drawing a red disc from the box is:

A $\frac{4}{23}$ **B** $-\frac{4}{23}$ **C** $\frac{19}{23}$ **D** $\frac{23}{4}$

9 A card is selected at random from a normal 52-card playing pack. Which of the following events are mutually exclusive?

A getting a queen, getting a red card
B getting a club, getting picture card
C getting a king, getting an ace
D getting a red 9, getting a diamond

10 The numbers 1 to 9 are written on cards and one card is selected at random. The event 'at least a 6' is equivalent to:

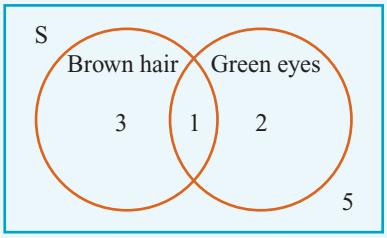
A 1, 2, 3, 4, 5 or 6 **B** 1, 2, 3, 4 or 5 **C** 6, 7, 8 or 9 **D** 7, 8 or 9

11 This table shows the possible results, including the order, when two children are born. The outcomes for the event 'at most one boy' are:

A {bg, gb} **B** {bg, gb, bb}
C {bg, gb, gg} **D** {bg, gb, bb, gg}

		1st child	
		Boy	**Girl**
2nd child	**Boy**	bb	gb
	Girl	bg	gg

The Venn diagram shows the number of people in a group of friends who have brown hair and green eyes. A person is selected at random from this group. Use the Venn diagram to answer questions **12** to **14**.



12 The number of people who have brown hair and green eyes is:

A 1 **B** 3
C 5 **D** 6

13 The number of people who do not have brown hair is:

A 2 **B** 5 **C** 7 **D** 8

14 The number of people who have brown hair or green eyes but not both is:

A 1 **B** 3 **C** 5 **D** 6

The table below shows information about a Year 8 class. Use the table to answer questions **15** to **17**.

	Passed Mathematics	Did not pass Mathematics	
Passed Science	22	2	24
Did not pass Science	x	y	5
	26	3	29

15 The missing numbers x and y are:

A 4, 1 **B** 19, 1 **C** 4, 23 **D** 19, 23

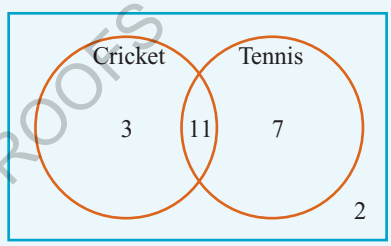
16 The number of students who passed mathematics but not science is:

A 2 **B** 4 **C** 33 **D** 26

17 The number of students who passed at least one subject is:

A 6 **B** 23 **C** 28 **D** 50

18 The information in the Venn diagram is to be represented in a two-way table like the one shown.



	Play tennis	Do not play tennis	
Play cricket		x	
Do not play cricket			
	y		

The values of x and y are:

A 3, 7 **B** 14, 7 **C** 3, 18 **D** 14, 18

If you have any difficulty with these questions, refer to the examples and questions in the section listed in the table.

Question	1–4	5, 6	7	8	9	10, 11	12–14	15–18
Section	A	B	C	D	E	F	G	H

12A Review set

1 The numbers 1 to 9 are written on cards, the cards shuffled and one is selected at random. Find the probability that the number is:

a 7 **b** 7 or 8 **c** odd **d** even **e** less than 4

2 A bag contains 4 oranges, 3 apples and 2 bananas. If one piece of fruit is chosen at random from the bag, find the probability that it is:

a an apple
b an orange
c an apple or a banana
d an orange or an apple or a banana
e a peach.

12053_Photo of fruit: oranges, apples and banannas

3 A normal six-sided die is rolled. Write down an event E for which:

a $P(E) = 0$ **b** $P(E) = 1$

4 Red, blue and green tickets are placed in a hat. If one ticket is chosen at random from this hat, the probability that it is red is $\frac{1}{2}$ and the probability that it is blue is $\frac{2}{5}$. Calculate the probability that it is green.

5 A die is rolled. Write down the complement of these events.

a a 4 **b** an even number **c** a number less than 4

6 The letters of a word are written on cards and the cards placed in a hat. When a card is chosen at random from this hat, the probability that it is the letter E is $\frac{2}{5}$. What is the probability that it is not the letter E?

7 A coin is tossed twice. The table shows all the possible outcomes.

a S = {_____}

b Use the table to list the outcomes for the event:

i a head and a tail, in any order **ii** not two heads

iii at least one head **iv** at most one head

		1st toss	
		H	**T**
2nd toss	**H**	HH	TH
	T	HT	TT

8 The numbers 1 to 9 are written on cards, the cards are shuffled and one card is selected at random. State whether the following events are mutually exclusive or non-mutually exclusive.

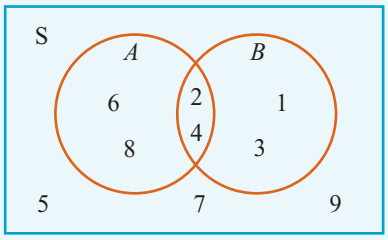
a selecting an even number, selecting a number > 4

b selecting an even number, selecting a multiple of 3

c selecting a number > 4, selecting a number < 4

d selecting a number > 3, selecting a number > 6

9 Numbers are written on cards, the cards are shuffled and one card is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram.



a What is the number of possible outcomes for this experiment?

b List the outcomes for the following events.

i A **ii** not A

iii B **iv** not B

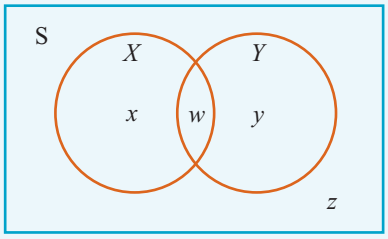
v A but not B **vi** B but not A

vii A and B **viii** A or B or both

ix A or B but not both **x** neither A nor B

10 In a class of 30 students, 15 have black hair, 12 have brown eyes and 7 have both black hair and brown eyes.

a Complete the Venn diagram representing this information, where X is the event 'black hair' and Y is the event 'brown eyes' by finding the numbers w, x, y and z. Note that the number of students, not the individual students, is shown in each region of the diagram.



b How many students:

i have black hair? **ii** do not have black hair?

iii have brown eyes? **iv** do not have brown eyes?

v have black hair but not brown eyes? **vi** have brown eyes but not black hair?

vii have black hair and brown eyes? **viii** have black hair or brown eyes or both?

ix have black hair or brown eyes but not both? **x** have neither black hair nor brown eyes?

11 The table shows the data collected from a group of students about the way they get to school.

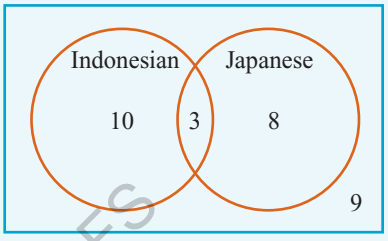
	Travel by bus	Do not travel by bus	
Walk	83	32	115
Do not walk	44	20	64
	127	52	179

How many students:

a travel by bus? **b** do not walk? **c** travel by bus or walk but not both?

d travel by bus and walk? **e** neither walk nor travel by bus?

12 The Venn diagram shows information about the languages studied by a group of students.



a Transfer this information to a two-way table like the one below.

	Study Indonesian	Do not study Indonesian	
Study Japanese			
Do not study Japanese			

b How many students study:

i Indonesian and Japanese? **ii** neither Indonesian nor Japanese?

iii Indonesian but not Japanese? **iv** Indonesian or Japanese but not both?

12B Review set

1 An ordinary six-sided die is thrown. Find the probability of getting:

a a 6 **b** an even number **c** a number > 3

d a number between 1 and 5 **e** a number divisible by 3

2 A letter is chosen at random from the letters of the word MISSISSIPPI. Find the probability that the letter chosen is:

a M **b** P **c** I **d** S **e** K

3 A spinner is divided into three equal parts coloured red, blue and green. Write down an event E for which:

a $P(E) = 0$ **b** $P(E) = 1$

4 When two coins are tossed the possible outcomes are exactly 2 heads, exactly 1 head or zero heads. Given that the probability of zero heads is $\frac{1}{4}$ and the probability of exactly 1 head is $\frac{1}{2}$, calculate the probability of getting exactly 2 heads.

5 A bag contains red, blue and green marbles. A marble is selected at random. Write down the complement of each event.

a a blue marble **b** a red or blue marble **c** not a blue marble

6 When a lolly is selected at random from a bag the probability that it is red is $\frac{6}{17}$. If a lolly is chosen at random from this bag, what is the probability that it is not red?

7 The numbers 1 to 9 are written on cards and one card is selected. List the outcomes for these events.

a at least 7

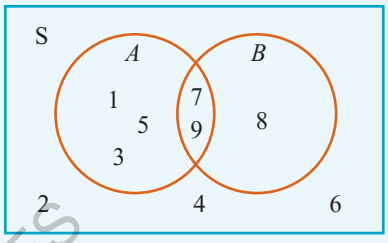
b at most 5

8 State with reasons whether the qualifiers 'or both' and 'but not both' are needed for the event 'A or B' if, when a die is rolled:

a A is the event 'an odd number', B is the event 'an even number'

b A is the event 'a number < 4', B is the event 'a number > 2'.

9 A spinner with nine equal parts numbered 1 to 9 is spun. A is the event 'the spinner lands on an odd number' and B is the event 'the spinner lands on a number > 6'. The outcomes of this experiment and events A and B are shown in the Venn diagram.



Use the diagram to list the outcomes in these events.

a an odd number

b *n*ot an odd number

c a number > 6

d a number not > 6

e a number > 6 that is not odd

f an odd number not > 6

g a number that is odd and > 6

h a number that is odd or > 6 or both

i a number that is odd or > 6 but not both

j a number that is neither odd nor > 6

10 In a group of 27 students, 12 play football, 11 play cricket and 6 play both football and cricket.

a Construct a Venn diagram to show this information, where F is the event 'plays football' and C is the event 'plays cricket'.

12054_Photo of students with blond hair and blue eyes or students playing cricket

b How many students:

i play football but not cricket?

ii play cricket but not football?

iii play football or cricket or both?

iv play football or cricket but not both?

v neither play football nor cricket?

11 The table shows the information collected from a group of people.

	Have blond hair	Do not have blond hair	
Have blue eyes	x	y	13
Do not have blue eyes	4	z	15
	11	17	28

a Find the missing numbers x, y and z.

b How many students have:

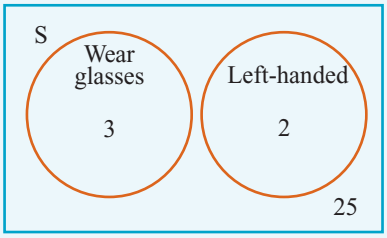
i blond hair and blue eyes?

ii neither blond hair nor blue eyes?

iii blond hair or blue eyes or both?

iv blond hair but not blue eyes?

12 The Venn diagram shows the information collected from a class of students about whether they wear glasses and whether they are left-handed.



Transfer this information to the two-way table below.

	Wear glasses	Do not wear glasses	
Left-handed			
Not left-handed			

12C Review set

1 The numbers 0 to 7 are written on cards, the cards are shuffled and one is selected at random. What is the probability that the number is:

a 7? **b** 2 or 3? **c** even?

d between 0 and 7? **e** greater than 4?

2 A card is selected at random from a normal playing pack of 52 cards. Find the probability that it is:

a the ace of spades **b** a king **c** a red 7 **d** a diamond

e black **f** red or black **g** a green 19

3 The numbers 1 to 5 are written on cards, the cards are shuffled and one is chosen at random. Write down an event E for which:

a $P(E) = 0$ **b** $P(E) = 1$

4 A purse contains 5-cent, 10-cent, 20-cent and 50-cent coins. When a coin is chosen at random from this purse, the probability of getting a 5-cent coin is $\frac{1}{8}$, the probability of getting a 10-cent coin is $\frac{2}{5}$ and the probability of getting a 50-cent coin is $\frac{1}{10}$. Calculate the probability of getting a 20-cent coin.

5 A card is selected at random from a normal playing pack. What is the complement of the event:

a a red card? **b** a spade? **c** an ace?

6 On a roulette wheel, the probability that the ball will stop in a black slot is $\frac{9}{17}$. What is the probability that it will not stop in a black slot?

7 A normal six-sided die is rolled. Write down two events A and B that are:

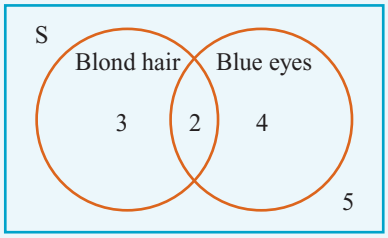
a mutually exclusive **b** non-mutually exclusive.

8 A card is chosen from a normal playing pack. List the outcomes for these events.

a a red card and a jack **b** a spade and a picture card

9 State with reasons if the qualifiers 'or both', 'but not both' are needed for the event 'A or B' given that a student is chosen from a class of boys and girls and A is the event 'a boy', B is the event 'has red hair'.

10 In a group of friends, some have blond hair and some have blue eyes. This information is shown in this Venn diagram.



a How many people are in this group?

b How many people:

i have blond hair?

ii do not have blond hair?

iii have blue eyes?

iv do not have blue eyes?

v have blond hair but not blue eyes?

vi have blue eyes but not blond hair?

vii have blond hair and blue eyes?

viii have blond hair or blue eyes or both?

xi have blond hair or blue eyes but not both?

x have neither blond hair nor blue eyes?

11 In a class of 30 students, 13 students study history, 15 study geography and 7 study both history and geography. Construct a Venn diagram to show this information. Let H be the event 'studies history' and G the event 'studies geography'.

12 **a** Represent the information in question **11** in a two-way table like the one below.

	Study History	**Do not study History**	
Study Geography			
Do not study Geography			

b How many students:

i do not study history?

ii do not study geography?

iii study history and geography?

iv study history or geography or both?

v study history or geography but not both?

vi study history but not geography?

vii study geography but not history?

viii study neither history nor geography?

ix study at least one of these subjects?

x study exactly one of these subjects?

xi study at most one of these subjects?

12D Review set

1 The numbers 1 to 21 are written on 21 cards. One card is selected at random. Find the probability that the number is:

a 10 **b** bigger than 10 **c** smaller than 10 **d** odd

e even **f** a multiple of 5 **g** contains the digit 5 **h** contains the digit 1.

2 The letters of the word PROBABILITY are written on cards and one is chosen at random. What is the probability that the letter is:

a B **b** I **c** R

d B or I **e** a vowel **f** K

3 A spinner is divided into three equal segments coloured red, blue and green. The spinner is spun. Write down an event E for which:

a $P(E) = 1$ **b** $P(E) = 0$

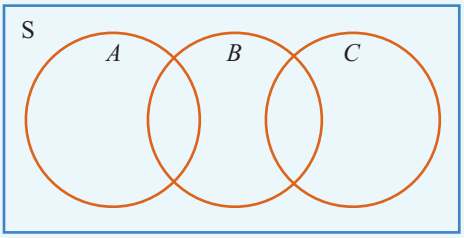
4 Black, white and purple tickets are placed in a hat and one ticket is drawn at random. Given that the probability of drawing a black ticket is $\frac{1}{3}$ and the probability of a white ticket is $\frac{1}{5}$, what is the probability of drawing a purple ticket?

5 A card is selected at random from a normal playing pack. Write down the complement of each event.

a a red card **b** a diamond

6 When a chocolate is selected at random from a box, the probability that it is a dark chocolate is 0.4. What is the probability that it is not a dark chocolate?

7 The Venn diagram shows three events A, B and C. State whether these statements are true or false.



a A and B are mutually exclusive.

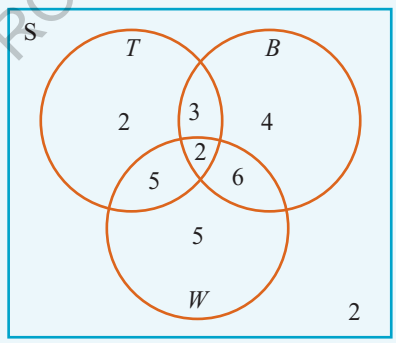
b A and C are mutually exclusive.

c B and C are non-mutually exclusive.

8 A normal six-sided die is thrown. List the outcomes for each event.

a a number > 3 **b** at least 3 **c** a number < 3 **d** at most 3

9 The Venn diagram represents the way in which the students in a class travel to school. T is the event 'travels by train', B is the event 'travels by bus' and W is the event 'walks to school'. How many students travel to school by:



a train? **b** train only?

c train and walk? **d** train or walk or both?

e train or walk but not both? **f** bus and walk?

g bus or walk but not both? **h** bus and train and walk?

i train but not by bus? **j** bus or train or both?

k none of these methods?

10 The two-way table shows the number of students in a class who study French and German.

	Study French	Do not study French	
Study German	3	w	11
Do not study German	x	13	19
	y	21	z

a Find the missing numbers w, x, y and z.

b Draw a Venn diagram to represent this information, using F is the event 'study French' and G is the event 'study German'.

c How many students:

i study French? **ii** do not study French?

iii study German? **iv** do not study German?

v study French and German? **vi** study French or German or both?

vii study French or German but not both? **viii** study neither French nor German?

ix study French but not German? **x** study German but not French?

xi study at least one of these languages? **xii** study at most one of these languages?

xiii study exactly one of these languages?