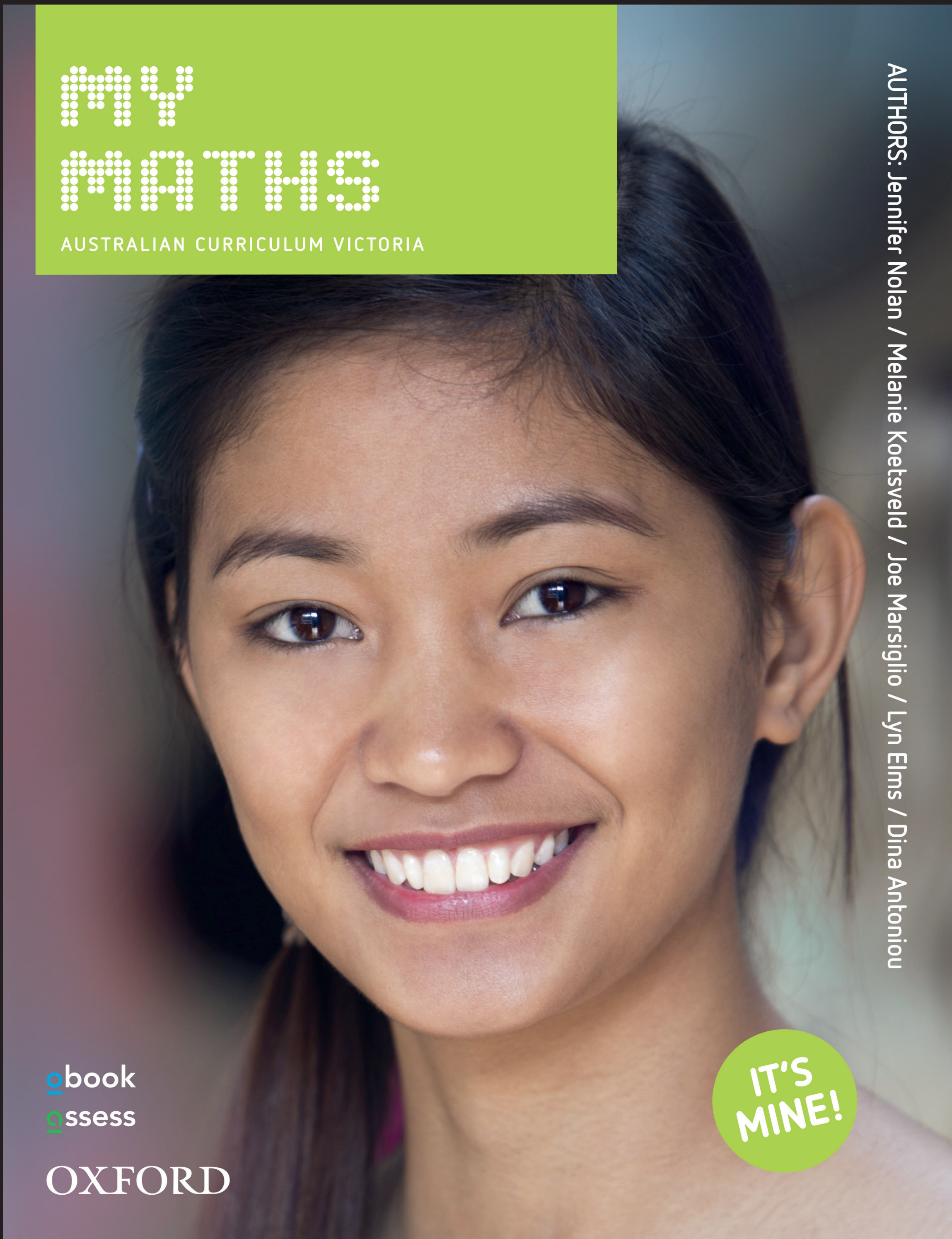


Helping you implement your product

OXFORD

Contents*

Teacher resources

- Curriculum grid
- Work programs for each chapter
- Differentiation tables for each chapter
- Teacher notes for each chapter
- Worksheet, Investigations and student book answers
- Assessment rubrics
- 80 chapter tests (2 per chapter) and answers
- Summative tests (3 per chapter) and answers
- 2000 testbank questions

Student resources

- Student book answers
- 227 interactive modules for exploration and play
- Video tutorials
- 307 SupportSheets for remediation and intervention
- 232 WorkSheets for practice
- 467 Investigations for problem solving and application
- 40 Connect tasks (3 per chapter)
- Additional resources, such as black line masters

* This is the contents from your Oxford Digital Dashboard.

Australian Curriculum: Mathematics Year 7

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the units.

Number and Algebra

Number and place value	**Elaborations**	**MyMaths 7**
Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)	• defining and comparing prime and composite numbers and explaining the difference between them • applying knowledge of factors to strategies for expressing whole numbers as products of powers of prime factors, such as repeated division by prime factors or creating factor trees • solving problems involving lowest common multiples and greatest common divisors (highest common factors) for pairs of whole numbers by comparing their prime factorisation	1H Multiples and factors 1I Prime and composite numbers
Investigate and use square roots of perfect square numbers (ACMNA150)	• investigating square numbers such as 25 and 36 and developing square-root notation • investigating between which two whole numbers a square root lies	1F Powers and square roots
Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)	• understanding that arithmetic laws are powerful ways of describing and simplifying calculations	1G Order of operations
Compare, order, add and subtract integers (ACMNA280)		4A Understanding negative numbers 4B Adding integers 4C Subtracting integers 4D Simplifying addition and subtraction of integers
Real numbers	**Elaborations**	**MyMaths 7**
Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)	• exploring equivalence among families of fractions by using a fraction wall or a number line (for example, by using a fraction wall to show that $\frac{2}{3}$ is the same as $\frac{4}{6}$ and $\frac{6}{9}$)	2A Understanding fractions 2B Equivalent fractions 4F Negative numbers and the Cartesian plane
Solve problems involving	• exploring and developing efficient strategies to solve additive problems involving fractions	2C Adding and subtracting

addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)	(for example, by using fraction walls or rectangular arrays with dimensions equal to the denominators)	fractions
Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)	• investigating multiplication of fractions and decimals, using strategies including patterning and multiplication as repeated addition, with both concrete materials and digital technologies, and identifying the processes for division as the inverse of multiplication	2D Multiplying fractions 2E Dividing fractions 2F Powers and square roots of fractions 3C Multiplying decimals 3D Dividing decimals by a whole number 3E Dividing decimals by a decimal
Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)	• using authentic examples for the quantities to be expressed and understanding the reasons for the calculations	2A Understanding fractions
Round decimals to a specified number of decimal places (ACMNA156)	• using rounding to estimate the results of calculations with whole numbers and decimals, and understanding the conventions for rounding	1A Understanding place value 3A Understanding decimals
Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)	• justifying choices of written, mental or calculator strategies for solving specific problems including those involving large numbers • understanding that quantities can be represented by different number types and calculated using various operations, and that choices need to be made about each • calculating the percentage of the total local municipal area set aside for parkland, manufacturing, retail and residential dwellings to compare land use	3F Converting between fractions and decimals 3H Converting between fractions, decimals and percentages
Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)	• using authentic problems to express quantities as percentages of other amounts	3G Understanding percentages 3I Calculating percentages
Recognise and solve problems involving simple ratios (ACMNA173)	• understanding that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem	2G Understanding ratios 2H Working with ratios

Money and financial mathematics	Elaborations	MyMaths 7
Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)	• applying the unitary method to identify ‘best buys’ situations, such as comparing the cost per 100 g	3D Dividing decimals by a whole number 3E Dividing decimals by a decimal
Patterns and algebra	**Elaborations**	**MyMaths 7**
Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)	• understanding that arithmetic laws are powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra	5A Understanding rules 5B Using pronumerals 5C Terms, expressions and equations
Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)	• using authentic formulas to perform substitutions	5D Evaluating expressions
Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)	• identifying order of operations in contextualised problems, preserving the order by inserting brackets in numerical expressions, then recognising how order is preserved by convention • moving fluently between algebraic and word representations as descriptions of the same situation	5B Using pronumerals 5C Terms, expressions and equations 5F Using flowcharts 5G Building expressions using flowcharts
Linear and non-linear relationships	**Elaborations**	**MyMaths 7**
Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)	• plotting points from a table of integer values and recognising simple patterns, such as points that lie on a straight line	4E Introducing the Cartesian plane 4F Negative numbers and the Cartesian plane 5B Using pronumerals
Solve simple linear equations (ACMNA179)	• solving equations using concrete materials, such as the balance model, and explain the need to do the same thing to each side of the equation using substitution to check solutions • investigating a range of strategies to solve equations	5E Strategies for solving equations 5H Solving equations using backtracking 5I Solving equations using a balance model
Investigate, interpret and analyse	• using travel graphs to investigate and compare the distance travelled to and from school	4G Interpreting graphs

graphs from authentic data (ACMNA180)	• interpreting features of travel graphs such as the slope of lines and the meaning of horizontal lines • using graphs of evaporation rates to explore water storage	

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 7**
Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)	• building on the understanding of the area of rectangles to develop formulas for the area of triangles • establishing that the area of a triangle is half the area of an appropriate rectangle • using area formulas for rectangles and triangles to solve problems involving areas of surfaces	9C Understanding area 9D Area of a rectangle 9E Area of a parallelogram 9F Area of a triangle 9G Surface area
Calculate volumes of rectangular prisms (ACMMG160)	• investigating volumes of cubes and rectangular prisms and establishing and using the formula $V = l \times b \times h$ • understanding and using cubic units when interpreting and finding volumes of cubes and rectangular prisms	9H Volume and capacity
Shape	**Elaborations**	**MyMaths 7**
Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)	• using aerial views of buildings and other 3-D structures to visualise the structure of the building or prism	7E Drawing 2D shapes and 3D objects 7F Planning and constructing 3D objects
Location and transformations	**Elaborations**	**MyMaths 7**
Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)	• describing patterns and investigating different ways to produce the same transformation such as using two successive reflections to provide the same result as a translation • experimenting with, creating and re-creating patterns using combinations of reflections and rotations using digital technologies	7G Symmetry of 2D shapes and 3D objects 7H Describing transformations 7I Performing transformations
Geometric reasoning	**Elaborations**	**MyMaths 7**
Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)	• defining and classifying pairs of angles as complementary, supplementary, adjacent and vertically opposite	6B Types of angles 6D Angles around a point 6E Angles and parallel lines
Investigate conditions for two	• constructing parallel and perpendicular lines using their properties, a pair of compasses and	6A Lines, rays and segments

lines to be parallel and solve simple numerical problems using reasoning (ACMMG164)	a ruler, and dynamic geometry software • defining and identifying the relationships between alternate, corresponding and co-interior angles for a pair of parallel lines cut by a transversal	6E Angles and parallel lines
Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)	• using concrete materials and digital technologies to investigate the angle sum of a triangle and quadrilateral	7A Classifying triangles 7B Classifying quadrilaterals
Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)	• identifying side and angle properties of scalene, isosceles, right-angled and obtuse-angled triangles • describing squares, rectangles, rhombuses, parallelograms, kites and trapeziums	7A Classifying triangles 7B Classifying quadrilaterals

Statistics and Probability

Chance	**Elaborations**	**MyMaths 7**
Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)	• discussing the meaning of probability terminology (for example, probability, sample space, favourable outcomes, trial, events and experiments) • distinguishing between equally likely outcomes and outcomes that are not equally likely	10H Describing probability 10J Experimental probability
Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)	• expressing probabilities as decimals, fractions and percentages	10H Describing probability 10I Theoretical probability 10J Experimental probability
Data representation and interpretation	**Elaborations**	**MyMaths 7**
Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)	• obtaining secondary data from newspapers, the Internet and the Australian Bureau of Statistics • investigating secondary data relating to the distribution and use of non-renewable resources around the world	10A Collecting data 10B Interpreting data
Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)	• understanding that some data representations are more appropriate than others for particular data sets, and answering questions about those data sets • using ordered stem-and-leaf plots to record and display numerical data collected in a class	10B Interpreting data 10C Dot plots, column and bar graphs 10D Pie graphs

	investigation, such as constructing a class plot of height in centimetres on a shared stem-and-leaf plot for which the stems 12, 13, 14, 15, 16 and 17 have been produced	10E Line graphs and scatterplots 10F Stem-and-leaf plots
Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)	• understanding that summarising data by calculating measures of centre and spread can help make sense of the data	10G Summary statistics
Describe and interpret data displays using median, mean and range (ACMSP172)	• using mean and median to compare data sets and explaining how outliers may affect the comparison • locating mean, median and range on graphs and connecting them to real life	10G Summary statistics

Year 7 achievement standard

By the end of Year 7, students solve problems involving the comparison, addition and subtraction of integers. They make the connections between whole numbers and index notation and the relationship between perfect squares and square roots. They solve problems involving percentages and all four operations with fractions and decimals. They compare the cost of items to make financial decisions. Students represent numbers using variables. They connect the laws and properties for numbers to algebra. They interpret simple linear representations and model authentic information. Students describe different views of three-dimensional objects. They represent transformations in the Cartesian plane. They solve simple numerical problems involving angles formed by a transversal crossing two lines. Students identify issues involving the collection of continuous data. They describe the relationship between the median and mean in data displays.

Students use fractions, decimals and percentages, and their equivalences. They express one quantity as a fraction or percentage of another. Students solve simple linear equations and evaluate algebraic expressions after numerical substitution. They assign ordered pairs to given points on the Cartesian plane. Students use formulas for the area and perimeter of rectangles and calculate volumes of rectangular prisms. Students classify triangles and quadrilaterals. They name the types of angles formed by a transversal crossing parallel line. Students determine the sample space for simple experiments with equally likely outcomes and assign probabilities to those outcomes. They calculate mean, mode, median and range for data sets. They construct stem-and-leaf plots and dot plots.

Number and Algebra

3 Decimals and percentages

3 Decimals and percentages

Teaching support for pages 120–1

Syllabus links

Content descriptions and elaborations

Real numbers

ACMNA154: Multiply and divide fractions and decimals using efficient written strategies and digital technologies

- investigating multiplication of fractions and decimals, using strategies including patterning and multiplication as repeated addition, with both concrete materials and digital technologies, and identifying the processes for division as the inverse of multiplication

ACMNA156: Round decimals to a specified number of decimal places

- using rounding to estimate the results of calculations with whole numbers and decimals, and understanding the conventions for rounding

ACMNA157: Connect fractions, decimals and percentages and carry out simple conversions

- justifying choices of written, mental or calculator strategies for solving specific problems including those involving large numbers
- understanding that quantities can be represented by different number types and calculated using various operations, and that choices need to be made about each
- calculating the percentage of the total local municipal area set aside for parkland, manufacturing, retail and residential dwellings to compare land use

ACMNA158: Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies

- using authentic problems to express quantities as percentages of other amounts

Money and financial mathematics

ACMNA174: Investigate and calculate 'best buys', with and without digital technologies

- applying the unitary method to identify ‘best buys’ situations, such as comparing the cost per 100 g

The proficiency strands **Understanding, Fluency, Problem solving** and **Reasoning** is fully integrated into the content of this chapter.

Teaching strategies

Discussion prompts

Direct students to examine the opening photo on pages 120 and 121 for this chapter in their Student Book.

- Ask students to suggest what they think the photograph could be. (A board at the Stock exchange)
- Can they identify the two parts represented in many of the numbers shown? (Whole numbers are represented by digits before the decimal point and the fractional or decimal part is represented by digits after the decimal point.)
- Ask students what they think the decimal numbers could represent. (Money, the prices of shares)
- Ask students how many decimal places are practical for money calculations, and what the decimal places in money represent. (Two decimal places; these represent cents)
- Ask students what might happen if somebody wanted to sell shares that are shown as a decimal number with three decimal places. (Decimal number would need to be rounded to two decimal places)
- Ask students to find an example of a decimal number with three decimal places from the photo, round this to two decimal places. (Answers will vary. An example is 5.985 = 5.99, remember 0, 1, 2, 3, 4 we round down and 5, 6, 7, 8, 9 we round up.)
- Ask students to find another two examples and round them to two decimal places.
- Brainstorm about other places where decimals are used in everyday life. (Examples could be measurements, house plans and recipes. Ensure students do not reach the conclusion that the main purpose of decimals is working with money and the economy.)

Essential question

Decimals and percentages are like fractions: they represent numbers that include whole numbers and parts of wholes. In what situations do decimals play a vital role?

Possible answers: Decimals play a vital role in money transactions and measurements. They are also used in scientific calculations, particularly for very large and very small measurements. House plans and cooking recipes often rely on decimal measurements.

Are you ready?

Prior knowledge and skills can be tested by completing **Are you ready?** This will give you an indication of the differentiated pathway each student should follow.

Students will demonstrate their ability to:

- understand place value and place-value notation from thousands to thousandths
- recognise place value of the digits in a decimal number
- round a number to its leading digit
- perform the operations of addition, subtraction, multiplication and division on whole numbers
- write the fractional equivalent of simple decimals
- write the decimal equivalent of simple fractions
- write proportions of a whole as a fraction
- write equivalent fractions with a denominator of 100
- calculate a fraction of a whole number.

It would be beneficial for students to have completed Chapter 1 *Whole numbers* and Chapter 2 *Fractions and ratios* before starting this chapter.

At the beginning of each topic there is a suggested differentiated pathway which allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to help them select the best pathway.

Support strategies and **SupportSheets** which may help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

ANSWERS

CHAPTER 3 DECIMALS AND PERCENTAGES

3 Are you ready?

1 a B	b C	c C	**2** D
3 a 0.7	b 0.01	**4** a 600	b 20
5 a B	b 67 136	**6** a 3515	b B
7 a B	b 314		
8 a C	b A	c 0.3	d C
9 a D	b $\frac{75}{100}$ or $\frac{3}{4}$	c $\frac{31}{100}$	
10 a D	b $\frac{70}{100}$	**11** a A	b 138

Resources

assess: assessments

Each topic of the *MyMaths 7* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students' confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

3A Understanding decimals

Teaching support for pages 122–7

Teaching strategies

Learning focus

To build on student understanding of place value in decimals.

Start thinking!

Have students think about how we can measure 6.022 metres using a tape measure. Does it just refer to centimetres and metres or does it also refer to other units (millimetres)?

Ensure students understand that decimal numbers are fractional parts of a number.

The task guides students to:

- write decimals in a place-value chart
- identify the number of decimal places in a decimal number
- understand the relationship between the place value of a decimal and its equivalent fractional value.

Differentiated pathways

Below Level	At Level	Above Level
1–3, 4a–f, 6a, 8b, 9c, 10a, d, 11–14, 16, 18a, b	2, 4g–l, 5a–f, 6b, c, 7–10, 13, 15–22	5g–i, 6c, d, 7, 8a, e, 10, 13, 15f–j, 16, 17, 19–21, 23
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q1–3

Focus: To demonstrate knowledge and understanding of place value

- Direct students to complete **SS 3A-1 Understanding place value** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how a place-value chart used for whole numbers can be extended to include decimal places.
- You may like to provide students with a copy of the BLM **Place-value chart – decimals** (see Resources) as an aid to completing further examples.

- Students should recognise the difference between the whole number and the decimal component of decimals. They need to be able to write a number in a place-value chart and also in expanded fractional notation.
- Students should have an understanding of placeholder zeros and trailing zeros, and their effect on the value of a decimal number.

At Level

At Level
2, 4g–l, 5a–f, 6b, c, 7–10, 13, 15–22

- Demonstrate **3A eTutor** or direct students to do this independently.
- Ensure students are able to:
  - write decimals in place-value notation and identify the number of decimal places
  - order numbers according to size and to distinguish between *greater than* and *less than*
  - consider placeholder zeros and trailing zeros
  - sketch a number line and mark the position of positive integers on the number line
  - round numbers to their leading digit.
- When using the < or > symbols to determine which is greater than or less than, have students first circle the larger number, and then insert the correct symbol. This process of always circling the larger number helps students to break questions down into parts and thus ensures a greater accuracy with these types of questions.
- When ordering decimal numbers, students may have a number of misconceptions which need to be monitored.

  (1) Students may think that the number of digits after the decimal point is an indication of how large the number is. For example, 3.123 33 is larger than 3.2 because 5 digits are more than 1 digit.

  (2) Students may mistakenly believe that 3.15 is larger than 3.5 because 15 is larger than 5.
- When comparing decimal numbers, have students place each number on a place-value chart, on top of one another (ensure they fill in all the trailing zeros before they start the comparison). Starting from the left side, students compare each digit in the place-value

chart.

- Q6 and Q7 could be run as a whole class activity. Laminate each of the numbers on large pieces of card. Hand out five decimal numbers from the Below Level to begin with and have students place themselves at the front of the class on an imaginary number line. Students who are not yet confident with decimal numbers could be given whole numbers and used as *anchors*. As students are not working independently they can participate in group discussions on which decimal numbers go where. It is easier to run this activity if in the beginning one student is designated the zero and another is a whole number which is larger than all the decimal numbers; this will provide a framework for the construction of the number line.
- When rounding numbers, students should draw a line after the final decimal place value before rounding. For example, 3.456 322 rounded to two decimal places would look like 3.45|6 322.
- When converting a decimal into a fraction, have students first identify the number of decimal places, as this will form the number of zeros in the denominator. Students then write the number in its fractional form before simplifying the fraction.

POTENTIAL DIFFICULTY
Understanding place value is essential to counting and comparing. Students should be aware that zero is a placeholder and that the decimal point separates the whole numbers from fractional numbers.

- For additional practice, students can complete **WS 3A-2 Comparing decimals** (see Resources). Students use a place-value chart to compare decimal number to see which number is larger.
- For more problem-solving tasks and investigations, direct students to **INV 3A-3 Target race** (see Resources). Students investigate representing decimals as a fraction of a whole on number lines in an attempt to reach $200 and $1000 targets. They interpret the results.

Below Level

Below Level
1–3, 4a–f, 6a, 8b, 9c, 10a, d, 11–14, 16, 18a, b

- Demonstrate **3A eTutor** or direct students to do this independently.
- When students transition from primary to secondary schooling, they come with knowledge of the region model for fractions, but may have little experience with symbols and language of fractions and thus decimal numbers.
- Cut out strips of paper which are 1 metre in length and around 5 cm in width. On each side, mark out intervals of 10 cm length. On one side these increments will represent

percentages, the other will be tenths. Over the course of this chapter, students can further break these 10-cm increments into 1-cm increments. When comparing fractions, decimals and percentages, students can use this one metre 'number line'. To compare a fraction with either a percentage or decimal, students can physically fold each number line into equal parts.

- It is not recommended that base 10 materials are used when exploring decimals numbers, as it is difficult to partition the one unit block into ten equal parts. A kinder square, which can be partitioned into 10 or 100 equal parts, is a far more accurate depiction of fractional numbers. Each kinder square represents 1 unit. The kinder square can be folded into 10 equal parts (vertically), each part representing one-tenth. The kinder square can be further folded into 10 more equal parts (horizontally), each part now representing one-hundredth. This model is good as it demonstrates the decrease in size of the portion when you move from 1 whole to 1 tenth to 1 hundredth. It also provides an accurate visual model. Students can shade in $\frac{10}{100}$ and see that it is equivalent to one-tenth.

- When comparing decimal numbers students can make a visual representation of each number as described above using a kinder square and compare each visual to determine which is larger.

POTENTIAL DIFFICULTY When comparing decimal numbers, students often mistakenly believe that the number of digits after the decimal point is indicative of the 'size of the number'.

- Q8 reviews rounding conventions and considers rounding to specific decimal places. Students are guided through the process.

POTENTIAL DIFFICULTY When we look at whole numbers, hundreds are larger than tens. When we look at decimal numbers, tenths are larger than hundredths.

Above Level

Above Level
5g–i, 6c, d, 7, 8a, e, 10, 13, 15f–j, 16, 17, 19–21, 23

- Demonstrate **3A eTutor** or direct students to do this independently.

- When converting decimal numbers into fractions, have students label the place value for each number after the decimal place value. The final decimal place value denotes the size of the denominator.

- Once students have mastered placing decimal numbers on a number line, have them include fractions and integers on the same number line. For additional practice, have students include positive and negative integers, fractions and decimals.

POTENTIAL DIFFICULTY
When introducing decimal number, some students think that decimal numbers are numbers which are less than one. When introducing decimal numbers ensure a range of numbers is introduced, including negative decimal numbers.

- To explore the use of the Dewey Decimal Classification system, refer students to **INV 3A-4 Decimals in the library** (see Resources). Students compare and order library books on a shelf according to a book's (decimal) call number and the standard procedure for book placement in libraries. As an extension, they can also go to the school library to investigate further call numbers.

- To recognise and write small quantities as decimals, refer students to **INV 3A-5 Dangerously small quantities** (see Resources). In this challenging task, students interpret the fractional and decimal meaning of parts per million, billion and trillion, as is sometimes scientifically reported. Students will use real data and information for this task and compare the sizes of the stated quantities.

Extra activities

1 In small groups, have students look through a newspaper and cut out all the decimals, percentages and fractions they can find. In each group, come up with reasons why some things are written as fractions while others are written as percentages or decimals in print media.

2 Consider the decimal number 9472.0564.

a How many decimal places are there? (4)

b What is the value of the digit 6? (6 thousandths)

c What is the value of the digit 9? (9 thousands)

d What does the 0 digit represent? (That there are no tenths)

e Is this decimal number less than or greater than 9472.7564? (Less than)

f Write a decimal that is greater than 9472.0564. (Answers will vary)

g Are 47.950 and 47.905 equivalent? Explain. (No; the zero in 47.905 is not a trailing zero and therefore affects the value of the decimal number.)

h Is there a different way of writing 47.950? Explain. (Yes, 47.95; the zero here is trailing and makes no difference to the value.)

3 Consider the decimal number 16.495.

a What is the whole number component? (16)

b What is the component that is less than one? (0.495)

c How many decimal places are there? (3)

d What is the value of the digit 1? (ten)

e What is the value of the digit 9? (9 hundredths)

f Is this decimal number less than or greater than 16.572? (Less than)

g Are 91.080 and 91.08 equivalent? Explain. (Yes; the trailing zero makes no difference to the value of the decimal number.)

Answers

3A Understanding decimals

3A Start thinking!

1

Thousands 1000	Hundreds 100	Tens 10	Ones 1	Decimal point .	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
5	2	1	8	.	3	4	7

2 It becomes smaller (by a factor of 10).

3 a $5 \times 1000 + 2 \times 100 + 1 \times 10 + 8 \times 1 + 3 \times \frac{1}{10} + 4 \times \frac{1}{100} + 7 \times \frac{1}{1000}$
$= 5000 + 200 + 10 + 8 + \frac{3}{10} + \frac{4}{100} + \frac{7}{1000}$

b Number of decimal places matches number of zeros in denominator.

4 a i 4 ii 7

b three

Exercise 3A Understanding decimals

1 a eight ones or 8
b eight thousandths or $\frac{8}{1000}$
c eight tenths or $\frac{8}{10}$
d eight ten-thousandths or $\frac{8}{10\,000}$
e eight tens or 80
f eight hundredths or $\frac{8}{100}$
g eight hundreds or 800
h eight hundredths or $\frac{8}{100}$
i eight ten-thousandths or $\frac{8}{10\,000}$
j eight tenths or $\frac{8}{10}$
k eight thousands or 8000
l eight tens or 80

2 a $28.61 = 2 \times 10 + 8 \times 1 + 6 \times \frac{1}{10} + 1 \times \frac{1}{100}$
b $304.128 = 3 \times 100 + 0 \times 10 + 4 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100} + 8 \times \frac{1}{1000}$
c $927.835 = 9 \times 100 + 2 \times 10 + 7 \times 1 + 8 \times \frac{1}{10} + 3 \times \frac{1}{100} + 5 \times \frac{1}{1000}$
d $16.0058 = 1 \times 10 + 6 \times 1 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 5 \times \frac{1}{1000} + 8 \times \frac{1}{10\,000}$
e $84.761 = 8 \times 10 + 4 \times 1 + 7 \times \frac{1}{10} + 6 \times \frac{1}{100} + 1 \times \frac{1}{1000}$
f $62.1852 = 6 \times 10 + 2 \times 1 + 1 \times \frac{1}{10} + 8 \times \frac{1}{100} + 5 \times \frac{1}{1000} + 2 \times \frac{1}{10\,000}$
g $821.793 = 8 \times 100 + 2 \times 10 + 1 \times 1 + 7 \times \frac{1}{10} + 9 \times \frac{1}{100} + 3 \times \frac{1}{1000}$
h $71.085 = 7 \times 10 + 1 \times 1 + 0 \times \frac{1}{10} + 8 \times \frac{1}{100} + 5 \times \frac{1}{1000}$
i $0.0018 = 1 \times \frac{1}{1000} + 8 \times \frac{1}{10\,000}$
j $3.8024 = 3 \times 1 + 8 \times \frac{1}{10} + 0 \times \frac{1}{100} + 2 \times \frac{1}{1000} + 4 \times \frac{1}{10\,000}$
k $8236.12 = 8 \times 1000 + 2 \times 100 + 3 \times 10 + 6 \times 1 + 1 \times \frac{1}{10} + 2 \times \frac{1}{100}$
l $183.049 = 1 \times 100 + 8 \times 10 + 3 \times 1 + 0 \times \frac{1}{10} + 4 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

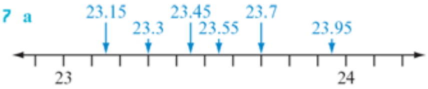
3 a 5785.195 b 496.283 c 0.006
d 5.703 e 36.07 f 40.1007

4 a 3 b 5 c 2 d 6 e 0
f 1 g 8 h 1 i 4 j 4
k 6 l 5

5 a > b < c > d < e >
f > g > h > i >

6 a 0.146, 0.238, 0.328, 0.461, 0.641, 0.823
b 1.234, 2.143, 2.341, 2.431, 3.412, 4.132
c 32.861, 36.182, 36.218, 36.812, 38.162, 38.261
d 18.1524, 18.2145, 18.2154, 18.4125, 18.4152, 18.4251

7 a



b 23 c 24

d If the tenths digit is 0, 1, 2, 3 or 4, round to the whole number part of the number. If the tenths digit is 5, 6, 7, 8 or 9, add 1 to the whole number and discard the remaining digits (the decimal place digits).

8 a The digit in the third decimal place is an 8. When rounding to two decimal places, if the digit in the third decimal place is 5 or more, then round up to the next digit in the second decimal place. This means that 4.5382 is closer to 4.54 than it is to 4.53.
b second, third, doesn't, one
c 4.538
d Look at the digit in the fourth decimal place (the next place value). Since it is a 2, the digit in the third decimal place doesn't change.
e Write the number and draw or imagine a vertical line after the required place value. Look at the digit to the right of this line. If the digit is 0, 1, 2, 3 or 4, then the digit in the required place does not change. If the digit is 5, 6, 7, 8 or 9, then add 1 to the digit in the required place value. Discard the digits after the line and write the answer.

9 a 3.547 b 3.5465 c 3.55

10 a 82.63 b 59.112 c 0.2634
d 0.27 e 15.000 f 2.175 499
g 0.219 h 165.5455 i 43.806 25

11 a 21.89 m b 22.81 m
c The largest value in the highest place value represents the largest distance.

Reflect

Possible answer: If there is no number in a place value between the first and last digit of a number, it should be filled with a placeholder zero so that other digits remain in their correct positions and the overall value of the number is not changed.

Resources

SupportSheet

SS 3A-1 Understanding place value

Focus: To interpret the place value associated with decimals

Resources: ruler

Students are guided through questions and activities that relate to recognising the place value of the digits in a decimal number using a place-value chart. They also write decimals in place-value notation (expanded fractional notation).

WorkSheet

WS 3A-2 Comparing decimals

Focus: To use a place-value chart to assist in comparing decimal numbers

Resources: ruler

Students use a place-value chart to compare decimal numbers. They establish a method to compare any two numbers by systematically comparing the digits of the same place value. Additional practice questions similar to those in Exercise 3A are also provided.

Investigations

INV 3A-3 Target race

Focus: To reach a target given that only small amounts of the target are reached on any day

Resources: ruler

Students investigate representing decimals as a fraction of a whole on number lines in an attempt to reach $200 and $1000 targets. They interpret the results.

INV 3A-4 Decimals in the library

Focus: To order and compare decimal call numbers in the Dewey Decimal Classification system

Students compare and order library books on a shelf according to a book's (decimal) call number and the standard procedure for book placement in libraries. As an extension, students can visit the school library to investigate further call numbers.

INV 3A-5 Dangerously small quantities

Focus: To recognise and write small quantities as decimals

In this challenging task, students interpret the fractional and decimal meaning of parts per million, billion and trillion, as is sometimes scientifically reported. Students will use real data and information for this task and compare the sizes of the stated quantities.

BLM

Place-value chart – decimals

Interactives

3A eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3B Adding and subtracting decimals

Teaching support for pages 128–33

Teaching strategies

Learning focus

To consolidate and apply understanding of the vertical method for adding and subtracting decimals.

Start thinking!

Students discover that is important to align place values when adding or subtracting decimal numbers.

In this task, students:

- estimate the cost of a meal of fish and chips
- estimate the change received from the order
- understand whether an answer is reasonable one
- add and subtract decimals correctly.

When looking at money can we have more than two decimal places?

Why is it important to be able to make a good estimate? (To know if your answer is reasonable)

When looking at Q2, students may benefit from placing each decimal number in a place-value chart.

Differentiated pathways

Below Level	At Level	Above Level
1a–d, 2, 4a–d, 5, 7, 9, 12, 16	1e–h, 2, 3, 4c–h, 5–8, 10, 13, 14, 16, 18, 19, 21	3, 6–8, 11, 13–15, 17, 19–23
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q4 & 5

Focus: To round whole numbers and decimal numbers to their leading digit; and to demonstrate knowledge and understanding of the vertical method of adding and subtracting whole numbers

- Direct students to complete **SS 3B-1 Estimating** (see Resources) if they had difficulty with Q4 or require more practice at this skill.
- You may need to undertake some explicit teaching so students not only understand the concept of rounding, but are also able to identify the leading digit.
- When reading a number aloud, it is read from left to right. The leading digit is the left-most digit.
- If rounding 562, the leading digit is 5 and is rounded to the nearest hundred. Is 562 closer to 500 or 600? It is closer to 600; therefore it is rounded up.
- If rounding 24.75, the leading digit is 2 and is rounded down to the nearest ten; that is, 20.
- Direct students to complete **SS 3B-2 Adding and subtracting whole numbers** (see Resources) if they had difficulty with Q5 or require more practice at this skill.
- Remind students of the importance of lining up numbers so digits with the same place value are directly underneath one another. Students can be provided with the BLM **Addition and subtraction grids** (see Resources) to assist in the correct alignment of digits.

At Level

At Level
1e–h, 2, 3, 4c–h, 5–8, 10, 13, 14, 16, 18, 19, 21

- Demonstrate **3B eTutor** or direct students to do this independently.
- Ensure students are able to:
  - round prices to the nearest whole dollar and then estimate a total cost. If they require additional support in estimating, direct students to **SS 3B-1 Estimating** (see Resources)
  - identify possible errors when an unreasonable answer is obtained
  - discover that lining up the digits according to place value when adding and subtracting is essential.
- Have students set out each addition and subtraction vertically. Some may benefit from

ruling up place-value columns. Line up each number according to its place value (similar to whole numbers) as opposed to asking students to match up the decimal point, as when we multiplying by a decimal number it is not necessary to line up each number under a decimal point. By teaching students only one rule for all setting out, we limit the opportunities for the development of misconceptions.

- To avoid confusion, ensure that the correct language is used when renaming. For example ten-*hundredths* is equivalent to one-*tenth*.
- When recording decimal numbers, the ones place is always recorded. When recording the number .72, we record it as 0.72.
- For Q21, an explanation of the difference between debits and credits may be necessary.

POTENTIAL DIFFICULTY
When adding and subtracting ragged decimal numbers, ensure all the trailing zeros are added to prevent place-value confusion.

- For additional practice, students can complete **WS 3B-3 Adding and subtracting decimal numbers** (see Resources). Students add and subtract decimal numbers using the vertical method.
- For more problem-solving tasks and investigations, direct students to **INV 3B-4 Takeaway decimals** (see Resources). Students explore the different possibilities of purchasing takeaway food and the associated costs, given certain requirement and financial constraints. They use a given menu and price list for their calculations.

Below Level

Below Level
1a–d, 2, 4a–d, 5, 7, 9, 12, 16

- Demonstrate **3B eTutor** or direct students to do this independently.
- When beginning to add and subtract decimal numbers some students with poor place value knowledge may find borrowing or carrying difficult. It is acceptable for them to begin adding and subtracting numbers with only one decimal place. These students may begin by making each decimal number out of kinder squares as described in *3A Understanding decimals*. Once they have mastered single digit place value, it is acceptable to stretch them to hundredths, introducing only one new place value at a time.
- To build on their understanding of subtraction, encourage students to progress from finding the difference between numbers to subtracting one from another.
- Direct students to the **examples. Example 3B-1** demonstrates an addition question,

while **Example 3B-2** demonstrates a subtraction question. Both show the importance of lining up the decimal point and the use of trailing zeros.

- Direct students to the **Key ideas**. You may like them to copy this summary.

Above Level

Above Level
3, 6–8, 11, 13–15, 17, 19–23

- Demonstrate **3B eTutor** or direct students to do this independently.
- Once students have mastered addition and subtraction of decimal numbers, have them perform the same operations with both positive and negative decimal numbers.
- Students may need to be reminded that written problems require a written answer, showing full working.

POTENTIAL DIFFICULTY
It is convention to add a space between each three decimal numbers. For example, 4.123 451.

- For more problem-solving tasks and investigations, direct students to **INV 3B-4 Takeaway decimals** and/or **INV 3B-5 Decimal deals** (see Resources).

  In **INV 3B-4 Takeaway decimals**, students explore the different possibilities of purchasing takeaway food and the associated costs, given certain requirement and financial constraints. They use a given menu and price list for their calculations.

  In **INV 3B-5 Decimal deals,** students fill in a grid with numbers obtained by rolling a ten-sided die. They either add or subtract numbers to achieve the biggest or smallest number possible. Students identify strategies to achieve the best outcome. This task is best done as a class or small group activity.

Extra activities

1 Students who require additional practice adding decimal numbers could measure the heights of all members of the class and then add their heights together to determine what the sum of all heights in the classroom would be.

2 In pairs, students roll four 10-sided dice, two of one colour and two of another. Each coloured pair of dice forms a decimal number. Depending on their level, students can form numbers with a different number of decimal places.

Below Level: Each pair of dice forms a number with one decimal place; for example, 4.2 and 3.3.

At Level: One pair of dice forms a number with two decimal places and the other pair forms a number with one decimal place; for example, 0.33 and 4.2.

Above Level: Each pair of dice forms a number with two decimal places; for example, 0.33 and 0.42.

Students race one another to add the numbers together.

3 Have students make their own addition and subtraction questions and then share them with a partner operating at a similar level. As an extension of this task, students can write the equation in words. For example, they could write question such as: ‘I finished with 2.34 and started with 5.20. What number did I subtract?’

4 Estimate the result of each calculation.

a 13.469 + 23.721 + 9 + 10.025 (≈ 56)

b 24.56 + 2.140 + 5 + 6.75 (≈ 39)

c 103.648 – 99.875 (≈ 4)

d 458.901 – 457.204 (≈ 2)

5 Use the vertical addition method to complete the additions.

a 13.469 + 23.721 + 9 + 10.025 (56.215)

b 24.56 + 2.140 + 5 + 6.75 (38.45)

6 Use the vertical subtraction method to complete the subtractions.

a 103.648 – 99.875 (3.773)

b 458.901 – 457.204 (1.697)

Answers

3B Adding and subtracting decimals

3B Start thinking!

1 a $13 b $14
2 According to estimates, $10 will not be enough to pay for either order.
3 a yes
b Either the chips, garden salad or whiting will reduce the value below $10 as each costs more than $1.65.
4 a no
b He added up the total in cents, not dollars.
c He added the calamari ring and onion ring as cents, not as dollars.
d $6.50 + $0.80 + $0.80 + $2.50 + $1.70
e $12.30

Exercise 3B Adding and subtracting decimals

1 a 90 b 310 c 200 d 290
e 150 f 1210 g 940 h 8100
2 a 88.21 b 316.47 c 192.273
d 292.065 e 149.601 f 1208.2163
g 944.911 h 8102.391
3 a 36.012 b 1519.797 c 66.017
d 3855.1036 e 177.9932 f 15.971
4 a 60 b 30 c 400 d 20
e 1470 f 110 g 10 h 1230
5 a 56.23 b 34.72 c 394.9
d 23.22 e 1476.68 f 113.27
g 15.588 h 1233.6118
6 a 123.926 b 7.487 c 15.47
d 25.14 e 6.675 f 10.338
8 a $43 b $7
c $42.51, change is $7.49
d only 49 cents away from actual cost
9 a 42.6 b 42.1
c Natasha received the highest score by 0.5.
10 17.6°C
11 Oliver 0.09 m, Sam 0.13 m, Than 0.14 m
12 Align place values correctly and leave zero place values correctly in place. Correct answer is 125.243.
13 66.68 m
14 a 12.49 L b three tins
15 Total monthly rainfall of 64.68 mm is 1.02 mm below September average.
16 7.5 kg
17 a Day 1: 187.2 km; Day 2: 148.6 km; Day 3: 249.8 km; Day 4: 144.8 km; Day 5: 251.5 km
b 28 071.5 c Day 4, 144.8 km
d Day 5, 251.5 km e 981.9 km
18 a 22.09 kg b more than 2.09 kg
19 a Total mass is 239.52 kg, so lift capacity has not been reached.
b Extra mass of 25.48 kg will take total mass to maximum capacity of lift.
20 a 5.47 km (or 5470 m)
b 1.4 km, 1.8 km, 1.96 km, 2.31 km
c 7.47 km (or 7470 m)
21 a i $453.98, $394.99, $329.99, $79.99, $51.46, $53.24, $48.24
ii $407.52 iii $127.20 iv $48.24
b Elisa has $48.24 in bank at end of month; not enough to purchase iPod nano.
22 a 283.15 b 62.369 c 15.413
d 617.58 e 0.23 f 158.7
23 Check totals are correct and correct signs are included.

Reflect

Possible answer: It is important to line up each number in the correct place value when adding and subtracting decimal numbers. If this is not the case, digits with different place values could be added or subtracted.

Resources

SupportSheets

SS 3B-1 Estimating

Focus: To estimate an answer after first rounding each number to its leading digit

Students are guided through a series of questions that relate to recognising the leading digit of a number and its place value, and then rounding it to its leading digit. They estimate the answer to different calculations after rounding each number to its leading digit.

SS 3B-2 Adding and subtracting whole numbers

Focus: To review the vertical method of adding and subtracting whole numbers

Students are guided through a series of questions to reinforce the vertical method of adding and subtracting whole numbers.

WorkSheet

WS 3B-3 Adding and subtracting decimal numbers

Focus: To add and subtract decimal numbers using the vertical method

Students use the vertical addition and subtraction methods with decimal numbers. Trailing zeros are used to ensure both numbers have the same number of decimal places. Additional practice questions similar to those in Exercise 3B are also provided.

Investigations

INV 3B-4 Takeaway decimals

Focus: To determine the costs of particular orders at a takeaway food outlet

Students explore the different possibilities of purchasing takeaway food and the associated costs, given certain requirements and financial constraints. They use a given menu and price list for their calculations.

INV 3B-5 Decimal deals

Focus: To determine the biggest and smallest number after adding and subtracting decimals

Resources: ten-sided die or a net of a **Ten-sided die** (BLM)

This task is best used as a class or small group activity. Students fill in a grid with numbers obtained by rolling a ten-sided die. They either add or subtract numbers to achieve the biggest or smallest number possible. Students identify strategies to achieve the best outcome.

BLMs

Addition and subtraction grids

Ten-sided die

Interactives

3B eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3C Multiplying decimals

Teaching support for pages 134–9

Teaching strategies

Learning focus

To consolidate and apply understanding of multiplying decimal numbers.

Start thinking!

Students:

- experiment with 'lots of' decimals such as 'lots of' 0.3 and 'lots of' 0.25 on a number line. Copies of the BLM **Number lines 1** (see Resources) can be supplied to students.
- equate the words 'lots of' with the multiplication sign (×)
- investigate the position of the decimal point in the answer when multiplying two decimal numbers.

Have students think about what 0.6 metres of material looks like. Is it close to a commonly used fraction? Is it more or less than $\frac{1}{2}$?

Can they make an estimate of how much material is needed for 14 costumes? Does the commonly used fraction described in the above suggestion help?

Have students list the circumstances when they would use estimation for multiplying by decimal numbers. (When they need an answer in a hurry, when they don't need to be accurate but close enough is good enough, and when they are checking an answer.)

Differentiated pathways

Below Level	At Level	Above Level
1a–f, 4a–d, 6, 10, 11a–d, g–i, 12	1a–f, 2, 3a–f, 4a–f, 5a–c, 6, 7, 8d, e, 10d, e, 11d–o, 13–16	1g–l, 3d–i, 4e–i, 5d–i, 6, 8, 9, 11j–o, 13, 15–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q6

Focus: To demonstrate knowledge and understanding of multiplying whole numbers

- Direct students to complete **SS 3C-1 Multiplying whole numbers** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so students remember how to multiply two numbers using long multiplication. Discussion of how to multiply by multiples of ten could also be included.
- Multiplication requires the quick recall of multiplication facts (or times tables facts). The BLM **Multiplication facts** (see Resources) can be laminated for students and retained for ready reference. Incorporating some quick times table games into your lesson plan, perhaps at the beginning of each lesson, can assist students to master these.

At Level

At Level
1a–f, 2, 3a–f, 4a–f, 5a–c, 6, 7, 8d, e, 10d, e, 11d–o, 13–16

- Demonstrate **3C eTutor** or direct students to do this independently.
- Ensure students are able to:
  - compare the answers they obtain when multiplying decimal numbers to the answers found when multiplying the same digits, but as whole numbers. They see that the digits in the answer are identical; however, they have a different place value.
  - estimate the product of decimals and determine where the decimal point is placed in the final answer
  - see that the number of decimal places in an answer is the same as the total number of decimal places in the original calculation.
- When multiplying by a large number, have students break the number up into more manageable parts. For example, when multiplying by 60, students can multiply first by 6 then by 10. This allows students to work with more manageable numbers and it provides a meaning to the term *multiple*.
- Another method of introducing multiplication is to use grid or graph paper. When modelling multiplication by decimal numbers use 1 mm grid paper and mark a square which is 10 mm by 10 mm. This represents an area of 100 mm^2. To model 0.7 multiplied by 0.3, colour 3 rows of 7 squares. The coloured squares represent 21 of the 100 squares; that is, $\frac{21}{100}$. So $0.7 \times 0.3 = 0.21$. This shows that area is a representation of multiplication. The BLM **1-mm grid paper** (see Resources) can be provided to students.

- For additional practice, students can complete **WS 3C-2 Multiplying decimals** (see Resources). Students are guided through the method of long multiplication of decimal numbers. You may like to provide students with the BLMs **Multiplication grid 1** and **Multiplication grid 2** (see Resources) to assist in setting out their problems. Some students may need to refer to the BLM **Multiplication facts** (see Resources). Additional practice questions are also provided.

- For more problem-solving tasks and investigations, direct students to **INV 3C-5 Best deal mobile** (see Resources). Students investigate different types of mobile phone plans and work out the different costs involved in each.

POTENTIAL DIFFICULTY
When introducing multiplication facts for whole numbers it is not uncommon to stress to students that when multiplying by ten we add a zero on the end; however, this fact does not stand up for multiplying decimal numbers.

Below Level

Below Level
1a–f, 4a–d, 6, 10, 11a–d, g–i, 12

- Demonstrate **3C eTutor** or direct students to do this independently.

- For students who find it difficult to remember if they round up or down, they should list how many numbers they need to add to round up and how many they need to take away to round down. Whichever is the least is the direction in which they should round.

- As a means of modelling decimal multiplication, students can use 1-mm grid paper and mark a square which is 10 mm by 10 mm. The BLM **1-mm grid paper** (see Resources) can be provided to students. Each small square of grid paper is equivalent to $\frac{1}{100}$th, each column in the square is equivalent to $\frac{1}{10}$th and each large square is equivalent to 1 unit or 1 ones. To model the multiplication of two single-digit numbers such as 2 × 4, have students colour in 2 rows of 4. Looking at the grid paper, students can identify that 2 × 4 is equivalent to 8 units or 8 ones. Once this method of multiplication is established, students can then move on to using it to multiply by decimal numbers. For example, 3 × 1.3 is shown as 3 rows of 1.3. Looking at the grid paper, students can identify that 3 × 1.3 is equivalent to 3.9.

- For Q11, students should see a pattern when multiplying decimal numbers by multiples of 10. Some students benefit from seeing that the decimal point is moved a corresponding number of places to the right with placeholder zeros filling the ‘gaps’. Others find it easier to refer to a place-value chart and see that the digits of the original number to be multiplied move to the left a corresponding number of places. Students requiring the chart can be supplied with the BLM **Place-value chart – decimals** (see Resources).

- Students who struggle with decimal multiplication are encouraged to attempt a question, then check their answer with a calculator.
- To explore amount of energy used when performing certain activities, refer students to **INV 3C-3 Burning kilojoules** (see Resources). Students explore the number of kilojoules used by doing certain activities for different periods of time. They compare activities and determine times and activities to burn off the kilojoule intake of certain foods. Students at this level should use a calculator to complete this task.

Above level

Above Level
1g–l, 3d–i, 4e–i, 5d–i, 6, 8, 9, 11j–o, 13, 15–18

- Demonstrate **3C eTutor** or direct students to do this independently.
- When multiplying decimal numbers, students should count the number of decimal numbers in the first number and add this to the number of decimal numbers in the second. This number represents the total number of decimal places in the answer.
- When using long multiplication, students should initially ignore the decimal place indicator and instead multiply the numbers as if they were whole numbers. Once the student obtains an answer, they then work out the number of decimal places that should be in the answer (based on the explanation above).
- Extend student knowledge by asking them to predict what effect a negative decimal number would have on the previous multiplications.

POTENTIAL DIFFICULTY
When multiplying a decimal of time, students may confuse 3.5 hours with 3 hours and 50 minutes and not the correct answer of 3 hours and 0.5 of an hour, which is equivalent to 30 minutes.

- When completing questions with a multiple of operations remind students to use the correct order of operations.
- Have students write a short paragraph on what happens to a number when we multiply it by a decimal number less than one. Does it get smaller or larger?
- To explore amount of energy used when performing certain activities, refer students to **INV 3C-3 Burning kilojoules** (see Resources). Students explore the number of kilojoules used by doing certain activities for different periods of time. They compare activities and determine times and activities to burn off the kilojoule intake of certain foods. As an extension, students calculate the number of kilojoules consumed (referring to a kilojoule counter) and those used in activities over 24 hours.

- Have students interpret and use a decimal calendar by completing the investigation **INV 3C-4 Decimal time and the revolutionary calendar** (see Resources). Students interpret the French Revolutionary calendar and decimal time. They compare and convert the current units of time with the French decimal time. This task is fairly challenging, allowing students to consider the advantages and disadvantages of both systems.
- Have students use and interpret large decimal numbers reported in the news by completing the investigation **INV 3C-6 Big news decimals** (see Resources). Students investigate how very large numbers are represented in the media. They will learn to interpret the information to determine the actual quantity these numbers represent.

Extra activities

1 In pairs, students roll four 10-sided dice, two of one colour and two of another. Each coloured pair of dice forms a decimal number. Depending on their level, students can form numbers with a different number of decimal places.

Below Level: Each pair of dice forms a number with one decimal place; for example, 4.2 and 3.3. An alternative activity for these students is using three dice instead of four. Two of the dice are one colour and one die is another. The single coloured die represents a whole number and the other two dice represent a one decimal number; for example, 3 and 4.2.

At Level: One pair of dice forms a number with two decimal places and the other pair forms a number with one decimal place; for example, 0.33 and 4.2.

Above Level: Each pair of dice forms a number with two decimal places; for example, 0.33 and 0.42.

Students race one another to multiply the numbers together.

2 Can you see a relationship between 0.9×0.2 and 9×2? Explain. (Both have 18 in their answer. $0.9 \times 0.2 = 0.18$ and $9 \times 2 = 18$)

3 Predict how many decimal places there will be in the answer to 12.746×7. (3)

4 Estimate an answer to 12.746×7. (≈ 91) Perform the calculation and check your answer on a calculator. (89.222). Ask students to explain in which situations an estimation would be acceptable and when it would not. (When buying lengths of wood to build something but not when measuring and cutting the wood)

5 How can you calculate the answer to 34.871×3.6? (Perform long multiplication as if the numbers are whole numbers and then place the decimal point in the correct position.) How many decimal places will there be in the answer? (4) Perform the calculation and check your answer on a calculator. (125.5356)

Answers

3C Multiplying decimals

3C Start thinking!

1 0.9; counting graduations on scale
2 2.0
3 4 × 0.6 =, 8 × 0.25 =
4 a 0.9 b 2.0 c 1.8 d 2.75
5 a 9 b 200 c 18 d 275
6 Digits in original calculations and answers are same, just place values are different.
7 same 8 8.4 m

Exercise 3C Multiplying decimals

1 a 5 b 7 c 6 d 6 e 6 f 6
g 8 h 7 i 8 j 10 k 9 l 8
2 a 0.12 b 0.24 c 0.42
d 0.072 e 0.0036 f 0.49
g 0.96 h 0.000 24 i 0.000 0007
j 0.000 15 k 0.0084 l 0.0088
3 a 26.6 b 39.2 c 3.15
d 9.12 e 36.0072 f 19.68
g 8.277 h 359.8 i 79.44
4 a 69.16 b 456.48 c 1258.32
d 28.644 e 130.974 f 964.73
g 47.7776 h 24.311 92 i 2251.1775
5 a 32.88 b 86.76 c 198.15
d 622.08 e 0.185 68 f 7.4684
g 12.4 h 3.702 i 1041.56
7 a $60 b $77.48
c Estimate is $17.48 lower.
8 a $9.74 b $6.08 c $9.73
d $8.28 e $8.97 f $10.09
9 a All these calculations involve the same digits.
b The digits represent different place values in the different numbers.
c, d

Original calculation	Estimated answer	Answer
38 × 21	40 × 20 = 800	798
3.8 × 21	4 × 20 = 80	79.8
38 × 2.1	40 × 2 = 80	79.8
3.8 × 2.1	4 × 2 = 8	7.98
380 × 210	400 × 200 = 80 000	79 800

e Number of decimal places in answer equals total number of decimal places in original calculation.

10 a i 20 ii 200 iii 2000 iv 20 000
b The original place value of 2 is 2 or 2 ones. After each calculation, place value of 2 increases by 10. That is, digit 2 moves to left by one place. Number of places the digit 2 is moved from original position equals number of zeros after the 1 in each number.
c i one, left ii two, left
iii three, left iv four, left
d i 52.634 ii 526.34
iii 5263.4 iv 52 634
e Decimal point moves to right (or digits move to left). Number of places moved equals number of zeros after the 1.
11 a 6000 b 4200 c 1.284
d 1572.1 e 0.4 f 7.02
g 61.0 h 150.0 i 310.0
j 6481.3 k 480 000 l 128 468 000
m 4.56 n 100.09 o 1003.01
12 a $713.40 b option 1 c $68.40
13 a $258 b $147.60
14 8 mm
15 a 9086 b 297 600 c 104 688
d 33 984 e 97 500 f 12 150
g 511 299 h 152 334 i 47 634.3
j 568 480
16 a $110 600 b $198 940 c $88 340
17 a 12.0807 b 36.62
c 12.958 d 0.481 74
18 a i 0.04 ii 0.16
iii 0.0625 iv 0.000 064
b The number of decimal places in the answer is twice the number of decimal places in the number to be squared.

Reflect

Possible answer: It is important to know the number of decimal places an answer should have before the calculation is completed. This way you can check your answer to see that it is the correct value. The number of decimal places in the answer is the same as the number of decimal places in the original calculation.

Resources

SupportSheet

SS 3C-1 Multiplying whole numbers

Focus: To review the method of long multiplication of whole numbers

Resources: Multiplication facts (BLM) (optional)

Students are guided through a series of questions to review the method of long multiplication of whole numbers. They answer questions independently to consolidate this process. Some students may find it helpful to refer to the BLM **Multiplication facts** (see Resources).

WorkSheet

WS 3C-2 Multiplying decimals

Focus: To multiply decimal numbers using the long multiplication method

Resources: The BLMs **Multiplication facts** (optional), **Multiplication grid 1** (optional), **Multiplication grid 2** (optional)

Students are guided through the method of long multiplication of decimal numbers. You may like to provide students with multiplication grids to assist in setting out their problems. Some students may need to refer to a sheet of multiplication facts. Additional practice questions similar to those in Exercise 3C are also provided.

Investigations

INV 3C-3 Burning kilojoules

Focus: To calculate the amount of energy used when performing certain activities

Resources: calculator (optional), kilojoule counter reference (optional)

Students explore the number of kilojoules used by doing certain activities for different periods of time. They compare activities and determine times and activities to burn off the kilojoule intake of certain foods. As an extension, students calculate the number of kilojoules consumed (referring to a kilojoule counter) and those used in activities over 24 hours. A calculator is not essential to complete this activity but would enable all students to complete this relevant task.

INV 3C-4 Decimal time and the revolutionary calendar

Focus: To interpret and use the revolutionary decimal calendar

Resources: calculator

Students interpret the French Revolutionary calendar and decimal time. They compare and convert the current units of time with the French decimal time. This task is fairly challenging, allowing students to consider the advantages and disadvantages of both systems.

INV 3C-5 Best deal mobile

Focus: To determine the best mobile phone plan using calculations involving decimals

Resources: calculator

Students investigate different types of mobile phone plans and work out the different costs involved in each.

INV 3C-6 Big news decimals

Focus: To use and interpret large decimal numbers reported in the news

Students investigate how very large numbers are represented in the media. They will learn to interpret the information to determine the actual quantity these numbers represent.

BLMs

Number lines 1

Multiplication facts

Multiplication grid 1

Multiplication grid 2

1-mm grid paper

Place-value chart – decimals

Interactives

3C eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3D Dividing a decimal by a whole number

Teaching support for pages 140–5

Teaching strategies

Learning focus

To consolidate and apply student understanding of using short division to divide decimals by whole numbers.

Start thinking!

Using a real-life context, students are guided to perform short-division problems. They are taken step by step through the addition of trailing zeros to obtain an answer that is exact with no remainder.

In this task, students:

- calculate the cost of a single cinema ticket after Tyler pays $57 for 4 tickets
- learn the terms *quotient* and *dividend*
- consider the number of trailing zeros necessary in the division problem to get an exact answer.

Some students may benefit from using play money to work out these problems. By holding the physical amount of money in their hands, students can then practise sharing this money out.

Using real-life applications, such as money, provides students with an opportunity to use prior knowledge to solve a problem.

Students need to be confident in setting out short-division problems. The BLM **Division grid** (see Resources) can be provided. Emphasise to students that the place values should align with one another.

Differentiated pathways

Below Level	At Level	Above Level
1, 3a, b, d, g, h, 5, 7, 10, 11, 17, 18, 21	1g–l, 2, 3c–j, 4a–f, 6, 8, 10, 12, 13, 16–23	4e–j, 8, 13–15, 17, 19, 21–26
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q7

Focus: To demonstrate knowledge and understanding of short division

- Direct students to complete **SS 3D-1 Dividing whole numbers** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so students remember how to perform short division with whole numbers.
- Division requires the quick recall of multiplication facts (times tables). The BLM **Multiplication facts** (see Resources) can be laminated for students and retained for ready reference.

At Level

At Level
1g–l, 2, 3c–j, 4a–f, 6, 8, 10, 12, 13, 16–23

- Demonstrate **3D eTutor** or direct students to do this independently.
- For those students who require further assistance with dividing decimals by a whole number, direct them to **WS 3D-2 Dividing decimals by a whole number** (see Resources). To encourage correct setting out, provide students with copies of the BLMs **1-cm grid paper** or **Division grid** (see resources).
- When dividing, ensure students set the question out correctly. They should set it out using short division conventions. Students should continue to add trailing zeros until they get an answer with no remainder.
- When dividing a decimal number by a whole number the answer will have the same number of digits after the decimal point as the number being divided by if it is not necessary to add any trailing zeros.

POTENTIAL DIFFICULTY
When completing cost per unit questions, students are performing a division. Division problems are much more difficult to do mentally. In fact, in most day to day operations when faced with questions that require a division, most numerate people will convert the question (possibly without even knowing it) to a multiplication question. For example, $12.50 divided by $2.50 is converted to 'How many $2.50s fit into $12.50? We know that 2 lots of $2.50 are $5.00, so 4 must be …' and so on.

- When calculating the cost of 100 grams of an item, students must first determine how many 'lots of' 100 grams there are in the total amount.

Below Level

Below Level
1, 3a, b, d, g, h, 5, 7, 10, 11, 17, 18, 21

- Demonstrate **3D eTutor** or direct students to do this independently.

- Direct students to complete **SS 3D-1 Dividing whole numbers** (see Resources) if they have difficulty with this concept or require more practice at this skill.
- When finding the cost of one item, some students may benefit from using play money. This will allow them to practise sharing out the money equally. Make sure the money is in a range of denominations so students can try a range of 'sharing' options.
- When comparing the cost of buying a weekly ticket or daily ticket, students should be encouraged to perform both calculations. Students will often try to perform an estimation of the calculation, and then only perform the calculation they believe is correct. While the use of estimation is a good strategy and one that is often encouraged, students should get into the habit of performing both calculations and then comparing them. For example: If I only use my ticket for 4 days out of the 7 ... These are the type of real-life applications they are more likely to encounter on standardised testing.

Above Level

Above Level
4e–j, 8, 13–15, 17, 19, 21–26

- Demonstrate **3D eTutor** or direct students to do this independently.
- When calculating the cost of 100 grams of an item, students must first determine how many 'lots of' 100 grams there are in the total amount.
- Collect recipes and multiply the ingredients to create catering quantities. Have students work out the total cost of all the ingredients and then determine the cost per serving. If the recipe is for chocolate biscuits they can then compare the price of the homemade chocolate chip biscuits with those purchased from a store.
- Have students look at a range of problems which require them to first convert between units. For example: grams and kilograms, metres and centimetres.
- To add, multiply and divide decimal numbers, refer students to **INV 3D-3 Decimals to go** (see Resources). Groups of two to four students play a game using the game board on the back of the card or the one supplied as the BLM **Decimals to go** (see Resources)**.** They explore the different possibilities of adding, multiplying and dividing decimals to achieve the highest and lowest possible answer.
- To determine the best buy when purchasing pasta sauce in different scenarios, refer students to **INV 3D-4 Best buys – pasta sauce** (see Resources). Students compare bottles of pasta sauce for price and quantity. Some implicit estimation techniques for mass and money are required to solve some everyday purchase problems, given certain constraints and conditions. Finally, students calculate the price per 100 g for different bottles of pasta sauce to determine the best buy.

Extra activities

1 Quick questions

a There are 15 pencils in the container. The container of pencils costs $25.05. How much will one pencil cost? ($1.67)

b There are 12 donuts in a box. The box of donuts costs $25.80. How much will one donut cost? ($2.15)

c There are 12 cans of drink in a box. The box of cans costs $23.76. How much will one can cost? ($1.98)

d Three kilograms of tomatoes cost $13.92. How much will 1 kg cost? ($4.64)

e Three kilograms of prawns cost $74.85. How much will 1 kg cost? ($24.95)

2 Collect junk mail over a number of weeks and bring them in to class. Have students compare which items are the best value for money and which are not. Have them identify strategies that supermarkets use to ‘trick’ you into thinking one item is better value than another.

Answers

3D Dividing a decimal by a whole number

3D Start thinking!

1 a no b 14 remainder 1 (or $14\frac{1}{4}$)
2 a $14.25
b To enable division calculation to continue until there is no remainder.
c No, there still would have been a remainder.
d Yes, but answer would contain trailing zeros. Exact decimal answer (without remainder) was obtained after only two decimal places so remaining trailing zeros in dividend are not required.
3 a i 3.5 ii 4.375 iii 7.25 iv 170.8
b i one ii three iii one iv none

Exercise 3D Dividing a decimal by a whole number

1 a 6.23 b 13.721 c 9.102 d 23.14
e 11.115 f 1.6398 g 2.9217 h 4.15
i 3.51 j 0.0256 k 0.1208 l 0.72804
2 a 2.5 b 6.75 c 12.6 d 4.35
e 2.05 f 0.4062 g 0.60375 h 39.05
i 14.0575 j 1.19375 k 1.145 l 3.2375
3 a $1.28 b $217.50 c $1.28 d $0.75
e $11.80 f $7.25 g $1.30 h $1.80
i $2.53 j $123.33
4 a $3.88 b $4.98 c $3.98 d $11.97
e $6.98 f $7.98 g $1.62 h $4.27
i $0.17 j $2.22
5 $186.25 6 $135.90 7 $24.65
8 a $2.49
b 2-L container, as it is cheaper per L.
9 $18.90 ÷ 6 = $3.15. Whole pizza is better value by $0.35 per slice.
10 a $2.75
b If using it five days per week, weekly option is cheaper by 20 cents per day.
c If plan to travel less than five days per week, buying daily tickets will cost less than a weekly ticket.
11 5 kg bag is the better buy as the cost is equivalent to $1.26 per kg, a saving of 19 cents per kg compared to loose potatoes.
12 a $8.99 ÷ 12 = $0.75. Carton of eggs is better value.
b $0.15
c Some people want to buy less than 12 eggs.
13 a $3.73 b $1.56 c $0.53 d $1.50
14 a Simon $0.80, Julia $0.66 b Julia
15 a Barilla $0.93, Leggos $0.82; Leggos is better value
b two, Barilla ($7.46 compared to $8.18)
16 $357 122.89; divide dividend by 18, then round to two decimal places.
17 Less than 1 L, as divisor (180) is larger than dividend (100); that is, dividing the smaller amount by the larger amount.
19 packing boxes at $8.13 an hour
21 a $2.10 b $21.00
c Cost of one mango found by 6.30 ÷ 3. Multiply by 10 for cost of 10 mangoes.
22 a $7.00 b $6.00 c $6.50 d $8.50
23 a $12.40 b $25.44 c $3.73 d $0.68
24 a Crinkle cut chips $2.54, small Pringles $2.85, large Pringles $2.81, Red Rock Deli $2.25
b Red Rock Deli c taste, texture
25 Ava at 6.08 km/h
26 21

Reflect

Possible answer: To find the best buy, we compare the cost of items for a unit amount such as 1 L, 1 kg or 100 g. The best buy is the one with the lowest price for the unit amount.

Resources

SupportSheet

SS 3D-1 Dividing whole numbers

Focus: To review the method of short division of whole numbers

Resources: BLM **Multiplication facts** (optional)

Students are guided through a series of questions to review the method of short division of whole numbers. Some students may find it helpful to refer to the BLM **Multiplication facts.**

Worksheet

WS 3D-2 Dividing decimals by a whole number

Focus: To divide a decimal by a whole number using short division

Resources: BLMs **Multiplication facts** (optional), **1-cm grid paper** (optional), **Division grid** (optional)

Students review the process and language used to divide a decimal by a whole number. Trailing zeros are used in the dividend when required. Students independently calculate several problems using the short-division process. To assist in setting out division problems, the BLMs **1-cm grid paper** or **Division grid** (see Resources) can be provided. Some students may find it helpful to refer to the BLM **Multiplication facts** (see Resources). Additional practice questions similar to those in Exercise 3D are also provided.

Investigations

INV 3D-3 Decimals to go

Focus: To add, multiply and divide decimals

Resources: counter, coin or token for each player, game board (Decimals to go)

Groups of between two and four students play a game using the game board on the back of the card or one supplied as **Decimals to go** (BLM). They explore the different possibilities of adding, multiplying and dividing decimals to achieve the highest and lowest possible answer.

INV 3D-4 Best buys – pasta sauce

Focus: To determine the best buy when purchasing pasta sauce in different scenarios

Resources: calculator (optional)

Students compare bottles of pasta sauce for price and quantity. Some implicit estimation techniques for mass and money are required to solve some everyday purchase problems, given certain constraints and conditions. Finally, students calculate the price per 100 g for different bottles of pasta sauce to determine the best buy.

BLMs

Division grid

Multiplication facts

1-cm grid paper

Decimals to go

Interactives

3D eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3E Dividing a decimal by a decimal

Teaching support for pages 146–51

Teaching strategies

Learning focus

To consolidate and apply understanding of dividing a decimal by a decimal

Start thinking!

Students are guided through different approaches to discover that an easy way to divide a decimal by a decimal is to first write an equivalent division problem where the divisor is a whole number.

In this task, students:

- investigate the result of dividing a decimal by a decimal using a number line
- progress to recognising a pattern using an equivalent division problem.

In the first approach (Q1), students build on the concept of using number lines for 'lots of' seen in 3C Start thinking!. When sketching the number line, ensure students set each increment an equal distance apart. Copies of the BLM **Number lines 2** (see Resources) can be provided for this task.

The second approach (Q2) involves observing a pattern in the results to given equivalent division problems where the divisors are multiples of 10, 100, 1000 and 10 000. They link this to the results for Q1.

Students are guided to see that they obtain the same result for all equivalent division problems and that the easiest one to calculate is where the divisor is the smallest whole number.

The concept of keeping a division problem equivalent by performing the same operation to both the dividend and the divisor is emphasised.

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 4, 8, 11, 12, 16, 20a–c	1, 2, 4, 5, 7, 8, 10–16, 20	3, 5–7, 9, 14, 17–19, 20c,d, 21–23
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2, 4, 5, 7, 8, 10–16, 20

- Demonstrate **3E eTutor** or direct students to do this independently.
- Ensure all students understand that to divide a decimal by a decimal, we first write an equivalent division problem with a whole number divisor before performing a short division. For equivalent division problems, students need to be confident in multiplying a decimal by a power to ten.
- When identifying the number of decimal places the decimal point should be moved to divide by a whole number, students first identify what the decimal number needs to be multiplied by to make it a whole number.
- When completing Q2, trailing zeros are not required to complete the short division. To help students to set out the division problem, the BLMs **1-cm grid paper** or **Division grid** (see Resources) can be provided. Some students may also benefit from using the BLM **Multiplication facts** (see Resources) to complete the division problems**.**
- When completing Q7, students divide numbers by a decimal but use multiplication facts rather than short division to obtain their answers.
- For questions involving cost per item, students may benefit from using play money to perform the calculation.
- Q12 and Q13 involve exploring patterns within equivalent division problems and using these patterns to predict further answers. Have students write in words the pattern they have observed.
- In Q20, students explore the shortcut that can be used when dividing by powers of 10.

POTENTIAL DIFFICULTY
Students can forget to multiply both the divisor and the dividend by the same power of 10. Emphasise that the division problem must be kept equivalent so that the overall value is unchanged.

- For additional practice, students can complete **WS 3E-1 Dividing by a decimal** (see Resources). Students initially establish the number of decimal places of the divisor and then find the equivalent short-division problem with a whole number division.

Below Level

Below Level
, 2, 4, 8, 11, 12, 16, 20a–c

- Demonstrate **3E eTutor** or direct students to do this independently.

- Direct students to the **examples**. **Example 3E-1** shows how to identify the number of decimal places the decimal point should be moved to divide by a whole number and to write the equivalent division problem obtained. **Example 3E-2** shows the steps involved in dividing a decimal by a decimal.

- When identifying the number of decimal places the decimal point should be moved to divide by a whole number, students first identify what the decimal number needs to be multiplied by to make it a whole number. Students who experience difficulty in progressing independently to dividing decimals by powers of ten (10, 100, 1000, etc.) may benefit from using a calculator so that the pattern relating the number of zeros in the power of ten to the answer can be easily seen.

- Some students benefit from seeing that the decimal point is moved a corresponding number of places to the left with placeholder zeros filling the ‘gaps’. Others find it easier to refer to a place-value chart and see that the digits of the original number to be divided move to the right a corresponding number of places. The BLM **Place-value chart – decimals** (see Resources) can be used.

- When completing Q2, trailing zeros are not required to complete the short division. To help students to set out the division problem, the BLMs **1-cm grid paper** or **Division grid** (see Resources) can be provided. Some students may also benefit from using the BLM **Multiplication facts** (see Resources) to complete the division problems.

Above Level

Above Level
3, 5–7, 9, 14, 17–19, 20c,d, 21–23

- Demonstrate **3E eTutor** or direct students to do this independently.

- When exploring ‘shortcuts’ that can be used when dividing by powers of ten (10, 100, 1000, etc.), students work through calculations involving the same digits but different numbers and then make judgements based on their observations. What do they notice happens to the answer as the number of decimal places in the divisor decreases?

- When completing calculations in which more than one operation is required, ensure students are using the correct order of operations.

- In Q21, students are provided an example showing the procedure involved in dividing by multiples of 10.

- To calculate exchange rates of the Australian dollar using the multiplication and division of decimals, refer students to **INV 3E-2 Dealing with the dollar** (see Resources). This is a more challenging activity. Students convert quantities of money between the Australian dollar and the US dollar, the euro and the yen. They may formulate a process for their conversions and consider under which conditions it is more favourable to convert currencies. As an extension, students explore a problem using current exchange rates.

Extra activities

1 What do you do when given a problem where the divisor is a decimal? (Find an equivalent division problem)

2 How do you write an equivalent division problem? (Multiply both the divisor and the dividend by 10, 100, 1000, etc. so that the divisor becomes a whole number)

3 Quick questions

a If the divisor was 0.002, what would you multiply by? (1000)

b If the divisor was 0.04, what would you multiply by? (100)

c Write an equivalent division problem for 150.756 ÷ 0.4. (1507.56 ÷ 4)

d Calculate 150.756 ÷ 0.4. (376.89)

4 Have students come up with real life scenarios where they would need to divide a decimal by another decimal number (unit price questions: If it cost $3.72 for 1.2 metres of material how much did it cost for 1 metre of material?).

Answers

3E Dividing a decimal by a decimal

3E Start thinking!

1 a 20 b 20
c Advantages include that it gives accurate results; disadvantages include it is time-consuming to draw number lines.
2 a i 20 ii 20 iii 20 iv 20
b Each answer is 20. Dividend and divisor are each reduced by same factor so calculations are equivalent.
c 20
d 20. The pattern continues as the divisor and dividend have both been reduced by the same factor.
e yes
3 a 60 ÷ 3 = 20 b 6 ÷ 0.3 = 20
c 0.6 ÷ 0.03 = 20 d 0.06 ÷ 0.003 = 20

Exercise 3E Dividing a decimal by a decimal

1 a Multiply by 10; 0.5 × 10 = 5
b Multiply by 100; 0.04 × 100 = 4
c Multiply by 10; 0.3 × 10 = 3
d Multiply by 10; 0.7 × 10 = 7
e Multiply by 100; 0.02 × 100 = 2
f Multiply by 10; 0.8 × 10 = 8
g Multiply by 1000; 0.006 × 1000 = 6
h Multiply by 10; 0.4 × 10 = 4
i Multiply by 100; 0.09 × 100 = 9
2 a 623.56 ÷ 8 b 9840 ÷ 8
c 1956.74 ÷ 6 d 4.62 ÷ 3
e 420 ÷ 4 f 0.5 ÷ 2
g 4620850 ÷ 9 h 0.56 ÷ 2
i 2108 ÷ 5
3 a 158.51 ÷ 5 b 1648.2 ÷ 4
c 1.9584 ÷ 3 d 3.6505 ÷ 7
e 5059.56 ÷ 2 f 304.56 ÷ 8
g 10 473.24 ÷ 6 h 51 250 ÷ 4
i 322.2 ÷ 9
4 a 19.3 b 281 c 84.7
d 119.7 e 162.8 f 185.3
g 34 150 h 47.1 i 364.8
5 a 11.905 b 43.375 c 5412.5
d 0.82 e 3187.895 f 642.175
g 188.25 h 2282.45 i 14.325
6 a 31.702 b 412.05 c 0.6528
d 0.5215 e 2529.78 f 38.07
g 1745.54 h 12 812.5 i 35.8
7 a 9 b 6 c 7
d 200 e 3 f 800
g 90 h 30 i 7100
8 a 8.40 ÷ 0.60 b 84 ÷ 6 c 14
9 86
10 a i 12.00 ÷ 0.50 ii 120 ÷ 5 iii 24
b i 12.00 ÷ 0.20 ii 120 ÷ 2 iii 60
c i 12.00 ÷ 0.80 ii 120 ÷ 8 iii 15
d i 12.00 ÷ 1.20 ii 120 ÷ 12 iii 10
11 a 240 000 ÷ 3000 = 80
b 24 000 ÷ 300 = 80
c 2400 ÷ 30 = 80 d 240 ÷ 3 = 80
e 24 ÷ 0.3 = 80 f 2.4 ÷ 0.03 = 80
g 0.24 ÷ 0.003 = 80 h 0.024 ÷ 0.0003 = 80
12 a i 90 ii 90 iii 90
b i 72 ÷ 0.8 = 90 ii 7.2 ÷ 0.08 = 90
iii 0.72 ÷ 0.008 = 90
c 0.99 ÷ 0.11 = 9 = 0.099 ÷ 0.011 = 9.9 ÷ 1.1 = 99 ÷ 11
13 a i 5 ii 50 iii 500 iv 5000
b The answer increases by a power of 10.
c The answer is a value larger than the dividend.
d larger
14 a A, B, E, G
b Dividend and divisor in each problem is multiplied (or divided) by same factor.
15 a no
b Check if dividend and divisor have both been multiplied (or divided) by same factor.
16 a $2.40 b $2.75 c 1.2-kg bag
17 a 144 pieces b yes, 0.1 m remaining
18 45.5 L
19 Divide cost by volume. Orange mineral water is the better buy ($1.68 per L compared to $1.80 per L for cola).
20 a i 3876.52 ii 387.652
iii 38.7652 iv 3.876 52
b Decimal point is moved left (or digits move right). Number of places moved equals number of zeros after the 1 in divisor.
c i one, left ii two, left
iii three, left iv four, left
d i 0.678 543 ii 9.53 iii 28.894
iv 0.157 21 v 0.2934 vi 0.702
vii 68.9421 viii 0.150 95 ix 0.0436
21 a 0.562 78 b 96.3 c 52.47
d 0.8326 e 6.8256 f 2.825
g 5.21 h 0.8921
22 a 15 960 b 22 c 2096.96
d 4 e 500 f 42.6768
23 32.4 ÷ 4.05 = 8

Reflect

Possible answer: It is not easy to divide by a decimal directly, so we change the problem to an equivalent calculation with a whole number divisor. Then we can perform the division in the same way.

Resources

WorkSheet

WS 3E-1 Dividing by a decimal

Focus: To divide a decimal by a decimal

Students initially establish the number of decimal places of the divisor and then find the equivalent short-division problem with a whole number division. Additional practice questions similar to those in Exercise 3E are also provided.

Investigation

INV 3E-2 Dealing with the dollar

Focus: To calculate exchange rates of the Australian dollar using the multiplication and division of decimals

Resources: calculator (optional), current exchange rates (optional)

This is a more challenging activity. Students convert quantities of money between the Australian dollar and the US dollar, the euro and the yen. They may formulate a process for their conversions and consider under which conditions it is more favourable to convert currencies. As an extension, students explore a problem using current exchange rates.

BLMs

Number lines 2

1-cm grid paper

Division grid

Multiplication facts

Place-value chart – decimals

Interactives

3E eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3F Converting between fractions and decimals

Teaching support for pages 152–7

Teaching strategies

Learning focus

To understand fractions and decimals and how to convert between them; and to apply understanding of the relationship to real-life contexts

Start thinking!

In this task, students:

- represent a given set of decimals on a number line
- use their understanding of place value to discover that each decimal can be expressed in hundredths and show this on the number line
- extend their understanding of place value and see that a decimal number can be expressed as more than one fraction of the same value (equivalent fractions)
- discover how to convert a decimal to a fraction by relating the place value of the last digit in a decimal to the denominator of the fraction

The BLM **Number lines 3** (see Resources) can be provided for this task.

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 4a–c, g, 10, 11, 15, 16a–d, 17	1–4, 5a–f, 10, 11, 12a–c, 15, 16e–h, 21, 23, 24	3, 5–9, 12–14, 16i–l, 18–26
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q8

Focus: To demonstrate knowledge and understanding of converting simple fractions to decimals and decimals to simple fractions

- Direct students to complete **SS 3F-1 Decimal equivalents to simple fractions** (see Resources) if they had difficulty with this question or require more practice.
- You may need to undertake some explicit teaching so students understand that the

place-value chart for decimals can be used to help them write decimals as fractions.

For example, to write 0.47 as a fraction, students need to look at the place value of the last digit in the decimal number (hundredths). So 0.47 is 47 hundredths or $\frac{47}{100}$. The BLM **Place-value chart – decimals** (see Resources) can be provided.

- Students can practise writing some additional examples as fractions, initially involving fractions that cannot be simplified and then some that can.

  For example, to write 0.645 as a fraction, look at the place value of the last digit in the decimal number (thousandths). So 0.645 is 645 thousandths or $\frac{645}{1000}$. Ask students if this can be simplified. (Both the numerator and the denominator can be divided by 5.) $0.645 = \frac{645}{1000} = \frac{129}{200}$. This cannot be simplified any further. Students may need to refer to 2B *Equivalent fractions*.

At Level

At Level
1–4, 5a–f, 10, 11, 12a–c, 15, 16e–h, 21, 23, 24

- Demonstrate **3F eTutor** or direct students to do this independently.
- Students should first write the fraction with a denominator of 10, 100, 1000 etc. and then simplify the fraction where possible.
- When portioning quantities, students may benefit from using counters.
- When identifying fractions between two decimal numbers, or decimal numbers between two fractions (as in Q23 and Q24), have students use their one metre percentage and decimal number line.
- As a class, a good introductory or end-of-topic activity is to play the game fraction/percentage/decimal bingo. Each pair of students receives a 3 × 3 grid. Students write 3 decimals, 3 percentages and 3 fractions on each grid. The class teacher or a nominated student reads out either a fraction, decimal or percentage and students mark off if they have an equivalent number. The first to fill their card calls out ‘bingo’.
- Students who require additional practice converting between fractions and decimals can play a game of dice. In one bag place only fraction dice; in another place only decimal dice. Students take turns to select one die from each bag. They then roll each die. The resulting numbers are either (At Level) added together, (Above Level) multiplied or (Below Level) one die is manipulated until it is equivalent to the other.

- When requiring additional practice, students can complete **WS 3F-2 Converting between fractions and decimals** (see Resources). Students are guided through a series of questions to establish the techniques used to convert between fractions and decimals. Additional practice questions similar to those in Exercise 3F are also provided.

- For more problem-solving tasks and investigations, direct students to **INV 3F-3 Brownlow best** (see Resources). Students calculate the average Brownlow Medal votes per game for some AFL legends. They round their answers and compare results. They also consider some anomalies and the possibilities this presents. As an extension, they research the statistics for a current player.

Below Level

Below Level
1, 2, 4a–c, g, 10, 11, 15, 16a–d, 17

- Demonstrate **3F eTutor** or direct students to do this independently.

- Direct students to the **examples. Example 3F-1** shows how to write a decimal as a fraction. **Example 3F-2** shows how to write a fraction as a decimal when the denominator is a power of 10. **Example 3F-3** shows how to write a fraction as a decimal by dividing the numerator by the denominator.

- Students who do not yet have an understanding of equivalence will find it difficult to convert between fractions and decimals. These students will benefit from creating visual models of each fraction or decimal and then comparing them. Either a kinder square (which can easily be folded into a fraction or divided up into hundredths) or a 10 × 10 grid is a good way to represent fractions and decimals. Later this same model can be used to represent and compare percentages.

- When comparing fractions and decimals as in Q10, students can draw the chocolate block on a 10 × 10 grid and colour in the two options. Which is larger? … 3 tenths or one quarter?

- When converting units of measure between fractions and decimals as in Q17, students may start by measuring out 4.25 m and then measuring how much more they need to make it 6 m. What is the difference? Alternatively, they can measure out both amounts, join them together, and then measure them and see if they are close to 6 m. What is the total length?

Above Level

Above Level
3, 5–9, 12–14, 16i–l, 18–26

- Demonstrate **3F eTutor** or direct students to do this independently.

- To convert a fraction with a denominator other than 100 to a decimal, students can either (1) multiply the fraction until the denominator is equivalent to 100 or (2) use short division as described in **Example 3F-3**.
- The most well-known non-recurring and non-terminating decimal number is π. Using the Internet, students can look up the first 1000 digits. Can they find their date of birth in it? Do they recognise any other number patterns in it? Look up who holds the current world record for reciting the most digits of π. What makes this so amazing? (There are no patterns to this decimal number, meaning each digit has to be memorised individually and cannot be related to an existing pattern.)

POTENTIAL DIFFICULTY
When converting a fraction to a decimal, encourage students to first see if the fraction can be written as an equivalent fraction with a denominator of 10, 100, 1000 etc. If this is not easy to do, they should perform a short division by dividing the numerator by the denominator.

- For more problem-solving tasks and investigations, direct students to **INV 3F-3 Brownlow best** (see Resources). Students calculate the average Brownlow Medal votes per game for some AFL legends. They round their answers and compare results. They also consider some anomalies and the possibilities this presents. As an extension, they research the statistics for a current player.
- To explore the relationship between imperial and metric measurements, refer students to **INV 3F-4 Working out the nuts and bolts** (see Resources). Students convert between the fractional imperial units used to identify the sizes of nuts and bolts and their metric decimal equivalents. They are introduced to the engineering terminology used to identify the sizes of nuts and bolts and the problems faced when universal products can be measured in two different systems.

Extra activities

1• Have students write this list of fractions ($\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{3}, \frac{2}{3}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$) in decimal form.
Which ones are recurring and which are terminating decimals?

2 Write each decimal as a fraction.

a 0.3 ($\frac{3}{10}$)

b 0.35 ($\frac{35}{100} = \frac{7}{20}$)

c 0.368 ($\frac{368}{1000} = \frac{46}{125}$)

3 Write each fraction as a decimal

a $\frac{5}{10}$ (0.5)

b $\frac{47}{100}$ (0.47)

c $\frac{5}{8}$ (0.625)

4 Look at examples of terminating decimals such as 0.25, 0.5 and 0.75. It may be useful to demonstrate the short division of the fractions $\frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$ so that students can see the use of the decimal point and trailing zeros, and that the division has no remainder. Fractions with denominators of 2, 4, 5, 8 and 10 can be used as additional examples. A fraction which has a denominator with prime factors of only 2 and/or 5 will form a terminating decimal. All other fractions form non-terminating decimals.

5 Look at examples of recurring decimals such as 0.333 333..., 0.222 222... and 0.454 545.... Perform divisions with $\frac{1}{3}, \frac{2}{9}$ and $\frac{5}{11}$ so students can see that, no matter how many trailing zeros are used, there is always a remainder. Emphasise the pattern of digits formed that is used to write the recurring decimal in an abbreviated form ($0.\dot{3}$, $0.\dot{2}$ and $0.\dot{4}\dot{5}$ or $0.\overline{45}$). A calculator could also be used to compare how the decimal answer is displayed.

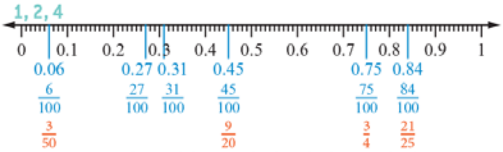
Answers

ANSWERS

3F Converting between fractions and decimals

3F Start thinking!

1, 2, 4



3 For a number with two decimal places, the last decimal place is hundredths.

4 $\frac{84}{100} = \frac{21}{25}, \frac{75}{100} = \frac{3}{4}, \frac{6}{100} = \frac{3}{50}, \frac{45}{100} = \frac{9}{20}$

5 A given fraction can have many equivalent fractions. These are fractions that represent the same value but have different numerators and denominators.

6 Some possible answers are:
$\frac{1}{2} = \frac{5}{10} = \frac{10}{20} = \frac{50}{100} = \frac{4}{8} = \frac{6}{12}$

7 The fourth decimal place is ten-thousandths, so $0.1587 = \frac{1587}{10\,000}$.

Exercise 3F Converting between fractions and decimals

1 a $\frac{25}{100}$ b $\frac{50}{100}$ c $\frac{35}{100}$ d $\frac{88}{100}$
e $\frac{23}{100}$ f $\frac{8}{100}$ g $\frac{12}{100}$ h $\frac{95}{100}$

2 a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{7}{20}$ d $\frac{22}{25}$
e $\frac{23}{100}$ f $\frac{2}{25}$ g $\frac{3}{25}$ h $\frac{19}{20}$

3 a $\frac{1}{5}$ b $\frac{7}{10}$ c $\frac{13}{100}$ d $\frac{13}{50}$
e $\frac{9}{1000}$ f $\frac{213}{500}$ g $\frac{64}{125}$ h $\frac{18}{25}$
i $\frac{1}{2500}$ j $\frac{14}{25}$ k $\frac{3}{8}$ l $\frac{7}{25}$

4 a 0.03 b 0.17 c 0.9
d 0.2010 e 0.375 f 0.001 56
g 0.07 h 0.921 i 0.042
j 0.0003 k 0.01981 l 0.0086

5 a 0.875 b 0.75 c 0.8 d 1.125
e 14.25 f 1.875 g 6.6 h 7.25
i 0.15 j 0.04 k 0.14 l 0.225

6 a Any decimal number with a finite number of decimal places.
b, c 0.2727… (the pattern continues indefinitely); you never get to a stage where there is no remainder.
d 27 e $0.\dot{2}\dot{7}$ or $0.\overline{27}$

7 a T b R c R d T e R f R
g R h T i T j R k T l R

8 a $0.\dot{8}$ b $0.\overline{21}$ c $0.\overline{467}$
d $0.127\dot{4}$ e $8.31\dot{6}$ f $1.\dot{3}$
g $9.102\overline{46}$ h $0.\overline{461538}$

9 a $0.\dot{2}$ b $0.8\dot{3}$ c $0.\overline{857143}$
d $0.\overline{72}$ e $1.1\dot{6}$ f $0.\dot{6}$
g $0.7\dot{3}$ h $4.41\dot{6}$

10 0.3 is greater than $\frac{1}{4}$ (which is equivalent to 0.25).

11 a 10 b $\frac{2}{10}$ c 0.2
d apple: $\frac{2}{10} = 0.2$; cola: $\frac{1}{10} = 0.1$; watermelon: $\frac{2}{10} = 0.2$; strawberries and cream: $\frac{3}{10} = 0.3$

12 a $\frac{1}{5} = \frac{2}{10}$ b $\frac{3}{4} = \frac{75}{100} = 0.75$
c $\frac{7}{20} = \frac{35}{100} = 0.35$ d $\frac{4}{5} = \frac{8}{10} = 0.8$
e $\frac{11}{50} = \frac{22}{100} = 0.22$ f $\frac{2}{4} = \frac{50}{100} = 0.50$
g $\frac{13}{20} = \frac{65}{100} = 0.65$ h $\frac{1}{2} = \frac{50}{100} = 0.50$
i $\frac{19}{50} = \frac{38}{100} = 0.38$ j $\frac{1}{8} = \frac{125}{1000} = 0.125$
k $\frac{4}{125} = \frac{32}{1000} = 0.032$
l $\frac{53}{2500} = \frac{212}{10\,000} = 0.0212$

13 Denominators are factors of a suitable power of 10.

14 a, e, g, i, l; $\frac{3}{5} = \frac{6}{10} = 0.6$, $\frac{5}{25} = \frac{20}{100} = 0.2$, $\frac{497}{500} = \frac{994}{1000} = 0.994$, $\frac{7}{25} = \frac{28}{100} = 0.28$, $\frac{103}{125} = \frac{824}{1000} = 0.824$. Others cannot be written as decimals without first dividing, as denominators are not factors of a suitable power of 10.

15 a $8.605 = 8 \times 1 + 6 \times \frac{1}{10} + 5 \times \frac{1}{1000}$
b decimal part c $\frac{605}{1000} = \frac{121}{200}$ d $8\frac{121}{200}$

16 a $5\frac{83}{100}$ b $3\frac{451}{500}$ c $\frac{9}{20}$ d $8\frac{7}{20}$
e $2\frac{1}{1250}$ f $3\frac{9}{10}$ g $12\frac{22}{55}$ h $1\frac{353}{500}$
i $40\frac{3}{8}$ j $1\frac{73}{100}$ k $3\frac{3}{20}$ l $26\frac{421}{500}$

17 a $4\frac{1}{4}$ m and $1\frac{4}{5}$ m (or 4.25 m and 1.8 m)
b $6\frac{1}{20}$ m (or 6.05 m) c yes

18 a i $0.\overline{142857}$ ii $0.\overline{285714}$
iii $0.\overline{428571}$ iv $0.\overline{571428}$
b Same digits are used in given order but with different starting point. Starting digit in recurring pattern for each fraction is in ascending order. That is, 1, 2, 4, 5.
c, d $0.\overline{714285}$ and $0.\overline{857142}$

19 $\frac{1}{9} = 0.\dot{1}, \frac{2}{9} = 0.\dot{2}, \frac{3}{9} = 0.\dot{3}, \frac{4}{9} = 0.\dot{4}, \frac{5}{9} = 0.\dot{5}, \frac{6}{9} = 0.\dot{6}, \frac{7}{9} = 0.\dot{7}, \frac{8}{9} = 0.\dot{8}$. Numerator of each fraction becomes recurring digit.

20 Denominators of 3 and 6 produce recurring fractions.

21 $\frac{28}{36} = 0.\dot{7}$, $\frac{38}{46} \approx 0.826$. Lou performed better this week.

22 Forces and motion: 0.8; Solids, liquids and gases: $0.8\dot{4}$; Separating mixtures: 0.875; Science at work: $0.\dot{8}$

23 Some possible answers are: $\frac{76}{100}, \frac{4}{5}, \frac{81}{100}$.

24 Some possible answers are: 0.25, 0.3, 0.35, 0.395.

25 $3\frac{162}{200} = 3\frac{81}{100}$

26 a $0.08, \frac{1}{10}, \frac{16}{34}, \frac{15}{25}, 0.62, \frac{14}{16}, 1.05, \frac{22}{20}$
b $0.28, \frac{62}{90}, \frac{58}{75}, 0.82, \frac{37}{17}, 2.21, 5\frac{12}{20}, 5.\dot{6}$

Reflect

Possible answer: The place values of decimals involve powers of ten and can be expressed as tenths, hundredths and so on. If a fraction has a denominator that is a power of 10 it can be easily converted to a decimal without the need to find equivalent fractions or perform a division calculation.

Resources

SupportSheet

SS 3F-1 Decimal equivalents to simple fractions

Focus: To visually understand the equivalence of simple fractions and decimals

Students work through a series of questions to help them identify the relationship between given simple fractions written as an equivalent tenth and their representation as a decimal.

WorkSheet

WS 3F-2 Converting between fractions and decimals

Focus: To convert between simple fractions and decimals

Students are guided through a series of questions to establish the techniques used to convert between fractions and decimals. Additional practice questions similar to those in Exercise 3F are also provided.

Investigations

INV 3F-3 Brownlow best

Focus: To calculate the average Brownlow votes per game

Resources: calculator (optional), Internet access (optional)

Students calculate the average Brownlow votes per game for some AFL legends. They round their answers and compare results. They also consider some anomalies and the possibilities this presents. As an extension, they research the statistics for a current player.

INV 3F-4 Working out the nuts and bolts

Focus: To convert between imperial and metric measurement to determine the size of nuts and bolts

Resources: calculator

Students convert between the fractional imperial units used to identify the sizes of nuts and bolts and their metric decimal equivalents. They are introduced to the engineering terminology used to identify the sizes of nuts and bolts and the problems faced when universal products can be measured in two different systems.

BLMs

Number lines 3

Place-value chart – decimals

Interactives

3F eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3G Understanding percentages

Teaching support for pages 158–63

Teaching strategies

Learning focus

To consolidate knowledge of the link between fractions and percentages; and to apply understanding of the relationship between them to application questions

Start thinking!

In this task, students:

- consider two squares which are each divided into 100 smaller squares.
- are introduced to the concept that 'per cent' means out of 100
- write the number of shaded and unshaded squares as a percentage
- discover that the percentage of shaded squares together with the percentage of unshaded squares represents 100%.

Differentiated pathways

Below Level	At Level	Above Level
1–3, 4a–e, 5, 6a, b, d, h, i, 7, 9–11, 13, 16	1–3, 4f–l, 5, 6, 8a–d, 10, 12–14, 16, 18	4i–j, 5e–h, 6e–l, 8d–f, 14–21
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q9 & Q10

Focus: To demonstrate knowledge and understanding of representing a portion of 100 as a fraction with a denominator of 100; and to demonstrate a knowledge and understanding of equivalent fractions

- Direct students to complete **SS 3G-1 Proportions of a whole as a fraction** (see Resources) if they had difficulty with Q9 or require more practice at this skill.
- You may need to undertake some explicit teaching so students remember how to write a fraction to represent a proportion of a whole.
- The BLMs **1-cm grid paper** or **100 grid** (see Resources) can be used.

Support strategies for Are you ready? Q10

- Direct students to complete **SS 3G-2 Equivalent fractions with a denominator of 100** (see Resources) if they had difficulty with Q10 or require more practice at this skill.

- You may need to undertake some explicit teaching so students understand shading on grids to represent equivalent fractions.

- Have students draw a square on grid paper (10 by 10) and divide it into ten equal columns. You may like to provide a copy of the BLMs **1-cm grid paper** or **100 grid** (see Resources)**.** Shade the square to represent $\frac{7}{10}$. Draw another square of the same size and divide it into ten equal columns and then ten equal rows so there are 100 small squares. Shade the large square to represent $\frac{70}{100}$. Compare the proportion of each large square that has been shaded. Repeat to demonstrate other fractions that can be written as an equivalent fraction with a denominator of 100. This will assist students in understanding the concept of percentages.

At Level

At Level
1–3, 4f–l, 5, 6, 8a–d, 10, 12–14, 16, 18

- Demonstrate **3G eTutor** or direct students to do this independently.

- Ensure students understand that for some situations where the proportion is not out of 100, they can still find the percentage by first writing an equivalent fraction with a denominator of 100.

- When writing a percentage as a fraction in its simplest form, have students turn the percentage into a fraction with a denominator of 100, and then simplify the fraction.

POTENTIAL DIFFICULTY
A mixed bag of buttons contains 56% red. This statement means for every 100 buttons 56 are red. It does not mean there are 56 red buttons in the bag, but indicates that the proportions of buttons remains fixed. If I had 100 buttons, 56 of them would be red. If I had 200 buttons, 112 of them would be red. If I had 350 buttons, 196 of them would be red.

- To find the percentage that remains as in Q10, encourage students to think of it as finding the difference between the percentage and 100%.

- When estimating percentages as in Q16, it is useful for students to mark approximately 50% and then talk about whether the line they are looking at is more or less than 50%.

- It may be useful to discuss Q17 as a whole class.

- When estimating the percentage of volume in a container, take in a range of containers and allow students to experiment with one another creating their own problems. This will allow students to see that the amount of liquid required to fill a container to 50%

capacity varies according to the dimensions of the container.

POTENTIAL DIFFICULTY
Some students may have difficulty in finding an equivalent fraction with a denominator of 100. Emphasise that the factor to be multiplied or divided by can be found by looking at the denominator first. What can you multiply or divide by to obtain 100? This factor then needs to be applied to both the numerator and the denominator of the original fraction.

- For additional practice, students can complete **WS 3G-3 Writing percentages** (see Resources). Students write a proportion out of 100 as a percentage. Students connect fractions with a denominator of 100 and percentages. They write percentages and investigate what 100% means. Additional questions are also provided.

Below Level

Below Level
1–3, 4a–e, 5, 6a, b, d, h, i, 7, 9–11, 13, 16

- Demonstrate **3G eTutor** or direct students to do this independently.
- For students who are struggling with the concept of percentages and require more practice and consolidation, direct them to **SS 3G-1 Proportions of a whole as a fraction** and **SS 3G-2 Equivalent fractions with a denominator of 100** (see Resources).
- To help students to develop a geometric understanding of percentages, have students make a percentage wheel. Take two different coloured circles of paper of the same size and use a protractor to divide each circle into 10 equal segments. Slice down the radius (following one of the tens lines). Now the two circles can be intertwined. Students can use this as a way of easily and quickly modelling different percentages.
- Percentages can be a difficult concept for some students, so visually representing each percentage can be useful. Provide each student with a copy of the BLM **100 grid** (see Resources) and ask them to shade one of the grids of 100 squares. Discuss how they have shaded one whole and that this represents 100%. Ask students to now shade two of the 100 grids and have them explain how this represents 200%. Continue shading 100 grids to show 300%, 400% and so on. Extend this to consider representing 150%, 250%, 320%, 175% and so on. Encourage students to develop the link between each percentage and the number of wholes; for example, 100% = 1, 200% = 2, 300% = 3 and so on.

POTENTIAL DIFFICULTY
Many students struggle with the concept of percentages. For students who are unfamiliar with the concept of percentages it is advisable to not begin with an emphasis on the algorithmic procedure.

- Have students add to their ‘percentage’ number line which was created in *3A Understanding decimals*.

- For additional support and guidance, students could attempt the **WS 3G-3 Writing percentages** (see Resources). Here they connect fractions with a denominator of 100 and percentages. They write percentages and investigate what 100% means.

Above Level

Above Level
4i–j, 5e–h, 6e–l, 8d–f, 14–21

- Demonstrate **3G eTutor** or direct students to do this independently.
- Some students may have difficulty in finding an equivalent fraction with a denominator of 100. Emphasise that the factor to be multiplied or divided by can be found by looking at the denominator first. What can you multiply or divide by to obtain 100? This factor then needs to be applied to both the numerator and the denominator of the original fraction.
- Have students try this problem: I recently printed 500 flyers for my son's dog washing business. Unfortunately, I did not have 500 sheets of yellow paper so I used two different coloured sheets. 60% of the flyers were printed on yellow paper and 40% were printed on blue sheets. How many sheets were printed on each colour? Extension: Unfortunately, I cannot use any sheets which ended up with smudges. If 10% of the blue sheets had smudges on them how many sheets would I need to dispose of?
- To investigate the use of percentages, refer students to **INV 3G-4 Calculating the football ladder** (see Resources). Students calculate the percentages for a league of football teams. They then explore the different possibilities of football ladder rankings depending on the score outcomes of games and the resulting team percentages.
- To explore the use of percentage, refer students to **INV 3G-5 Percentages with punch** (see Resources). Students calculate the percentages of ingredients in various punch recipes and solve problems raised in different scenarios. As an extension, they research other non-alcoholic punch recipes and determine the percentage composition of the ingredients.

Extra activities

1 What does 'per cent' mean? (out of 100)

2 What is the equivalent percentage to each fraction?

a $\frac{1}{2}$ (50%)

b $\frac{1}{4}$ (25%)

c $\frac{3}{4}$ (75%)

3 Write each fraction as a percentage.

a $\frac{17}{100}$ (17%)

b $\frac{4}{20}$ (20%)

c $\frac{16}{20}$ (80%)

d $\frac{3}{5}$ (60%)

Answers

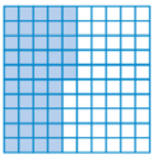
3G Understanding percentages

3G Start thinking!

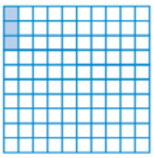
1 a 23 b 65
2 a 23% b 65%
3 a 77% b 35%
4 a i 23% ii 77% b Add to 100%.

Exercise 3G Understanding percentages

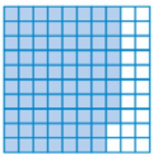
1 a 52% b 50% c 36%
2 a 48% b 50% c 64%
3 a $\frac{7}{100}$ b $\frac{19}{100}$ c $\frac{43}{100}$ d $\frac{57}{100}$
e $\frac{3}{100}$ f $\frac{63}{100}$ g $\frac{11}{100}$ h $\frac{91}{100}$
i $\frac{79}{100}$ j $\frac{21}{100}$ k $\frac{99}{100}$ l $\frac{33}{100}$
4 a $\frac{16}{25}$ b $\frac{9}{50}$ c $\frac{27}{100}$ d $\frac{4}{5}$
e $\frac{1}{20}$ f $\frac{19}{20}$ g $\frac{1}{10}$ h $\frac{9}{20}$
i $\frac{3}{5}$ j $\frac{3}{4}$ k $\frac{7}{20}$ l $\frac{22}{25}$
5 a 23% b 78% c 35% d 99%
e 13% f 7% g 82% h 4%
6 a 45% b 22% c 75% d 68%
e 60% f 20% g 60% h 92%
i 36% j 25% k 80% l 20%
7 a i 14 sections shaded of total 20 sections
ii $\frac{14}{20}$ iii $\frac{70}{100}$ iv 70%
b i 5 sections shaded of total 10 sections
ii $\frac{5}{10}$ iii $\frac{50}{100}$ iv 50%
c i 1 section shaded of total 4 sections
ii $\frac{1}{4}$ iii $\frac{25}{100}$ iv 25%
8 a 76% b 40% c 75%
d 30% e 35% f 6%
9 a i 20% ii 10% iii 30%
b i 80% ii 90% iii 70%
c Express number of pieces eaten out of total number of pieces as equivalent fraction with denominator of 100.
10 a 55% b 82% c 25% d 33%
11 a



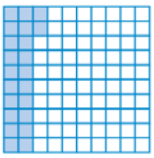
b



c



d



12 a see part c
b i 10 is factor of 100 so denominator of 10 can easily be expressed as denominator of 100.
ii Each cm will represent 10%.
iii Each mm will represent 1%.
c $15\% = \frac{15}{100}$ so 15 mm out of 100 mm (or 1.5 cm out of 10 cm) needs to be shaded.

d i

ii

iii

iv

13 a C b A c D d B
14 a 40% b 90% c 4% d 20%
15 a 40% b 30% c 65% d 75%
16 a 25% b 45% c 70%
17 Percentage is calculated by finding volume of liquid in each container as a fraction of total volume that container holds. However, 50% of a large container is still going to be more that 50% of a small container.
18 a three times whole amount
b i 2 ii 3 iii 4 iv 1.5
19 a 4.5 b 525%
20 Multiply denominator by 100 (change any mixed number to improper fraction first).
21 a $\frac{1}{700}$ b $\frac{1}{450}$ c $\frac{1}{125}$ d $\frac{1}{200}$
e $\frac{7}{1000}$ f $\frac{3}{400}$ g $\frac{2}{325}$ h $\frac{1}{120}$
i $\frac{33}{800}$ j $\frac{23}{300}$ k $\frac{37}{200}$ l $\frac{41}{400}$
m $\frac{12}{125}$ n $\frac{61}{700}$ o $\frac{67}{600}$ p $\frac{23}{550}$

Reflect

Possible answer: The number 100 is important because 'per cent' means out of 100.

Resources

SupportSheets

SS 3G-1 Proportions of a whole as a fraction

Focus: To find the fractional proportion where the denominator is 100

Resources: 1-cm grid paper (BLM) or **100 grid** (BLM), ruler, coloured pencils

Students are guided through questions to understand and write fractions with a denominator of 100. This is in preparation for understanding the concept of percentages.

SS 3G-2 Equivalent fractions with a denominator of 100

Focus: To express a fraction as an equivalent fraction with a denominator of 100

Students use diagrams to help them write a fraction as an equivalent fraction with a denominator of 100. They then establish a numeric process to form equivalent fractions with a denominator of 100.

WorkSheet

WS 3G-3 Writing percentages

Focus: To write a proportion out of 100 as a percentage

Students connect fractions with a denominator of 100 and percentages. They write percentages and investigate what 100% means. Additional questions are also provided.

Investigations

INV 3G-4 Calculating the football ladder

Focus: To calculate percentages and determine a football ladder

Resources: calculator

Students calculate the percentages for a league of football teams. They then explore the different possibilities of football ladder rankings depending on the score outcomes of games and the resulting team percentages.

INV 3G-5 Percentages with punch

Focus: To determine the percentages of ingredients in different punch recipes

Resources: calculator (optional), non-alcoholic punch recipes (optional)

Students calculate the percentages of ingredients in various punch recipes and solve problems raised in different scenarios. As an extension, they research other non-alcoholic punch recipes and determine the percentage composition of the ingredients.

BLMs

1-cm grid paper

100 grid

Interactives

3G eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3H Converting between fractions, decimals and percentages

Teaching support for pages 164–9

Teaching strategies

Learning focus

To consolidate student understanding of converting between fractions, decimals and percentages.

Start thinking!

In this task, students:

- build on their understanding of expressing a quantity as a fraction of a whole and writing this as an equivalent fraction with a denominator of 100
- demonstrate that they can express this fraction as a percentage (the numerator with a % sign) and a decimal (hundredths).
- write the number of red balloons in a bunch of balloons as a fraction, a percentage and a decimal
- explain why the number of purple balloons in the bunch can be written correctly in three different ways

Students discover that there is a link between writing a number as a percentage and as a decimal.

They are guided to explain how to convert a percentage to a decimal. Similarly, they explain how to convert a decimal to a percentage.

Guide students to understand that a proportion of a whole can be expressed in different forms (fraction, decimal, percentage) but still have the same value.

Differentiated pathways

Below Level	At Level	Above Level
1a–f, 3, 8, 9, 10a, 11	1, 2, 4a–f, 8–16	1g–i, 2g–l, 4g–l, 5–11, 13–15, 17–21
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2, 4a–f, 8–16

- Demonstrate **3H eTutor** or direct students to do this independently.
- When converting between percentages and decimals students may, for percentages less than 100, use the 1-metre ruler they created in *3A Understanding decimals* or a 10 × 10 grid. Have students colour in the percentage and then identify the total number of tenths and hundredth to convert the percentage into a fraction.
- When completing Q9, students can make the table on a computer and print it out as a one page document which can be laminated for future reference.

POTENTIAL DIFFICULTY
When comparing decimals, percentages and fractions, students should convert all into the same form, thus ensuring students are comparing like for like.

- For students who need additional practice converting between an equivalent percentage, decimal and fraction form, direct them to **WS 3H-1 Numbers as fractions, decimals and percentages** (see Resources). In this WorkSheet, they are guided to write a proportion of a whole from a diagram as a percentage, decimal and fraction.
- For more problem-solving tasks and investigations, direct students to **INV 3H-2 Designing plans** and **INV 3H-3 Dominations** (see Resources).

  In **INV 3H-2 Designing plans,** students explore the different possibilities of designing the plans of a school and a neighbourhood. They design their own plan and express various areas as fractions, percentages and decimals.

  For **INV 3H-3 Dominations,** a group of up to four students play a game of Dominos, recognising equivalent forms of fractions, percentages and decimals. The domino cards are provided in the BLM **Dominations** (see Resources). The game is intended to support the recall and identification of the common benchmark fractions in equivalent decimal and percentage forms.

Below Level

Below Level
1a–f, 3, 8, 9, 10a, 11

- Demonstrate **3H eTutor** or direct students to do this independently.
- Many students at this level will need additional practice converting between an equivalent percentage, decimal and fraction form. Direct them to **WS 3H-1 Numbers as fractions, decimals and percentages** (see Resources). In this WorkSheet, they are guided to write a proportion of a whole from a diagram as a percentage, decimal and fraction.

- Students who do not yet have an understanding of equivalence will find it difficult to convert between fractions, decimals and percentages. These students will benefit from creating visual models of each fraction, decimal or percentage and then compare them. Either a kinder square (which can easily be folded into a fraction or divided up into hundredths) or the 10 × 10 grid in the BLM **100 grid** (see Resources) can be used to represent fractions, decimals and percentages.

- When converting decimals to a fraction have students use the BLM **100 grid** as described above before encouraging them to multiply the number by 100.

- Create a set of laminated cards which have a fraction, a decimal or a percentage on one side. Make sure that each card has a pair. For example: the decimal 0.500 can be matched up with the percentage 50%. The decimal 0.25 can be matched up with the fraction $\frac{1}{4}$. Students use these cards to play a game of either Snap or Memory.

Above Level

Above Level
1g–i, 2g–l, 4g–l, 5–11, 13–15, 17–21

- Demonstrate **3H eTutor** or direct students to do this independently.

- When comparing percentages, decimals and fractions, have students convert them all to one form before comparing.

- When comparing percentages that contain fractions, first convert the fractional part to a decimal number.

- Have students try this extension question: I fill a glass to the very top with milk. I take a large sip. Which happens first? The glass is $\frac{5}{16}$ empty or the glass is 60% full. ($\frac{5}{16}$ empty)

- For more problem-solving tasks and investigations, direct students to **INV 3H-2 Designing plans** (see Resources). Students explore the different possibilities of designing the plans of a school and a neighbourhood. They design their own plan and express various areas as fractions, percentages and decimals.

- For further problem-solving tasks and investigations, direct students to **INV 3H-3 Dominations** (see Resources). A group of up to four students play a game of Dominos, recognising equivalent forms of fractions, percentages and decimals. The domino cards are provided in the BLM **Dominations** (see Resources). The game is intended to support the recall and identification of the common benchmark fractions in equivalent decimal and percentage forms.

Extra activities

1 In pairs, students write 6 different numbers, either fractions, decimals or percentages on different pieces of card. Pairs then swap their cards with another. Students place these numbers on a number line made out of rope. The first team to place their numbers in the correct place on the number line wins. **Note:** Students can only write numbers that could be arranged on a number line.

2 As a class, students can play a game of *Memory*. Print a copy of the cards in the BLM **Fractions, decimals and percentages** (see Resources) for each pair of students. Simplify for struggling students by deleting the first column. Extend by asking students to add further cards of their own to the game.

3 **a** What shortcut can be used to convert a percentage into a decimal? (÷100)

b What shortcut can be used to convert a decimal into a percentage? (× 100)

c When changing a fraction into a percentage, what must the denominator be? (100)

d If a denominator cannot be expressed as 100, what method can be used to change a fraction into a percentage? (Short division and then × 100)

Answers

3H Converting between fractions, decimals and percentages

3H Start thinking!

1 10 2 3 3 $\frac{3}{10}$ 4 $\frac{30}{100}$

5 a 30% b 0.3

6 The percentage value is the same as the number of hundredths in the decimal.

7

Balloon colour	Fraction of total	Fraction with a denominator of 100	Percentage	Decimal
Red	$\frac{3}{10}$	$\frac{30}{100}$	30%	0.3
Blue	$\frac{2}{10}$	$\frac{20}{100}$	20%	0.2
Yellow	$\frac{1}{10}$	$\frac{10}{100}$	10%	0.1
Green	$\frac{4}{10}$	$\frac{40}{100}$	40%	0.4

8 A percentage and a decimal number that are equivalent have same digits. Both can be related to a fraction with denominator 100. Percentage amount is same as numerator and decimal amount is number of hundredths or result of dividing numerator by 100.

9 All descriptions are equivalent but they are expressed in different forms: fraction (Stephanie), percentage (Maria) and decimal (Ben).

Exercise 3H Converting between fractions, decimals and percentages

1 a 0.46 b 0.13 c 0.99
d 0.25 e 0.2 f 0.5
g 0.05 h 0.08 i 0.01

2 a 0.2384 b 0.1965 c 0.467
d 0.0309 e 5.674 f 0.00467
g 0.128 95 h 0.7328 i 2.005
j 0.1092 k 4.0404 l 0.000 101

3 a 28% b 88% c 15% d 62%
e 46% f 72% g 9% h 4%

4 a 51.8% b 902% c 10.5%
d 70% e 2150% f 841%
g 710% h 0.9% i 300.1%
j 500% k 17.654% l 0.0045%

5 a 37.5% b 25% c 43.75%
d 52.5% e 87.5% f 31.25%
g 4.8% h 11.25%

6 a 33.33% b 42.86% c 83.33%
d 81.82% e 55.56% f 71.43%
g 61.54% h 58.33%

8 a $\frac{2}{5}$ b i 40% ii 60%
c i 0.4 ii 0.6

9

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	0.3	33.3.%
$\frac{1}{8}$	0.125	12.5%
$\frac{5}{8}$	0.625	62.5%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%

10 a 72% b 76.6.%
c second test, as percentage is higher

12 a $\frac{1}{8}$ b 0.125 c 12.5%

13 a 24.4% b $\frac{61}{250}$ c 0.244

14 a $\frac{47}{100}$ b $\frac{53}{100}$

15 a $\frac{71}{100}$, 0.71
b i 0.09% ii 0.0009 iii $\frac{9}{10\,000}$
c 99.853%; values may be rounded and there may be other substances in Sun.

16 20%

17 Some examples are provided.
a $\frac{101}{1000}$, $\frac{105}{1000}$, $\frac{1075}{10\,000}$
b 72%, 72.65%, 73%, 73.25%
c 0.887, 0.8888, 0.888 987, 0.888 999 999

18 a 0.122 b 0.2825 c 0.057
d 0.2024 e 0.1726 f 0.0835
g 0.755 h 0.218

19 Both methods of working correct.

20 Nick

21 a $\frac{6}{31}$, $\frac{3}{7}$, 0.4231, 0.249, 12.8%, $\frac{9}{7}$%
b 388%, $3\frac{7}{8}$, 3.87, $385\frac{2}{3}$%, 3.826, $3\frac{9}{11}$

Reflect

Possible answer: Fractions, decimals and percentages are all related as they can each represent a proportion of a whole.

Resources

WorkSheet

WS 3H-1 Numbers as fractions, decimals and percentages

Focus: To convert numbers to equivalent fractions, decimals or percentages

Students are guided through questions to recognise and convert between an equivalent percentage, decimal and fraction. They write a proportion of a whole from a diagram as a percentage, decimal and fraction.

Investigations

INV 3H-2 Designing plans

Focus: To design various plans using percentage, fraction and decimal constraints

Resources: coloured pencils, computer, *Design a school* (TLF-IDL127), *Design a neighbourhood* (TLF-IDL122), **1-cm grid paper** (BLM)

Students explore the different possibilities of designing the plans of a school and a neighbourhood using the The Learning Federation files *Design a school* (TLF-IDL127) and *Design a neighbourhood* (TLF-IDL122). They design their own plan and express various areas as fractions, percentages and decimals.

INV 3H-3 Dominations

Focus: To recognise equivalent percentages, fractions and decimals in different forms

Resources: Dominations (BLM)

A group of up to four students play a game of Dominos, recognising equivalent forms of fractions, percentages and decimals. The domino cards are provided in the BLM **Dominations.** The game is intended to support the recall and identification of the common benchmark fractions in equivalent decimal and percentage forms.

BLMs

100 grid

Dominations

Fractions, decimals and percentages

1-cm grid paper

Interactives

3H eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3I Calculating percentages

Teaching support for pages 170–5

Teaching strategies

Learning focus

To consolidate student understanding of how to calculate percentages of different amounts.

Start thinking!

In this task, students:

- build on their understanding of how to shade a percentage on a 100 grid using the
- are guided to understand that 15% of 100 means ‘15 out of every 100’
- apply this to showing how to represent 15% of 200 and 15% of 600 using a number of 100 square grids
- use the concept that 'of' in mathematics can be replaced with the operation of × to rewrite percentage statements as multiplication problems and then perform the calculations
- discover a method that can be used to calculate the percentage of any amount

The BLM **100 grid** (see Resources) can be provided to students for this task.

Differentiated pathways

Below Level	At Level	Above Level
1b–d, l, 5, 8, 13, 16, 20	1, 2, 3a–f, 4a–f, 5, 6a–c, 7, 10, 13, 16, 17, 19–21	1e–l, 2a–g, 3g–l, 4g–l, 5, 6, 9, 11, 12, 13e, f, 14, 15, 18–24
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q11

Focus: To demonstrate knowledge and understanding of multiplying a fraction by a whole number

- Direct students to complete **SS 3I-1 Finding the fraction of a whole number** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so students understand how to write a whole number as a fraction and remember that ‘of’ can be replaced with ‘×’.

At Level

At Level
1, 2, 3a–f, 4a–f, 5, 6a–c, 7, 10, 13, 16, 17, 19–21

- Demonstrate **3I eTutor** or direct students to do this independently.
- Students who struggle with the algorithm for calculating percentages should work on strategies to mentally calculate common percentages. For example, students who know that 10% of 350 is 35 can use this knowledge to work out what 15% is (10% plus $\frac{1}{2}$ of 10%), what 30% is (3 lots of 10%) and what 90% is (10% less than 100% of the total amount). As the majority of everyday experiences we have with percentages involves multiples of 5s or 10s, these strategies are invaluable.
- Throughout the year have students work out their own percentage on each assessment. This could not only apply to mathematics but to other curriculum areas within the school. This could form part of your school's numeracy policy. The more often students are exposed to real-life examples of mathematical problems, the more likely they are to make lasting mental connections.
- When calculating the percentage of a dollar amount, remind students that the answer should be written in a dollar amount.
- For additional practice, students can complete **WS 3I-3 Calculating the percentage of a quantity** (see Resources). Students are guided through a series of questions to consolidate the process of calculating the percentage of a quantity. Additional practice questions are also provided.
- Those students seeking further problem-solving and investigation activities can be directed to **INV 3I-3 Shopping with 10%** (see Resources). Students investigate the method of finding 10% of a number as dividing the number by 10. They then explore other percentages that can be determined from 10% and apply them to estimate and calculating common percentages encountered when shopping.

Below Level

Below Level
1b–d, I, 5, 8, 13, 16, 20

- Demonstrate **3I eTutor** or direct students to do this independently.
- Students at this level generally require additional practice at percentage calculations. They can be directed to complete **WS 3I-3 Calculating the percentage of a quantity** (see Resources) where they are guided through a series of questions to consolidate the process of calculating the percentage of a quantity.

- Direct students to **Example 3I-1** which shows how to calculate the percentage of a given dollar amount. They are reminded that, when working with money, the final answer should contain only two decimal places (if the answer is in $).
- Direct students to the **Key ideas**. You may like them to copy this summary.
- When calculating percentages such as 20% and 30%, have students first work out 10% then multiply this amount by 2 or 3.
- An additional activity for students operating at this level is to collect brochures from local stores and calculate the price of items during sale times, as items are typically reduced by regular amounts such as 10%, 20% and 50%.

Above Level

Above Level
1e–l, 2a–g, 3g–l, 4g–l, 5, 6, 9, 11, 12, 13e, f, 14, 15, 18–24

- Demonstrate **3I eTutor** or direct students to do this independently.
- When calculating the percentage of an amount where the percentage contains one decimal place (as in Q3), students should first multiply both the numerator and denominator of the fraction by ten, so that the decimal is not remaining in the fraction. Alternatively, the percentage can be expressed as a decimal rather than a fraction in the multiplication problem.
- The more able students may recognise there are two possible methods for completing the calculations in Q12: calculate 25% of each price and then add it to the original wholesale price or calculate 125% of each original wholesale price. It provides an opportunity for you to discuss these two methods, showing that the final answer is the same. This is discussed later in Q22 and Q23.

POTENTIAL DIFFICULTY
When completing fraction calculations, students can simplify by cancelling to make the numbers smaller and easier to manage. Remind students about the restrictions on cancelling: you cannot cancel numerator to numerator, or denominator to denominator.

- For more problem-solving tasks and investigations, direct students to **INV 3I-3 Shopping with 10%** (see Resources). Students investigate the method of finding 10% of a number as dividing the number by 10. They then explore other percentages that can be determined from 10% and apply them to estimate and calculating common percentages encountered when shopping.
- For a further problem-solving task, direct students to **INV 3I-4 Growing money** (see Resources). Students use a computer-based spreadsheet program to calculate the amount of interest earned over a set period of time.

Extra activities

1 What mathematical operation replaces ‘of’? (×)

2 How can you make a whole number in a calculation look like a fraction? (Write the whole number with a denominator of 1)

3 Does putting a number over 1 change its value? (No)

4 In the calculation $\frac{4}{7} \times \frac{14}{28}$, which numbers can be cancelled? (4 and 28, 7 and 14, 14 and 28)

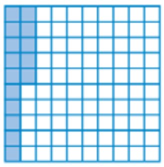
5 In the calculation $\frac{4}{7} \times \frac{14}{28}$, is it possible to cancel 4 and 14? (No, you cannot cancel numerator to numerator, or denominator to denominator.)

Answers

3I Calculating percentages

ANSWERS

3I Start thinking!

1 a


b 15 c 15

2 a Draw two 10 × 10 grids and shade 15 squares in each grid.
b 30

3 a Draw six 10 × 10 grids and shade 15 squares in each grid.
b 90

4 90; same result 5 $90 6 $510

Exercise 3I Calculating percentages

1 a 195 b 12 c 104 d 40
e 7.5 f 18 g 39 h 12
i 100 j 480 k 200 l 132

2 a $4.80 b $76.50 c $55
d $198.90 e $700 f $168.30
g $1485 h $3.98 i $8.95

3 a 4.5 b 3.85 c 90.96 d 30.6
e 27.82 f 288.75 g 50.88 h 25.2
i 80.94 j 21.12 k 12.375 l 7.956

4 a $68\frac{3}{5}$ b $67\frac{1}{5}$ c $57\frac{3}{5}$ d $14\frac{2}{5}$
e $67\frac{1}{5}$ f $807\frac{1}{2}$ g $11\frac{3}{5}$ h $43\frac{3}{4}$
i $141\frac{3}{4}$ j $83\frac{3}{10}$ k $294\frac{2}{5}$ l $101\frac{1}{4}$

5 a 5980 b 520

6 a 44 b 73.5 c 45.6
d 400 e 10.68 f 450.8
g 148.5 h 308 i 1134

7 a $659.70 b $1539.30 c no

8 a $114 b $456

9 a Each percentage is greater than 100%.
b Final amount is greater than original amount.

10 a Ripper Reptiles: $108, Enclosed: $162.50
b Ripper Reptiles: $492, Enclosed: $487.50
c Enclosed

11 a water: 29.25 kg, protein: 8.1 kg, fat: 4.5 kg, minerals and vitamins: 2.7 kg, carbohydrates: 0.45 kg
b water: 40.95 kg, protein: 11.34 kg, fat: 6.3 kg, minerals and vitamins: 3.78 kg, carbohydrates: 0.63 kg
c water: 63.7 kg, protein: 17.64 kg, fat: 9.8 kg, minerals and vitamins: 5.88 kg, carbohydrates: 0.98 kg

12 a $3.75 b $1.30 c $7.35
d $9.20 e $1.75 f $1.20

13 a i $119.90 ii $1318.90
b i $20.90 ii $229.90
c i $49.90 ii $548.90
d i $0.30 ii $3.29
e i $3.00 ii $32.99
f i $5.40 ii $59.40

14 Move decimal point one place to left of original price value to obtain GST amount.

15 a i $69.60 ii $510.40
b i $1125 ii $3375
c i 2.11 km ii 40.09 km
d i 0.86 L ii 7.74 L
e i 22.2 kg ii 125.8 kg
f i 35.2 kg ii 140.8 kg

16 $55.99

17 Prices may be increased by any amount. Reduction is limited because 100% reduction means $0.

18 a 15 cents b $2.65
c $1\frac{3}{50}$ d 106%

19 a marked price should be $6.24 (rounded incorrectly)
b marked price should be $2.58 (rounded to the next 5 cents)
c marked price should be $3.61 (rounded incorrectly)
d marked price should be $4.63 (no increase calculated)
e marked price should be $1 (calculated 25% of 80 dollars, not 80 cents)

20 a halve b triple c quadruple

21 1%: find 10% of amount and then 10% of this result (or move decimal point two places to left); 50%: halve original amount

22 Answers are the same.

23 Add 10% to 100%, then calculate 110% of pre-tax amount to determine final amount.

24 Some possible answers are provided.
a First calculate 10% of amount and then 5% of amount. Add two results together.
b First calculate 10% of amount and then 1% of amount. Add 10% result and three lots of 1% result.
c Halve amount to calculate 50% and then add 5% of original amount.
d Subtract two lots of 10% of amount.

Reflect

Possible answer: Percentages are useful to compare amounts that are not out of the same total.

Resources

SupportSheet

SS 3I-1 Finding the fraction of a whole number

Focus: To review the process of multiplying a fraction by a whole number

In a series of questions, students are guided through the steps involved in multiplying a fraction by a whole number.

WorkSheet

WS 3I-2 Calculating the percentage of a quantity

Focus: To find the percentage of a quantity

Students are guided through a series of questions to consolidate the process of finding the percentage of a quantity. Additional practice questions similar to those in Exercise 3I are also provided.

Investigations

INV 3I-3 Shopping with 10%

Focus: To estimate common percentage quantities using a 10% rule

Students investigate the method of finding 10% of a number as dividing the number by ten. They then explore other percentages that can be determined from 10% and apply them to estimate and calculate common percentages encountered when shopping.

INV 3I-4 Growing money

Focus: To use a spreadsheet to explore money growing at a given percentage

Resources: Computer with *Microsoft Excel* or another spreadsheet application

Students use a computer-based program to calculate the amount of interest earned over a set period of time. They are guided to construct a spreadsheet using *Microsoft Excel*. As an extension, they change the values of the variables to determine changes in the end results.

BLM

100 grid

Interactives

3I eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 176–9

Additional teaching strategies

Multiple-choice

1 Answer: C. The fourth (next) decimal place is greater than 5, so round up.
13.054 872 6 ≈ 13.055
A: mistakenly and incorrectly rounded to one decimal place or three significant figures.
B: incorrectly rounded.
D: mistakenly rounded to four decimal places.

2 Answer: B. 183.506 is not less than 183.099

3 Answer: B. The result is 3.3989, which is less than the other results.
A: result is 3.586, which is larger than 3.3989.
C: result is 5.6556, which is larger than 3.3989.
D: result is 3.917, which is larger than 3.3989.

4 Answer: B. 604 × 21 = 12 684. The answer will have 3 decimal places, so
6.04 × 2.1 = 12.684

5 Answer: A. 8.21 ÷ 4 = 2.0525

6 Answer: C. One apple costs \$9.85 ÷ 10 = \$0.985.
Six apples cost \$0.985 × 6 = \$5.91
A: \$9.85 has been divided by 10, but not multiplied by 6.
B: \$9.85 was divided by 6.
D. \$9.85 was divided by 6 and multiplied by 10.

7 Answer: A. 23 ÷ 0.4 = 57.5, which is more than the other results.
B: result is 40, which is smaller than 57.5.
C: result is 55, which is smaller than 57.5.
D: result is 50, which is smaller than 57.5.

8 Answer: A. 1 ÷ 5 = 1.0 ÷ 5 = 0.2
B: denominator has been taken to make 0.5
C: combined numerator and denominator to make 1.5.
D: combined numerator and denominator to make 0.15.

9 Answer: B. $0.26 = \frac{26}{100} = \frac{13}{50}$
A: chose a decimal answer and not a fraction.
C: correct fraction but has not been simplified.

D: equivalent value but 26% is not a fraction.

10 Answer: D. $\frac{5}{8}$ of the shape is shaded and $\frac{5}{8} \times \frac{100\%}{1} = 62.5\%$

A: correct fraction, but a percentage was asked for.

B: chose 5% because 5 parts were shaded; however, 5 out of 8 parts are shaded and percentages are out of 100 parts.

C: incorrectly calculated.

11 Answer: D. $48\% = \frac{48}{100} = \frac{12}{25}$

A: correct fraction but has not been simplified.

C: correct fraction but has not been fully simplified.

D: chose an option that is not a fraction.

12 Answer: C. $\frac{9}{20} \times \frac{100\%}{1} = 9 \times 5\% = 45\%$

A: chose the numerator to make 9%.

B: chose the given fraction with a percentage sign.

D: multiplied the numerator by the denominator.

13 Answer: D. $7 \div 8 = 7.000 \div 8 = 0.875$

$0.875 \times 100\% = 87.5\%$

A: combined numerator and denominator to make 78%.

B: combined numerator and denominator to make 7.8.

C: incorrectly chose 0.75 which is the decimal for $\frac{3}{4}$, not $\frac{7}{8}$.

14 Answer: A. $64.5\% = 64.5 \div 100 = 0.645$

B: same number is given, without dividing by 100.

C: divided 64.5 by 1000, or the decimal point has been moved an extra place.

D: multiplied 64.5 by 1000, or the decimal point has been moved an extra place in the wrong direction.

15 Answer: B. $0.1265 \times 100\% = 12.65\%$

A: same number is given, without multiplying by 100.

C: multiplied 0.1265 by 100 twice.

D: digits of 0.1265 are taken as a whole number and multiplied by 100.

16 Answer: D

$$\frac{25}{100} \times \frac{220}{1} = \frac{25}{5} \times \frac{11}{1} = 55$$

A: multiplied 25 by 220 to obtain 5500.

B: added 25 to 220 to obtain 245.

C: subtracted 25 from 220 to obtain 195.

Short answer

1 **a** $\frac{7}{100}$

b $\frac{7}{100}$

c $\frac{7}{1\,000\,000}$

d $\frac{7}{10}$

2 **a** **i** 5

ii 1589.23

b **i** 5

ii 12.71

c **i** 4

ii 0.93

d **i** 7

ii 152.88

3 18.0958, 12.8905, 12.5089, 12.0985, 10.9852

4 Set out with the decimal points under each other, adding zeros at the end if needed.

a 120.46

b 9.532

c 1.693

d 4.2885

5 **a** $12.63 \times 100 = 1263$

b $15.3298 \times 10 = 153.298$

c $4.2 \times 50 = 4.2 \times 5 \times 10 = 21 \times 10 = 210$

d $3.047 \times 200 = 3.047 \times 2 \times 100 = 6.094 \times 100 = 609.4$

6 **a** $7 \times 3 = 21$

The answer must have 2 decimal places.
$0.7 \times 0.3 = 0.21$

b $9 \times 4 = 36$
The answer must have 3 decimal places.
$0.9 \times 0.04 = 0.036$

c $11 \times 8 = 88$
The answer must have 3 decimal places.
$1.1 \times 0.08 = 0.088$

d $8 \times 4 = 32$
The answer must have 4 decimal places.
$0.08 \times 0.04 = 0.0032$

7 **a** $258 \times 1 = 258$
The answer will have 2 decimal places.
$25.8 \times 0.1 = 2.58$
Alternatively, treat multiplying by 0.1 as dividing by 10 and move the decimal point one place left.

b $468 \times 1 = 468$
The answer will have 3 decimal places.
$46.8 \times 0.01 = 0.468$
Alternatively, treat multiplying by 0.01 as dividing by 100 and move the decimal point two places left.

c $65 \times 72 = 4680$
Answer will have 2 decimal places.
46.80 or 46.8

d $423 \times 19 = 8037$
Answer will have 3 decimal places.
$4.23 \times 1.9 = 8.037$

8 Digits move to the right or decimal point moves to the left, to match the number of zeros.

a $78.94 \div 10 = 7.894$

b $1256.359 \div 100 = 12.563\ 59$

c $124.56 \div 10 = 12.456$

d $189\ 765.6 \div 1000 = 189.7656$

9 **a** 9.327

b 15.76

c 29.726

d 40.931 25

10 **a** \$7.80 ÷ 5 = \$1.56
\$12.60 ÷ 7 = \$1.80

b The 500 gram block is the better buy as it is cheaper per 100 grams.

11 **a** $6.2 \div 0.5 = 62 \div 5 = 12.4$

b $0.75 \div 0.2 = 7.5 \div 2 = 3.75$

c $8.62 \div 0.4 = 86.2 \div 4 = 21.55$

d $0.0081 \div 0.05 = 0.81 \div 5 = 0.162$

12 **a** $\frac{75}{100} = \frac{3}{4}$

b $\frac{18}{100} = \frac{9}{50}$

c $\frac{489}{1000}$

d $\frac{3}{1000}$

13 **a** 0.87

b 0.117

c 0.3

d 0.07

14 **a** Either $16 \div 25 = 0.64$ or $\frac{16}{25} = \frac{64}{100} = 0.64$

b Either $19 \div 50 = 0.38$ or $\frac{19}{50} = \frac{38}{100} = 0.38$

c Either $15 \div 20 = 0.75$ or $\frac{15}{20} = \frac{75}{100} = 0.75$

d Either $3 \div 8 = 0.375$ or $\frac{3}{8} = \frac{3\times125}{8\times125} = \frac{375}{1000} = 0.375$

15 **a** **i** $\frac{49}{100}$

ii $\frac{49}{100}$

b **i** $\frac{22}{100}$

ii $\frac{11}{50}$

c **i** $\frac{75}{100}$

ii $\frac{3}{4}$

c **i** $\frac{5}{100}$

ii $\frac{1}{20}$

16 100% – 23% – 31% = 46% of the pie remains.

17 **a** $0.56 \times 100 = 56\%$

b $0.08 \times 100 = 8\%$

c $1.79 \times 100 = 179\%$

d $0.2748 \times 100 = 27.48\%$

18 **a** $78 \div 100 = 0.78$

b $46.5 \div 100 = 0.465$

c $9 \div 100 = 0.09$

d $10.679 \div 100 = 0.106\ 79$

19 **a** $\frac{3}{5}\times100\% = 60\%$ or $\frac{3}{5} = \frac{60}{100} = 60\%$

b $\frac{7}{8} \times 100\% = 87.5\%$ or $\frac{7}{8} = \frac{875}{1000} = \frac{87.5}{100} = 87.5\%$

c $1 = \frac{100}{100} = 100\%$

d $2\frac{3}{4} = \frac{11}{4} \times 100\% = 275\%$ or $2\frac{3}{4} = \frac{11}{4} = \frac{275}{100} = 275\%$

20 **a** $450 \div 10 = 45$ or $\frac{10}{100} \times \frac{450}{1} = \frac{1}{1} \times \frac{45}{1} = 45$

b $\frac{30}{100} \times \frac{500}{1} = \frac{30}{1} \times \frac{5}{1} = 150$

c $\frac{45}{100} \times \frac{300}{1} = \frac{45}{1} \times \frac{3}{1} = 135$

d $\frac{85}{100} \times \frac{650}{1} = \frac{85}{2} \times \frac{13}{1} = 552.5$

e $\frac{6}{100} \times \frac{1499}{1} = \frac{8994}{100} = 89.94$

f $\frac{41}{100} \times \frac{347}{1} = \frac{14\ 227}{100} = 142.27$

21 **a** $\frac{272}{100} \times \frac{550}{1} = \frac{136}{1} \times \frac{11}{1} = \frac{1496}{1} = 1496$ km

b 0.3475×950 g $= 330.125$ g

c $0.058 \times \$120 = \$6.96$

d 0.162×450 L $= 72.9$ L

NAPLAN-style practice

Multiple-choice options have been listed as A, B, C and D for ease of reference.

1 Answer: D. Place value is hundredths.
Refer to *3A Understanding decimals*.

2 Third decimal place is 6 so 0 in second place is rounded up to 1.
Answer is 164.51.
Refer to *3A Understanding decimals*.

3 Answer: A.

B: incorrectly chose a list with jumbled order.
C: chose list that is almost ascending except for the last number.
D: chose list that is in descending order.
Refer to *3A Understanding decimals*.

4 Answer is 6.528.
Refer to *3B Adding and subtracting decimals*.

5 Answer: D.
A: ignored placeholder zeros.
B: zeros should be added after the 3 and the 6.
C: mistakenly included a zero in the wrong place in the first number.
Refer to *3B Adding and subtracting decimals*.

6 \$16.55 + \$13.20 + \$12.60 = \$42.35. Money needed = \$49 – \$42.35 = \$6.65
Refer to *3B Adding and subtracting decimals*.

7 Answer: A.
126 × 55 = 6930
There are four decimal places in the answer.
1.26 × 0.55 = 0.6930 = 0.693
B: ignored the 0 at the end of 6930 when calculating 4 decimal places.
C: digits of 55 were written at the end of 1.26 to obtain 1.2655.
D: mistakenly calculated 1.26 + 0.55 to obtain 1.81.
Refer to *3C Multiplying decimals*.

8 \$5.99 × 3 = \$17.97
Refer to *3C Multiplying decimals*.

9 Answer: D. 0.548 ÷ 0.2 = 5.48 ÷ 2 = 2.74
A: mistakenly calculated 0.548 × 0.2 to obtain 0.1096.
B: mistakenly calculated 0.548 ÷ 2 to obtain 0.274.
C: mistakenly calculated 0.548 × 2 to obtain 1.096.
Refer to *3D Dividing decimals by a whole number*.

10 Cost of one DVD = \$47.92 ÷ 8 = \$5.99
Cost of ten DVDs = \$5.99 × 10 = \$59.90
Refer to *3D Dividing decimals by a whole number*.

11 Answer: D. Compare for 100 teabags.
A: \$1.77 × 10 = \$17.70
B: \$2.98 × 4 = \$11.92
C: \$4.08 × 2 = \$8.16
D: \$7.48
Refer to *3C Multiplying decimals*.

12 Answer: C. Compare for 1 L.
A: $1.98 ÷ 0.6 = $19.80 ÷ 6 = $3.30
B: $2.98
C: $5.92 ÷ 2 = $2.96
D: $8.90 ÷ 3 ≈ $2.97 ($2.9666…)
Refer to *3E Dividing decimals by a decimal*.

13 Either 11 ÷ 20 = 0.55 or $\frac{11}{20} = \frac{55}{100} = 0.55$
Refer to *3F Converting between fractions and decimals*.

14 Answer: D. $0.875 = \frac{875}{1000} = \frac{7}{8}$
A: incorrect denominator used, should be 1000.
B: incorrect numerator and fraction not simplified.
C: correct fraction but not simplified to the simplest form.
Refer to *3F Converting between fractions and decimals*.

15 Three pairs for $9.99 is $3.33 per pair, cheaper than $3.99.
$35.20 ÷ $9.99 = 3, with $5.23 left over.
3 lots of 3 pairs for $9.99 can be bought.
One more pair of tights for $3.99 can be bought with the $5.23.
Total pairs of tights = 3 × 3 + 1 = 10
Refer to *3C Multiplying decimals* and *3E Dividing decimals by a decimal*.

16 Answer: A.
Total cost of tights = $29.97 + $3.99 = $33.96
Change = $35.20 – $33.96 = $1.24
B: chose the amount left after purchasing 9 pairs.
C: chose the cost of 9 pairs of tights.
D: chose the cost of 10 pairs of tights.
Refer to *3B Adding and subtracting decimals*.

17 Answer: C.
Total cost for 10 pairs of tights = $33.96
Average cost = $33.96 ÷ 10 = $3.396 ≈ $3.40
Refer to *3D Dividing decimals by a whole number*.

18 Answer: D. The amount unshaded is 4 triangles out of 10 = 40%
A: amount unshaded is 6 squares out of 10 = 60%. 40% is shaded.
B: amount unshaded is 1 part in 4 = 25%.
C: amount unshaded is 25 squares out of 100 = 25%.
Refer to *3G Understanding percentages*.

19 100% – 75% – 12.5% = 12.5%
12.5% of the necklace is made of copper.
Refer to *3G Understanding percentages*.

20 30% are red so 100% – 30% = 70% are not red.
Refer to *3G Understanding percentages*.

21 $\frac{52}{80}\times\frac{100}{1}=\frac{13}{1}\times\frac{5}{1}=65\%$
Refer to *3H Converting between fractions, decimals and percentages*.

22 59.23% = 59.23 ÷ 100 = 0.5923
Refer to *3H Converting between fractions, decimals and percentages*.

23 Answer: A. Convert all to decimals, or convert all to percentages.
A: 0.8, 0.76, 0.45, 0.125, 0.12, 0.09 or 80%, 76%, 45%, 12.5%, 12%, 9%
B: 0.76, 0.8, 0.45, 0.12, 0.125, 0.09 or 76%, 80%, 45%, 12%, 12.5%, 9%
C: 0.09, 0.12, 0.125, 0.45, 0.76, 0.8 or 9%, 12%, 12.5%, 45%, 76%, 80%
This is ascending (smallest to largest), not descending order.
D: 0.45, 0.76, 0.8, 0.12, 0.125, 0.09 or 45%, 76%, 80%, 12%, 12.5%, 9%
Refer to *3H Converting between fractions, decimals and percentages*.

24 $\mathbf{\frac{35}{100}\times\frac{\$620}{1}=\frac{7}{1}\times\frac{\$31}{1}=\$217}$
Refer to *3I Calculating percentages*.

25 Answer: C. 25% of \$49.99 = $\frac{25}{100}\times\frac{\$49.99}{1}=\frac{1}{4}\times\frac{\$49.99}{1}\approx\$12.50$

\$49.99 – \$12.50 = \$37.49
Alternatively, 75% of \$49.99 = 0.75 × \$49.99 ≈ \$37.49.
A: calculated the amount of discount to obtain \$12.50.
B: mistakenly discounted by 50%.
D: mistakenly calculated 25 × \$49.99 = \$1249.75.
Refer to *3I Calculating percentages*.

26 Answer: B.
Cost before discount = 25 × \$2.10 = \$52.50
Discount = $\frac{30}{100}\times\frac{\$52.50}{1}=\frac{3}{10}\times\frac{\$52.50}{1}=\frac{3}{1}\times\frac{\$5.25}{1}=\$15.75$
Cost after discount = \$52.50 – 15.75 = \$36.75
Alternatively, cost after discount = 70% of \$52.50 = 0.7 × \$52.50 = \$36.75.
A: calculated the full cost of the cupcakes.
C: applied the wrong discount.
D: calculated the discount to obtain \$15.75.

Refer to *3I Calculating percentages*.

27 Answer: C.
Cost before discount = 3 × \$14.99 = \$44.97
Discount = $\frac{20}{100}\times\frac{\$44.97}{1}=\frac{1}{5}\times\frac{\$44.97}{1}=\$44.97\div5\approx\$8.994$
Cost after discount = \$44.97 – \$8.994 = \$35.976
Alternatively, cost after discount = 80% of \$44.97 = 0.8 × \$44.97 = \$35.976.
Cost per cupcake = \$35.976 ÷ 30 = \$1.1992 ≈ \$1.20
A: chose the discounted cost for 30 cupcakes (\$35.98).
B: mistakenly calculated the cost per cupcake using the full cost (\$44.97 ÷ 30 = \$1.499 ≈ \$1.50).
D: mistakenly used a discount of 30% rather than 20%.
Refer to *3I Calculating percentages*.

Analysis

a \$10.35 + \$8.25 + \$9.90 + \$10.80 = \$39.30

b Number of roses
= \$39.30 ÷ \$2.99 ≈ \$39.30 ÷ \$3 = 13 with \$0.30 remainder.
They can buy 13 roses.
Cost = \$2.99 × 13 = \$38.87
Change = \$39.30 – \$38.87 = \$0.43 or 43 cents

c \$2.99 × 12 = \$35.88

d Discount = $\frac{25}{100}\times\frac{\$35.88}{1}=\frac{1}{1}\times\frac{\$8.97}{1}=\$8.97$
Discounted cost = \$35.88 – 8.97 = \$26.91
Alternatively, discounted cost = 75% of \$35.88 = 0.75 × \$35.88 = \$26.91

e 37.5% = 37.5 ÷ 100 = 0.375

f \$2.99 × 18 = \$53.82

g 37.5% of \$53.82 = 0.375 × \$53.82 = \$20.1825

h \$20.1825 has 4 decimal places.
\$20.1825 ≈ \$20.18

i Discounted cost = \$53.82 – \$20.18 = \$33.64
Change = \$39.30 – \$33.64 = \$5.66

j Each person receives \$5.66 ÷ 4 = \$1.415 ≈ \$1.42

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 3 Chapter test A

Chapter 3 Chapter test B

Summative tests

The three tests, A, B and C, for each chapter accommodate different student ability levels, with one section of overlap in each (the ‘Proficient’ part). These tests have been carefully mapped against AUSVELS and the Australian Curriculum in order to provide an accurate assessment of each student's level of achievement. When a student’s marks are entered into the provided spreadsheet calculator, a letter grade is calculated based upon a weighted average of percentages according to the type of test completed.

Chapter 3 Summative test A: Modified

Aimed at the lower level of student ability.

The top mark a student can achieve in a modified test is a **C**.

Chapter 3 Summative test B: Core

Aimed at the middle level of student ability.

The top mark a student can achieve in a core test is a **B**.

Chapter 3 Summative test C: Extension

Aimed at the upper level of student ability.

The top mark a student can achieve in an extension test is an **A**.

Test answers

Chapter 3 Chapter test answers

Chapter 3 Summative test answers

Summative test spreadsheet calculator

Connect

Teaching support for pages 180–1

Teaching strategies

And the winner is …

Focus: To use fractions, decimals and percentages in a real-life context

- Students explore the scoring system of Olympic gymnastics. Gymnasts are scored out of ten, with scores dependent on the degree of difficulty and the form and execution of their performance. Six judges give each performer a score out of ten.
- Students are guided through an investigation where the performance of two teams is compared.
- Students could discuss the task requirements in small groups to:
  - identify the degree of difficulty and execution scores for each gymnast from the table
  - understand how to calculate the average of a set of scores
  - understand how to work out a gymnast's final score.
- As a further task, students can investigate the scoring system for diving. It has some similarities to gymnastics. Three judges score each component of the dive and give a score out of 10. The highest and lowest scores are discarded, as with gymnastics. The remainder of the scores are added and multiplied by the degree of difficulty. This final answer is then multiplied by 0.6.
- Direct students to complete the appropriate section of the **Connect worksheet** (see Resources). This section provides scaffolding to the task to guide students through the problem-solving process. They can use this as a foundation to presenting their findings in a report.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided to the **Connect worksheet**.
- An assessment rubric is available (see Resources)
- Two additional Connect investigations are provided **CI 3-1 Hot dog stand** and **CI 3-2 Nutrition label facts.**

Additional Connect investigations

CI 3-1 Hot dog stand

Focus: To connect the key ideas of decimals and percentages to organise a hot dog stand

Resources: calculator (optional)

Students apply their knowledge of decimals and percentages to answer a series of questions about the most economical way to purchase the items and requirements for a school hot dog stand. They consider the selling price of the hot dogs given certain constraints and conditions to achieve the greatest possible profit.

Students consider:

- a range of hot dog packages to find the best buy
- the number of hot dog packages that will be required for the number of customers expected
- the best buy for a range of hot dog roll options
- the cost of the condiments required
- the total cost of producing 400 hot dogs
- the best price for selling the hot dogs
- the amount made on the day
- the profit or loss made in various scenarios.

Students also explore a different number of sales over the course of a week.

As an extension, students set up a business plan to run a hot dog stall at school.

An assessment rubric is also available (see Resources).

CI 3-2 Nutrition label facts

Focus: To connect the key ideas of decimals and percentages to understand and prepare a food nutrition label

Students are given information about some of the nutrition components of food. They use their decimal and percentage skills to interpret and analyse the information from a nutrition food label to make healthy food choices.

Students consider a well-known breakfast cereal in terms of:

- fat content
- energy content
- carbohydrate content
- fibre content
- protein content.

Students analyse the ingredients of this cereal and calculate the recommended daily intake of different components of the cereal.

As an extension, students examine the labels of different foods and compare them, putting them in order from healthiest to least healthy.

An assessment rubric is also available (see Resources).

Resources

Connect worksheet

CW 3 And the winner is …

Additional Connect investigations

CI 3-1 Hot dog stand

CI 3-2 Nutrition label facts

Assessment rubrics

And the winner is …

Hot dog stand

Nutrition label facts

oxforddigital

OXFORD UNIVERSITY PRESS AUSTRALIA & NEW ZEALAND

Australian Curriculum: Mathematics Year 8

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the units.

Number and Algebra

Number and place value	**Elaborations**	**MyMaths 8**
Use index notation with numbers to establish the index laws with positive integral indices and the zero index (ACMNA182)	• evaluating numbers expressed as powers of positive integers	1H Powers and roots 1I Index laws 3G Powers of directed numbers
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)	• using patterns to assist in finding rules for the multiplication and division of integers • using the number line to develop strategies for adding and subtracting rational numbers	1B Order of operations 1D Operations with fractions 1F Operations with decimals 3B Adding integers 3C Subtracting integers 3D Simplifying addition and subtraction of integers 3E Multiplying and dividing integers 3F Operations with directed numbers
Real numbers	**Elaborations**	**MyMaths 8**
Investigate terminating and recurring decimals (ACMNA184)	• recognising terminating, recurring and non-terminating decimals and choosing their appropriate representations	1G Terminating, non-terminating and recurring decimals
Investigate the concept of irrational numbers, including π (ACMNA186)	• understanding that the real number system includes irrational numbers	1G Terminating, non-terminating and recurring decimals
Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187)	• using percentages to solve problems, including those involving mark-ups, discounts and GST • using percentages to calculate population increases and decreases	2C Percentage calculations
Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188)	• understanding that rate and ratio problems can be solved using fractions or percentages and choosing the most efficient form to solve a particular problem • calculating population growth rates in Australia and Asia and explaining their difference	2E Understanding ratios 2F Working with ratios 2G Dividing a quantity in a given ratio 2H Understanding rates

Money and financial mathematics	**Elaborations**	**MyMaths 8**
Solve problems involving profit and loss, with and without digital technologies (ACMNA189)	• expressing profit and loss as a percentage of cost or selling price, comparing the difference • investigating the methods used in retail stores to express discounts	2D Financial calculations
Patterns and algebra	**Elaborations**	**MyMaths 8**
Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)	• applying the distributive law to the expansion of algebraic expressions using strategies such as the area model	4F Working with brackets
Factorise algebraic expressions by identifying numerical factors (ACMNA191)	• recognising the relationship between factorising and expanding • identifying the greatest common divisor (highest common factor) of numeric and algebraic expressions and using a range of strategies to factorise algebraic expressions	4G Factorising expressions
Simplify algebraic expressions involving the four operations (ACMNA192)	• understanding that the laws used with numbers can also be used with algebra	4C Simplifying expressions containing like terms 4D Multiplying algebraic terms 4E Dividing algebraic terms
Linear and non-linear relationships	**Elaborations**	**MyMaths 8**
Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)	• completing a table of values, plotting the resulting points and determining whether the relationship is linear • finding the rule for a linear relationship	3H The Cartesian plane 5A Understanding equations 5H Plotting graphs of linear relationships
Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution. (ACMNA194)	• solving real life problems by using variables to represent unknowns	5B Solving equations using tables 5D Solving equations using backtracking 5E The balance model and equivalent equations 5F Solving equations by performing the same operation on both sides 5G Solving equations with the unknown on both sides 5I Solving linear equations using graphs

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 8**
Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195)	• choosing units for area including mm^2, cm^2, m^2, hectares, km^2, and units for volume including mm^3, cm^3, m^3 • recognising that the conversion factors for area units are the squares of those for the corresponding linear units • recognising that the conversion factors for volume units are the cubes of those for the corresponding linear units	8C Area of rectangles and triangles 8D Area of other quadrilaterals 8E Area of circles 8F Surface area 8G Volume of prisms 8H Converting units of area and volume
Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196)	• establishing and using formulas for area such as trapeziums, rhombuses and kites	8A Length and perimeter 8D Area of other quadrilaterals 8F Surface area
Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)	• investigating the circumference and area of circles with materials or by measuring, to establish an understanding of formulas • investigating the area of circles using a square grid or by rearranging a circle divided into sectors	8B Circumference of circles 8E Area of circles 8F Surface area
Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198)	• investigating the relationship between volumes of rectangular and triangular prisms	8G Volume of prisms
Solve problems involving duration, including using 12- and 24- hour time within a single time zone (ACMMG199)	• identifying regions in Australia and countries in Asia that are in the same time zone	6F Angles and time zones 6G Working with time zones
Geometric reasoning	**Elaborations**	**MyMaths 8**
Define congruence of plane shapes using transformations (ACMMG200)	• understanding the properties that determine congruence of triangles and recognising which transformations create congruent figures • establishing that two figures are congruent if one shape lies exactly on top of the other after one or more transformations (translation, reflection, rotation), and recognising that the matching sides and the matching angles are equal	7F Translations, rotations and reflections 7G Understanding congruence 7H Using congruence

Develop the conditions for congruence of triangles (ACMMG201)	• investigating the minimal conditions needed for the unique construction of triangles, leading to the establishment of the conditions for congruence (SSS, SAS, ASA and RHS) • solving problems using the properties of congruent figures • constructing triangles using the conditions for congruence	7G Understanding congruence 7H Using congruence
Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202)	• establishing the properties of squares, rectangles, parallelograms, rhombuses, trapeziums and kites • identifying properties related to side lengths, parallel sides, angles, diagonals and symmetry	7A Triangle properties 7B Quadrilateral properties 7I Dilations

Statistics and Probability

Chance	**Elaborations**	**MyMaths 8**
Identify complementary events and use the sum of probabilities to solve problems (ACMSP204)	• identifying the complement of familiar events • understanding that probabilities range between 0 to 1 and that calculating the probability of an event allows the probability of its complement to be found	10B Theoretical probability 10C Tree diagrams
Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’ (ACMSP205)	• posing ‘and’, ‘or’ and ‘not’ probability questions about objects or people	10B Theoretical probability 10F Experimental probability 10G Simulations and long term trends
Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292)	• using Venn diagrams and two-way tables to calculate probabilities for events, satisfying ‘and’, ‘or’ and ‘not’ conditions • understanding that representing data in Venn diagrams or two-way tables facilitates the calculation of probabilities • collecting data to answer the questions using Venn diagrams or two-way tables	10D Two-way tables 10E Venn diagrams 10F Experimental probability 10G Simulations and long term trends
Data representation and interpretation	**Elaborations**	**MyMaths 8**
Investigate techniques for collecting data, including census, sampling and observation (ACMSP284)	• identifying situations where data can be collected by census and those where a sample is appropriate	9A Sampling data
Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206)	• investigating the uses of random sampling to collect data	9A Sampling data 9B Collecting data 9C Presenting data in graphs

		9D Stem-and-leaf plots and dot plots 9E Presenting grouped data
Explore the variation of means and proportions in random samples drawn from the same population (ACMSP293)	• using sample properties to predict characteristics of the population	9F Summary statistics 9G Analysing data
Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207)	• using displays of data to explore and investigate effects	9F Summary statistics 9G Analysing data

Year 8 achievement standard

By the end of Year 8, students solve everyday problems involving rates, ratios and percentages. They describe index laws and apply them to whole numbers. They describe rational and irrational numbers. Students solve problems involving profit and loss. They make connections between expanding and factorising algebraic expressions. Students solve problems relating to the volume of prisms. They make sense of time duration in real applications. They identify conditions for the congruence of triangles and deduce the properties of quadrilaterals. Students model authentic situations with two-way tables and Venn diagrams. They choose appropriate language to describe events and experiments. They explain issues related to the collection of data and the effect of outliers on means and medians in that data.

Students use efficient mental and written strategies to carry out the four operations with integers. They simplify a variety of algebraic expressions. They solve linear equations and graph linear relationships on the Cartesian plane. Students convert between units of measurement for area and volume. They perform calculations to determine perimeter and area of parallelograms, rhombuses and kites. They name the features of circles and calculate the areas and circumferences of circles. Students determine the probabilities of complementary events and calculate the sum of probabilities.

Number and Algebra

3 Positive and negative numbers

3 Positive and negative numbers

Teaching support for pages 120–1

Syllabus links

Content descriptions and elaborations

Number and place value

ACMNA183: Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies

- Using patterns to assist in finding rules for the multiplication and division of integers
- Using the number line to develop strategies for adding and subtracting rational numbers

Linear and non-linear relationships

ACMNA193: Plot linear relationships on the Cartesian plane, with and without the use of digital technologies

- Completing a table of values, plotting the resulting points and determining whether the relationship is linear

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Before the introductory lesson for this section, write each of the following statements onto cards. Photographs or diagrams can be added to each card to assist visual learners.
  – A submarine can dive to 250 metres below sea level.
  – The wreck of the *Titanic* lies below sea level at approximately 3800 metres.
  – Eagles can fly up to approximately 3000 metres above sea level.

  - Commercial aircraft cruise at approximately 9000 metres above sea level.
  - The highest mountain in Australia is Mt Kosciusko at approximately 2250 metres above sea level.
  - A 4.8 m great white shark nicknamed ‘Shack’ has set the deepest dive ever recorded by a great white shark at 1200 metres.
- Direct students to examine the opening photo for this chapter on pages 120 and 121 of their Student Book.
- Brainstorm as a class what it means to be at 1500 metres below sea level.
- Discuss with the class whether a vertical or horizontal number line would be most appropriate to represent a scale which could be used to show measures both above and below sea level.
- On the board, draw a vertical number line.
- Discuss a possible scale with the class and add this to the diagram. (A scale with a major interval of 200 may be useful.)
- Ask students where sea level would be represented and discuss how this is a reference point.
- Read the pre-prepared cards to the class, one at a time, and ask students to volunteer to come forward and place each card in an appropriate position on the number line. These cards can be attached to the board using Blu-Tack® or a magnet.

  Note: It may be necessary to adjust the scale; cards already placed may need to be adjusted.
- As a class, discuss the placement of each card.
- If appropriate, the number line can remain on the board and each student can research information for an extra card. These cards can all be added to the number line during future lessons. Encourage students to use diagrams or photographs on their cards to make the number line visually appealing.

Essential question

Other possible reference points could be ground level, zero on a number line or the origin of a Cartesian plane.

Are you ready?

Prerequisite knowledge and skills can be tested by completing **Are you ready?** This will give you an indication of the differentiated pathway each student can follow.

Students need to be able to:

- solve problems involving all four operations with whole numbers
- solve problems involving all four operations with fractions and decimals
- use fractions and decimals, and their equivalents
- construct a simple number line
- plot points on a number line
- move left or right (or up and down) on a number line
- calculate the average of a set of values
- convert between index form and expanded notation
- calculate the value of the basic numeral for expressions in index form
- assign and plot ordered pairs for given points in the first quadrant of the Cartesian plane.

At the beginning of each topic there is a suggested differentiated pathway which allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to select the best pathway.

Support Strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

ANSWERS

CHAPTER 3 POSITIVE AND NEGATIVE NUMBERS

3 Are you ready?

1 A: +3; B: −2; C: 0; D: −4; E: +1.5

2	a C	b −3	c +3				
3	a C	b A		4	a 815	b 22	
5	a B	b $\frac{1}{20}$		6	a D	b $\frac{1}{4}$	
7	a 12.1	b 2.9		8	a 2.88	b 21	
9	7.2						
10	a B	b A	c C				
11	a D	b B	c C				

Resources

assess: assessments

Each topic of the *MyMaths 8* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students' confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

3A Understanding negative numbers

Teaching support for pages 122–7

Teaching strategies

Learning focus

To use understanding of directed numbers and a number line to compare the size of directed numbers.

To apply understanding of directed numbers and the visualising of the location of them on a number line in order to compare them.

Start thinking!

The task guides students to:

- review the notation, reference on a number line, and compare the sizes of positive and negative numbers
- consider temperature values and indicate temperature readings on a vertical number line. They review notation; that is, that positive numbers are indicated with a '+', or with no sign; and negative numbers are indicated with a '–'.
- consider the reference point for directed numbers on the number line
- discover that directed numbers can include whole numbers (integers), fractions and decimals
- consider how to compare directed numbers using a number line, and to rank the temperatures in order of their size.

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 3, 6a–f, 7a–d, 8a, 10, 13a, b, 17	1, 2d–f, 3d–f, 4–6, 7a–g, 8a,b, 9a,b, 11–15, 17	4, 5, 6d–l, 7d-i, 8, 9, 11, 14–16, 18, 19
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q1

Focus: To review directed numbers and to identify values on a number line

- Direct students to complete **SS 3A-1 Points on a number line** (see Resources) if they had difficulty with this question or require more practice at this skill.

- You may need to undertake some explicit teaching so students understand how to read a number line. It may be appropriate to provide students with items of different scale and explicitly demonstrate how to read a value from these items. For example, provide students with a ruler, a protractor, a measuring cylinder and a thermometer and allow them to explore reading values from different types of scales.

At Level

At Level
1, 2d–f, 3d–f, 4–6, 7a–g, 8a,b, 9a,b, 11–15, 17

- Demonstrate **3A eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
Reading fractional or extrapolating unmarked numbers from a number line is cognitively demanding for many students.

- Support students who are challenged when reading unmarked numbers from a number line by verbally counting on or back from zero. Emphasise the mirror qualities of the negative side and the positive side.

- Be clear when asking which number is bigger. Students initially believe when you are asking this of two negative numbers that you are asking which number is more negative whereas in maths the convention is that bigger means more positive. For example, a student who believes that –8 is bigger than –5 sees no difference between the concepts of more negative and more positive and equates them. They will then also have great difficulty in comparing a negative number with a positive number. To minimise this confusion always clarify that by bigger you mean more positive or which number is located further to the right on the number line.

- You may wish students to copy the summary of directed numbers from the **Key ideas** section,

- Direct students to **WS 3A-2 Comparing directed numbers** (see Resources) if they need support or more practice with comparing directed numbers. Direct them to **Example 3A-2** which shows this concept.

- For more problem-solving tasks and investigations, direct students to **INV 3A-4 Up, down, left and right** and/or **INV 3A-5 Comparing world temperatures** (see Resources).

Below Level

Below Level
1, 2, 3, 6a–f, 7a–d, 8a, 10, 13a, b, 17

- Demonstrate **3A eTutor** or direct students to do this independently.
- Direct students to **SS 3A-1 Points on a number line** (see Resources) if they have difficulty locating the position of points on a number line, or require more practice at this skill.
- Help students to visualise positive numbers by always colouring the number line blue (or black) at the positive end and red at the negative end.

> **POTENTIAL DIFFICULTY**
> Many students who are not yet abstract mathematicians have trouble visualising the interval that is implied between each scale mark on the number line. This is compounded when a scale other than a unit scale or fractions are added to the number line.

- Consider supporting student use of the number line by showing the quantity at each position.
- Some students may experience difficulty in visualising where to place integers which are too large or too small to indicate on a given number line. Encourage these students to sketch a blank number line and only write in the zero. Guide students to consider where each of the larger/smaller integers will be placed. They may like to mark in a location using a cross, allowing them to model which will be furthest to the left or right.
- Direct students to **WS 3A-2 Comparing directed numbers** (see Resources) if they need support or more practice with comparing directed numbers. Direct them to **Example 3A-2** which shows this concept.
- For problem-solving tasks and investigations, direct students to **INV 3A-4 Up, down, left and right** and/or **INV 3A-5 Comparing world temperatures** (Q1, 2) (see Resources).

Above Level

Above Level
4, 5, 6d–l, 7d-i, 8, 9, 11, 14–16, 18, 19

- Demonstrate **3A eTutor** or direct students to do this independently.

- Q18 involves the real-life context of prescription glasses. Students explore the differences between prescriptions which are written as directed numbers, and determine which values mean that a person is long-sighted and which indicates that a person is short-sighted. Students could be encouraged to research their own prescription, or the prescription of family members, and compare their real-life findings with their answers.

- The activity on **INV 3A-5 Comparing world temperatures** (see Resources) has a diagram of thermometers calibrated both in $^{\circ}$C (Celsius) and $^{\circ}$F (Fahrenheit). Students should be encourage to find out which countries still use Fahrenheit to record their temperatures and give an equivalent temperature for those given in $^{\circ}$C. As an additional challenge, some students may be able to determine how to convert from Celsius to Fahrenheit. ($\frac{5}{9}$ $^{\circ}$C +32 = $^{\circ}$F)

- For an additional problem-solving task and investigation, direct students to **INV 3A-3 What's your business?** (see Resources).

Extra activities

1 Quick questions

a How are –23 and +45 read? ('negative twenty-three'; 'positive forty-five', or 'forty-five')

b On a horizontal number line, are negative numbers on the left or the right of where zero is marked? (left)

c What are integers? (positive and negative whole numbers and zero)

d Which is larger?

i 0 or 12 (12)

ii –5 or 0 (0)

iii –3 or –9 (–3)

e Which is smaller?

i –2 or –10 (–10)

ii –15 or –12 (–15)

iii 0 or –1 (–1)

2 A good way to review the material covered in this section is to play a game of 'Heads down, thumbs up' in which students are asked to compare integers without a number line. By playing this with the students, you are able to select appropriate questions for specific students and to also assess their understanding.

Answers

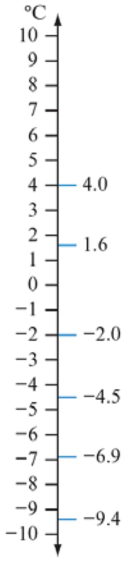
3A Understanding negative numbers

3A Start thinking!

1 a $-2°C, -4.5°C, -6.9°C, -9.4°C$. Negative sign $(-)$ indicates number is negative.
b $1.6°C, 4.0°C$

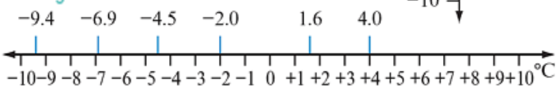
2 $-2°C, 4°C$

3 a see diagram
b highest $4.0°C$, lowest $-9.4°C$
c 0



4 Some possible answers for a vertical number line are: $+7$ (7 units up from 0), -5 (5 units down from 0), -0.15 (0.15 units down from 0), $+6\frac{2}{5}$ ($6\frac{2}{5}$ units up from 0).

5



6 a A number is larger if it is higher on a vertical number line, and smaller if it is lower.
b A number is larger if it is further right on a horizontal number line, and smaller if it is further left.

7 $-9.4°C, -6.9°C, -4.5°C, -2°C, 1.6°C, 4.0°C$

8 January

Exercise 3A Understanding negative numbers

1 a negative integers: $-1, -2, -3, -4, -5, -6, -7, -8, -9, -10$; positive integers: $+1, +2, +3, +4, +5, +6, +7, +8, +9, +10$
b 0

2 a 9 b 5 a 1 d 5 e 0 f -4

3 a 5 b 0 c -2 d -5 e -1 f -9

4 a 5.2 b $\frac{1}{2}$ c 4.3 d $\frac{4}{7}$ e -1.5 f $-\frac{2}{5}$

5 a $-4\frac{1}{2}$ b -2.1 c -6.34 d $-8\frac{6}{11}$
e -3.54 f $-\frac{3}{4}$

6 a $-3 < 2$ b $-8 < -4$ c $0 > 1$
d $9 > -9$ e $-3 > -5$ f $-7 < -2$
g $1.4 > -2.6$ h $-1.5 < -1.2$ i $4.3 > -4.3$
j $-3.8 > -5.8$ k $-0.6 > -0.7$ l $-8.2 < 0$

7 a 35 b -24 c 0 d 47
e $-50\frac{1}{4}$ f 113.2 g $-94\frac{1}{3}$ h -89.3
i $-2000\frac{1}{7}$

8 a $-20, -12, -11, -7, 8, 10, 14$
b $-41, -33, -19, 0, 6, 29, 42$
c $-88, -48, -28, -18, -8, 8, 68$
d $-140, -126, -82, 3, 73, 104, 145$

9 a $71.9, 4.3, -9.7, -10.8, -15.4, -27.6$
b $10, 5\frac{1}{2}, -\frac{1}{2}, -2\frac{1}{2}, -5, -11\frac{1}{2}$
c $21.6, 5.34, 0, -9.04, -9.4, -14.2$
d $-7\frac{2}{3}, -7\frac{1}{3}, -6\frac{2}{3}, -1\frac{1}{3}, 1\frac{2}{3}, 5\frac{1}{3}, 7\frac{1}{3}$

10 $-6, -5, -4, -3, -2, -1, 0, +1, +2, +3$

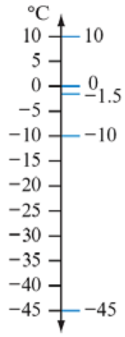
11 Some possible answers are: $-\frac{90}{4}, -25\frac{1}{2}, -\frac{115}{4}, -\frac{593}{20}$

12 Some possible answers are: $-4.75, -3.845, 0.1235, 2.9578, -3.1$.

13 a 0
b i 4 units to the left of 0
ii 5 units to the right of 0
iii 3 units right of 0 iv 5 units left of 0
v 3 units to the left of 0
vi 9 units to the right of 0
c $+5$ and -5; $+3$ and -3
d i $+8$ ii -6 iii -1
iv $+11$ v $+32$ vi -17
e Some possible answers are: $+4$ and -4, -10 and $+10$, -7 and 7.

14 a -3 b $+2174.30$ c -18
d -408 e $+5895$ f $+500$
g -46.55 h -40

15 a see diagram
b higher
c $-45°C, -10°C, -1.5°C, 0°C, 10°C$
d warmer



16 a i \$34.45 ii \$13.40
b i \$51.30 ii \$35.00
c Account is overdrawn; \$13.25 owed to bank.

17 a $+86$ b \$14 c -14

18 a long sighted
b short sighted
c Elle has weaker vision.
d normal vision; that is, neither long nor short sighted

19 a three
b i two blue counters ii four red counters
iii six blue counters iv five red counters
v one red counter vi eight blue counters
c The reference point for opposite integers is always zero as both integers are the same number of units away from 0. Moving forward to the positive integer and then moving back as indicated by the opposite integer will always result in zero. When combined, opposite integers 'cancel' each other to result in zero. This can be modelled by equal numbers of blue and red counters.
d Two blue counters and two red counters produce two zero pairs; remaining three red counters represent integer -3.
e i $+3$ ii -1 iii $+2$
f One possible answer is given.
i five blue counters, three red counters
ii two blue counters, six red counters
iii nine red counters, two blue counters
iv three red counters, six blue counters
v two red counters, two blue counters
vi two blue counters, four red counters

Reflect

Possible answer: Negative numbers are useful because they can indicate a loss or a value which is less than zero.

Resources

SupportSheet

SS 3A-1 Points on a number line

Focus: To describe and plot integer and non-integer values on a number line

Resources: ruler

Students consider the features of a number line and are reminded that the scale divisions must be equally spaced. They consider zero as the reference point and describe the numbers found to the left and right of zero. They have the opportunity to both read and represent values on a number line.

WorkSheet

WS 3A-2 Comparing directed numbers

Focus: To review positive and negative numbers and use a number line to help compare the size of directed numbers

Resources: ruler (optional)

Students first practise representing integers on a horizontal number line and then consider directed numbers. They complete statements using inequality signs. Students also reorder lists into both ascending and descending order.

Investigations

INV 3A-3 What's your business?

Focus: To rewrite numbers given in real-life situations as directed numbers

Students apply their knowledge of positive and negative numbers to read a ledger. They may need to be given definitions of the terms *income*, *expense* and *balance*. As an extension, students examine a bank statement and write questions relating to directed numbers.

INV 3A-4 Up, down, left and right

Focus: To explore movement along horizontal and vertical number lines

Resources: white cards or white cardboard, scissors, partner

Students make a set of cards and play a game of 'Snap' with a partner. They match directed numbers with descriptions of the position of the number on either a vertical or horizontal number line. As an extension, students write instructions to produce words from letters shown on a number line.

INV 3A-5 Comparing world temperatures

Focus: To use directed numbers to compare and order given temperatures

Resources: access to the Internet

Students compare recorded temperatures from a variety of locations around the world and rank them from lowest to highest. As an extension, students select five places around the world and determine their current temperatures. They investigate the best time of year to visit these places.

Interactives

3A eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3B Adding integers

Teaching support for pages 128–33

Teaching strategies

Learning focus

To use a number line to add integers.

To apply understanding of adding integers.

Start thinking!

The task guides students to:

- discover the strategy of using a number line to add integers
- discover that moving to the right on a number line is moving in the positive direction. Conversely, moving to the left is moving in the negative direction.
- discover that, when adding integers, they need to locate the starting integer on the number line and move the appropriate number of steps left or right depending on the sign of the second integer
- discover that it is the sign in front of the integer being added to the start number that indicates the direction to be moved
- summarise their findings at the end of the task.

Differentiated pathways

Below Level	At Level	Above Level
1, 2a–d, 3a–f, 5a–f, 11	1d–f, 2d–i, 3f–l, 4a,b, 5d–l, 6, 8a–f, 9, 10a–j, 11–15	1f, 2g–l, 4, 5g–r, 6, 7, 8d–l, 9, 10, 12, 14–17
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q2

Focus: To identify a starting point on a number line and to move left or right to identify an end point

- Direct students to complete **SS 3B-1 Moving left and right along a number line** (see Resources) if they had difficulty with this question or require more practice at this skill.

- You may need to undertake some explicit teaching so students understand that a positive number may be written with a positive sign (as in the number line in Q2 of Are you ready?), or without a positive sign; that is +2 and 2 represent the same value.

- It may be beneficial to draw a number line on the ground outside and demonstrate how students can move along a number line, by stepping left and right. Students could use this life-size number line to complete the questions from Are you ready?, (Q2) but may also like to complete extra questions of a similar type.

At Level

At Level
1d–f, 2d–i, 3f–l, 4a,b, 5d–l, 6, 8a–f, 9, 10a–j, 11–15

- Demonstrate **3B eTutor** or direct students to do this independently.

- Language which supports the understanding of the concept of negative integers is to say more positive when adding positives and more negative when adding negatives.

- Some students may experience difficulty with the use of a number line and may prefer to use the zero pair model (using counters). If students prefer the zero pair model, they can be provided with two different colours of counters and they can model the additions to find the results for each.

- Direct students to **WS 3B-2 Adding integers using a number line** (see Resources) if they need support or more practice using a number line to add integers. Direct them to **Example 3B-1** which shows this concept.

- For more problem-solving tasks and investigations, direct students to **INV 3B-3 Magic squares** and/or **INV 3B-4 Additional match up** (see Resources).

Below Level

Below Level
1, 2a–d, 3a–f, 5a–f, 11

- Demonstrate **3B eTutor** or direct students to do this independently.

- Direct students to complete **SS 3B-1 Moving left and right along a number line** (see Resources) if they have difficulty moving along a number line, or require more practice at this skill.

- Allow students to work within a range of numbers which can be included on a number line (i.e. numbers from –20 to +20).

- To minimise possible confusion for some students, avoid moving left and right on the number line. Instead, consolidate conceptual understanding by referring to the directions as more positive (moving right) and more negative (moving left). Once this is understood introduce the idea of a less positive (move left) and less negative (move right).
- Direct students to **WS 3B-2 Adding integers using a number line** (see Resources) if they need support or more practice using a number line to add integers. Direct them to **Example 3B-1** which shows this concept.
- For a problem-solving task and investigation, direct students to **INV 3B-4 Additional match up** (see Resources).

Above Level

Above Level
1f, 2g–l, 4, 5g–r, 6, 7, 8d–l, 9, 10, 12, 14–17

- Demonstrate **3B eTutor** or direct students to do this independently.
- Even if directed to do so in a particular question, students who demonstrate competency in addition of negative integers, fractions and decimal numbers should only use a number line if it is required to scaffold their learning.
- In Q16, students complete addition questions where larger numbers are used. Completion of these without a number line is recommended. It may be an opportunity to introduce the use of a calculator to check their answers.
- For more problem-solving tasks and investigations, direct students to **INV 3B-4 Additional match up** and/or **INV 3B-5 And the winner is …** (see Resources).

Extra activities

Quick questions:

1 Which direction on a horizontal number line is the negative direction? (Left)

2 For a horizontal number line, are the positive numbers on the left or the right of zero? (Right)

3 Complete the following additions:

a (+10) + (+7) (+17)

b (–9) + (+4) (–5)

c	(+13) + (–7)	(+6)
d	(–5) + (–5)	(–10)
e	(–7) + (– 2)	(–9)
f	(–6) + (+5)	(–1)
g	(–2) + (–1)	(–3)
h	(+10) + (–15)	(–5)

Answers

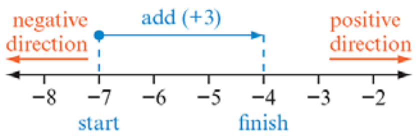
3B Adding integers

ANSWERS

3B Start thinking!

1 a 7 b right c +7; they are the same

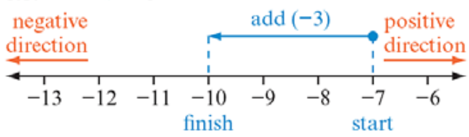
2 a



b −4 c right

3 a left b +3

4 a



b −10 c left

5 a Start at first number and move in positive direction (right) by number of units indicated by second integer.

b Start at first number and move in negative direction (left) by number of units indicated by second integer.

Exercise 3B Adding integers

1 a (−5) + (+3) = −2 b (+6) + (−2) = +4
c (−4) + (−5) =−9 d (−2) + (+5) = +3
e (+7) + (−5) = +2 f (−9) + (+4) = −5

2 a +3 b +6 c +5 d −8 e −4 f −3
g −6 h +5 i −4 j 0 k 0 l 0

3 a +2 b +3 c −5 d −2 e +5 f +1
g +9 h −9 i 0 j 0 k −2 l 0

4 a (−11) + (−6) = −17 b (+17) + (−4) = +13
c (−19) + (+5) = −14 d (−13) + (+6) = −7

5 a −16 b +8 c +7 d +18 e 0
f −8 g −7 h −14 i −19 j +15
k +2 l −9 m 0 n −12 o +20
p −4 q −20 r +5

6 a positive b negative
c positive, negative

7 If size (number of units from 0) of positive number is larger than size of negative number, sum will be positive. If size of negative number is larger than size of positive number, sum will be negative. If positive number and negative number have same size (opposite numbers), sum will be zero.

8 a positive b negative c zero
d negative e negative f positive
g positive h positive i zero
j negative k positive l negative

9 a +17 b −19 c 0 d −7 e −17
f +4 g +5 h +41 i 0 j −5
k +3 l −3

10 a +28 b −29 c +8 d −5 e 0
f −30 g +7 h −18 i −4 j −47
k +12 l −23 m −17 n 19 o 62
p −180

11 a D b 2°C

12 a C b fourth level c ground level

13 a (−21) + (+12) b −9°C

14 a −3 b (−3) + (+17)c 14 m

15 a −60 b (−60) + (+35) c $25

16 a −123 b +180 c −112 d −150
e +29 f −7 g −170 h +13
i 0 j +180 k +125 l −24

17 a −198°C; (−212) + (+14) = −198
b −193°C; (−212) + (+19) = −193

Reflect

Possible answer: If adding two positive integers, the result will be positive. If adding two negative integers, the result will be negative. When you are adding opposite integers the result will be zero. When adding a positive and a negative integer, the result can be positive or negative depending on the size of the numbers.

Resources

SupportSheet

SS 3B-1 Moving left and right along a number line

Focus: To determine the position of points on a number line after movements left or right

Resources: ruler

Students read and represent values on a number line. They then locate values on the number line resulting from a movement left or right of a given point.

WorkSheet

WS 3B-2 Adding integers using a number line

Focus: To use a number line to represent the addition of integers and determine the result

Resources: ruler

Students are guided through addition problems modelled on a number line. Some students may be more comfortable using counters and the zero pair model. They could be encouraged to use both models to consolidate their understanding.

Investigations

INV 3B-3 Magic squares

Focus: To add integers in order to complete a magic square

Students add integers to complete a '4 by 4' magic square. As an extension, students create their own magic squares.

INV 3B-4 Additional match up

Focus: To use dice to play a 'Bingo' game that involves players adding integers

Resources: three different coloured dice, partner, six counters per player

Students play a game like 'Bingo' which involves adding the numbers shown on a positive die and a negative die, and matching the result to a chosen number. As an extension, students play another game with an increased range of integers and all three dice.

INV 3B-5 And the winner is …

Focus: To use addition of integers to solve a problem

Students apply their understanding of adding integers to determine lap times of race participants. As an extension, students consider a problem in which the times were recorded incorrectly and how this impacts upon the final outcome of the race.

Interactives

3B eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3C Subtracting integers

Teaching support for pages 134–9

Teaching strategies

Learning focus

To use a number line to subtract integers.

To apply understanding of subtracting integers.

Start thinking!

The task guides students to:

- discover the strategy of using a number line to subtract integers
- discover that moving to the right on a number line is moving in the positive direction. Conversely, moving to the left is moving in the negative direction.
- discover that the subtraction sign means ‘do the opposite’ of the sign on the number being subtracted
- discover that, when subtracting integers, they need to locate the starting integer on the number line and move the appropriate number of steps left or right, depending on the sign of the second integer
- discover that it is the sign in front of the integer to be added to the start number combined with the subtraction sign that means ‘do the opposite’ and indicates the direction to be moved. If the sign of the second integer is positive, students discover that they need to do the opposite of moving in the positive direction; that is, they move in the negative direction (to the left). If the sign of the second integer is negative, students need to do the opposite of moving in the negative direction; that is, they move in the positive direction (to the right).
- summarise their findings at the end of the task.

Differentiated pathways

Below Level	At Level	Above Level
1b, d, 2a, c, f, g, j, k, 3, 4a, b, 5a–f, 11	1, 2a–f, 3a–f, 4–6, 8a–f, 9, 10a–f, 11–14	2g–l, 3g–l, 5j–r, 6–9, 10g–p, 12, 14–17
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2a–f, 3a–f, 4–6, 8a–f, 9, 10a–f, 11–14

- Demonstrate **3C eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
When using a number line, students will often mistakenly count the lines which mark the numbers instead of counting the spaces or 'jumps' between each number.

- Use language that supports conceptual understanding so subtracting a positive becomes less positive and subtracting a negative becomes less negative which is more positive. Students generally assimilate more and less positive directions on the number line first.
- Direct students to **WS 3C-1 Subtracting integers using a number line** (see Resources) if they need support or more practice using a number line to subtract integers. Direct them to **Example 3C-1** which shows this concept.
- You may like students to copy the **Key ideas** as a summary. They could add diagrams to help their understanding.
- For a problem-solving task and investigation, direct students to **INV 3C-3 Create a subtraction** (see Resources).

Below Level

Below Level
1b, d, 2a, c, f, g, j, k, 3, 4a, b, 5a–f, 11

- Demonstrate **3C eTutor** or direct students to do this independently.
- Some students have great difficulty in visualising negative numbers and therefore operating with these numbers proves to be extremely difficult. Spend time building conceptual knowledge of these students and encourage them to 'see' +3 as three ones and –3 as a space for three ones. This helps with zero pairing as a space for three ones is filled by three ones leaving none left over.
- As in Exercise 3B, maintain the colour of positive and negative number lines. Keep counters or numbers as consistent as possible, where red is negative and blue is positive.
- Direct students to **WS 3C-1 Subtracting integers using a number line** (see Resources) if they need support or more practice using a number line to subtract

integers. Direct them to **Example 3C-1** which shows this concept.

- You may like students to copy the **Key ideas** as a summary. They could add diagrams to help their understanding.
- For a problem-solving task and investigation, direct students to **INV 3C-3 Create a subtraction** (Q1, 2) (see Resources).

Above Level

Above Level
2g–l, 3g–l, 5j–r, 6–9, 10g–p, 12, 14–17

- Demonstrate **3C eTutor** or direct students to do this independently.
- Even if stated in the question, students should only use a number line if it is required to support their learning. (For example, Q2 and Q3).
- Students should be operating with numbers which cannot be easily accommodated by a number line, ensuring students operate abstractly with integers and negative fractional numbers.
- Deepen student understanding of subtraction with negative numbers by asking them to articulate *why* subtraction of a negative number results in a positive move on a number line. This ensures that capable students do not just rely on efficient recall of shortcuts but build depth in their understanding of concepts.
- For more problem-solving tasks and investigations, direct students to **INV 3C-2 Subtraction match up, INV 3C-3 Create a subtraction** and/or **INV 3C-4 Boris' baffle** (see Resources).

Extra activities

Quick questions:

1 In which direction do you move on a horizontal number line when subtracting a negative number? (positive/right)

2 In which direction do you move on a horizontal number line when subtracting a positive number? (negative/left)

3 Complete the following subtractions:

a (–5) – (+2) (–7)

b (+10) – (+7) (+3)

c	(+13) – (–7)	(+20)
d	(–5) – (–5)	(0)
e	(–7) – (– 2)	(–5)
f	(–8) – (+2)	(–10)
g	(–2) – (–1)	(–1)
h	(+10) – (–15)	(+25)

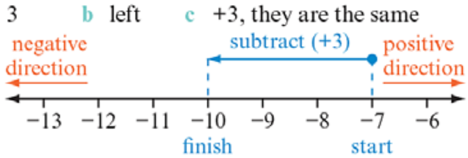
Answers

3C Subtracting integers

3C Start thinking!

1 a 3 b left c +3, they are the same

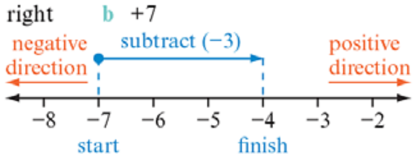
2 a



b −10 c left

3 a right b +7

4 a



b −4 c right

5 a Start at position of first integer and move in negative direction (left) by number of units indicated by second integer.

b Start at position of first integer and move in positive direction (right) by number of units indicated by second integer.

Exercise 3C Subtracting integers

1 a (+2) − (−4) = +6 b (+1) − (+5) = −4
c (−4) − (−3) = −1 d (−6) − (+3) = −9
e (+3) − (−2) = +5 f (+4) − (+4) = 0

2 a +2 b −7 c 0 d −1 e +8 f −6
g −7 h +10 i +3 j −5 k −3 l +8

3 a −5 b −10 c +6 d +7 e −9 f 0
g +4 h −9 i +1 j +4 k +4 l 0

4 a (+16) − (+3) = +13 b (−19) − (−5) = −14
c (−12) − (+4) = −16 d (−8) − (−10) = +2

5 a +5 b +19 c +20 d −18 e 0 f −20
g +18 h −20 i +7 j +2 k 0 l +16
m −11 n +3 o +2 p −6 q −20 r −4

6 a positive b negative c positive, negative

7 If two integers are identical, subtracting them will always result in zero. When subtracting numbers that are both positive, if first number is larger, result will always be positive. If second number is larger, result will always be negative. When subtracting numbers that are both negative, if first number is larger, result will be positive, otherwise result will be negative.

8 a positive b negative c zero
d negative e negative f negative
g positive h negative i zero
j negative k negative l positive

9 a +19 b −20 c 0 d −3 e −3
f −28 g +22 h −7 i 0 j −1
k −5 l +7

10 a −32 b +25 c +14 d 0 e +11
f −20 g −69 h +140 i −4 j +91
k +90 l −86 m −70 n +92 o −166
p +27

11 a B b −10°C

12 a A b 5 m c D, 5 m
d the water level

13 51°C; (+33) − (−18) = 51

14 a +28 b shark −3; sting ray −11
c i (+28) − (−3) ii (+28) − (−11)
iii (−3) − (−11)
d i 31 m ii 39 m iii 8 m

15 a −124 b −80 c −140 d +170 e −368
f +135 g +5 h +220 i −480 j +92
k +595 l −915

16 a $63; (−247) − (−310) = 63
b Initially, account is overdrawn by $310. During the month, $63 is added to the account and overdrawn amount is reduced from $310 to $247.

17 One possible answer is: −7 and −3.
(−7) + (−3) = −10, (−7) − (−3) = −4

Reflect

Possible answer: You move to the left on a number line when subtracting, but reverse the direction (move right) when subtracting a negative number.

Resources

WorkSheet

WS 3C-1 Subtracting integers using a number line

Focus: To use a number line to represent the subtraction of integers and determine the result

Resources: ruler

Students are guided through subtraction problems modelled using a number line. They state the problem and the answer for a selection of questions. They complete further subtraction problems using this model. Some students may be more comfortable using counters and the zero pair model and could be encouraged to use both models to consolidate their understanding.

Investigations

INV 3C-2 Subtraction match up

Focus: To use dice to play a ‘Bingo’ game that involves players subtracting integers

Resources: three different coloured dice, partner, six counters per player

Students play a game like ‘Bingo’ which involves subtracting the numbers shown on a positive die and a negative die, and matching the result to a chosen number. As an extension, students play another game with an increased range of integers and all three dice. They analyse their choice of numbers.

INV 3C-3 Create a subtraction

Focus: To use the skill of subtracting integers to solve a given problem

Resources: pencil, eraser, ruler

Students subtract positive and negative integers to achieve a prescribed total given in a problem-solving format. They also find an alternative solution to the problem which gives the same result. As an extension, students create their own puzzle.

INV 3C-4 Boris’ baffle

Focus: To use the skill of subtracting integers to determine temperatures

Resources: access to the Internet (optional), calculator (optional)

Students apply their knowledge of subtracting positive and negative integers in determining temperatures at the South Pole. As an extension, students collect and compare temperatures for each capital city in Australia.

Interactives

3C eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3D Simplifying addition and subtraction of integers

Teaching support for pages 140–5

Teaching strategies

Learning focus

To practise the simplification of addition and subtraction of integers.

To apply understanding of the simplification of addition and subtraction of integers by writing a simpler equivalent calculation.

Start thinking!

The task guides students to:

- discover how to simplify the way in which additions and subtractions of integers are written
- consider the addition of a positive integer and the subtraction of a negative integer and discover that, in both cases, they move to the right on the number line. They discover that they can write an equivalent calculation that is simpler.
- consider the subtraction of a positive integer and the addition of a negative integer and discover that, in both cases, they move to the left on the number line. An equivalent calculation that is simpler can again be written.
- discover that, when the adjacent signs are the same (both + or both –), the signs can be rewritten as a single +
- discover that, when the adjacent signs are different (one negative and one positive, in any order), the operation sign and the sign from the integer can be written as a single –. It is important to note that the order of the signs is not important.

Differentiated pathways

Below Level	At Level	Above Level
1–3, 5a–g, 6–9, 13a, 21	1, 2c–f, 3e–j, 4a–d, 5h–l, 6–10, 13b, 16a–d, 17–19, 21	1, 4, 10–12, 14, 15, 16e–h, 17–20, 22–26
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2c–f, 3e–j, 4a–d, 5h–l, 6–10, 13b, 16a–d, 17–19, 21

- Demonstrate **3D eTutor** or direct students to do this independently.
- Students who understand why the shortcuts work will have a better chance of retaining the shortcuts in long term memory. To support understanding it may help to allow students to derive the shortcuts themselves than to teach them explicitly.

  This can be done by encouraging students to attempt the first few questions in this exercise without any explicit teaching and ask them to consider which direction they move on the number line for each combination of operation and sign.
- Direct students to **WS 3D-1 Simplifying addition and subtraction calculations** (see Resources) if they need support or more practice with this concept. Direct them to **Example 3D-1** which shows the thinking involved.
- You may like students to copy the **Key ideas** as a summary. This shows all the simplifying combinations.
- For more problem-solving tasks and investigations, direct students to **INV 3D-2 Subtract 'n' total** and/or **INV 3D-3 Ben's budget** (see Resources).

Below Level

Below Level
1–3, 5a–g, 6–9, 13a, 21

- Demonstrate **3D eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
Some students will struggle to correctly locate negative numbers on the number line. This knowledge is a prerequisite to being able to then perform operations on these numbers.

- Help students to differentiate between the subtraction sign and the negative sign which denotes a negative number. This can be done easily be using language which clarifies this difference.

  For example 4 – (–3) 'Four take away 3 negatives, what happens when you take away negatives?' (It makes something less negative.) 'If you make something less negative is that the same as something else?' (The same as making it more positive.)

- Q13 requires students to complete addition tables. The BLM **Addition tables** (see Resources) can be used when completing this question. It contains both tables ready for students to fill in. Tables can then be pasted into the student's workbook. Some students may find it beneficial to write the question into each box within the tables.
- Direct students to **WS 3D-1 Simplifying addition and subtraction calculations** (see Resources) if they need support or more practice with this concept. Direct them to **Example 3D-1** which shows the thinking involved.
- You may like students to copy the **Key ideas** as a summary. This shows all the simplifying combinations.
- For a problem-solving task and investigation, direct students to a simplified version of **INV 3D-2 Subtract 'n' total** (see Resources).

Above Level

Above Level
1, 4, 10–12, 14, 15, 16e–h, 17–20, 22–26

- Demonstrate **3D eTutor** or direct students to do this independently.
- Q26 explores the informal use of backtracking to calculate the missing value in given equations. The more able students will recognise that they can use the balance method to calculate the missing value. Other students may need to use a trial and error approach and can be encouraged to use their calculators.
- For more problem-solving tasks and investigations, direct students to **INV 3D-2 Subtract 'n' total, INV 3D-3 Ben's budget** and/or **INV 3D-4 Nomographs** (see Resources).

Extra activities

Quick questions:

1 Simplify the following problems and calculate a result for each.

a	(+8) + (+4)	(8 + 4 = 12)
b	(–2) + (+2)	(–2 + 2 = 0)
c	(+7) + (–8)	(7 – 8 = –1)
d	(–12) – (+4)	(–12 – 4 = –16)

e	(–3) – (–7)	(–3 + 7 = 4)
f	(+4) – (–6)	(4 + 6 = 10)

2 Highlight the signs that can be simplified in the following. Write an equivalent simplified problem and calculate a result for each.

a	(+6) + (–2)	(+6) + (–2)	(6 – 2 = 4)
b	(–14) – (–8)	(–14) – (–8)	(–14 + 8 = –6)
c	(–1) + (+3)	(–1) + (+3)	(–1 + 3 = 2)
d	(+9) – (+2)	(+9) – (+2)	(9 – 2 = 7)

3 If the highlighted signs between two integers combine in the simplified expression to be +, in which direction do you move on a number line? (positive, right)

4 If the highlighted signs between two integers combine in the simplified expression to be –, in which direction do you move on a number line? (negative, left)

Answers

3D Simplifying addition and subtraction of integers

3D Start thinking!

1 a i +7 ii +7 b right
  c You move in same direction along number line (to right) when adding a positive number or subtracting a negative number.
  d They produce same result.
2 The two operation signs next to each other (either + + or − −) can be replaced with one addition operation.
3 a i +3 ii +3 b left
  c You move in same direction along number line (to left) when subtracting a positive number or adding a negative number.
  d They produce same result.
4 The two operation signs next to each other (either + − or − +) can be replaced with one subtraction operation.
5 For two operations next to each other that are the same, replace with +.
For two operations next to each other that are different, replace with −.

Exercise 3D Simplifying addition and subtraction of integers

1 −, negative, left; −, negative, left; +, positive, right
2 a (−3) − (+7) = −3 − 7
  b (+1) + (+6) = 1 + 6
  c (−4) + (−5) = −4 − 5
  d (+2) − (−4) = 2 + 4
  e (+5) − (+9) = 5 − 9
  f (−6) − (−8) = −6 + 8
3 a −1 + 3 b 8 − 4 c −5 − 2 d 6 + 1
  e 4 + 3 f 3 − 9 g −5 + 8 h −3 + 1
  i −7 − 3 j −6 + 7
4 a 3 b 6 c −4 d 0 e −1 f 5
  g 1 h −5 i 9
5 a 5 b −2 c 6 d −10 e −3 f −7
  g 4 h 2 i 0 j −9 k −9 l 7
6 a −10 b 7 c −9 d 6 e −4 f 2
7 a 2 b 4 c −7 d 7 e 7 f −6
  g 3 h −2 i −10 j 1
8 a C b A c B
9 a −15 b −9 c 9
10 a 3 b −6 c −4 d 16 e −19 f −10
  g −17 h −2 i −13 j 0 k −20 l 2
11 a negative b positive c negative
  d zero e positive f positive
  g negative h negative i negative
12 a −10 b 30 c −7 d 0 e 7 f 7
  g −13 h −24 i −3
13 a

+	−5	−3	0	1	4
−3	−8	−6	−3	−2	1
−2	−7	−5	−2	−1	2
−1	−6	−4	−1	0	3
3	−2	0	3	4	7
6	1	3	6	7	10

  b

+	−10	−14	−7	13	19
11	1	−3	4	24	30
14	4	0	7	27	33
−10	−20	−24	−17	3	9
−22	−32	−36	−29	−9	−3
−18	−28	−32	−25	−5	1

14 a −5 b −99 c −32 d 9
  e −40 f −275 g 242 h 118
  i 0 j −2000 k −2980 l −31
  m 27 n −640 o −5685
15 a 10 b −4 c −3 d −13 e 0
  f −33 g −10 h −26 i −30 j −17
16 a −6 b 15 c 0 d 12 e −14
  f −31 g 8 h 2
17 a A: +50, B: −2, C: +32
  b i (+50) − (−2) = 50 + 2
    ii (+32) − (−2) = 32 + 2
    iii (+50) − (+32) = 50 − 32
  c i 52 m ii 34 m iii 18 m
18 four levels below ground; 15 − 19 = −4
19 a +285 b 285 − 300
  c −15; account is overdrawn by $15
20 a (−900) − (+15) = −915; 915 m down
  b (−2500) − (−900) = −1600; 1600 m down
  c (+15) − (−2500) = +2515; 2515 m up
21 first level underground; 3 − 9 + 5 = −1
22 One possible answer is: (+21) + (−15) + (−8).
23 a 24 b −312 c −5 d 70
  e 1018 f −4218
24 a 100 b −21 c 366 d −249
  e −4000 f 9007
25 a 45 b −30 c 1261 d −367
  e −1020 f −3260
26 a 33 b 77 c −125 d −40

Reflect

Possible answer: The same adjacent signs are equivalent to a + sign and different adjacent signs are equivalent to a – sign.

Resources

WorkSheet

WS 3D-1 Simplifying addition and subtraction calculations

Focus: To write addition and subtraction problems involving integers in a simpler equivalent form and then use a number line to calculate the result

Resources: ruler

Students consider the use of a number line in addition and subtraction problems. They also simplify problems using equivalent operations, so that pairs of signs are replaced by single signs. Students then summarise their findings. These could be made into a poster and displayed in the classroom for students to reference.

Investigations

INV 3D-2 Subtract 'n' total

Focus: To use a deck of playing cards to add and subtract integers

Resources: deck of playing cards, partner

Students practise the addition and subtraction of positive and negative integers as part of a card game. As an extension, students play an alternative version of the game, using different rules.

Simplified version

Use only the black cards to represent positive integers.

INV 3D-3 Ben's budget

Focus: To use addition and subtraction of integers in budgeting

Students apply their understanding of adding and subtracting integers to an application task based upon Ben's budget. As an extension, students construct their own budget and consider their saving and spending over the period of a week.

INV 3D-4 Nomographs

Focus: To introduce the use of nomographs for adding and subtracting integers

Resources: ruler

Students perform addition and subtraction of integers using a nomograph as a tool. They consider patterns in the results achieved.

BLM

Addition tables

Interactives

3D eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3E Multiplying and dividing integers

Teaching support for pages 146–51

Teaching strategies

Learning focus

To use patterns that can be used to predict the direction sign for the outcome of multiplication and division calculations.

To apply understanding of patterns to predict the direction sign for the outcome of multiplication and division calculations.

Start thinking!

The task guides students to:

- discover patterns to predict the direction sign for the outcome of multiplication calculations. Students may like to use counters to follow through this task. If this is the case, provide students with both red (negative) and blue (positive) counters.
- consider multiplication of two positive integers and model this with counters
- consider the scenario in which one positive integer and one negative integer are multiplied and model the result of such calculations using counters
- consider the multiplication of two negative integers
- summarise their findings at the conclusion of this task.

Differentiated pathways

Below Level	At Level	Above Level
1, 2a,b,f,h,j, 4c,d,g,j, 5a,b, 7a, 12a,d, 16a–f	2–5, 7b, 8a–c, 10, 12–16	3, 4, 5f–l, 6, 8, 9, 11, 13–19
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q3 and Q4

Focus: To review multiplication and division of whole numbers

- Direct students to complete **SS 3E-1 Multiplying and dividing whole numbers** (see Resources) if they had difficulty with these questions or require more practice at this skill.

- You may need to undertake some explicit teaching so students are reminded of different strategies that can be used when multiplying and dividing whole numbers, as listed below.
  - Incorporate some tables races into the start or end of lessons.
  - Model multiplication and division, exploring the concept of 'lots of' using counters.
  - Discuss multiplication strategies such as:

    Informal use of the distributive law
    For example:
    36 × 4 = 30 × 4 + 6 × 4
    = 120 + 24
    = 144

    Doubling
    For example:
    85 × 2 = double 80 + double 5
    = 160 + 10
    = 170

    If the question was 85 × 4 the components would each be doubled twice and then added.

    Students may need to be reminded of strategies when multiplying by powers of ten. These can be explained using colour coding.
    For example:
    10 × 10 = 100
    100 × 10 = 1000 and so on.

  - Discuss division strategies such as:

    Grouping
    For example, 126 ÷ 9:
    10 'lots of' 9 = 90
    126 – 90 = 36
    36 ÷ 9 = 4
    10 + 4 = 14
    So: 126 ÷ 9 = 14

    Students may need to be reminded of strategies when dividing by powers of ten. Again, these can be explained using colour coding.
    10 ÷ 10 = 1
    100 ÷ 10 = 10

1000 ÷ 10 = 100
10 000 ÷ 100 = 100

At Level

At Level
2–5, 7b, 8a–c, 10, 12–16

- Demonstrate **3E eTutor** or direct students to do this independently.
- Support student visualisation of the concepts of multiplication and division by demonstrating the area or array model and connecting division facts to multiplication facts. Many students miss this crucial connection between multiplication and division in primary school so it is worthwhile ensuring it is understood for positive integers before continuing with negative integers.
- Use red counters or dots in an array model to represent negative integers in the equations 2 × (–3) = –6 and –6 ÷ 2 = –3. Proceed from this point to describe the commutation that if 2 × (–3) = –6 then –2 × 3 = –6.
- Only once students are happily operating with these concepts should you introduce the multiplication of a negative number by a negative multiplier.

  (If students understand the reason for a shortcut they are more likely to understand what to do and not have to rely on memory recall to perform calculations.)
- In Q10, students should be encouraged to show all of their working, using highlighting to clearly identify each step.

  For example: Q10a

  –4 × +3 × –5

  = –12 × –5

  = +60

  Students who are struggling with the arithmetic should be encouraged to use their calculator.
- Direct students to **WS 3E-2 Multiplying and dividing integers** (see Resources) if they need support or more practice with this concept. Direct them to **Example 3E-1** and **Example 3E-2** which show the thinking involved.

Below Level

Below Level
1, 2a,b,f,h,j, 4c,d,g,j, 5a,b, 7a, 12a,d, 16a–f

- Demonstrate **3E eTutor** or direct students to do this independently.
- Direct students to complete **SS 3E-1 Multiplying and dividing whole numbers** (see Resources) if they have difficulty multiplying or dividing whole numbers, or require more practice at this skill.

POTENTIAL DIFFICULTY
Most students have been shown how to perform division and multiplication algorithms using a calculator whilst in primary school; however, this does not mean that they necessarily understand either concept.

- For students who are not yet multiplicative thinkers, support their understanding with physical models of the calculations which you want them to perform. If the student has accepted the premise that red counters are negative then it is possible to have a concrete model of the multiplication 4 × (–2) is 4 groups of negative two. However, this is not comparable to –4 × 2 as negative 4 groups of two cannot be modelled physically with the red counters to create the answer of –8. Often as teachers we underestimate what a difficult concept negative groups is. Only students who understand the Commutative Law will be able to understand that 4 × (–2) is equivalent to –4 × 2.
- Q7 requires students to complete multiplication tables. The BLM **Multiplication tables** (see Resources) can be provided to students for this question. It contains both tables ready for students to fill in. Tables can then be pasted into the student’s workbook. Encourage students to write each calculation into the appropriate box within the table.
- Direct students to **WS 3E-2 Multiplying and dividing integers** (see Resources) if they need support or more practice with this concept. Direct them to **Example 3E-1** and **Example 3E-2** which show the thinking involved.
- You may like students to copy the **Key ideas** as a summary. This shows all the multiplying and dividing sign combinations.

Above Level

Above Level
3, 4, 5f–l, 6, 8, 9, 11, 13–19

- Demonstrate **3E eTutor** or direct students to do this independently.
- Some students display very easy recall of mathematical shortcuts and can operate efficiently with any values which are given in a similar problem. However, they can

function as much more effective mathematicians if they have a deep understanding of the concept as this understanding will allow the student to operate with and manipulate the concept outside of the familiar parameters. To support this learning, encourage students to investigate why a shortcut works and discuss their findings with a partner.

- In Q11, students calculate multiplication and division problems involving three or more integers. Remind them of the order of operations, **BIDMAS**, and that because all of the operations are multiplication or division, they should work from left to right.

  For example: Q11a

  $-2 \times 2 \times -3 \times 3$

  $= -4 \times -3 \times 3$

  $= -4 \times -3 \times 3$

  $= 12 \times 3$

  $= 36$

  Students may need to be reminded that positive numbers are often written without the + sign. Students should also be encouraged to show all of their working, using highlighting to clearly identify each step, as shown above.

- Students are required to write their own problems for Q17, Q18 and Q19. Advise them that a factor tree could be useful here.

- For more problem-solving tasks and investigations, direct students to **INV 3E-3 Chocolates** and/or **INV 3E-4 Multiplication triangulation** (see Resources).

Extra activities

1 Describe how highlighting can be used to identify the difference between addition and subtraction, and multiplication and division calculations. (When adding and subtracting, highlighting adjacent signs indicates where signs can be combined. When multiplying and dividing, the highlighting of signs is used to predict the sign of the result.)

2 Review the concepts covered in this topic by playing a game of ‘Heads down, thumbs up’ in which students are asked to complete multiplication and division problems.

Answers

3E Multiplying and dividing numbers

3E Start thinking!

1 a There are six blue counters. b +6
2 +6 3 no 4 positive
5 a There are six red counters. b −6
6 negative 7 −6 8 negative
9 a +6 b positive

Exercise 3E Multiplying and dividing numbers

1 a +12 b −14
2 a −12 b +35 c −12 d +27 e −8
f −24 g +72 h +20 i −44 j −7
k +8 l +45
3 a i +2 × +3 = +6 so +6 ÷ +2 = +3 or +6 ÷ +3 = +2
ii +2 × −3 = −6 so −6 ÷ +2 = −3 or −6 ÷ −3 = +2
iii −2 × +3 = −6 so −6 ÷ −2 = +3 or −6 ÷ +3 = −2
iv −2 × −3 = +6 so +6 ÷ −2 = −3 or +6 ÷ −3 = −2
b yes, same pattern
c i positive ii positive
iii negative iv negative
4 a −5 b +4 c +6 d −7 e +17 f −8
g −7 h +9 i −6 j −12 k +10 l −9
5 a −35 b −3 c 7 d −60 e −4 f 60
g −5 h −36 i 90 j −1 k −38 l 45
6 a −3 b −7 c −9 d 100 e −70 f −3
7 a

×	−2	−1	0	+1	+2
−4	8	4	0	−4	−8
−2	4	2	0	−2	−4
0	0	0	0	0	0
+2	−4	−2	0	2	4
+4	−8	−4	0	4	8
+6	−12	−6	0	6	12

b

×	−25	−20	−10	10	20
7	−175	−140	−70	70	140
5	−125	−100	−50	50	100
0	0	0	0	0	0
−2	50	40	20	−20	−40
−4	100	80	40	−40	−80
−6	150	120	60	−60	−120

8 a −8 b −12 c 17 d 15 e 8 f −3
9 a −1 × −7 = 7 b −1 × +12 = −12
c −1 × −88 = 88 d −1 × 25 = −25
e −1 × (8 − 5) = −1 × 3 = −3
f −1 × (−3 + 2) = −1 × −1 = 1
10 a 60 b −42 c −54 d −48 e −130
f 84 g −160 h 110 i −180
11 a 36 b −5 c −24 d −6 e 30
f −42
12 a 10, 8, 6, 4, 2, 0, −2, −4, −6, −8, −10; decreases by 2, or you get the next number by subtracting 2 (or adding −2)
b −15, −12, −9, −6, −3, 0, 3, 6, 9, 12, 15; increases by 3, or you get the next number by adding 3 (or subtracting −3)
c 20, 16, 12, 8, 4, 0, −4, −8, −12, −16, −20; decreases by 4, or you get the next number by subtracting 4 (or adding −4)
d −25, −20, −15, −10, −5, 0, 5, 10, 15, 20, 25; increases by 5, or you get the next number by adding 5 (or subtracting −5)
13 a i 0 ii 0 iii 0 iv 0
b Answer is always zero.
c i 0 ii 0 iii undefined iv undefined
d Answer is always zero.
e Result is undefined. It is not possible to divide a number by zero.
14 Some possible answers are given.
a i −2 × 12, 4 × −6, −3 × 8
ii −2 × −9, 6 × 3, −18 × −1
b i −12 ÷ 3, 36 ÷ −9, −8 ÷ 2
ii 10 ÷ 2, −20 ÷ −4, −35 ÷ −7
c −2 × 0, 0 × 6, −200 × 0
d 0 ÷ 3, 0 ÷ −6, 0 ÷ 100
e i −2 × 6 × 4, −3 × −8 × −2, 12 × 4 × −1
ii −10 × −2 × 6, 24 × −5 × −1, −12 × 5 × −2
15 a −6, −3 b 9, −5 c −11, 6 d −4, 14
e 4, 6 or −6, −4 f −9, −1 or 1, 9
g −16, 2 or −2, 16
16 a −120 b +80 c 200 m d +110 e +40
f i 40 m ii 230 m g −160 h −40
i Hayden. He needs to walk 20 m west.
j Natalie: 320 m; Tyler: 240 m; Hayden: 120 m; Rhys: 320 m; Sophie: 160 m; Imogen: 220 m
17 One possible answer is: −2 × 7 × 3.
18 One possible answer is: −4 × 6 ÷ −3.
19 One possible answer is: 2 × −5 × 25 × 4.

Reflect

Possible answer: When multiplying or dividing two integers with like signs, the result will be positive. When multiplying or dividing two integers with unlike signs, the result will be negative. When multiplying an integer by zero, the result will be zero. When dividing zero by an integer, the result will be zero. It is not possible to divide an integer by zero as the result is undefined.

Resources

SupportSheet

SS 3E-1 Multiplying and dividing whole numbers

Focus: To review the methods of multiplying and dividing a whole number by a single-digit number and review numbers written using index notation

Students complete both a multiplication problem and a division problem in which a whole number is multiplied or divided by a single digit. Students also consider repeated multiplication and the use of index form.

They then complete multiplication and division calculations independently.

WorkSheet

WS 3E-2 Multiplying and dividing integers

Focus: To perform calculations involving multiplication and division of integers

Students use a concrete model to represent multiplication with integers and summarise any existing patterns. Students also consider division and any patterns which exist in these types of problems.

Investigations

INV 3E-3 Chocolates

Focus: To multiply and divide integers related to a real-life example

Students apply their understanding of multiplying and dividing integers in the real-life context of producing chocolates. Students consider aspects of chocolate production, such as chocolates rejected due to poor quality, total sales made and profit.

INV 3E-4 Multiplication triangulation

Focus: To multiply and divide integers and use the answers to solve a puzzle

Students practise the skill of multiplying and dividing integers and then use their answers to solve a multiplication puzzle. *Note:* Not all of the answers are used to solve the puzzle. As an extension, students create their own puzzles with prescribed features.

BLM

Multiplication tables

Interactives

3E eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3F Operations with directed numbers

Teaching support for pages 152–7

Teaching strategies

Learning focus

To use understanding of operations with integers to perform operations on directed fractions and decimals.

To apply understanding of working with integers to perform operations involving directed fractions and decimals.

Start thinking!

The task guides students to:

- discover that operations and patterns discussed so far also apply to directed fractions and decimals
- place directed fractions and decimals on a number line
- use the number line to add and subtract directed fractions and decimals.

Students may find it useful to refer to the number line they made earlier in this chapter.

Differentiated pathways

Below Level	At Level	Above Level
1a–d, 7b,d, 8a, 10a, 11a,c, 16	1, 2a–d, 3a–c, 4–6, 7a–f, 8b, 10a–d, 11, 12a–c, 13a–c, 14–17	2d–i, 3d–i, 4d–i, 5d–f, 6d–f, 7d–o, 8, 10c–f, 12, 13, 15, 17–21
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q5–9

Focus: To review operations with fractions and decimals, and to review the calculation of the average of a group of numbers

- Direct students to complete **SS 3F-1 Adding and subtracting fractions** (see Resources) if they had difficulty with Q5, or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded that they can only add or subtract fractions that have the same denominator. Discussion of

equivalent fractions so that common denominators can be obtained may be useful.

- Some students find it difficult to visualise the addition and subtraction of mixed numbers, and could be encouraged to change mixed numbers to improper fractions. Changing mixed numbers to improper fractions can be visualised using diagrams, or a fraction wall.
- Direct students to complete **SS 3F-2 Multiplying and dividing fractions** (see Resources) if they had difficulty with Q6, or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded of the strategies used to multiply and divide fractions. Students who are experiencing difficulty multiplying fractions can be encouraged to use the area model.
- Students may need to be reminded that there is no need to find a common denominator when completing these questions, and that they should change mixed numbers to improper fractions.
- Direct students to complete **SS 3F-3 Adding and subtracting decimals** (see Resources) if they had difficulty with Q7, or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded of the importance of aligning decimal numbers when adding and subtracting. Students can be encouraged to complete their working for these types of questions in a grid, aligning the numbers on the decimal point. Remind students that they can insert trailing zeros to fill in the spaces of a grid to eliminate confusion.
- Direct students to complete **SS 3F-4: Multiplying and dividing decimals** (see Resources) if they had difficulty with Q8, or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded that they can complete a multiplication calculation by initially ignoring the decimal points. The number of decimal places in the answer is the same as the total number of decimal places in the original calculation.
- Students can also be reminded to change a division problem to an equivalent calculation with a whole number divisor by multiplying by a suitable power of 10 (10, 100, 1000, …).
- Sections *1C Understanding fractions* through to and including *1F Operations with decimals* are excellent if students need to review material, should further skills practice for these concepts be required.
- Direct students to complete **SS 3F-5 Finding the average** (see Resources) if they had difficulty with Q9 or require more practice at this skill.

- You may need to undertake some explicit teaching so students are reminded that the average of a group of scores is the sum of all the scores divided by the number of scores.

At Level

At Level
1, 2a–d, 3a–c, 4–6, 7a–f, 8b, 10a–d, 11, 12a–c, 13a–c, 14–17

- Demonstrate **3F eTutor** or direct students to do this independently.

- This exercise relies on the student's ability to manipulate calculations with fractions and decimals accurately. This work has been covered in detail in Chapter 1. If students have difficulty or require more practice at these skills, they should be directed to **SS 3F-1 Adding and subtracting fractions**, **SS 3F-2 Multiplying and dividing fractions**, **SS 3F-3 Adding and subtracting decimals** and **SS 3F-4: Multiplying and dividing decimals** (see Resources).

POTENTIAL DIFFICULTY
Finding the lowest common multiple does not, in itself, support student understanding of the concept of equivalent fractions.

- Ensure students can identify equivalent fractions and can articulate that equivalence can be found in fractions from the same family (thirds, sixths, ninths etc., are all in the same family) then relate this concept to the short cut of finding a different denominator. Only once students demonstrate understanding of this concept should the shortcut of finding the Lowest Common Denominator be introduced.

- For Q15, direct students to complete **SS 3F-5 Finding the average** (see Resources) if they have difficulty calculating averages, or require more practice at this skill.

- Direct students to **WS 3F-6 Four operations with directed numbers** (see Resources) if they need support or more practice manipulating operations with directed numbers. This WorkSheet deals with directed fractions and directed decimals.

- For a problem-solving task and investigation, direct students to **INV 3F-8 In a spin** (see Resources).

Below Level

Below Level
1a–d, 7b,d, 8a, 10a, 11a,c, 16

- Demonstrate **3F eTutor** or direct students to do this independently.

- Direct students to complete **SS 3F-3 Adding and subtracting decimals** (see Resources) if they have difficulty adding or subtracting decimals, or require more practice at this skill. This may help them with Q7.

POTENTIAL DIFFICULTY
Students who are adding or subtracting the denominators as well as the numerators are attempting to emulate the processes of addition and subtraction of whole numbers with fractions. They do not yet see the denominator as a descriptor of the type of fraction.

- Support conceptual understanding of fractions by referring always to the fraction by name, for example:

  $3 \div 4$ should be written as $\frac{3}{4}$ and verbally described as 3 fourths, never 3 over 4.

- Ensure competency is demonstrated with fractions with the same denominator before introducing fractions of a different denominator.

- Use fractions with related denominators so that only one equivalent fraction has to be found (e.g. fourths and eighths). For some students, avoid the use of finding the lowest common denominator as a strategy but instead help students to identify an equivalent fraction by using a **Fraction wall** BLM (see Resources).

- For a problem-solving task and investigation, direct students to **INV 3F-8 In a spin** (Q1, 2) (see Resources).

Above Level

Above Level
2d–i, 3d–i, 4d–i, 5d–f, 6d–f, 7d–o, 8, 10c–f, 12, 13, 15, 17–21

- Demonstrate **3F eTutor** or direct students to do this independently.

- When multiplying by decimal numbers, students will benefit from visualising what happens during the multiplication.

- Start by reminding students of the area model of multiplication and then ask them to draw 2×3.1 (or 2 groups of 3.1) on graph or grid paper.

  In this way, students can see that $2 \times 3.1 = 6.2$

  Extend this to 2.3×3.1

  In this way, students can see that 2.3×3.1 = 6 ones and 11 tenths and 3 hundredths or 7.13

Once students have completed several multiplications in this way, they are more able to use the shortcut of counting the number of places after the decimal points as a strategy for multiplying decimals.

- Direct students to **WS 3F-6 Four operations with directed numbers** (see Resources) if they need support or more practice manipulating operations with directed numbers. This WorkSheet deals with directed fractions and directed decimals.
- For more problem-solving tasks and investigations, direct students to **INV 3F-7 Converting temperatures** and/or **INV 3F-8 In a spin** (see Resources).

Extra activities

1 What are directed numbers? (Integers which include positive and negative whole numbers and zero; positive and negative fractions and decimals)

2 What patterns can be used to predict the direction of the result when multiplying or dividing directed numbers? (When multiplying or dividing two numbers with like signs, the result will be positive. When multiplying or dividing two numbers with unlike signs, the result will be negative.)

3 Describe a strategy which can be used to identify the difference between addition and subtraction, and multiplication and division. (Using a highlighter when completing operations with directed numbers will highlight which strategy needs to be used. When adding and subtracting directed numbers, adjacent signs can be highlighted and combined to form one sign. When multiplying or dividing, signs which are not adjacent can be highlighted and used to predict the direction of the result.)

Answers

3F Operations with directed numbers

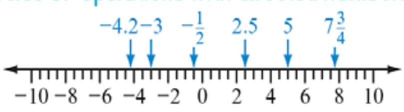
3F Start thinking!

1 a The first number is written more simply as $\frac{1}{7}$ and the operations + (−) can be replaced with −.
  b $-\frac{2}{7}$

2 a $-0.4 + 0.9$ b 0.5

3 a positive b positive c negative
  d negative e negative f negative
  g positive h positive

Exercise 3F Operations with directed numbers

1


2 a −2.5 b −5.6 c 7.2 d −10.3 e −0.2
  f 1.3 g 3.4 h 1.7 i 5.3

3 a $\frac{4}{7}$ b $-2\frac{1}{3}$ c $-\frac{2}{8} = -\frac{1}{4}$
  d $-\frac{2}{5}$ e $-1\frac{2}{3}$ f $-1\frac{1}{4}$
  g $\frac{2}{9}$ h $-1\frac{1}{10}$ i $-1\frac{1}{6}$

4 a $\frac{4}{11}$ b $-\frac{1}{3}$ c $-1\frac{1}{5}$ d $\frac{4}{15}$ e $-\frac{7}{12}$
  f $-\frac{5}{27}$ g $-1\frac{1}{5}$ h $-\frac{40}{81}$ i $\frac{2}{9}$

5 a 34.8 b −140.1 c −79.55 d −3.8
  e 8.7 f −3.24

6 a $-\frac{1}{5}$ b $-\frac{3}{7}$ c $-\frac{9}{11}$ d $-1\frac{1}{6}$ e $-\frac{3}{14}$ f $\frac{1}{8}$

7 a −1.51 b −1.7 c −78.9 d −7.496
  e −0.405 f 3.92 g 4.4 h 4.5
  i −1.4 j −18.2 k $-\frac{1}{3}$ l 0
  m $-\frac{2}{3}$ n $-\frac{3}{20}$ o $\frac{7}{18}$

9 a

−4	6	−8
−6	−2	2
4	−10	0

b

−6.8	17	0
10.2	3.4	−3.4
6.8	−10.2	13.6

c

$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$
0	$-\frac{5}{6}$	$\frac{1}{3}$

10 a $-\frac{2}{3}$ b $-5\frac{7}{10}$ c $\frac{11}{12}$ d −15 e $\frac{2}{3}$ f $-1\frac{1}{2}$

11 a −9 b 0 c −5 d 7 e 22 f 5

12 a 1.25 b −0.26 c −1.88 d −1.9
  e −1 f −7.2

13 a −1 b $\frac{3}{7}$ c $-\frac{13}{14}$ d −1 e $\frac{1}{3}$ f $-\frac{3}{14}$

14 a 12.3 b −5.5
  c $-5.5 - 12.3$ or $12.3 - (-5.5)$ d 17.8 m

15 a

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Difference	6.7	6.3	2.3	3.1	1.9	1.3	2.2

  b Sunday c 1.2°C d −2.2°C e 3.4°C

16 71.7°C; $(+65.2) - (-6\frac{1}{2})$

17 a deposit b withdrawal
  c i He has money in the bank.
    ii He owes the bank money.
  d −$95.35 e $134.65 f −$25.90
  g Yes. A fee is charged.

18 Some examples are: −3.2 and 3.2, $\frac{1}{8}$ and $-\frac{1}{8}$.

19 a Some examples are: 3.86 and −10.36, −3.2 and −3.3.
  b Some examples are: $-\frac{1}{6}$ and $-\frac{1}{3}$, $\frac{3}{4}$ and $-\frac{5}{4}$.

20 Some examples are given.
  a −5 and −4, −12.1 and −2.1
  b −4 and 2, −5 and 3

21 Negative. For sum of two numbers to be zero, they must be opposite numbers. As one number is positive and the other negative, product will be negative (+ × − → −).

Reflect

Possible answer: When performing operations on directed numbers the BIDMAS order of operations apply.

Resources

SupportSheets

SS 3F-1 Adding and subtracting fractions

Focus: To review the methods for adding and subtracting fractions with different denominators

Students work through the addition and subtraction of fractions which have different denominators.

SS 3F-2 Multiplying and dividing fractions

Focus: To review the methods for multiplying and dividing fractions

Resources: coloured pencils

Students work through the multiplication and division of fractions. Students review the process of cancelling and consider the conversion of mixed numbers to improper fractions before calculation.

SS 3F-3 Adding and subtracting decimals

Focus: To review the methods for adding and subtracting decimals

Students work through addition and subtraction of decimals and are reminded of the importance of aligning place values when performing these operations.

SS 3F-4 Multiplying and dividing decimals

Focus: To review the methods for multiplying and dividing decimals

Resources: calculator

Students are guided through the multiplication and division of decimals by whole numbers and decimals.

SS 3F-5 Finding the average

Focus: To calculate the average of a list of positive whole numbers

Resources: calculator (to check working)

Students are reminded of the process required to calculate the average of a group of numbers.

WorkSheet

WS 3F-6 Four operations with directed numbers

Focus: To perform operations on directed numbers

Resources: ruler

Students review the definition of a *directed number* and are guided through each of the four operations, working with either directed fractions or directed decimals.

Investigations

INV 3F-7 Converting temperatures

Focus: To use order of operations rules with directed numbers to convert between degrees Celsius and degrees Fahrenheit

Resources: access to the Internet (optional), calculator (optional)

Students are required to use a nomograph for this task. It would be beneficial if they have completed **INV 3D-4** before commencing the task. Students apply their knowledge of order of operations in a real-life context to convert temperatures in degrees Celsius into degrees Fahrenheit, and vice versa.

INV 3F-8 In a spin

Focus: To use order of operations rules to compare given scoring systems for a game

Students apply their understanding of order of operations to compare scoring systems for a spinning game. As an extension, students consider different spinners and select a suitable one to construct to play the spinning game.

Interactives

3F eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3G Powers of directed numbers

Teaching support for pages 158–63

Teaching strategies

Learning focus

To calculate powers of directed numbers.

To apply understanding of raising directed numbers to a power.

Start thinking!

The task guides students to:

- discover powers of integers
- compare a positive and negative integer raised to the power of 3. They write each in expanded form and discover that the rules for multiplying positive and negative numbers apply.
- explore further examples of integers raised to a power. They can be provided with the BLM **Index and expanded form** (see Resources), which contains the table for this task.
- recognise patterns that can be used to determine whether the result will be positive or negative when raising a negative number to a power.

Differentiated pathways

Below Level	At Level	Above Level
1–3, 4b,c,d,f,g, 6a,b	2, 3, 4a–f, 5, 6a–e, 7, 9, 10, 12, 13, 15, 16	4e–l, 5, 6e–l, 7, 8, 11–21
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q10

Focus: To review index notation and to convert between index notation and expanded notation to calculate the basic numeral

- Direct students to complete **SS 3G-1 Powers of positive whole numbers** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded of the

definition for, and are able to identify, the base and the power (index). They also need to be able to recognise the difference between expressing a number in index notation, expanded notation and as a basic numeral.

- It may be beneficial to refer students to *1H Powers and roots* and *1I Index laws* in order to review these definitions.

At Level

At Level
2, 3, 4a–f, 5, 6a–e, 7, 9, 10, 12, 13, 15, 16

- Demonstrate **3G eTutor** or direct students to do this independently.
- When introducing the topic-specific vocabulary for indices, use vocabulary building exercises such as word walls, student definition posters, Hangman etc. to help students with acquisition of the language.
- Connect powers of negative integers to student prior knowledge of multiplication of two negative numbers (a positive product) and multiplication of a positive number and a negative one (a negative product). Therefore an even power means an even number of multiplications and a positive product whereas an odd power means an odd number of multiplications and a negative product.
- Recording index form and the equivalent expanded form of a number in a table can help students to visualise how these two forms are related and help with memory retention.
- Direct students to **WS 3G-2 Calculating powers of directed numbers** (see Resources) if they need support or more practice calculating powers of directed numbers. **Example 3G-2** shows the thinking process involved in these calculations.
- For Q16, you can assist students to understand the multiplication of numbers in index form with the same base by modelling each index with counters. Numbers with the same base will have the same coloured counters. (This is similar to a strategy shown in Chapter 1.)

  Students can explore what happens in a calculation such as $(–3)^2 \times (–3)^3$, by examining how many counters there are.

  (–3) ●● × (–3) ●●● = (–3) ●●●●●

  or just:

  ●● + ●●● = ●●●●●

To reinforce that the base needs to be the same before indices can be added, use different coloured counters to represent the indices for different bases. Explain that only counters of the same colour can be added.

Students can also explore the index law for division using counters. For $(–3)^5 \div (–3)^2$, they could show a fraction with five counters as the index for the numerator and two counters as the index for the denominator. Students remove as many pairs of the same colour from the top and bottom as possible. Then they count the number of counters remaining.

$$\frac{(-3)\bullet\bullet\bullet\bullet\bullet}{(-3)\bullet\bullet} = (-3)\bullet\bullet\bullet$$

or

$$\bullet\bullet\bullet\bullet\bullet - \bullet\bullet = \bullet\bullet\bullet$$

Students can also use counters of different colours to represent the indices of numbers having different bases, to show that these indices cannot be subtracted.

- For a problem-solving task and investigation, direct students to **INV 3G-3 Dice cubed** (see Resources).

Below Level

Below Level
1–3, 4b,c,d,f,g, 6a,b

- Demonstrate **3G eTutor** or direct students to do this independently.
- Direct students to complete **SS 3G-1 Powers of positive whole numbers** (see Resources) if they have difficulty converting powers to basic numerals, or require more practice at this skill.

POTENTIAL DIFFICULTY
Students often mistake the power as a multiplier of the base number. For example, 4^2 is believed to be 4 × 2 instead of 4 × 4.

- Allow students to work with powers of positive integers if they are not yet able to understand the concept that multiplication of two negative integers generates a positive result.
- In real life, powers exist often in measurement and are used to describe either really large (positive powers) or really small (negative powers) numbers. Support student understanding by asking students to choose whether they would prefer to have $1 000 000 today or $1 today and doubled every day for 1 month. Record the doubling

as consecutive multiplication by 2 to emphasise the size of the number.

Above Level

Above Level
4e–l, 5, 6e–l, 7, 8, 11–21

- Demonstrate **3G eTutor** or direct students to do this independently.
- Encourage students to create their own rule regarding the result of raising negative integers to even powers (positive result) or to odd powers (negative result). Articulating understanding helps students to embed conceptual knowledge.

  It will support student solving of algebraic equations in the future if they are able to.
- When completing Q20, advise students to use a calculator to check their answers. It's important they understand the implications of inserting/not inserting brackets in such calculations.
- For more problem-solving tasks and investigations, direct students to **INV 3G-3 Dice cubed** and/or **INV 3H-4 Number cruncherama** (see Resources).

Extra activities

1 Write the rule which applies to multiplying two numbers each with a different sign. (The result will be negative.)

2 Write the rule which applies to raising a negative integer to an odd power. (The result will be negative.)

3 Write the rule which applies to multiplying two numbers that are both negative. (The result will be positive.)

4 Write the rule which applies to raising a negative integer to an even power. (The result will be positive.)

5 Calculate the basic numeral for each of the following:

a $(-3)^4$ (81)

b $\left(\frac{1}{4}\right)^2$ $\left(\frac{1}{16}\right)$

c $\left(-\frac{2}{3}\right)^3$ $\left(-\frac{8}{27}\right)$

d $(-0.75)^6$ (≈ 0.178)

e $(-5)^3$ (-125)

f $\left(\frac{1}{2}\right)^4$ $\left(\frac{1}{16}\right)$

g $\left(-\frac{2}{7}\right)^2$ $\left(\frac{4}{49}\right)$

h $(-0.2)^3$ (-0.008)

Answers

3G Powers of directed numbers

3G Start thinking!

1 a $2 \times 2 \times 2$ b 8

2 a $(-2) \times (-2) \times (-2)$ b -8

c negative; product of first pair is positive, then multiplying this by a negative produces a negative result.

3, 4

Index form	Base	Index or power	Expanded form	Basic numeral
5^2	5	2	5×5	25
$(-5)^2$	-5	2	-5×-5	25
4^3	4	3	$4 \times 4 \times 4$	64
$(-4)^3$	-4	3	$-4 \times -4 \times -4$	-64
2^4	2	4	$2 \times 2 \times 2 \times 2$	16
$(-2)^4$	-2	4	$-2 \times -2 \times -2 \times -2$	16
3^5	3	5	$3 \times 3 \times 3 \times 3 \times 3$	243
$(-3)^5$	-3	5	$-3 \times -3 \times -3 \times -3 \times -3$	-243

5 A negative base raised to an even power gives a positive result. A negative base raised to an odd power gives a negative result.

6 Because 5 and -5 are raised to an even power (same positive result) while 4 and -4 are raised to an odd power (one positive result, one negative result).

7 yes, as the power is even

Exercise 3G Powers of directed numbers

1 a 125 b 49 c -27 d 10 000 2 E

3 a 5^3 b $(-7)^2$ c $(-3)^3$ d $(-10)^4$

4 a $-9 \times -9 = 81$ b $8 \times 8 = 64$

c $-6 \times -6 \times -6 = -216$

d $7 \times 7 \times 7 = 343$

e $-5 \times -5 \times -5 \times -5 = 625$

f $1 \times 1 \times 1 \times 1 = 1$

g $4 \times 4 \times 4 \times 4 \times 4 = 1024$

h $-10 \times -10 \times -10 \times -10 \times -10 = -100\,000$

i $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

j $-3 \times -3 \times -3 \times -3 \times -3 \times -3 = 729$

k $-2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 = -128$

l $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1 = 1$

5 a negative b positive c positive

d positive

6 a $-1.2 \times -1.2 = 1.44$ b $0.9 \times 0.9 = 0.81$

c $-0.4 \times -0.4 \times -0.4 = -0.064$

d $-0.6 \times -0.6 \times -0.6 \times -0.6 = 0.1296$

e $-0.2 \times -0.2 \times -0.2 \times -0.2 \times -0.2 = -0.000\,32$

f $-0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 = -0.000\,000\,1$

g $-\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2} = -\frac{1}{8}$ h $-\frac{5}{9} \times -\frac{5}{9} = \frac{25}{81}$

i $-\frac{4}{7} \times -\frac{4}{7} \times -\frac{4}{7} = -\frac{64}{343}$

j $-\frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3} = \frac{1}{81}$

k $-\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = -\frac{32}{243}$

l $-\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} = \frac{729}{64} = 11\frac{25}{64}$

6 a $-1.2 \times -1.2 = 1.44$ b $0.9 \times 0.9 = 0.81$

c $-0.4 \times -0.4 \times -0.4 = -0.064$

d $-0.6 \times -0.6 \times -0.6 \times -0.6 = 0.1296$

e $-0.2 \times -0.2 \times -0.2 \times -0.2 \times -0.2 = -0.000\,32$

f $-0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 = -0.000\,000\,1$

g $-\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2} = -\frac{1}{8}$ h $-\frac{5}{9} \times -\frac{5}{9} = \frac{25}{81}$

i $-\frac{4}{7} \times -\frac{4}{7} \times -\frac{4}{7} = -\frac{64}{343}$

j $-\frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3} = \frac{1}{81}$

k $-\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = -\frac{32}{243}$

l $-\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} = \frac{729}{64} = 11\frac{25}{64}$

7 negative, positive

8 a $(-4)^5, (-2)^3, (-3)^2, (+5)^4$

b $(-2)^7, (-1)^{10}, (-7)^2, 10^2$

c $(-0.1)^3, (0.2)^3, (-0.2)^2, (-0.3)^2$

d $\left(-\frac{1}{2}\right)^5, \left(-\frac{1}{3}\right)^3, \left(-\frac{1}{2}\right)^4, \left(-\frac{1}{3}\right)^2$

9 a $(-5)^3 \times (-9)^4$ b $(-4)^6 \times 3^3$

c $7^2 \times (-6)^6$ d $(-8)^4 \times (-10)^2$

10 a $-2 \times -2 \times -2 \times -4 \times -4 = -128$

b $-5 \times -5 \times 3 \times 3 \times 3 \times 3 = 2025$

c $-10 \times -10 \times -10 \times -2 \times -2 \times -2 \times -2 \times -2 = 32\,000$

d $-3 \times -3 \times -3 \times -3 \times -3 \times -1 \times -1 = -243$

e $-6 \times -6 \times -2 \times -2 \times -2 = -288$

f $-1 \times -1 \times -1 \times -1 \times 3 \times 3 = 9$

g $-7 \times -7 \times -1 \times -1 \times -1 \times -1 \times -1 = -49$

h $-3 \times -3 \times -3 \times -2 \times -2 = -108$

i $-1 \times -1 \times -1 \times -2 \times -2 \times -2 \times -2 \times -2 = 32$

11 a 17 b 10 049 c 96 d -7

e 125 f 4 g 48 h 0.159

i -1.218 j $-\frac{1}{72}$ k $-\frac{19}{32}$

l $-0.000\,002\,7$ m 4 n $-\frac{2}{27}$ o $-\frac{45}{32}$

12 a i -1 ii 1 iii -1 iv 1 v -1 vi 1

b those with even index or power (ii, iv, vi)

c those with odd index or power (i, iii, v)

d Negative numbers raised to an even power will give a positive result and negative numbers raised to an odd power will give a negative result.

e i negative ii positive iii positive

iv negative v positive vi negative

13 a i 9 ii -27 iii -243

b -243 c same result

d i -8 ii -32 iii 256

e 256 f same result

g To multiply numbers in index form with the same base, add the powers.

h i $(-4)^3 \times (-4)^2, (-4)^5, -1024$

ii $(-2)^4 \times (-2)^3, (-2)^7, -128$

iii $(-0.1)^5 \times (-0.1)^3, (-0.1)^8, 0.000\,000\,01$

iv $\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{2}\right)^2, \left(-\frac{1}{2}\right)^5, -\frac{1}{32}$

i Bases must be the same.

14 a i 729 ii 81 iii 9
b 9 c same result
d i −128 ii 16 iii −8
e −8 f same result
g To divide numbers in index form with the same base, subtract the powers.
h i $(-5)^8 \div (-5)^5, (-5)^3, -125$
ii $(-6)^9 \div (-6)^7, (-6)^2, 36$
iii $(-0.2)^{10} \div (-0.2)^6, (-0.2)^4, 0.0016$
iv $\left(-\frac{4}{5}\right)^{12} \div \left(-\frac{4}{5}\right)^9, \left(-\frac{4}{5}\right)^3, -\frac{64}{125}$
i Bases must be the same.

15 a $-8 \div -8 = 1$ b $(-2)^0$ c yes; $(-2)^0 = 1$

16 a −8 b −216 c 10 000 d 1
e −27 f 49 g 1 h −3

17 a 5 and −5; both numbers give the result of 25 when squared.
b No; to square a number is to multiply the number by itself. Either a negative number is squared ($- \times - \rightarrow +$) or a positive number is squared ($+ \times + \rightarrow +$). In both cases the result is positive.
c only one number (2)
d only one number (−2)
e There will be two numbers that when squared (or raised to an even power) give the same positive result but no number can be squared to give a negative result. There will only be one number that when cubed (or raised to an odd power) results in a given answer.

ANSWERS

f Two; raising both 2 and −2 to the power of 4 gives the result of 16. No number can be raised to the power of 4 to give a negative result.
g There will be two numbers that when raised to an even power (such as 2, 4, 6, 8, etc.) give the same positive result. For example, $2^6 = 64$ and $(-2)^6 = 64$. No number raised to an even power gives a negative result. There will only be one number that when raised to an odd power (such as 3, 5, 7, etc.) results in a given answer. For example,
$2^5 = 32$ and $(-2)^5 = -32$.

18 a 7 and −7 b 9 and −9 c 1 and −1
d 2 and −2 e 8 and −8 f 10 and −10

19 a 3 b −5 c 4 d −1 e −4 f −10

20 a $(+3)^2 = 9$, $(-3)^2 = 9$, $3^2 = 9$ and $-3^2 = -9$
b $(-3)^2 = -3 \times -3 = +9$
while $-3^2 = -(3^2) = -(3 \times 3) = -9$
c i yes ii no iii no iv yes
d A number of the form $-x^a$ will always produce a negative result regardless of whether the power is odd or even. A negative base raised to an odd power will also produce a negative result.

21 a −73 b −10.99 c $\frac{4}{27}$

Reflect

Possible answer: When raising a positive number to a power, the basic numeral will be positive. When raising a negative number to an even power, the basic numeral will be positive. When raising a negative number to an odd power, the basic numeral will be negative.

Resources

SupportSheet

SS 3G-1 Powers of positive whole numbers

Focus: To work with numbers in index form and determine the numerical value they each represent

Resources: coloured pencils

Students review the terms *base*, *power* and *basic numeral*. They are guided through a review of index notation being a series of repeated multiplications. Students complete practice questions independently, including questions in which they calculate the basic numeral to compare numbers written in index form.

WorkSheet

WS 3G-2 Calculating powers of directed numbers

Focus: To calculate powers of positive and negative whole numbers, fractions and decimals

Students convert between expanded form and index form, with bases that are positive and negative whole numbers, fractions or decimals. Students observe the relationship between an odd or even power and the direction of the basic numeral.

Investigations

INV 3G-3 Dice cubed

Focus: To use dice to generate numbers involving decimals and fractions raised to a power

Resources: three dice

Students consider decimals raised to a given power within the context of a dice game. As an extension, students play another game using three dice rather than two, to create fractions raised to a power.

INV 3G-4 Number Cruncherama

Focus: To use the idea of a number crunching machine to perform operations with powers

Students perform calculations involving powers using the idea of a 'number crunching machine'. They start with a given number and progress through a sequence of steps to produce a result. As an extension, students design their own number crunching sequence.

BLM

Index and expanded form

Interactives

3G eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

3H The Cartesian plane

Teaching support for pages 164–9

Teaching strategies

Learning focus

To use understanding of the four quadrants of the Cartesian plane to plot and describe points on the Cartesian plane.

To apply understanding of the four quadrants of the Cartesian plane to plot and describe points on the plane.

Start thinking!

The task guides students to:

- review the four quadrants of the Cartesian plane to plot and describe points. Students can be provided with the BLM **Cartesian plane** (see Resources) needed for the task.
- consider the name for each of the axes and also the coordinates of the origin
- review how to plot points on the Cartesian plane and how to state the coordinates of a plotted point.

Differentiated pathways

Below Level	At Level	Above Level
1, 3, 5, 6b, 7	1–5, 6a–c, 7, 8, 10	5, 6, 8–12
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q11

Focus: To identify and write the coordinates of points located in the first quadrant of the Cartesian plane

- Direct students to complete **SS 3H-1 The Cartesian plane (positive whole numbers)** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so students are reminded of numbering and labelling conventions for the first quadrant of the Cartesian plane. It may be beneficial to complete the following activity.

Draw a life-size Cartesian plane (first quadrant only) on the ground, with the scale on each axis extending to 10. Ask a student to stand at the origin and say the coordinates of this point. Have each student 'plot' a point by walking along the horizontal axis, and then parallel to the vertical axis. Plot a range of points. Discuss each point with reference to its coordinates.

Still using the life-size Cartesian plane, stand on the point (3, 9). Ask students to tell you the coordinates of the point. This is a very powerful learning tool if the coordinates are given in the incorrect order.

- Remind students that, when writing coordinates, they need to 'run before they jump'. In other words, they need to move horizontally along the *x*-axis, before they move up or down parallel to the vertical axis.

At Level

At Level
1–5, 6a–c, 7, 8, 10

- Demonstrate **3H eTutor** or direct students to do this independently.

POTENTIAL DIFFICULTY
Some students do not 'see' that a point on the *x*-axis has a value of zero for its *y*-coordinate or that a point on the *y*-axis has a value of zero for its *x*-coordinate.

- Draw a life-size Cartesian plane (all four quadrants) on the ground. Starting at zero, number the positive *x*-axis, and then number the positive *y*-axis. Number the negative *x*-axis, explaining that the numbering corresponds to the number of steps taken away from zero. Repeat for the negative *y*-axis. Once the axes are numbered, walk to the point (–5, –7). Discuss the coordinates of the point. Have students 'plot' a range of points on your life-size Cartesian plane.

- You can also stand on the Cartesian plane and ask the students to give the coordinates for your position. Ensure that the coordinates are given in the correct order.

- Remind students of a strategy used to remember the order of coordinates: 'run left or right before you jump up or down'.

- Help students to identify fractional numbers between grid lines by asking students to verbally count in halves or quarters to see which fractional number comes first. For example: 2, $2\frac{1}{4}$, $2\frac{2}{4}$, $2\frac{3}{4}$, 3.

- Direct students to **WS 3H-2 The Cartesian plane (directed numbers)** (see Resources) if they need support or more practice reading and plotting points on the Cartesian plane. **Example 3H-1** and **Example 3H-2** show the thinking process involved.

- For some problem-solving tasks and investigations, direct students to **INV 3H-3 Cartoon images** and/or **INV 3H-4 Cartesian targets** (see Resources).

Below Level

Below Level
1, 3, 5, 6b, 7

- Demonstrate **3H eTutor** or direct students to do this independently.
- Direct students to complete **SS 3H-1 The Cartesian plane (positive whole numbers)** (see Resources) if they have difficulty identifying or plotting points in the first quadrant of the Cartesian plane, or require more practice at this skill.

POTENTIAL DIFFICULTY
When graphing, there are four mistakes that students commonly make; the *x*-axis is confused for the *y*-axis; the axes are not numbered from the origin; the scales for the axis are wrong or inconsistent and the space between the grid lines is numbered instead of the lines themselves as if a column graph is being generated.

- Ensure that students can operate effectively in the first quadrant of the Cartesian plane before introducing negative numbers.
- Use a unit scale on both axes as much as possible to support students for whom the cognitive load of determining missing numbers is too great.
- Direct students to **WS 3H-2 The Cartesian plane (directed numbers)** (see Resources) if they need support or more practice reading and plotting points on the Cartesian plane. **Example 3H-1** and **Example 3H-2** show the thinking process involved.
- For some problem-solving tasks and investigations, direct students to **INV 3H-3 Cartoon images** and/or **INV 3H-4 Cartesian targets** (see Resources).

Above Level

Above Level
5, 6, 8–12

- Demonstrate **3H eTutor** or direct students to do this independently.
- Students who are already able to competently use Cartesian graphs should be encouraged to use the information on the graph to extrapolate or predict coordinates outside this information.
- Increase cognitive difficulty by asking students to use scaled axes.
- Pair students and ask them to describe what effect reflection in the *x*-axis or the *y*-axis

has on a point or all of the coordinates of a given shape.

- As an extension to Q9, have students choose their own image to produce a list of instructions which can be used to produce an enlarged drawing.
- Q12 introduces the concept of quadratic relationships. Students should be encouraged to think beyond the familiar linear relationships they have been exposed to so far.
- For more problem-solving tasks and investigations, direct students to **INV 3H-3 Cartoon images** and/or **INV 3H-4 Cartesian targets** (see Resources).

Extra activities

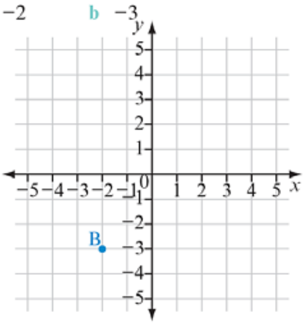
As a whole class activity, students could play a game of ‘Battleships’. The following ‘ships’ are used: 3 tugboats (2 crosses), 3 battleships (3 crosses), 2 gunboats (4 crosses) and 2 aircraft carriers (5 crosses). Players take turns at saying coordinates to each other, trying to hit their opponent’s hidden fleet, until one player has lost all of their vessels.

Answers

3H The Cartesian plane

3H Start thinking!

1 a i x-axis ii y-axis b (0, 0)
2 a 3 b −4 c (3, −4) d 4
3 a −2 b −3
c

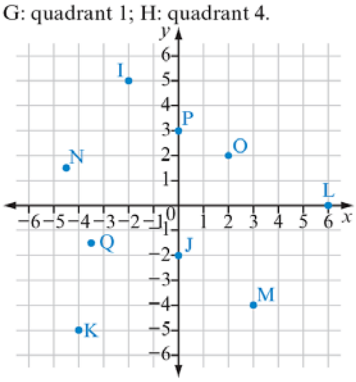


d 3

4 One possible answer is: quadrant 1: (1, 2), quadrant 2: (−1, 2); quadrant 3: (−1, −2); quadrant 4: (1, −2).

Exercise 3H The Cartesian plane

1 A(5, −4), B(−2, 0), C(−3, 3), D(−4, −3), E(0, 5), F(−6, 4), G(4.5, 2), H(1.5, −4.5)
2 A: quadrant 4; B: x-axis; C: quadrant 2; D: quadrant 3; E: y-axis; F: quadrant 2; G: quadrant 1; H: quadrant 4.
3



4 I: quadrant 2; J: y-axis; K: quadrant 3; L: x-axis; M: quadrant 4; N: quadrant 2; O: quadrant 1; P: y-axis; Q: quadrant 3.
5 a

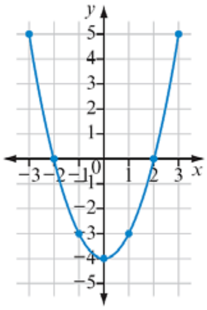
x-coordinate	−6	−4	−2	0	2	4	6
y-coordinate	−3	−2	−1	0	1	2	3
coordinates	(−6, −3)	(−4, −2)	(−2, −1)	(0, 0)	(2, 1)	(4, 2)	(6, 3)

b

x-coordinate	−2	−1	0	1	2	3	4
y-coordinate	−7	−5	−3	−1	1	3	5
coordinates	(−2, −7)	(−1, −5)	(0, −3)	(1, −1)	(2, 1)	(3, 3)	(4, 5)

6 a i (−3, 5), (−2, 0), (−1, −3), (0, −4), (1, −3), (2, 0), (3, 5)

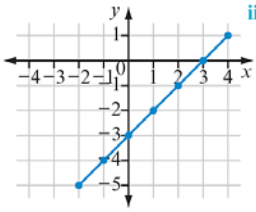
ii



iii curved line

b i (−2, −5), (−1, −4), (0, −3), (1, −2), (2, −1), (3, 0), (4, 1)

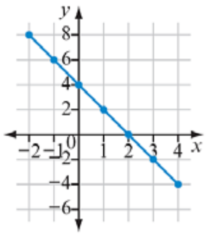
ii



iii straight line

c i (−2, 8), (−1, 6), (0, 4), (1, 2), (2, 0), (3, −2), (4, −4)

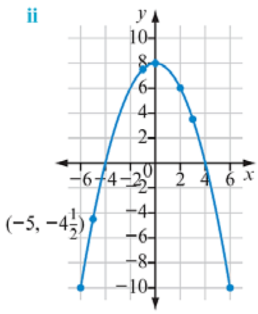
ii



iii straight line

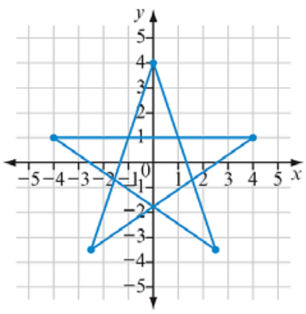
d i (−6, −10), (−5, $-4\frac{1}{2}$), (−1, $7\frac{1}{2}$), (0, 8), (2, 6), (3, $3\frac{1}{2}$), (6, −10)

ii

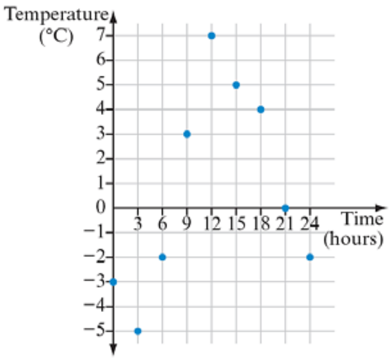


iii curved line

7



10 a–d



e −5°C; at 3 hours after midnight (3.00 am)
f 7°C; at 12 hours after midnight (12.00 pm or midday)
g 12°C h 0.78°C

11 a i

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−1	0	1	2	3	4	5

ii

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−6	−4	−2	0	2	4	6

iii

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−6	−5	−4	−3	−2	−1	0

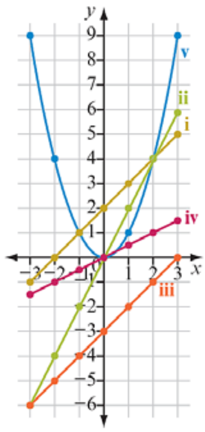
iv

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−1.5	−1	−0.5	0	0.5	1	1.5

v

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	9	4	1	0	1	4	9

b



12 a Each x-coordinate is squared and then has 4 subtracted from it to produce the matching y-coordinate.
b Each x-coordinate has 3 subtracted from it to produce the matching y-coordinate.
c Each x-coordinate is multiplied by −2 and then has 4 added to it to produce the matching y-coordinate.
d Each x-coordinate is squared, then divided by −2 and then has 8 added to it to produce the matching y-coordinate.

Reflect

Possible answer: Positive numbers are used to indicate how far right along the x-axis or, alternatively, how far up the y-axis a point is located from the origin. Negative numbers are used to indicate how far left along the x-axis or, alternatively, how far down the y-axis a point is located from the origin.

Resources

SupportSheet

SS 3H-1 The Cartesian plane (positive whole numbers)

Focus: To identify and plot whole-number coordinates of points in the first quadrant of the Cartesian plane

Resources: ruler, **1-cm grid paper** (BLM)

Students review a Cartesian plane on which there are positive numbers only. The plotting and reading of coordinate points are reviewed. Students are also required to specify each of the numbers given in the coordinate pair as being either the x-coordinate or the y-coordinate.

WorkSheet

WS 3H-2 The Cartesian plane (directed numbers)

Focus: To read and plot points involving directed numbers in all four quadrants of the Cartesian plane

Resources: ruler, **1-cm grid paper** (BLM)

Students review the structure of the Cartesian plane and identify the *x*-axis, the *y*-axis and the origin. Students plot and read points that involve directed numbers. Classification of each quadrant is also required.

Investigations

INV 3H-3 Cartoon images

Focus: To use the Cartesian plane to enlarge and reduce a picture by plotting points in all four quadrants

Resources: tracing paper, ruler, coloured pencils, grid or graph paper, sticky tape

Students apply their understanding of plotting points in the four quadrants on a Cartesian plane to create a list of instructions for a classmate to follow. These instructions give the coordinates and the order in which points are to be plotted to produce the image of a cartoon figure. As an extension, they consider producing instructions to enlarge or reduce the size of the cartoon image.

INV 3H-4 Cartesian targets

Focus: To use the Cartesian plane to locate points and play a game of ‘Target’

Resources: coloured pencils, grid or graph paper, partner, two decks of playing cards

Before commencing this game, remove jacks, queens and kings from the decks of cards. An ace represents the number one. Students plot points in the four quadrants of the Cartesian plane within the context of playing a card game. A partner is required. As an extension, students play another game, with a different aim and new rules.

BLMs

1-cm grid paper

Cartesian plane

Interactives

3H eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 170–3

Additional teaching strategies

Multiple-choice

1 Answer: C. –2 is an integer between –3.5 and 3.5.
A: –4 is less than –3.5.
B: –2.5 is not an integer because it has a decimal component.
D: 5 is greater than 3.5.

2 Answer: A. (+12) + (–19) = –7
B: (+12) + (–5) = +7
C: (+12) + (+5) = +17
D: (+12) + (+19) = +31

3 Answer: C. (–24) – (+20) = –24 – 20 = –44
A: may have completed the incorrect calculation (–24) + (+20).
B: may have completed the incorrect calculation (+24) – (+20).
D: may have completed the incorrect calculation (+24) + (+20).

4 Answer: D. (–35) – (–47) = –35 + 47
A: may have incorrectly combined highlighted signs: (–35) – (–47) as a negative sign.
B: may have incorrectly combined highlighted signs: (–35) – (–47) to make 35 positive
C: may have incorrectly combined highlighted signs: (–35) – (47) making 35 and 47 positive

5 Answer: D. 5 – 12 + 3 – 7 + 6 = –5
A: –4 – 11 + 16 – 2 + 6 = 5
B: 8 – 13 – 1 + 17 – 6 = 5
C: –7 + 5 – 9 + 22 – 6 = 5

6 Answer: B. Dividing a positive number by a negative number will give a negative number and 8 ÷ 2 = 4, so 8 ÷ –2 = –4.

7 Answer: B. –9 + 4 = –5, 4 × (–9) = –36
A: –4 + 9 = +5, –4 × 9 = –36
C: 4 + 9 = +13, 4 × 9 = +36
D: (–9) + (–4) = –13, (–9) × (–4) = +36

8 Answer: B.

$\frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$

9 Answer: D. Dividing a negative number by a negative number will give a positive number.

A: $(-1.2) + (-2.4) = -1.2 - 2.4 = -3.6$

B: Multiplying a negative number by a positive number will give a negative number.

C: $(-4.9) - (-1.1) = -4.9 + 1.1 = -3.8$

10 Answer: A. $(-2)^6 = 64$

B: $(+6)^2 = 36$

C: $(-1)^{10} = 1$

D: $(-4)^3 = -64$

11 Answer: C. $(-5)^8 \times (-5)^4 = (-5)^{8+4} = (-5)^{12}$

A: divided indices instead of adding indices

B: subtracted indices instead of adding indices

D: multiplied indices instead of adding indices

12 Answer: C. A point with both coordinates negative lies in quadrant 3.

A: may have incorrectly considered the point (+3, +8). A point with both coordinates being positive will be plotted in quadrant 1.

B: may have incorrectly considered the point (+3, –8). A point with the *x*-coordinate being positive and the *y*-coordinate being negative will be plotted in quadrant 4.

D: may have incorrectly considered the point (–3, +8). A point with the *x*-coordinate being negative and the *y*-coordinate being positive will be plotted in quadrant 2.

Short answer

1 **a** $-5 > +2$: False, +2 is greater than –5.

b $-10 < -8$: True, –10 is less than –8.

c $0 > -3.5$: True, 0 is greater than –3.5

d $7\frac{1}{2} < -7\frac{1}{2}$: False, any negative number will be less than any positive number.

2 Ascending means from smallest to largest.

a –8, –4, –2, 0, 4, 8

b –19, –9, –1, 0, $5\frac{1}{3}$, 5.5

3 **a** $(-5) + (-4) = -9$

b (+2) + (+7) = +9

c (+22) + (–34) = –12

d (–50) + (+69) = +19

4 –$28 + $150 = +$122, therefore her new account balance is $122.

5 **a** (+3) – (+8) = +3 – 8 = –5

b (–1) – (–9) = +3 – 8 = 1 + 9 = +8 or 8

c (–46) – (+35) = +3 – 8 = –46 – 35 = –81

d (+71) – (+53) = +71 – 53 = +18 or 18

6 18 – (–3) = 18 + 3 = 21, therefore the temperature difference is 21°C.

7 **a** –8 + 7 = –1

b –4 – 6 = –10

c 17 – 25 = –8

d –44 +34 = –10

e –66 + 66 = 0

f –50 – 50 = –100

8 **a** +7 × –8 = –56

b –5 × –9 = +45

c –36 ÷ +4 = –9

d –100 ÷ –20 = +5

e –12 × 0 = 0

f 4 × –15 = –60

g 18 ÷ –6 = –3

h –42 ÷ –3 = +14

9 **a** –6 × 3–5= –18 × –10 = 180

b 20 ÷ –4 × 7 = –5 × 7 = –35

c $18 \div -3 \times 2 = -6 \times 2 = -12$

d $-5 \times -2 \times -4 = 10 \times -4 = -40$

10 **a** $\left(-\frac{3}{4}\right)-\left(-\frac{1}{2}\right)=-\frac{3}{4}+\frac{2}{4}=-\frac{1}{4}$

b $\left(+\frac{2}{5}\right)-\left(+\frac{1}{3}\right)=\frac{6}{15}-\frac{5}{15}=\frac{1}{15}$

c $\frac{3}{7}-1\frac{2}{7}=\frac{3}{7}-\frac{9}{7}=-\frac{6}{7}$

d $-2\frac{1}{6}+6\frac{2}{3}=-2-\frac{1}{6}+6+\frac{2}{3}=-2-\frac{1}{6}+6+\frac{4}{6}=+4+\frac{3}{6}=4\frac{1}{2}$

e $\frac{1}{2}\div\frac{4}{7}-\frac{2}{5}=\frac{1}{2}\times\frac{4}{7}-\frac{2}{5}=\frac{2}{7}-\frac{2}{5}=\frac{10}{35}-\frac{14}{35}=-\frac{4}{35}$

f $-\frac{1}{8}+\frac{2}{3}\times-\frac{9}{8}=-\frac{1}{8}-\frac{3}{4}=-\frac{1}{8}-\frac{6}{8}=-\frac{7}{8}$

11 **a** $(+5.7) + (-6.2) = 5.7 - 6.2 = -0.5$

b $(-0.9) - (-0.64) = -0.9 + 0.64 = -0.26$

c $7.45 - 9.38 = -1.93$

d $-12.5 - 11.6 = -24.1$

e $-3.2 \div -0.2 - 16 = -32 \div -2 - 16 = 16 - 16 = 0$

f $0.7 \times -0.2 - 0.6 \times -0.3 = -0.14 - (-0.18) = -0.14 + 0.18 = 0.04$

12 Average = $(-4.2 + 5.6 + 7.1 + -9.3) \div 4 = (-0.8) \div 4 = -0.2$

13 **a** $(-2)^5 = -32$

b $(-3)^2 = +9$ or 9

c $(-1)^9 = -1$

d $(-10)^4 = +10\,000$ or $10\,000$

14 **a** $(-9)^8 \times (-9)^5 \div (-9)^{11} = (-9)^{13} \div (-9)^{11} = (-9)^2 = +81$ or 81

b $(-6)^7 \times (-6)^4 \div (-6)^{11} = (-6)^{11} \div (-6)^{11} = (-6)^0 = +1$ or 1

15 A $(3, -4)$

B (0, 4)

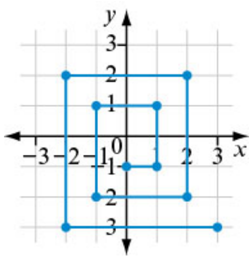
C (–2, 0)

D (–3, –2)

E $(1\frac{1}{2}, 2\frac{1}{2})$

F $(-3\frac{1}{2}, 3\frac{1}{2})$

16



The next four points are (3, 3), (–3, 3), (–3, –4), (4, –4).

NAPLAN-style practice

Multiple-choice options have been listed as A, B, C and D for ease of reference.

1 Answer: A. 3 is the largest number. On a number line 3 is furthest to the right.
B: incorrectly selected the smallest number.
C: incorrectly identified the smallest positive number.
D: incorrectly identified the largest negative number.
Refer to *3A Understanding negative numbers*.

2 Answer: B. –4.7 is smaller than –4.5. On a number line –4.7 is furthest to the left.
$-4\frac{2}{5}$ = –4.4 and is not as negative as –4.5.
A: incorrectly identified this as being smaller as it is further to the right
C: incorrectly selected the largest number
D: incorrectly identified this as being smaller as it is further to the right, or may have incorrectly converted the fraction to a decimal. (–4.4)
Refer to *3A Understanding negative numbers*.

3 Answer: B. (+5) + (–9) = 5 – 9 = –4
A: incorrectly calculated (–5) + (–9) = –14
C: incorrectly calculated (–5) + (+9) = +4
D: incorrectly calculated (+5) + (+9) = +14

Refer to *3B Adding integers*.

4 Answer: C. (–6) – (–7) = –6 + 7 = +1 or 1
A: incorrectly calculated (–6) + (–7) = –13
B: incorrectly calculated (+6) + (–7) = –1
D: incorrectly calculated (+6) + (+7) = +13
Refer to *3C Subtracting integers*.

Q5–9 refer to *3D Simplifying addition and subtraction of integers*.

5 Answer: B. (+3) – 5 = 3 – 5 = –2, or second level below ground level.
A: incorrectly calculated (+5) + (–3) = 5 – 3 = 2 (second level above ground level)
C: incorrectly calculated (+5) + (+3) = 5 + 3 = 8 (eighth level above ground level)
D: incorrectly calculated (–5) + (–3) = –5 – 3 = –8 (eighth level below ground level)

6 17 + (–42) = –25

7 –36 – 44 = –80

8 50 m + 12 m = 62 m or 50 m – (–12 m) = 50 m + 12 m = 62 m

9 Answer: C. –16°C + 24°C = 8°C
A: incorrectly calculated (–16) + (–24) = –40°C
B: incorrectly calculated (+16) + (–24) = –8°C
D: incorrectly calculated (+16) + (+24) = 40°C

Q10–12 refer to *3E Multiplying and dividing integers*.

10 –4 × –3 = 12

11 x × –5 = –30; x = –30 ÷ –5 = 6

12 Answer: C. –4 + 7 = +3, –4 × 7 = –28
A: –6 + 5 = –1, –6 × 5 = –30
B: –8 + (–3) = –11, –8 × –3 = +24
D: 2 + 9 = 11, 2 × 9 = 18

13 Average = (–5°C + 2°C + 4°C + (–3°C) + (–1°C)) ÷ 5 = –3°C ÷ 5 = –0.6°C
Refer to *3F Operations with directed numbers*.

14 \$124.80 + \$75.00 – \$42.75 – \$149.95 – \$52.20 + \$150 = \$104.90
There is \$104.90 in the bank at the end of the month.
Refer to *3F Operations with directed numbers*.

15 $-\frac{3}{8}\times\frac{4}{9}\times-\frac{2}{7}=-\frac{1}{1}\times\frac{1}{3}\times-\frac{1}{7}=\frac{1}{21}$

Refer to *3F Operations with directed numbers*.

Q16–18 refer to *3G Powers of directed numbers*.

16 $(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$

17 Answer: A. $(-5)^6 = -5 \times -5 \times -5 \times -5 \times -5 \times -5 = 15\ 625$ (The power is even.)
All the other powers are odd and therefore will give a negative result.
B: $(-3)^5 = -3 \times -3 \times -3 \times -3 \times -3 = -243$ (Power is odd)
C: $(-2)^7 = -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 = -128$ (Power is odd)
D: $(-4)^3 = -4 \times -4 \times -4 = -64$ (Power is odd)

18 Answer: D. $(-3)^{10} \div (-3)^7 = (-3)^3 = -27$
A: incorrectly raised (–3) to the power of zero
B: incorrectly multiplied –3 by 3, rather than raising – 3 to the power of 3
C: incorrectly written the response as positive; when the power is odd, the result will be negative.

Q19–27 refer to *3H The Cartesian plane*.

19 The *x*-coordinate is always first in a pair of coordinates; that is, –5.

20 The origin is where the *x*-axis and *y*-axis meet, at (0, 0).

21 Answer: C. Point F has the coordinates (2, –2).
A: A is at (–3, 3).
B: B is at (–2, –2).
D: G is at (3, 3).

22 Point A has the coordinates (–3, 3).

23 Point E has the coordinates (1, –5).

24 Point D has the coordinates (0, –6).

25 Points B and C are in the third quadrant.

26 Point G(3, 3) has the same *y*-coordinate (3) as point A(–3, 3).

27 Table 4 matches the points plotted on the Cartesian plane.

x	–3	–2	–1	0	1	2	3
y	3	–2	–5	–6	–5	–2	3
(*x*, *y*)	(–3, 3)	(–2, –2)	(–1, –5)	(0, –6)	(1, –5)	(2, –2)	(3, 3)

Analysis

a **i** The highest temperature was on Wednesday. (Maximum: 7.8°C)

ii The lowest temperature was on Friday. (Minimum: –5.1°C)

b 6.5°C – 0.7°C = 5.8°C

c Tuesday: 4.1°C – (–2.2°C) = 4.1°C + 2.2°C = 6.3°C
Wednesday: 7.8°C – (–0.9°C) = 7.8°C + 0.9°C = 8.7°C
Thursday: 2.5°C – (–3.4°C) = 2.5°C + 3.4°C = 5.9°C
Friday: –0.6°C – (–5.1°C) = –0.6°C + 5.1°C = 4.5°C
Saturday: 1.8°C – (–3.9°C) = 1.8°C + 3.9°C = 5.7°C
Sunday: 5.3°C – 0.2°C = 5.1°C

d The biggest range of temperatures was on Wednesday.

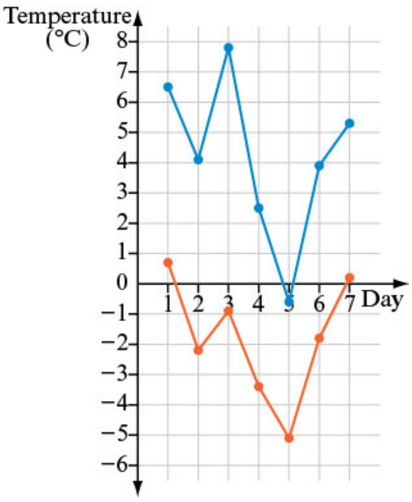
e **i** Average minimum temperature:
[0.7 + (–2.2) + (–0.9) + (–3.4) + (–5.1) + (–1.8) + 0.2] ÷ 7 = (–12.5) ÷ 7 ≈ –1.8°C

ii Average minimum temperature:

[6.5 + 4.1 + 7.8 + 2.5 + (–0.6) + 3.9 + 5.3] ÷ 7 = (295) ÷ 7 ≈ 4.2°C

f 4.2°C – (–1.8°C) = 4.2°C + 1.8°C = 6.0°C

g, i, j



h (1. 0.7), (2, –2.2), (3, –0.9), (4, –3.4), (5, –5.1), (6, –1.8), (7, 0.2)

k Maximum daily temperatures show similar trend to minimum daily temperatures. Temperature was coldest on Friday before starting to increase again over next two days.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 3 Chapter test A

Chapter 3 Chapter test B

Summative tests

The three tests, A, B and C, for each chapter accommodate different student ability levels, with one section of overlap in each (the 'Proficient' part). These tests have been carefully mapped against AUSVELS and the Australian Curriculum in order to provide an accurate assessment of each student's level of achievement. When a student's marks are entered into the provided spreadsheet calculator, a letter grade is calculated based upon a weighted average of percentages according to the type of test completed.

Chapter 3 Summative test A: Modified

Aimed at the lower level of student ability.

The top mark a student can achieve in a modified test is a **C**.

Chapter 3 Summative test B: Core

Aimed at the middle level of student ability.

The top mark a student can achieve in a core test is a **B**.

Chapter 3 Summative test C: Extension

Aimed at the upper level of student ability.

The top mark a student can achieve in an extension test is an **A**.

Test answers

Chapter 3 Chapter test answers

Chapter 3 Summative test answers

Summative test spreadsheet calculator

Connect

Teaching support for pages 174–5

Teaching strategies

Playing golf

Focus: To use a familiar context to connect the key ideas of directed numbers and the Cartesian plane

- Students analyse golf scores to determine if a golfer could apply to become a professional golfer.
- They also design a golf course, showing the tee-off positions and the position of each hole on a Cartesian plane.
- This is an open-ended task in which more capable students should be encouraged to include an interpretation of real-life golf events and results into their task report.
- The task requirements are expressed using everyday language so that students need to recognise the operation required.
- You may like students to discuss the task requirements in small groups to identify:
  - the meaning of *par*
  - how to work out the number of strokes required for a par round of 18 holes
  - how directed numbers are used to describe a score
  - how to calculate an average score
  - what the golf terms *bogey*, *eagle*, etc. mean.
- Direct students to complete the matching **Connect worksheet** (see Resources). This provides scaffolding for the task, to guide students through the problem-solving process. Students can use this as a foundation for presenting their findings in a report.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- As an extension, students can design a game where rolling a die simulates the number of strokes taken at each hole for a round of golf.
- Sample answers are provided to **Connect worksheet**.

- An assessment rubric is available (see Resources).
- Two additional Connect investigations are provided: **CI 3-1 Shares** and/or **CI 3-2 Simultaneous machines** (see Resources).

Additional Connect investigations

CI 3-1 Shares

Focus: To perform operations involving directed numbers, and plot coordinates for a real-life scenario

Resources: grid or graph paper, ruler, calculator, newspapers (five consecutive days), access to the Internet

Students apply their understanding of operations with directed numbers by using the real-life scenario of the share market. They are guided to determine the value of a portfolio.

Students plot values and read information from a graph relating to the movement of a given stock. They have the option of selecting a stock and following its performance over a week. Students consider the value of the stock and determine if it has increased or decreased in value.

An assessment rubric is available (see Resources).

CI 3-2 Simultaneous machines

Focus: To perform operations involving directed numbers and plot points to determine a solution to a problem

Resources: grid or graph paper, ruler, calculator (optional)

Students perform operations with directed numbers and plot two linear relationships. They consider the coordinates of the point of intersection and interpret its meaning in relation to a given problem regarding ‘number crunching machines’.

As an extension, students consider the effect of a new machine, and consider whether this machine will achieve the same outcome as each of the other machines.

An assessment rubric is available (see Resources).

Resources

Connect worksheet

CW 3 Playing golf

Additional Connect investigations

CI 3-1 Shares

CI 3-2 Simultaneous machines

Assessment rubrics

Playing golf

Shares

Simultaneous machines

Australian Curriculum: Mathematics Year 9

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Number and Algebra

Real numbers	**Elaborations**	**MyMaths 9**
Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)	• identifying direct proportion in real-life contexts	1A Working with whole numbers 1B Working with decimals 1C Working with ratios 4G Relationships and direct proportion
Apply index laws to numerical expressions with integer indices (ACMNA209)	• simplifying and evaluating numerical expressions, using involving both positive and negative integer indices	2B Index laws
Express numbers in scientific notation (ACMNA210)	• representing extremely large and small numbers in scientific notation, and numbers expressed in scientific notation as whole numbers or decimals	2D Scientific notation
Money and financial mathematics	**Elaborations**	**MyMaths 9**
Solve problems involving simple interest (ACMNA211)	• understanding that financial decisions can be assisted by mathematical calculations	1D Percentage of an amount 1E Writing one quantity as a percentage of another 1F Understanding simple interest 1G Working with simple interest

Patterns and algebra	**Elaborations**	**MyMaths 9**
Extend and apply the index laws to variables, using positive integer indices and the zero index (ACMNA212)	• understanding that index laws apply to variables as well as numbers	2B Index laws 2C Negative indices
Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)	• understanding that the distributive law can be applied to algebraic expressions as well as numbers • understanding the relationship between expansion and factorisation and identifying algebraic factors in algebraic expressions	2A Working with algebraic terms 2E Expanding algebraic expressions 2F Factorising using common factors 2G Factorising quadratic expressions
Linear and non-linear relationships	**Elaborations**	**MyMaths 9**
Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214)	• investigating graphical and algebraic techniques for finding distance between two points • using Pythagoras' theorem to calculate distance between two points	3G Midpoint and length of line segments
Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294)	• investigating graphical and algebraic techniques for finding midpoint and gradient • recognising that the gradient of a line is the same as the gradient of any line segment on that line	3D Gradient and intercepts 3G Midpoint and length of line segments

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)	• determining linear rules from suitable diagrams, tables of values and graphs and describing them using both words and algebra	3A Solving linear equations 3B Solving linear equations with the unknown on both sides 3C Plotting linear graphs 3E Sketching linear graphs using gradient and *y*-intercept 3F Sketching linear graphs using *x*- and *y*-intercepts
Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (ACMNA296)	• graphing parabolas, and circles connecting *x*-intercepts of a graph to a related equation	4A Solving quadratic equations 4B Plotting quadratic relationships 4C Parabolas and transformations 4D Sketching parabolas using transformations 4E Sketching parabolas using intercepts 4F Circles and other non-linear relationships 4G Relationships and direct proportion

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 9**
Calculate the areas of composite shapes (ACMMG216)	• understanding that partitioning composite shapes into rectangles and triangles is a strategy for solving problems involving area	7C Area of simple shapes 7D Area of composite shapes
Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)	• analysing nets of cylinders to establish formulas for surface area • connecting the volume and capacity of a cylinder to solve authentic problems	7F Surface area of cylinders 7G Volume
Solve problems involving the surface area and volume of right prisms (ACMMG218)	• solving practical problems involving surface area and volume of right prisms	7E Surface area 7G Volume

Investigate very small and very large time scales and intervals (ACMMG219)	• investigating the usefulness of scientific notation in representing very large and very small numbers	7A Understanding and representing measurement
Geometric reasoning	**Elaborations**	**MyMaths 9**
Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220)	• establishing the conditions for similarity of two triangles and comparing this to the conditions for congruence • using the properties of similarity and ratio, and correct mathematical notation and language, to solve problems involving enlargement (for example, scale diagrams) • using the enlargement transformation to establish similarity understanding that similarity and congruence help describe relationships between geometrical shapes and are important elements of reasoning and proof	5C Transformations 5D Congruent figures 5E Dilation and scale factor 5F Similar figures 5G Similar triangles
Solve problems using ratio and scale factors in similar figures (ACMMG221)	• establishing the relationship between areas of similar figures and the ratio of corresponding sides (scale factor)	5E Dilation and scale factor 5F Similar figures 5G Similar triangles 5H Scale factor and area
Pythagoras and trigonometry	**Elaborations**	**MyMaths 9**
Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles (ACMMG222)	• understanding that Pythagoras' Theorem is a useful tool in determining unknown lengths in right-angled triangles and has widespread applications • recognising that right-angled triangle calculations may generate results that can be integers, fractions or irrational numbers	6A Understanding Pythagoras' theorem 6B Using Pythagoras' theorem to find the length of the hypotenuse 6C Using Pythagoras' theorem to find the length of a shorter side
Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223)	• developing understanding of the relationship between the corresponding sides of similar right-angled triangles	6D Understanding trigonometry
Apply trigonometry to solve right-angled triangle problems (ACMMG224)	• understanding the terms 'adjacent' and 'opposite' sides in a right-angled triangle • selecting and accurately using the correct trigonometric ratio to find unknown sides (adjacent, opposite and hypotenuse) and angles in right-angled triangles	6E Using trigonometry to find lengths 6F Using trigonometry to find angles 6G Applications involving right-angled triangles

Statistics and Probability

Chance	**Elaborations**	**MyMaths 9**
List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225)	• conducting two-step chance experiments • using systematic methods to list outcomes of experiments and to list outcomes favourable to an event • comparing experiments which differ only by being undertaken with replacement or without replacement	9A Theoretical probability 9C Tree diagrams 9D Two-way tables 9F Experiments with replacement 9G Experiments without replacement
Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (ACMSP226)	• using Venn diagrams or two-way tables to calculate relative frequencies of events involving ‘and’, ‘or’ questions • using relative frequencies to find an estimate of probabilities of ‘and’, ‘or’ events	9B Experimental probability and relative frequency 9D Two-way tables 9E Venn diagrams
Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (ACMSP227)	• investigating a range of data and its sources, for example the age of residents in Australia, Cambodia and Tonga; the number of subjects studied at school in a year by 14-year-old students in Australia, Japan and Timor-Leste	8G Comparing data

Data representation and interpretation	**Elaborations**	**MyMaths 9**
Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources (ACMSP228)	• comparing the annual rainfall in various parts of Australia, Pakistan, New Guinea and Malaysia	8E Collecting data
Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282)	• using stem-and-leaf plots to compare two like sets of data such as the heights of girls and the heights of boys in a class • describing the shape of the distribution of data using terms such as ‘positive skew’, ‘negative skew’ and 'symmetric' and 'bimodal'	8A Understanding and representing data 8B Grouped data 8F Describing data
Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283)	• comparing means, medians and ranges of two sets of numerical data which have been displayed using histograms, dot plots, or stem and leaf plots	8C Summary statistics 8D Summary statistics from displays 8G Comparing data

Year 9 achievement standard

By the end of Year 9, students solve problems involving simple interest. They interpret ratio and scale factors in similar figures. Students explain similarity of triangles. They recognise the connections between similarity and the trigonometric ratios. Students compare techniques for collecting data from primary and secondary sources. They make sense of the position of the mean and median in skewed, symmetric and bi-modal displays to describe and interpret data.

Students apply the index laws to numbers and express numbers in scientific notation. They expand binomial expressions. They find the distance between two points on the Cartesian plane and the gradient and midpoint of a line segment. They sketch linear and non-linear relations. Students calculate areas of shapes and the volume and surface area of right prisms and cylinders. They use Pythagoras’ Theorem and trigonometry to find unknown sides of right-angled triangles. Students calculate relative frequencies to estimate probabilities, list outcomes for two-step experiments and assign probabilities for those outcomes. They construct histograms and back-to-back stem-and-leaf plots.

Number and Algebra

4 Non-linear relationships

4 Non-linear relationships

Teaching support for pages 152–153

Syllabus links

Content descriptions and elaborations

Linear and non-linear relationships

ACMNA296: Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations

- graphing parabolas, and circles connecting *x*-intercepts of a graph to a related equation

Real numbers

ACMNA208: Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems

- identifying direct proportion in real-life contexts

The proficiency strands **Understanding, Fluency, Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Direct students to examine the opening photo for this chapter.
- Ask students to consider a basketball game and a player taking a free throw. The path of the ball is curved but is it possible to write an equation to track the path of the ball?
- In Chapter 3, we looked at linear relationships but the path of the basketball will need a different type of relationship.
- If a relationship is not linear, then it is called non-linear; there are several different types of non-linear relationships that are considered in this chapter.

Essential question

The path of the basketball will be curved. The two variables to be compared will be horizontal distance travelled and height of the ball.

Are you ready?

Prerequisite knowledge and skills can be tested by completing **Are you ready?**. This will give you an indication of the differentiated pathway each student can follow.

Students need to be able to:

- factorise quadratic expressions
- substitute into quadratic expressions
- simplify algebraic expressions
- complete a table of values for a relationship between x and y
- identify the rule for graphs of vertical and horizontal lines
- perform translation and reflection on a point on the Cartesian plane
- find the x- and y-intercepts for graphs of linear relationships
- find the gradient of a linear graph.

At the beginning of each topic, there is a suggested differentiated pathway that allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to select the best pathway.

Support Strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

CHAPTER 4 NON-LINEAR RELATIONSHIPS

ANSWERS

4 Are you ready?

1 a $x(x + 7)$ b $(x - 3)(x + 3)$
c $(x + 3)(x + 2)$ d $-x(x + 3)$

2 a i -4 ii 21 iii 0
b i 0 ii 5 iii -4
c i 1 ii 16 iii 1
d i -10 ii 0 iii -12

3 a $4x^2 + 4x + 1$ b $x^2 - 5x - 5$
c $-x^2 - 5x - 6$ d $2x^2$

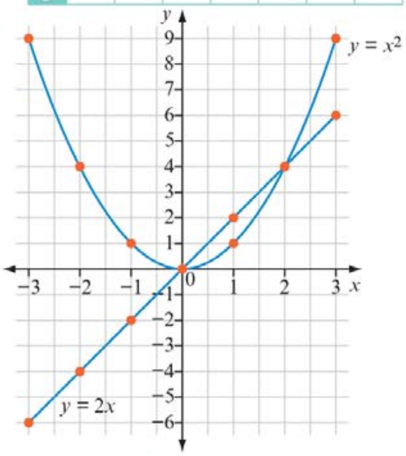
4 a i

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

ii

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b



c i linear ii non-linear

5 a $y = 1$ b $x = -3$ c $x = 2$ d $y = -2$

6 a $(2, -3)$ b $(6, 3)$ c $(2, -2)$
d $(2, 4)$ e $(-4, 3)$ f $(0, 6)$

7 a $x = 4; y = -4$ b $x = -3; y = 6$

8 gradient = 2

Resources

assess: assessments

Each topic of the *MyMaths 9* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students' confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

4A Solving quadratic equations

Teaching support for pages 154–159

Teaching strategies

Learning focus

To recognise a quadratic equation and solve them using the Null Factor Law

Start thinking!

The task guides students to:

- identify both a quadratic expression and a quadratic equation
- develop the use of the Null Factor Law to solve a factorised quadratic equation.

Differentiated pathways

Below Level	At Level	Above Level
1–5, 7a–d, 10a–c, 11, 13a–f, 14–16	1–4, 5a–c, 6–12, 13a–i, 14–18	1, 3, 4, 7–9, 10d–f, 11–13, 16, 18–23
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q1–3

Focus: To develop an understanding of what constitutes a quadratic expression and revise the skills of factorising and substitution into these expressions.

- Direct students to complete **SS 4A-1 Factorising quadratic expressions** (see Resources) if they had difficulty with Q1 or require more practice at this skill.
- Direct students to complete **SS 4A-2 Substitution into quadratic expressions** (see Resources) if they had difficulty with Q2 or require more practice at this skill.
- Direct students to complete **SS 4A-3 Simplifying algebraic expressions** (see Resources) if they had difficulty with Q3 or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to remember the basic rules of factorisation for quadratic expressions.
  - Always first look for a common factor.

- If the remaining expression has three terms and is a quadratic trinomial, look for two numbers that multiply to give the constant term and add to give the coefficient of ‘x’.
- If there are only two terms, look to factorise as the difference of two squares.

At Level

At Level
1–4, 5a–c, 6–12, 13a–i, 14–18

- Demonstrate **4A eTutor** or direct students to do this independently.
- The Null Factor Law needs to be clearly understood.by students. That is, if two (or more) terms multiply to give a result of zero, at least one of those terms must equal zero.
- Factorising changes the quadratic expression into the product of two linear factors. These linear factors are solved as two separate linear equations.
- Most quadratic equations in this exercise will give two solutions. Quadratic equations that factorise as perfect squares will give one solution.
- Some quadratic equations have no solutions and these will not be able to be factorised.
- Just as with all other types of equations, remind students that they can check their solutions by substituting their solution into the equation.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- For Q1, remind students that:
  - a quadratic equation is of degree 2; that is, the term with the greatest power in the equation is x^2
  - to be an equation rather than an expression, there must be an equals sign.
- Direct students to **Example 4A-1**. It shows how to solve quadratic equations, which are already factorised, using the Null Factor Law and will help students to complete Q2 and Q3.
- In Q2, pay particular attention to part b. The single term factor of x still needs to be set equal to zero and becomes a solution. This method is used for all quadratic equations where ‘x’ is a common factor.
- **Example 4A-2** shows how to solve quadratic equations after first factorising. This will help students to complete Q4.

- In Q8, students need to understand that the only difference between $-x(x-3)=0$ and $x(x-3)=0$ is that the first equation has been produced by multiplying both sides of the second equation by –1. Similarly, in Q9, the first equation has been produced by multiplying both sides of the second equation by –2.
- **Example 4A-3** shows how to solve quadratic equations after first dividing both sides by a negative number. This will help students to complete Q10.
- In Q11, students look at the number of solutions to a quadratic equation.
  - In part d, discuss the concept that there is one solution to the equation because the quadratic is a perfect square.
  - In part e, students will be unable to factorise the expression although some may confuse it for the difference of two squares. Explain that if they cannot factorise a quadratic equation, it does not necessarily mean it has no solutions. They will learn other techniques in the future. To see that this equation (Q11e) has no solutions, students need to consider the equation $x^2 = -4$.
- For Q13, remind students that quadratic equations are solved using the Null Factor Law. This law can only be applied if the quadratic expression is equal to zero and so all equations need to be rearranged in this form first.
- Students will need a graphics calculator or similar software to complete Q14.
- From Q16 onwards, students need to consider when a negative answer has meaning and when it does not.
- For additional practice, students can complete Q1 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4A Q3 and Q4 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

Below Level

Below Level
1–5, 7a–d, 10a–c, 11, 13a–f, 14–16

- Students may need to complete **SS 4A-1 Factorising quadratic expressions** (see Resources).
- Students may need to complete **SS 4A-2 Substitution into quadratic expressions** (see Resources).
- Students may need to complete **SS 4A-3 Simplifying algebraic expressions** (see Resources).

- For Q1, remind students that:
  - a quadratic equation is of degree 2; that is, the term with the greatest power in the equation is x^2
  - to be an equation rather than an expression, there must be an equals sign.
- In Q2, pay particular attention to part b. The single term factor of *x* still needs to be set equal to zero and becomes a solution. This method is used for all quadratic equations where '*x*' is a common factor.
- Students may wish to do Q5 simultaneously with Q4. They should check each solution by substituting the value in the left side of the equation and showing that it gives a result of zero; that is, the same as the value on the right side.

POTENTIAL DIFFICULTY
In questions such as Q10 where a numerical factor can be taken out, some students put that numerical factor equal to zero and then perceive that factor to be a solution to the equation.

- In Q11, students look at the number of solutions to a quadratic equation.
  - In part d, discuss the concept that there is one solution to the equation because the quadratic is a perfect square.
  - In part f, students will be unable to factorise the expression although some may confuse it for the difference of two squares. Explain that a quadratic equation does not factorise does not prevent it from having solutions. To see that this equation has no solutions students need to consider the equation $x^2 = -4$.
- For Q13, remind students that quadratic equations are solved using the Null Factor Law. This law can only be applied if the equation is equal to zero and so all equations need to be rearranged in this form first.
- Students will need a graphics calculator or similar software to complete Q14.
- For students who do not progress past Q4, direct them to Q1 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- This exercise will prove very difficult for students with below average algebra skills.
- If students have software that will assist with factorisation, this can be provided for support.

Above Level

Above Level

1, 3, 4, 7–9, 10d–f, 11–13, 16, 18–23

- For Q1, remind students that:
  - a quadratic equation is of degree 2; that is, the term with the greatest power in the equation is x^2
  - to be an equation rather than an expression there must be an equal to sign.
- In Q2, pay particular attention to part b. The single term factor of x still needs to be set equal to zero and becomes a solution. This method is used for all quadratic equations where 'x' is a common factor.
- Above Level students have not been given Q5 to do but should be routinely checking solutions by substitution.
- In Q8, students need to understand that the only difference between $-x(x - 3) = 0$ and $x(x - 3) = 0$ is that the first equation has been produced by multiplying both sides of the second equation by –1. Similarly, in Q9, the first equation has been produced by multiplying both sides of the second equation by –2.
- In Q11, students look at the number of solutions to a quadratic equation.
  - In part d, discuss the concept that there is one solution to the equation because the quadratic is a perfect square.
  - In part e, students will be unable to factorise the expression although some may confuse it for the difference of two squares. Explain that if they cannot factorise a quadratic equation, it does not necessarily mean it has no solutions. They will learn other techniques in the future. To see that this equation (Q11e) has no solutions, students need to consider the equation $x^2 = -4$.
- For Q13, remind students that quadratic equations are solved using the Null Factor Law. This law can only be applied if the quadratic expression is equal to zero and so all equations need to be rearranged in this form first.
- From Q16 onwards, students need to consider when a negative answer has meaning and when it does not.

Extra activities

1 Quick questions

a Solve $x - 9 = 0$. $(x = 9)$

b Factorise $x^2 - 9$. $[(x - 3)(x + 3)]$

c Factorise $x^2 - 9x$. $[x(x - 9)]$

d Factorise $x^2 - 6x + 9$. $[(x - 3)^2]$

2 Use the Null Factor Law to solve each equation.

a $x(x + 1) = 0$ $(x = 0 \text{ or } x = -1)$

b $x(x + 1)(x - 1) = 0$ $(x = 0, x = -1 \text{ or } x = 1)$

c $x^2(x - 1) = 0$ $(x = 0 \text{ or } x = 1)$

d $(x - 1)(x + 2)(x - 3) = 0$ $(x = 1, x = -2 \text{ or } x = 3)$

e $x^3 - 4x = 0$ $(x = 0, x = -2 \text{ or } x = 2)$

Answers

4A Solving quadratic equations

4A Start thinking!

1 A quadratic expression has 2 as the highest power of the variable, e.g. $x^2 + 3x$, $3x^2$, $5 - x^2$.

2 a A quadratic equation has an equals sign, quadratic expression does not.

b i, iii

c Highest power in a quadratic equation is 2; highest power in a linear equation is 1.

3 a $x = -2$ or $x = 2$ b $x = 3$ or $x = 5$

c $x = 0$ or $x = -7$ d $x = -4$ or $x = 1$

4 a i 0 ii 0 iii 0 iv 0 v 0 vi 0

b i $x = 0$ ii $x = 0$ iii $x = 1$ iv $x = -5$

c zero; zero

5 a $x - 3$ and $x - 5$

b One factor or the other must equal zero.

c $(x - 3)(x - 5) = 0$

$x - 3 = 0$ or $x - 5 = 0$

$x = 3$ or $x = 5$

d Substitute each x value into the original equation $(x - 3)(x - 5) = 0$ and show that each value makes the equation a true statement.

Exercise 4A Solving quadratic equations

ANSWERS

1 a, c, e and h

2 a $(x+7)(x-4)=0$
$x+7=0$ or $x-4=0$
$x=-7$ or $x=4$
b $x(x-2)=0$
$x=0$ or $x-2=0$
$x=0$ or $x=2$
c $(x+5)(x-5)=0$
$x+5=0$ or $x-5=0$
$x=-5$ or $x=5$

3 a $x=-2$ or $x=3$ b $x=1$ or $x=7$
c $x=-4$ or $x=4$ d $x=0$ or $x=6$
e $x=-5$ or $x=-1$ f $x=-2$ or $x=0$
g $x=-8$ or $x=8$ h $x=-1$ or $x=7$
i $x=0$ or $x=11$ j $x=-3$ or $x=5$
k $x=2$ l $x=-5$

4 a $x=-2$ or $x=5$ b $x=1$ or $x=2$
c $x=0$ or $x=-5$ d $x=0$ or $x=3$
e $x=-6$ or $x=6$ f $x=-7$ or $x=-3$
g $x=-2$ or $x=4$ h $x=-1$ or $x=1$
i $x=0$ or $x=-8$ j $x=1$ or $x=3$
k $x=-3$ l $x=1$

6 a $x=-2$ or $x=2$ b $x=3$ or $x=5$
c $x=0$ or $x=-7$ d $x=-4$ or $x=1$

7 a yes b no c no d yes
e no f yes g yes h no

8 a Dividing both sides of equation by -1 gives an identical equation.
b $x=0$ or $x=3$

9 a Dividing both sides of equation by -2 gives an identical equation.
b $x=-4$ or $x=5$

10 a $x=-9$ or $x=0$ b $x=-8$ or $x=2$
c $x=1$ or $x=4$ d $x=-6$ or $x=6$
e $x=-7$ or $x=-3$ f $x=-2$ or $x=1$

11 a two b linear equation has one solution
c no
d one; the two factors produce the same solution.
e In part d the two factors are the same, whereas in part a they are different.
f Zero; not possible to factorise x^2+4.
g A quadratic equation can have zero, one or two solutions.

12 a $x=-2$ or $x=2$ b $x=-5$ or $x=2$
c $x=3$ d no solutions
e $x=-7$ or $x=0$ f $x=-8$ or $x=-4$
g $x=-8$ or $x=9$ h no solutions

13 a $x=-3$ or $x=1$ b $x=-4$ or $x=5$
c $x=-5$ or $x=5$ d $x=-2$
e $x=-8$ or $x=0$ f $x=-6$ or $x=6$
g $x=0$ or $x=3$ h $x=6$
i $x=-7$ or $x=-3$ j $x=-1$ or $x=8$
k $x=2$ or $x=4$ l $x=-2$ or $x=6$

15 a \$100 b 10 weeks
c No; if amounts were the same, relationship would be linear

16 a i 9 m ii 8 m b 8 m c 4 s
d Not possible to have negative time values.

17 a 10 b 2 c 2 and 6 d 4

18 a $x(x+8)$ cm^2 b x^2+8x cm^2
c $x^2+8x=560$ d $x^2+8x-560=0$
$(x+28)(x-20)=0$
e $x=20$ or $x=-28$. $x=20$ is the feasible solution and $x=-28$ is not, as it is not possible to have a negative length.
f width = 20 cm and length = 28 cm

19 a $x^2+2x=35$ b $x=-7$ or $x=5$
c length = 7 m, width = 5 m

20 $x(x-12)=640$
length = 32 cm, width = 20 cm

21 a $x^2-3x+2=0$ b $x^2-10x=0$
c $x^2+2x-15=0$

22 Multiplying an equation by any constant factor will result in the same solution. For example, the solution $x=1$ or $x=2$ matches the equation $(x-1)(x-2)=0$, which is equivalent to $x^2-3x+2=0$.
Multiplying by a constant (say, 3) results in an equation $3(x^2-3x+2=0)$ or $3x^2-9x+6=0$, which still has the same solutions, $x=1$ or $x=2$.

23 a $x=0$ or $x=6$ b $x=0$ or $x=1$
c $x=-3$ or $x=1$ d $x=2$ or $x=3$

Reflect

Possible answer: The Null Factor Law makes it possible to solve quadratic equations. Without setting the quadratic expression equal to zero, we would be unable to determine with certainty what any one factor of the expression is equal to.

Resources

SupportSheets

SS 4A-1 Factorising quadratic equations

Focus: To use a variety of techniques to factorise quadratic equations

Resources: ruler

Students revise what is meant by factorising and look at removing a common factor from an expression. They then look at quadratic expressions and identify the appropriate method of factorisation:

- common factors (for expressions of the form $x^2 + bx$)
- difference of two squares (for expressions of the form $x^2 – c$)
- looking for two numbers that multiply to c and add to b (for expressions of the form $x^2 + bx + c$).

SS 4A-2 Substitution into quadratic expressions

Focus: To substitute values into a variety of quadratic expressions

Students explore substitution into quadratic expressions in both expanded and factored form. Students will pay particular attention to the substitution of negative values and understanding that a negative value squared will always be positive.

SS 4A-3 Simplifying algebraic expressions

Focus: To simplify algebraic expressions by collecting like terms

Students consider like terms and in particular like terms that are used in quadratic expressions. Students will simplify expressions involving x^2, x and constant terms that will have a standard quadratic solution.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q1 relates to Exercise 4A.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x – h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

BLM

1-cm grid paper

Interactives

4A eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4B Plotting quadratic relationships

Teaching support for pages 160–165

Teaching strategies

Learning focus

To introduce the parabola as the graph of a quadratic relationship

Start thinking!

The task guides students to:

- plot a quadratic relationship from a table of values
- see that a quadratic relationship is not linear
- recognise that the graph of a quadratic relationship is called a parabola
- identify the turning point and axis of symmetry as key features of a parabola
- recognise that a parabola has a single *y*-intercept and list its coordinates
- recognise that a parabola may have two *x*-intercepts and list their coordinates.

Differentiated pathways

Below Level	At Level	Above Level
1–6, 8, 9a, b, 10, 12	1–8, 9a–c, 10–13	2, 3, 5–8, 9d–f, 11–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q4 and Q5

Focus: To revise substituting into a table of values, recognising linear and non-linear relationships and writing the rule for horizontal and vertical lines.

- Direct students to complete **SS 4B-1 Plotting relationships** (see Resources) if they had difficulty with Q4 or require more practice at this skill.
- Direct students to complete **SS 4B-2 Writing the rule for horizontal and vertical linear graphs** (see Resources) if they had difficulty with Q5 or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.

- Students need to:
  - substitute for x into an algebraic rule
  - recognise that when a negative number is squared, the result is positive
  - see that if a straight line cannot be drawn through points on a Cartesian plane, the relationship is not linear
  - recognise that a vertical line has a rule or equation of the form $x = c$, and a horizontal line has a rule or equation of the form $y = c$.

At Level

At Level
1–8, 9a–c, 10–13

- Demonstrate **4B eTutor** or direct students to do this independently.
- Provide students with copies of the BLM **Cartesian plane grids** (see Resources) to help them complete this topic.
- Students will need calculator, pencil, ruler and eraser to complete this topic.
- Define a parabola as the graph of a quadratic relationship.
- Discuss some examples of applications of parabolas:
  - the path of a projectile
  - the design of a headlight
  - satellite dishes etc.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4B-1**. It shows how to plot a parabola and will help students to complete Q1 and Q2.
- After completing Q1, ask students to discuss the most significant difference between the two graphs which is whether the graph is upright or inverted.
- After completing Q2, students should notice that upright parabolas have a positive x^2 term, while inverted parabolas have a negative x^2 term.
- **Example 4B-2** shows how to identify features of a parabola. This will help students to complete Q3–5.

- In Q7, discuss with students what features of the parabola will enable them to most quickly match the graph to the equation.
  - Positive or negative x^2 term will identify whether the parabola is upright or inverted.
  - The *y*-intercept will be equal to the constant term.
  - Parabolas without an '*x*' term will be symmetrical about the *y*-axis.
  - Only then would it be necessary to find the *x*-intercepts.
- For additional practice, students can complete Q2 and Q3 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4B Q1–5 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Quadratics and types of numbers** (see Resources).

Below Level

Below Level
1–6, 8, 9a, b, 10, 12

- Students may need to complete **SS 4B-1 Plotting relationships** (see Resources).
- Students may need to complete **SS 4B-2 Writing the rule for horizontal and vertical linear graphs** (see Resources).
- After completing Q1, ask students to discuss the most significant difference between the two graphs which is whether the graph is upright or inverted.
- After completing Q2, students should notice that upright parabolas have a positive x^2 term, while inverted parabolas have a negative x^2 term.
- For Q4, help students to see that:
  - the axis of symmetry is an imaginary vertical line halfway between the two *x*-intercepts
  - the turning point lies on the axis of symmetry and the *y* value at this point is found by substituting the *x* value for this axis into the equation.
- For Q6, students will need a graphics calculator or access to graphical software such as *GeoGebra*.

- To complete Q8, students should easily see which parabolas are upright and which are inverted but they will need to be guided to see that the rule of an inverted parabola has a negative x^2 term.
- In Q10, discuss part b with students. In many practical questions, negative values of the independent variable have no meaning.
- For students who do not progress past Q5, direct them to Q2 and Q3 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- Students may lack the motor skills to draw many of these graphs. Allow as much technology as needed to support them.
- The main concepts that it is hoped Below Level students can attain from this exercise are to:
  - recognise a parabola as a non-linear relationship and as such the graph is not a straight line
  - be able to draw a parabola by plotting points
  - understand that the orientation of the parabola depends on whether the coefficient of x^2 is positive or negative.

Above Level

Above Level
2, 3, 5–8, 9d–f, 11–18

- After completing Q2, students should notice that upright parabolas have a positive x^2 term, while inverted parabolas have a negative x^2 term.
- In Q7, discuss with students what features of the parabola will enable them to most quickly match the graph to the equation.
  - Positive or negative x^2 term will identify whether the parabola is upright or inverted.
  - The y-intercept will be equal to the constant term.
  - Parabolas without an 'x' term will be symmetrical about the y-axis.
  - Only then would it be necessary to find the x-intercepts.

- In part d of Q12, students are considering whether it is possible to draw a parabola with two y-intercepts. If they say that it is not, ask them to consider a sideways parabola and what the equation would look like.
- In Q16, relate the fact that the graph has only one x-intercept to its equation. The quadratic expression will be a perfect square.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Quadratics and types of numbers** (see Resources).

Extra activities

1 Quick questions

Consider the quadratic relationship $y = x^2 + 4x - 12$.

a Find y when $x = 0$. (–12)

b Find x when $y = 0$. (–6 or 2)

c Find y when $x = -2$. (–16)

d Find y when $x = -4$. (–12)

2 A golf ball is hit and the path of the golf ball follows the relationship $h = -\frac{1}{500}d(d - 200)$, where d is the horizontal distance in metres that the ball has travelled from the point it was hit and h is the height of the ball in metres.

a What horizontal distance does the ball travel? How do you know this? (200 m, as $h = 0$)

b What is the greatest height that the ball reaches? (20 m)

c On a Cartesian plane, draw a graph to represent the path of the ball.

Answers

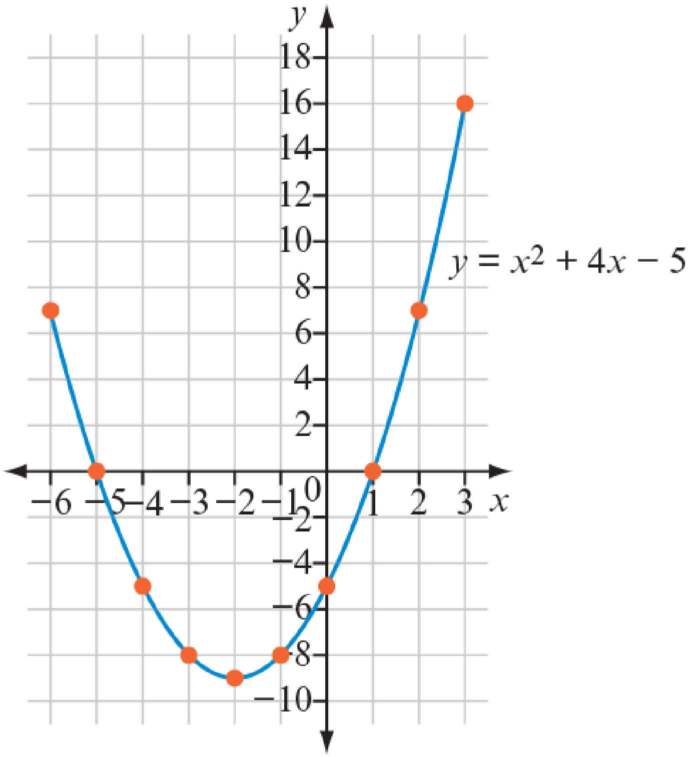
4B Plotting quadratic relationships

4B Start thinking!

1

x	−6	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7	16

2



3 The points form a symmetrical curve that changes direction at the point (−2, −9).

4 Non-linear relationship, as the points do not form a straight line.

5 a minimum b (−2, −9)

6 a It acts like a mirror line so that the left side is symmetrical to the right side.

b $x = -2$

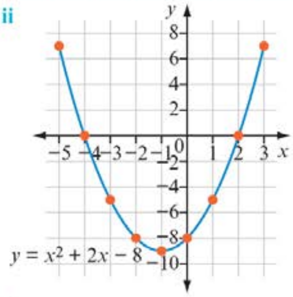
7 a one; (0, −5) b two; (−5, 0) and (1, 0)

Exercise 4B Plotting quadratic relationships

1 a i

x	−5	−4	−3	−2	−1	0	1	2	3
y	7	0	−5	−8	−9	−8	−5	0	7

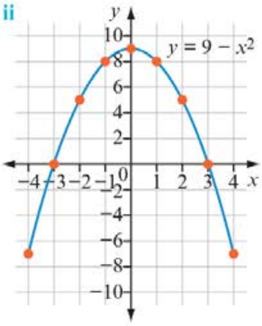
ii



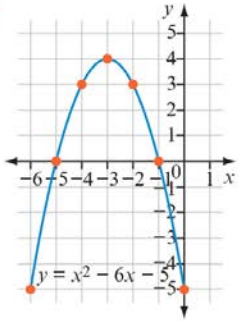
b i

x	−4	−3	−2	−1	0	1	2	3	4
y	−7	0	5	8	9	8	5	0	−7

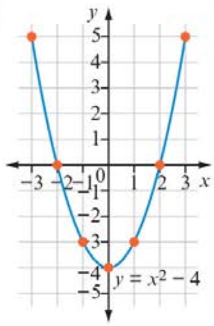
ii



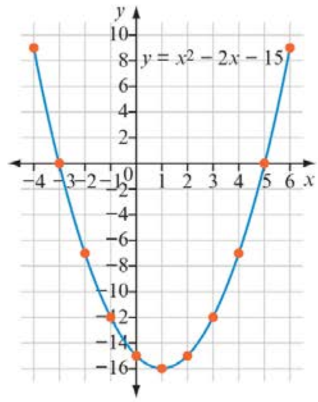
2 a



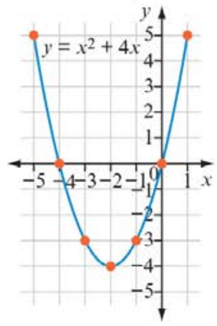
b



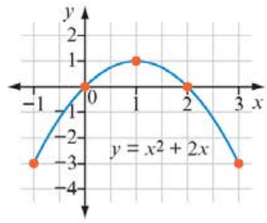
c



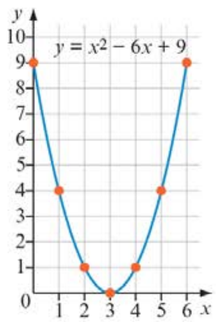
d



e



f



3 a minimum b $(2, -1)$ c $x = 2$
d $(0, 3)$ e $(1, 0)$ and $(3, 0)$

4 a i minimum ii $(-1, -9)$ iii $x = -1$
iv -8 v -4 and 2
b i maximum ii $(0, 9)$ iii $x = 0$
iv 9 v -3 and 3

5 a i maximum ii $(-3, 4)$ iii $x = -3$
iv -5 v -5 and -1
b i minimum ii $(0, -4)$ iii $x = 0$
iv -4 v -2 and 2
c i minimum ii $(1, -16)$ iii $x = 1$
iv -15 v -3 and 5
d i minimum ii $(-2, -4)$ iii $x = -2$
iv 0 v -4 and 0
e i maximum ii $(1, 1)$ iii $x = 1$
iv 0 v 0 and 2
f i minimum ii $(3, 0)$ iii $x = 3$
iv 9 v 3

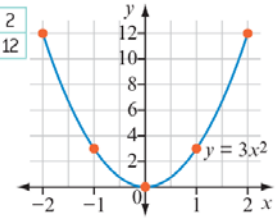
7 a F b E c B d A e C f D

8 a upright b inverted c upright
d upright e upright f inverted

9 a

x	−2	−1	0	1	2
y	12	3	0	3	12

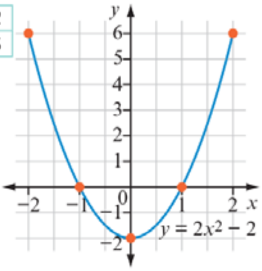
i upright
ii minimum; (0, 0)
iii 0
iv 0



b

x	−2	−1	0	1	2
y	6	0	−2	0	6

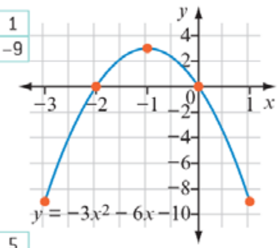
i upright
ii minimum; (0, −2)
iii −2
iv −1 and 1



c

x	−3	−2	−1	0	1
y	−9	0	3	0	−9

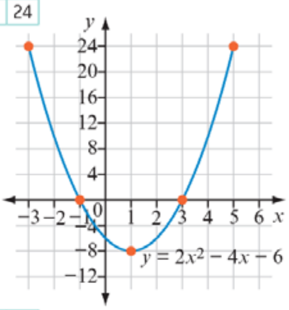
i inverted
ii maximum; (−1, 3)
iii 0
iv −2 and 0



d

x	−3	−1	1	3	5
y	24	0	−8	0	24

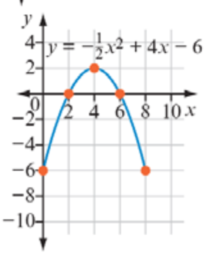
i upright
ii minimum; (1, −8)
iii −6
iv −1 and 3



e

x	0	2	4	6	8
y	−6	0	2	0	−6

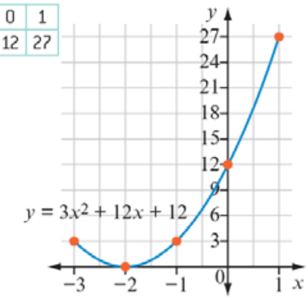
i inverted
ii maximum; (4, 2)
iii −6
iv 2 and 6



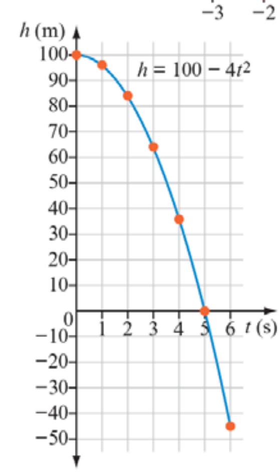
f

x	−3	−2	−1	0	1
y	3	0	3	12	27

i upright
ii minimum; (−2, 0)
iii 12
iv −2



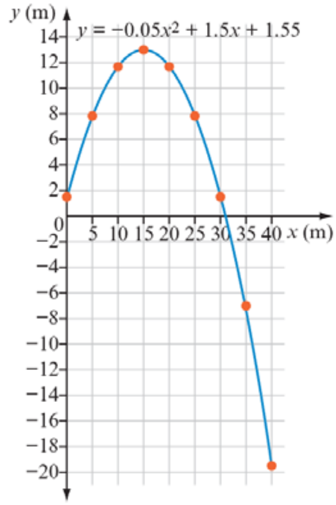
10 a



b not possible to have negative time values

c i 84 m ii 64 m **d** 100 m **e** 5 s

11 a



b 12.8 m **c** 1.55 m **d** 31 m

12 **a** **i** two **ii** one **iii** zero

b A parabola changes direction only once, so there can only be a maximum of two x-intercepts.

c one

d No; a parabola only intersects the y-axis once.

13 **a** -2 and 5 **b** $x = -2$ or $x = 5$

c They are the same.

d The x-intercepts of a parabola represent the solutions to a quadratic equation.

14 **a** $x = -5$ or $x = 1$ **b** $x = 0$ or $x = 4$

c $x = -2$ or $x = 2$ **d** $x = -5$ or $x = 1$

e $x = -2$ or $x = 2$ **f** $x = -1$ or $x = 5$

15 $x = 5$ **14** $x = -4$

17 no x-intercepts, one y-intercept

18 one x-intercept and one y-intercept

Reflect

Possible answer: A parabola can be recognised from its equation as it will be in the form $y = ax^2 + bx + c$.

Resources

SupportSheets

SS 4B-1 Plotting relationships

Focus: To plot relationships presented in a table of values and observe the patterns formed by the points on a Cartesian plane

Resources: 1-cm grid paper (BLM) or graph paper, ruler

Students use an extended table of values to first revise plotting simple linear relationships and then use similar tables of values to draw simple parabolas.

SS 4B-2 Writing the rule for horizontal and vertical linear graphs

Focus: To write the rules used to describe a horizontal line and a vertical line

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students consider rules of the form $x = b$ and recognise this as representing all points with this x value, generating a vertical line. Similarly, they consider rules of the form $y = c$ and recognise this as representing all points with this y value, generating a horizontal line.

Students practise writing the rule for different vertical and horizontal lines shown on a Cartesian plane.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q2 and Q3 relate to Exercise 4B.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

Investigation

INV 4-1 Quadratics and types of numbers

Focus: To investigate the relationship between quadratics and sets of numbers like square, triangular, pentagonal, hexagonal and octagonal numbers

Resources: 1-cm grid paper (BLM) or graph paper, calculator

Students look at square, triangular, pentagonal and hexagonal numbers and consider algebraic expressions that generate each number sequence. They see that each algebraic expression is quadratic and by solving quadratic equations for the term number, identify where particular numbers fit into the number pattern. They also draw graphs of these relationships.

As an extension, students investigate octagonal numbers.

BLMs

Cartesian plane grids

1-cm grid paper

Interactives

4B eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4C Parabolas and transformations

Teaching support for pages 166–171

Teaching strategies

Learning focus

To consider the graph of $y = x^2$ and see how other numbers in the equation change this most basic parabola

Start thinking!

The task guides students to:

- draw the graph of $y = x^2$ and identify the features of this basic parabola
- draw graphs of $y = ax^2$ for different values of a
- see the effect that different values of 'a' have on the graph of $y = x^2$.

Differentiated pathways

Below Level	At Level	Above Level
1–4, 7–9, 13	1–5, 6a–d, 7–11, 12a–d, 13–15, 16a–d, 17	1–5, 6e,f, 9–11, 12e–h, 13–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q6

Focus: To see how a point on a number plane will move under the transformations of reflection and translation

- Direct students to complete **SS 4C-1 Performing transformations on a coordinate point** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to understand that:
  - reflection means to move the object to the other side of a line of reflection inverting all its characteristics

  - translation means to move an object in a given direction or directions while maintaining its orientation.

At Level

At Level
1–5, 6a–d, 7–11, 12a–d, 13–15, 16a–d, 17

- Demonstrate **4C eTutor** or direct students to do this independently.
- The concepts covered in this section are:
  - dilation of $y = x^2$ by a coefficient of x^2. If $y = ax^2$ and $a > 1$, the graph will be narrower than $y = x^2$, and if $0 < a < 1$, the graph will be wider than $y = x^2$
  - a negative coefficient of x^2 will invert the graph of $y = x^2$
  - adding or subtracting a constant from $y = x^2$ will move the graph up (adding) or down (subtracting) the y-axis
  - adding or subtracting a constant from x before squaring (creating a perfect square expression) will move the graph of $y = x^2$ horizontally along the x-axis.
- Students will need calculator, pencil, ruler and eraser to complete this topic.
- Provide students with copies of the BLM **Cartesian plane grids** (see Resources) to help them.
- For tasks that ask students to compare graphs to $y = x^2$ they can use the BLM $\boldsymbol{y = x^2}$ (see resources). This master has $y = x^2$ drawn as a starting point so students can use the graph drawn as the basis for their transformations.
- If you can use some dynamic graphic software to demonstrate the transformations, this will help with student understanding.
- Provide some graphics calculators or graphing software to assist students who find this task too difficult.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **example 4C-1**. It shows how to describe a transformation used to produce a graph from the graph of $y = x^2$ and will help students to complete Q2–4.
- After completing Q1 and Q2, ensure that students understand the concept and definition of dilation. Establish that dilation by a factor greater than 1 makes the graph *narrower*, while a factor between 0 and 1 makes the graph *wider*.

- In Q3, ensure students understand that a reflection is due to the effect of the negative x^2 term and that the reflection takes place in the y-axis.
- **Example 4C-2** shows how to describe a transformation used to produce a graph from the graph of $y = -x^2$. This will help students to complete Q5.
- **Example 4C-3** shows how to identify transformations to produce a graph from the graph of $y = x^2$. This will help students to complete Q6–8.
- In Q5–8, students will need to combine the concepts of dilation and reflection to draw the graphs and describe the transformations.
- In Q9 –11, students need to look at the graph of $y = x^2$ and see that adding or subtracting a constant moves the graph up or down the y-axis in the same direction as the addition or subtraction.
- In Q13–15, students are looking at graphs that are of the form $y = (x - h)^2$. When students graph these from a table of values they will see that there is a horizontal movement of the graph. Unlike graphs of the form $y = x^2 + k$, the movement of the graph is counter-intuitive to some students. Explain that the movement is to the point such that $x - h = 0$, hence the movement is opposite to the sign shown.
- For additional practice, students can complete Q4 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4C Q17 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

Below Level

Below Level
1–4, 7–9, 13

- Students may need to complete **SS 4C-1 Performing transformations on a coordinate point** (see Resources).
- After completing Q1 and Q2, ensure that students understand the concept and definition of dilation. Establish that dilation by a factor greater than 1 makes the graph *narrower*, while a factor between 0 and 1 makes the graph *wider*.
- In Q3, ensure students understand that a reflection is due to the effect of the negative x^2 term and that the reflection takes place in the y-axis.
- In Q7 and Q8, students will need to combine the concepts of dilation and reflection to match the graphs to the equations.

- In Q9, students need to look at the graph of $y = x^2$ and see that adding or subtracting a constant moves the graph up or down the y-axis in the same direction as the addition or subtraction.

- In Q13, students are looking at graphs that are of the form $y = (x – h)^2$. When students graph these from a table of values they will see that there is a horizontal movement of the graph. Unlike graphs of the form $y = x^2 + k$, the movement of the graph is counter-intuitive to some students. Explain that the movement is to the point such that $x – h = 0$, hence the movement is opposite to the sign shown.

- Below Level students will find the concept of transforming a graph very difficult. To support and consolidate understanding of these concepts, allow students to draw graphs by first plotting a table of values. The BLM **Quadratic table of values** (see resources) can be provided to help them do this quickly.

- Allowing students to complete questions using a graphics calculator or graphing software will also assist them to have some success with this content.

Above Level

Above Level
1–5, 6e,f, 9–11, 12e–h, 13–18

- After completing Q1 and Q2, ensure that students understand the concept and definition of dilation. Establish that dilation by a factor greater than 1 makes the graph *narrower*, while a factor between 0 and 1 makes the graph *wider*. Ask: what happens when the dilation factor is 1?

- In Q3, ensure students understand that a reflection is due to the effect of the negative x^2 term and that the reflection takes place in the y-axis.

- In Q5–8, students will need to combine the concepts of dilation and reflection to draw the graphs and describe the transformations.

- In Q10, students need to look at the graph of $y = x^2$ and see that adding or subtracting a constant moves the graph up or down the y-axis in the same direction as the addition or subtraction.

- In Q13–16, students are looking at graphs that are of the form $y = (x – h)^2$. When students graph these from a table of values they will see that there is a horizontal movement of the graph. Unlike graphs of the form $y = x^2 + k$, the movement of the graph is counter-intuitive to some students. Explain that the movement is to the point such that $x – h = 0$, hence the movement is opposite to the sign in the question.

- In Q18, students will bring all possible transformations together to look at a parabola of the form $y =a(x – h)^2 + k$. To summarise the transformations, the graph of $y = x^2$ is:

– dilated by a factor of ‘a’

– moved vertically ‘k’ units (down if k is negative)

– moved horizontally ‘h’ units in the direction of the solution to $x - h = 0$.

Extra activities

1 Quick questions

Find the value of y when $x = 0$ for each quadratic relationship.

a $y = x^2 + 4$ ($y = 4$)

b $y = x^2 - 5$ ($y = -5$)

c $y = (x + 4)^2$ ($y = 16$)

d $y = (x - 5)^2$ ($y = 25$)

2 **Above Level students:** Consider the graph of $x = y^2$.

a What would the graph look like? (parabola on its side)

b There are two transformations that could be performed on $y = x^2$ to produce this graph. What are they? (90° clockwise rotation or reflection in the line $y = x$)

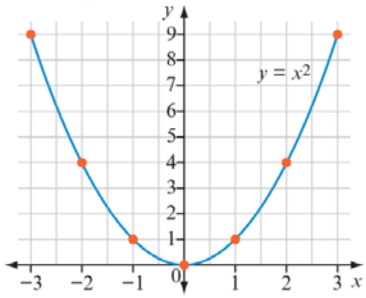
c How might the graph of $x = y^2$ differ from $y = \sqrt{x}$? (square root sign means positive square root so $y = \sqrt{x}$ is only the top half of the graph of $x = y^2$)

Answers

4C Parabolas and transformations

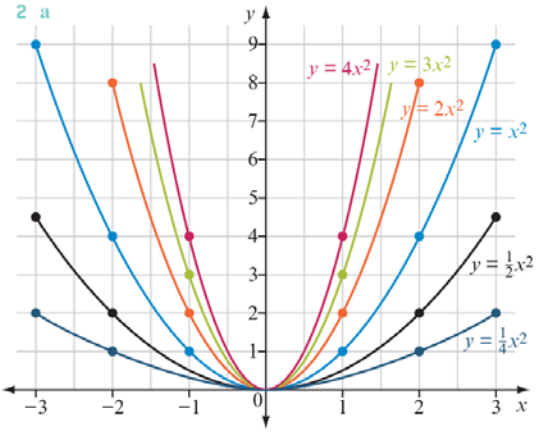
4C Start thinking!

1 a



b minimum turning point at (0, 0), x-intercept at 0 and y-intercept at 0, axis of symmetry $x = 0$

2 a



b All graphs have same minimum turning point, axis of symmetry and x- and y-intercepts but different shapes (some wider than $y = x^2$ and some narrower).

c i $y = 2x^2$, $y = 3x^2$ and $y = 4x^2$

ii $y = \frac{1}{2}x^2$ and $y = \frac{1}{4}x^2$

d For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$, dilation produces a wider graph than $y = x^2$.
For $a > 1$, dilation produces a narrower graph than $y = x^2$.

e ii dilated by factor of 3, narrower

iii dilated by factor of 4, narrower

iv dilated by factor of $\frac{1}{2}$, wider

v dilated by factor of $\frac{1}{4}$, wider

3 For rules of the form $y = ax^2$ where a is positive, there is dilation only (dilation factor is a). For $0 < a < 1$, dilation produces a wider graph than $y = x^2$. For $a > 1$, dilation produces a narrower graph than $y = x^2$.

Exercise 4C Parabolas and transformations

1 **a** $y = \frac{1}{2}x^2$

b One possible answer is $y = \frac{1}{4}x^2$.

c $y = 2x^2$ **d** One possible answer is $y = 4x^2$.

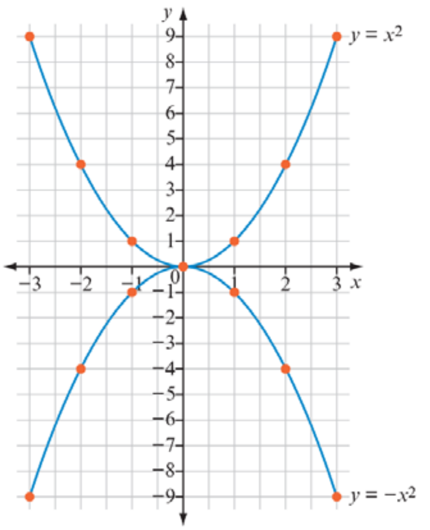
2 **a** dilate by factor of 2, narrower

b dilate by factor of $\frac{1}{2}$, wider

c dilate by factor of $\frac{1}{4}$, wider

d dilate by factor of 4, narrower

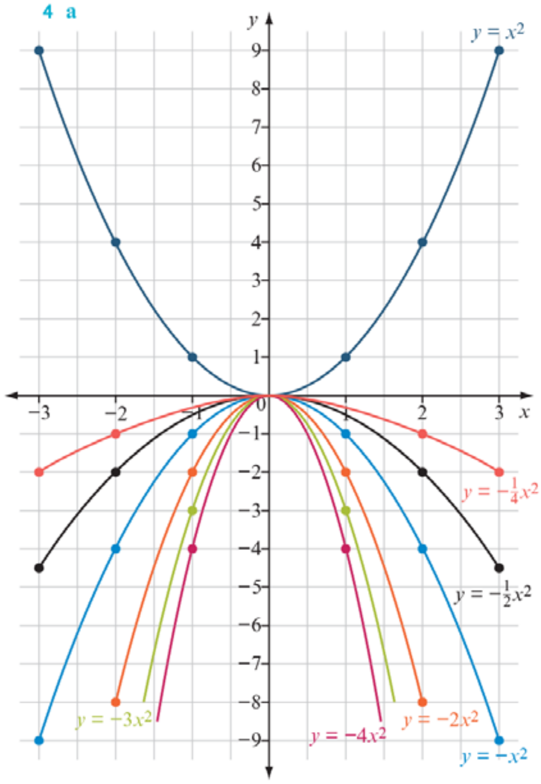
3 **a**



b Mirror image of $y = x^2$ (reflected in x-axis); no dilation; reflection has been performed.

c x-axis; exact image appears beneath x-axis, which acts as mirror line.

4 **a**



b All graphs have same maximum turning point, axis of symmetry and x- and y-intercepts, but different shapes (some wider than $y = -x^2$ and some narrower)

c **i** $y = -2x^2$, $y = -3x^2$ and $y = -4x^2$

ii $y = -\frac{1}{2}x^2$ and $y = -\frac{1}{4}x^2$

5 **a** dilate by factor of 2 (narrower)

b dilate by factor of $\frac{1}{2}$ (wider)

c dilate by factor of $\frac{1}{4}$ (wider)

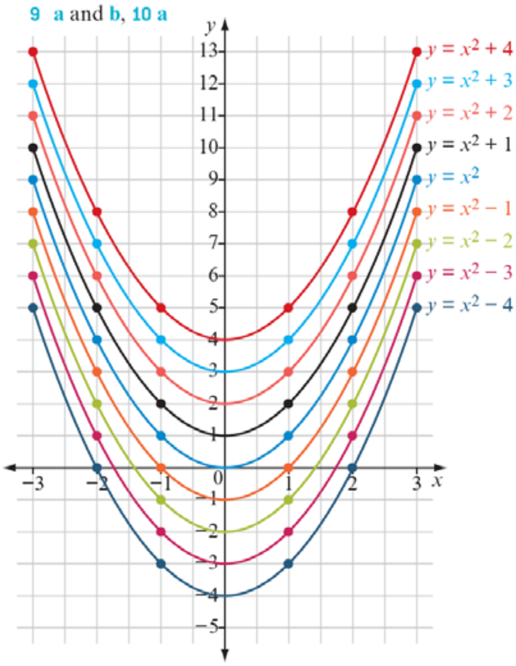
d dilate by factor of 4 (narrower)

6 a dilate by factor of 5 (narrower)
b reflect in x-axis
c dilate by factor of 4 (narrower) and reflect in x-axis
d dilate by factor of $\frac{1}{4}$ (wider)
e dilate by factor of 10 (narrower)
f dilate by factor of $\frac{1}{7}$ (wider) and reflect in x-axis
g dilate by factor of 8 (narrower) and reflect in x-axis
h dilate by factor of $\frac{2}{3}$ (wider) and reflect in x-axis

7 a dilation and reflection
b i $y = -2x^2$: dilate by factor of 2 (narrower) and reflect in x-axis
ii $y = -3x^2$: dilate by factor of 3 (narrower) and reflect in x-axis
iii $y = -4x^2$: dilate by factor of 4 (narrower) and reflect in x-axis
iv $y = -\frac{1}{2}x^2$: dilate by factor of $\frac{1}{2}$ (wider) and reflect in x-axis
v $y = -\frac{1}{4}x^2$: dilate by factor of $\frac{1}{4}$ (wider) and reflect in x-axis

8 a E **b** A **c** D **d** C **e** F **f** B

9 a and **b**, **10 a**



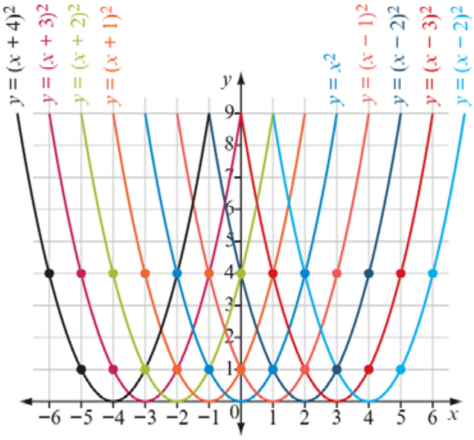
9 c i All graphs have same shape as $y = x^2$, a minimum turning point and same axis of symmetry.
ii Graphs have different turning point coordinates and different y-intercepts; graphs have no x-intercepts.
d Graph shifts up by same number of units as constant term.
e i translated 1 unit up
ii translated 2 units up
iii translated 3 units up
iv translated 4 units up

10 a see above
b i All graphs have same shape as $y = x^2$, a minimum turning point and same axis of symmetry.
ii Graphs have different turning point coordinates and different y-intercepts. Graphs all have two x-intercepts.
c Graph shifts down by same number of units as constant term.
d i translated 1 unit down
ii translated 2 units down
iii translated 3 units down
iv translated 4 units down

11 **a** shifts the graph vertically (translation)
b If k is positive, graph shifts up and if k is negative, graph shifts down.
c up, down
d Graph has no vertical movement.

12 **a** **i** $k = 6$ **ii** translated 6 units up
b **i** $k = -7$ **ii** translated 7 units down
c **i** $k = -5$ **ii** translated 5 units down
d **i** $k = 8$ **ii** translated 8 units up
e **i** $k = 9$ **ii** translated 9 units up
f **i** $k = -11$ **ii** translated 11 units down
g **i** $k = 1.5$ **ii** translated 1.5 units up
h **i** $k = -7.2$ **ii** translated 7.2 units down

13 **a, b, 14** **a**



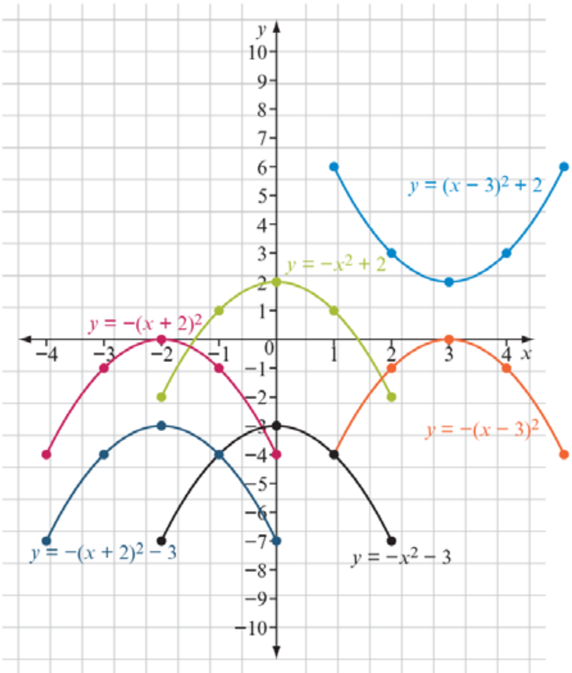
13 **c** **i** All graphs have the same shape as $y = x^2$ and a minimum turning point.
ii Graphs have different turning point coordinates, axes of symmetry and y-intercepts. Graphs all have one x-intercept (they sit on the x-axis).
d Shifts graph horizontally to right.
e **i** translated 1 unit right
ii translated 2 units right
iii translated 3 units right
iv translated 4 units right

14 **a** see above
b **i** All graphs have same shape as $y = x^2$, and a minimum turning point.
ii Graphs have different turning point coordinates, axes of symmetry and y-intercepts. Graphs all have one x-intercept (they sit on the x-axis).
c Shifts graph horizontally to left.
d **i** translated 1 unit left
ii translated 2 units left
iii translated 3 units left
iv translated 4 units left

15 **a** Shifts graph horizontally (translation).
b If h positive, graph moves to right; if h negative, graph moves to left.
c right, left
d Graph has no horizontal movement.

16 **a** **i** $h = 5$ **ii** translated 5 units right
b **i** $h = -7$ **ii** translated 7 units left
c **i** $h = 6$ **ii** translated 6 units right
d **i** $h = -9$ **ii** translated 9 units left
e **i** $h = -8$ **ii** translated 8 units left
f **i** $h = 12$ **ii** translated 12 units right
g **i** $h = 2.5$ **ii** translated 2.5 units right
h **i** $h = -6.7$ **ii** translated 6.7 units left

17 **a**



b **i** (0, 2) **ii** (0, −3) **iii** (3, 0) **iv** (−2, 0) **v** (3, 2) **vi** (−2, −3)
c **i** inverted **ii** inverted **iii** inverted **iv** inverted **v** upright **vi** inverted
d A iii, B i, C v, D iv, E vi, F ii

18 **a** **i** $a = -1; h = 0; k = 2$
ii $a = -1; h = 0; k = -3$
iii $a = -1; h = 3; k = 0$
iv $a = -1; h = -2; k = 0$
v $a = 1; h = 3; k = 2$
vi $a = -1; h = -2; k = -3$
b Turning point of $y = x^2$ (0, 0) shifts horizontally h units and vertically k units and end result is coordinates (h, k).
c If a greater than 0, parabola will be upright and if a less than 0, parabola will be inverted.
d Dilation: narrower when $a < -1$ or $a > 1$ and wider when $-1 < a < 0$ or $0 < a < 1$. For $a > 0$, upright parabola and for $a < 0$, inverted parabola (reflection in x-axis). Horizontal translation (h units). For $h > 0$, move right and for $h < 0$, move left. Vertical translation (k units). For $k > 0$, move up and for $k < 0$, move down.

Reflect

Possible answer: $y = x^2$ is the most basic graph because it has its turning point at the origin. All transformations can then be referenced to this most important point on the Cartesian plane.

Resources

SupportSheet

SS 4C-1 Performing transformations on a coordinate point

Focus: To perform reflections and translations on a coordinate point

Resources: ruler, **1-cm grid paper** (BLM) or graph paper

This task will help students to define key terms associated with transformations including translation, reflection and image. Students will identify the coordinates of an image point that has been translated, vertically, horizontally or both as well as reflected in a given line.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q4 relates to Exercise 4C.

Students solve a variety of quadratic equations in preparation for determining *x*-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of *x*- and *y*-intercepts and the turning point.

BLMs

Cartesian plane grids

$y = x^2$

1-cm grid paper

Interactives

4C eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4D Sketching parabolas using transformations

Teaching support for pages 172–177

Teaching strategies

Learning focus

To use the knowledge gained about transformations to sketch parabolas

Start thinking!

The task guides students to:

- consider a series of quadratic relationships
- think about how each rule is different to $y = x^2$ and what transformation that will cause
- recognise that:
  - a dilation is produced by the coefficient of x^2
  - a reflection in the x-axis is produced by a negative coefficient of x^2
  - a vertical movement is produced by the addition of a constant
  - a horizontal movement is produced by the subtraction of a constant from x before squaring the result.

Differentiated pathways

Below Level	At Level	Above Level
1–4, 7, 8, 10, 12–14	1, 2a–c, 3a–c, 4–6, 7a–c, 8a–c, 9–16	1, 2d–e, 3d–e, 4–6, 7d–e, 8d–e, 9, 11, 12e–h, 15–22
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 2a–c, 3a–c, 4–6, 7a–c, 8a–c, 9–16

- Demonstrate **4D eTutor** or direct students to do this independently.
- Students will need calculator, pencil, ruler and eraser to complete this topic.

- Provide students with copies of the BLM $\boldsymbol{y = x^2}$ (see Resources). Students can then apply the transformations to this graph to obtain each of their answers.
- This topic asks students to sketch parabolas using some or all of the transformations covered in topic 4C.
- Remind students of each of the transformations individually and how they are recognised.
  - Dilation is produced by the coefficient of x^2
  - Reflection in the x-axis is produced by a negative coefficient of x^2
  - Vertical movement is produced by the addition of a constant. The vertical movement is in the direction of the constant.
  - Horizontal movement is produced by subtracting a constant from x before squaring the result. The horizontal movement is opposite to the sign in the brackets.
- Once students understand each of the individual transformations they will need to combine them using $y = a(x – h)^2 + k$. This is known as turning point form of a parabola.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4D-1**. It shows how to sketch a parabola by performing a vertical translation and will help students to complete Q2.
- **Example 4D-2** shows how to sketch a parabola by performing a horizontal translation. This will help students to complete Q3.
- After completing Q4 and Q5, students should be able to see that, when the rule for a parabola is written in the form $y = a(x – h)^2 + k$, the coordinates of the turning point of the parabola are (h, k). Hence, this is why it is called turning point form.
- **Example 4D-3** shows how to sketch a parabola by performing more than one transformation. This will help students to complete Q7–9.
- In Q8, ask students to consider whether the order in which the reflection and translation are performed makes a difference to the answer. Students should understand that if the translation is performed first the reflection that needs to occur is no longer in the x-axis.
- Q11 requires students to substitute the coordinates of the turning point into $y = a(x – h)^2 + k$. You may need to explain that since no dilation has been performed:
  - $a = 1$ if the parabola is upright

  - $a = –1$ if the parabola is inverted.

- In Q13 and Q14, students may need to draw a sketch of the graph to find the smallest and largest possible values of y and see that these are the y-coordinates of the turning point. Help students to understand that:
  - all upright parabolas have a minimum y value but no maximum
  - all inverted parabolas have a maximum y value but no minimum.

- For additional practice, students can complete Q5–7 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4D Q4, Q7 and Q12 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

Below Level

Below Level
1–4, 7, 8, 10, 12–14

- After completing Q4, students may be able to see that, when the rule for a parabola is written in the form $y = a(x – h)^2 + k$, the coordinates of the turning point of the parabola are (h, k). Hence, this is why it is called turning point form.

- The graphs in Q7 are obtained using both a horizontal and vertical translation. Remind students that the:
  - vertical translation is the value of k in $y = a(x – h)^2 + k$
  - horizontal translation is the value of h in $y = a(x – h)^2 + k$.

- Students at this level will most likely deal with the vertical translation correctly but find the direction of the horizontal translation counter-intuitive. Reinforce that the direction of the horizontal translation is such that $x – h = 0$.

- In Q8, students need to perform a reflection in the y-axis and then either a vertical or horizontal translation. As the reflection needs to be in the x-axis, students will cope better if they are taught to complete the reflection first.

- In Q13 and Q14, students may need to draw a sketch of the graph to find the smallest and largest possible values of y and see that these are the y-coordinates of the turning point. Help students to understand that:
  - all upright parabolas have a minimum y value but no maximum
  - all inverted parabolas have a maximum y value but no minimum.

- For students who do not progress past Q12, direct them to Q5–7 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.

- Below Level students may find it very difficult to draw graphs where they need to perform more than one transformation. Allowing them to use graphing software to obtain their answers will help them to achieve some success. If they do this, have them copy their answers onto the BLM $\boldsymbol{y = x^2}$ (see resources) and ask them to describe the transformation.

- Although the purpose of this exercise is teach students to sketch graphs without the need for a table of values, students operating at this level may need to use this method to obtain graphs.

Above Level

Above Level
1, 2d–e, 3d–e, 4–6, 7d–e, 8d–e, 9, 11, 12e–h, 15–22

- After completing Q4 and Q5, students should be able to see that, when the rule for a parabola is written in the form $y = a(x - h)^2 + k$, the coordinates of the turning point of the parabola are (h, k). Hence, this is why it is called turning point form.

- In Q8, ask students to consider whether the order in which the reflection and translation are performed makes a difference to the answer. Students should understand that if the translation is performed first, the reflection that needs to occur is no longer in the x-axis.

- Q11 requires students to substitute the coordinates of the turning point into $y = a(x - h)^2 + k$. You may need to explain that since no dilation has been performed:
  - $a = 1$ if the parabola is upright
  - $a = -1$ if the parabola is inverted.

- In Q15, students sketch the graph of a parabola that represents the height of a basketball in terms of time. Discuss with students that the graph drawn does not represent the flight path of the ball. Ask them to consider why not? (Answer: the relationship only considers the vertical position of the ball and makes no reference to the horizontal position.)

Extra activities

1 Quick Questions

State the coordinates of the turning point of each parabola with these rules.

a $y = x^2$ (0, 0)

b $y = x^2 + 2$ (0, 2)

c $y = 3x^2$ (0, 0)

d $y = (x + 4)^2$ (–4, 0)

2 The prefix ‘para’ means ‘almost’. For example, a paramedic means almost a medical practitioner but not actually a doctor. As such, a parabola is almost a bowl shape. There are many daily objects that take this shape. Identify as many objects as possible that are parabolic in shape. (Answers may include, but not be limited to, the path of a projectile, a headlight in a car, a satellite dish, a household vase.)

Answers

4D Sketching parabolas using transformations

4D Start thinking!

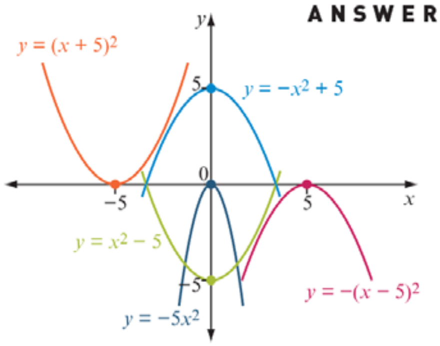
1 a i translate 5 units down
ii translate 5 units left
iii dilate by factor of 5 and reflect in x-axis
iv reflect in x-axis and translate 5 units up
v reflect in x-axis and translate 5 units right

b i upright ii upright iii inverted
iv inverted v inverted
Compare the rule to $y = a(x - h)^2 + k$. Look at sign of a. If negative, graph is inverted and if positive, graph is upright.

c Use (h, k) values.
i (0, −5) ii (−5, 0) iii (0, 0)
iv (0, 5) v (5, 0)

ANSWERS

d



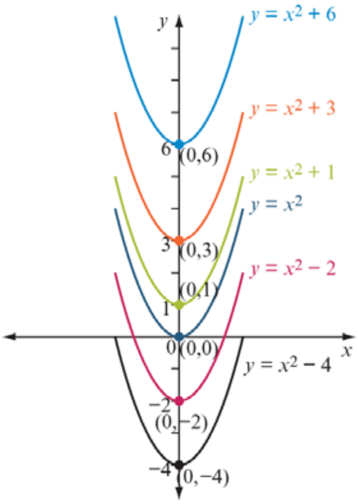
e Use values for a, h and k.

2 Use value of a to determine if there is dilation (narrower or wider) or reflection.
For $a > 0$, upright parabola and for $a < 0$, inverted parabola (reflection in the x-axis). Horizontal translation (h units). For $h > 0$, move right and for $h < 0$, move left. Vertical translation (k units). For $k > 0$, move up and for $k < 0$, move down.

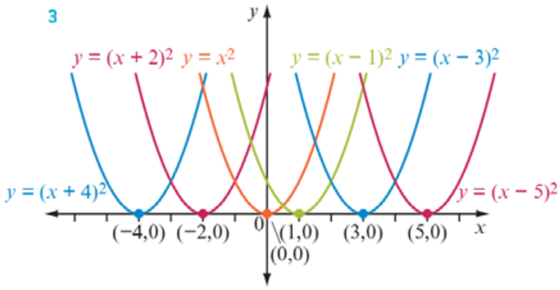
Exercise 4D Sketching parabolas using transformations

1 **a** **i** upright **ii** (2, 0)
iii translated 2 units right **iv** C
b **i** inverted **ii** (0, 0)
iii dilation by factor of 2, reflection in x-axis
iv A
c **i** upright **ii** (0, 2)
iii translated 2 units up **iv** B

2



3

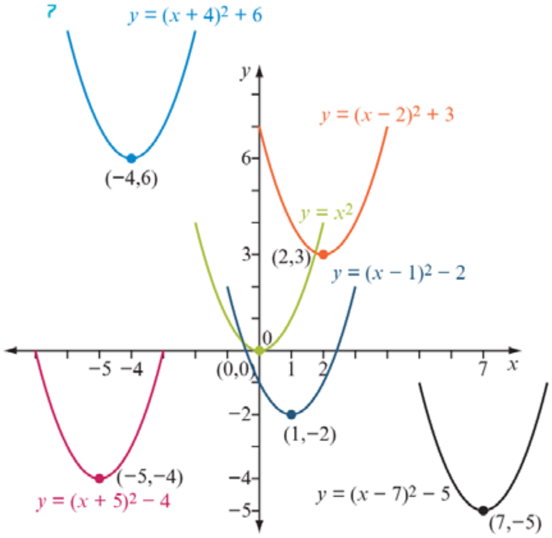


4 **a** upright **b** (4, −2)
c C. Turning point has moved right 4 units ($h = 4$) and down 2 units ($k = -2$).

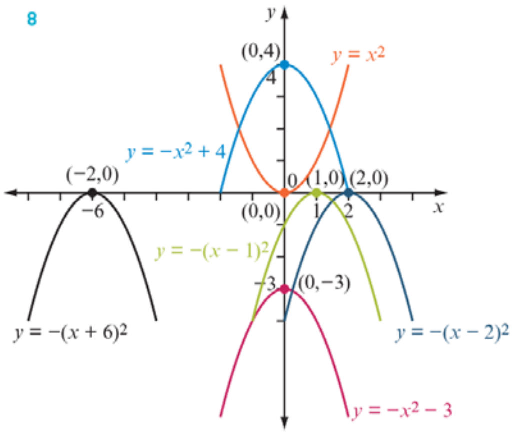
5 **a** $a = 1$, $h = 3$ and $k = 4$
b no dilation or reflection; horizontal translation (3 units right) and vertical translation (4 units up)
c (3, 4)
d Graph shifts right 3 units and up 4 units, resulting in turning point coordinates (3, 4)

6 **a** $a = 1$, $h = 4$ and $k = -2$
b no dilation or reflection; horizontal translation (4 units right) and vertical translation (2 units down)
c (4, −2)

7



8

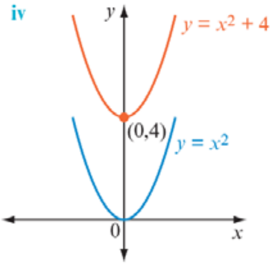


9 **a** E **b** D **c** A **d** F **e** C **f** B

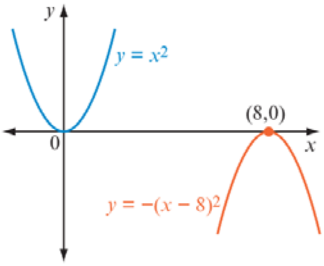
10 **a** **i** upright **ii** $(1, -4)$
iii y-intercept: $(0, -3)$; x-intercepts: $(-1, 0)$ and $(3, 0)$
b **i** inverted **ii** $(-3, 1)$
iii y-intercept: $(0, -8)$; x-intercepts: $(-4, 0)$ and $(-2, 0)$
c **i** upright **ii** $(3, -9)$
iii y-intercept: $(0, 0)$; x-intercepts: $(0, 0)$ and $(6, 0)$

11 **a** $y = (x - 3)^2 + 7$ **b** $y = (x + 2)^2 + 5$
c $y = -(x - 2)^2 + 4$ **d** $y = -(x - 6)^2 - 1$
e $y = (x - 9)^2$ **f** $y = -x^2 + 4$
g $y = -(x + 1)^2 - 2$ **h** $y = x^2 - 5$

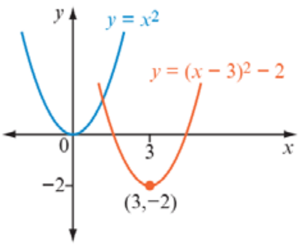
12 **a** **i** translate 4 units up
ii upright
iii $(0, 4)$
iv



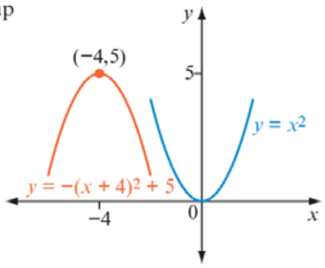
b **i** reflect in x-axis and translate 8 units right
ii inverted
iii $(8, 0)$
iv



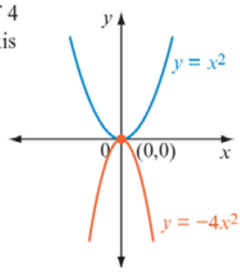
c **i** translate 3 units right and 2 units down
ii upright
iii $(3, -2)$
iv



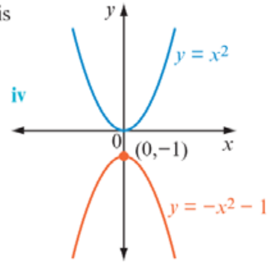
d **i** reflect in x-axis and translate 4 units left and 5 units up
ii inverted
iii $(-4, 5)$
iv



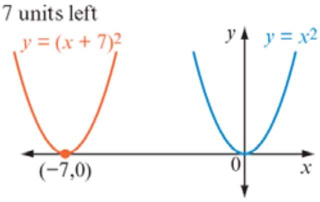
e **i** dilate by factor of 4 and reflect in x-axis
ii inverted **iv**
iii $(0, 0)$



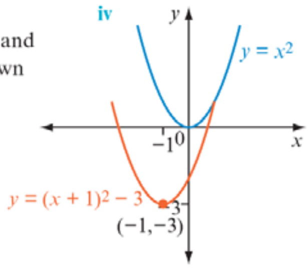
f **i** reflect in x-axis and translate 1 unit down
ii inverted **iv**
iii $(0, -1)$



g **i** translate 7 units left
ii upright
iii $(-7, 0)$
iv



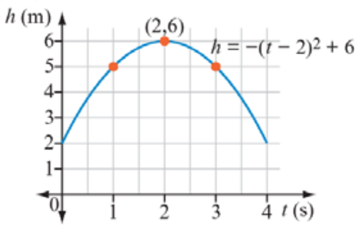
h **i** translate 1 unit left and 3 units down
ii upright
iii $(-1, -3)$
iv



13 **a** $(4, 5)$ **b** 5

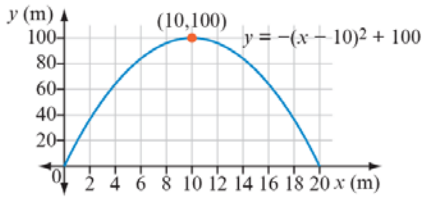
14 2. Graph is inverted so has maximum turning point. No possible larger y value than the y coordinate of the turning point, 2.

15 **a** $(2, 6)$ **b** **i** 2 m **ii** 2 m
c



d **i** 2 m **ii** 6 m

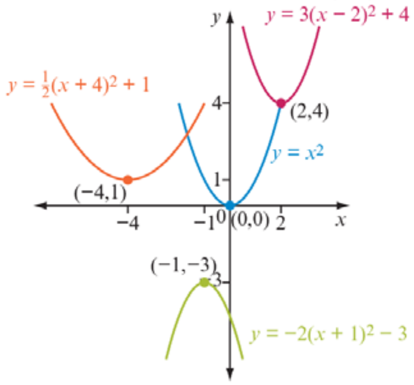
16 a



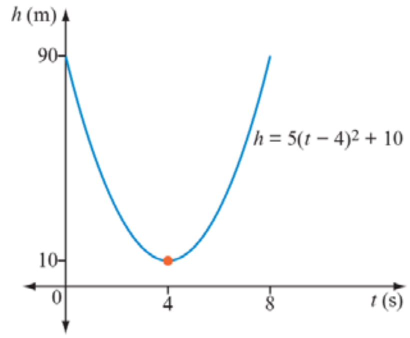
b 100 m **c** 20 m

17	**a**	**i** narrower	**ii** upright	**iii** $(4, -3)$
	b	**i** narrower	**ii** inverted	**iii** $(-1, 5)$
	c	**i** narrower	**ii** upright	**iii** $(-2, 0)$
	d	**i** narrower	**ii** inverted	**iii** $(0, -4)$
	e	**i** the same	**ii** inverted	**iii** $(5, 4)$
	f	**i** wider	**ii** upright	**iii** $(-2, 6)$
	g	**i** wider	**ii** inverted	**iii** $(3, -4)$
	h	**i** wider	**ii** upright	**iii** $(0, 3)$
	i	**i** wider	**ii** inverted	**iii** $(7, 0)$

18



19 a



b 90 m **c** 10 m

20 The graph of $y = 2x^2$ is the result of multiplying each y value in the graph of $y = x^2$ by a factor of 2. The graph of $y = -2x^2$ is the result of multiplying each y value in the graph of $y = 2x^2$ by a factor of -1. This results in an 'upside down' or inverted graph. An inverted graph is also a reflection in the x-axis.

21 a $y = -4x^2$ **b** $y = \frac{1}{3}x^2$
c $y = (x + 2)^2$ **d** $y = -x^2 - 5$
e $y = (x + 1)^2 + 4$ **f** $y = -2(x - 5)^2 + 3$

22 a $y = 3(x - 2)^2$ **b** $y = -(x + 5)^2 - 4$
c $y = -\frac{1}{2}x^2$ **d** $y = -4(x + 2)^2$

Reflect

Possible answer: Writing the quadratic relationship in turning point form allows us to find the coordinates of the turning point and identify the number of units horizontally and vertically to translate the graph of $y = x^2$.

Resources

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q5–7 relate to Exercise 4D.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

BLMs

$y = x^2$

1-cm grid paper

Interactives

4D eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4E Sketching parabolas using intercepts

Teaching support for pages 178–183

Teaching strategies

Learning focus

To be able to sketch a parabola showing the critical features of x- and y-intercepts as well as the turning point

Start thinking!

The task guides students to:

- see that a parabola whose rule is in the form $y = x^2 + bx + c$ cannot be sketched easily by using transformations
- find the y-intercept of a parabola by substituting $x = 0$
- find the x-intercepts of a parabola by substituting $y = 0$ and solving the resulting quadratic equation
- consider the coefficient of x^2 in determining whether the graph is upright or inverted
- think about using the x-intercepts of the graph to help determine the coordinates of the turning point.

Differentiated pathways

Below Level	At Level	Above Level
1–9, 10a–c, 14	1–3, 4a, c, f, 5a, c, f, 6–9, 10d–i, 11a, b, 12, 13a–c, 14, 15a–c, 16–18	1–3, 6–9, 10d–l, 11c, d, 12, 13, 15c–e, 16–20
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q7

Focus: To revise the method of finding the x- and y-intercepts of a linear function

- Direct students to complete **SS 4E-1 Finding *x*- and *y*-intercepts** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to know that:

– to find the y-intercept, they must substitute $x = 0$ into the rule

– to find the x-intercept, they must substitute $y = 0$ into the rule and solve the resulting equation.

At Level

At Level
1–3, 4a ,c, f, 5a, c, f, 6–9, 10d–i, 11a, b, 12, 13a–c, 14, 15a–c, 16–18

- Demonstrate **4E eTutor** or direct students to do this independently.
- Students will need calculator, pencil, ruler and eraser to complete this topic.
- Provide students with copies of the BLM **Cartesian plane grids** (see resources). This will assist students in drawing up a large number of Cartesian planes for their sketches.
- Students should be aware of the key features that need to be shown when sketching a parabola:
  – x-intercepts (found by solving the quadratic equation formed when $y = 0$)
  – y-intercept (found by substituting $x = 0$)
  – axis of symmetry (found by halving the distance between the x-intercepts)
  – coordinates of the turning point (y value found by substituting the x value of the axis of symmetry into the equation).
- Students may need to be reminded of what the x- and y-intercepts are and how each is found.
- In finding the x-intercepts for a parabola, students will need to solve a quadratic equation. They may need to revise that for a quadratic equation of the form:
  – $x^2 + bx = 0$ will be factorised by taking out a common factor of x
  – $x^2 – m^2 = 0$ will be factorised using the difference of two squares rule
  – $x^2 + bx + c = 0$ will be factorised using the quadratic trinomial method.

  After the quadratic is factorised, the Null Factor Law is used to find the solutions.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4E-1**. It shows how to find coordinates of the x- and y-intercepts of a quadratic relationship and will help students to complete Q1 and Q2.

- **Example 4E-2** shows how to find coordinates of the turning point using *x*-intercepts. This will help students to complete Q3 and Q4.
- **Example 4E-3** shows how to sketch a parabola using *x*- and *y*-intercepts. This will help students to complete Q6–10.
- For Q7, remind students that it is the coefficient of x^2 that determines if a parabola is upright or inverted.
- In parts f, h and i of Q10, students need to take out –1 as a common factor first before completing the factorisation.

    e.g. $y = -x^2 - 8x - 12$ when $y = 0$

    $0 = -(x^2 + 8x + 12)$

    $0 = -(x + 6)(x + 2)$

- For Q11, to find the *x*-intercepts, students will most likely need to expand and simplify the expression in order to solve the quadratic equation.
- From completing Q13, students should recognise that if the quadratic expression can be factorised as a perfect square, there will only be one *x*-intercept.
- Q15 demonstrates parabolas that have no *x*-intercepts. Students will have to draw the graphs showing the turning point and *y*-intercept only.
- In Q16, explain that the parabola drawn is a representation of the path of the arrow and questions can be answered from the graph.
- For additional practice, students can complete Q8–10 of **WS 4-1 Quadratic relationships** (see Resources). Additional questions similar to Exercise 4E Q2, Q4, Q6 and Q10 are provided. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- For more problem-solving tasks and investigations, direct students to **INV 4-2 Fencing the chicken run** (see Resources).

Below Level

Below Level
1–9, 10a–c, 14

- Students may need to complete **SS 4E-1 Finding *x*- and *y*-intercepts** (see Resources).
- For Q5, when students are sketching parabolas use the BLM **Cartesian plane grids** (see Resources). After they have found the *x*- and *y*-intercepts, assist them in working

out an appropriate scale for their axes. Remind them that the scale on the *x*- and *y*-axes need not be the same.

- For Q7, remind students that it is the coefficient of x^2 that determines if a parabola is upright or inverted.
- For students who do not progress past Q6, direct them to Q8–10 of **WS 4-1 Quadratic relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A, 4B, 4C, 4D and 4E, and can be completed progressively or as a skills review of Exercises 4A–E.
- The idea behind this exercise is to sketch parabolas showing the *x*- and *y*-intercepts. This does not require graphs to be drawn to scale. However, students working at this level achieve more when using a scale. A scale allows students to check points on the parabola by substitution.
- Some students will still need to produce their graphs using a table of values. If students are doing this, they will easily find the *y*-intercept as $x = 0$ should be a value in their table. However, they will need more support to ensure that their graph shows any *x*-intercepts.
- A graphics calculator or graphing software may be helpful to assist students in obtaining their graphs. Use this software where appropriate to display the value of all intercepts.

Above Level

Above Level
1–3, 6–9, 10d–l, 11c, d, 12, 13, 15c–e, 16–20

- For Q7, remind students that it is the coefficient of x^2 that determines if a parabola is upright or inverted.
- In Q11, ask students to explore the *x*-intercepts in two ways.
  - Expand and simplify the expression into the form $x^2 + bx + c = 0$ and solve the equation.
  - Find the coordinates of the turning point from the turning point form that the rule is given in. Then consider the shape of $y = x^2$ and the number of units that *y* is below the *x*-axis. The square root of this number will be the number of units either side of the turning point that the *x*-intercepts will lie. For example, if the vertex is at (2, –9) the *x*-intercepts will be 3 units either side of $x = 2$ (i.e. $x = –1$ and $x = 5$).

- From completing Q13, students should recognise that if the quadratic expression can be factorised as a perfect square, there will only be one x-intercept.
- Q15 demonstrates parabolas that have no x-intercepts. Students will have to draw the graphs showing the turning point and y-intercept only.
- In Q18, explain that the parabola drawn is a representation of the path of the soccer ball and questions can be answered from the graph.
- In Q20, students can only write more quadratic rules by multiplying the entire rule by a constant.
- For more problem-solving tasks and investigations, direct students to **INV 4-2 Fencing the chicken run** (see Resources).

Extra activities

1 Quick questions

Solve each quadratic equation.

a $x^2 + 4x = 0$ ($x = 0$ or $x = –4$)

b $x^2 – 16 = 0$ ($x = –4$ or $x = 4$)

c $x^2 = 25$ ($x = –5$ or $x = 5$)

d $x^2 + 6x + 9 = 0$ ($x = –3$)

e $x^2 – 8x – 20 = 0$ ($x = –2$ or $x = 10$)

2 Some parabolas have two x-intercepts, some have one x-intercept and others have none at all. Consider the relationship $y = x^2 + 6x + 3$.

a Find the y-intercept. (3)

b Attempt to find the x-intercepts. Consider if being unable to factorise the expression on the right side means that the x-intercepts do not exist. (x-intercepts do exist but expression on RS not easy to factorise)

c The following process will convert the equation $y = x^2 + 6x + 3$ into turning point form.

Create a perfect square by halving 6 and squaring the result. Subtract 9 to compensate. The constant term should still total 3. Factorise the perfect square expression.

$y = x^2 + 6x + 3$

$= (x^2 + 6x + 9) – 9 + 3$

$= (x^2 + 6x + 9) - 6$

$= (x + 3)^2 - 6$

d What are the coordinates of the turning point? [(–3, –6)]

e Sketch the graph of $y = x^2 + 6x + 3$ showing the turning point and y-intercept.

f How many x-intercepts does the parabola have? (2)

g Using digital technology, draw the parabola. Find two whole numbers between which each x-intercept lies. (–6 and –5, –1 and 0)

h Repeat this process with the following parabolas to determine if they have two, one or no x-intercepts.

i $y = x^2 + 8x + 20$ (no x-intercepts)

ii $y = x^2 - 4x - 2$ (two x-intercepts)

iii $y = x^2 - 6x + 9$ (one x-intercept)

Answers

4E Sketching parabolas using intercepts

ANSWERS

4E Start thinking!

1 When a quadratic relationship is in turning point form, it is easy to identify the transformations applied to $y = x^2$ to result in the given relationship. The general form of a quadratic, that is, $y = ax^2 + bx + c$, does not follow the same rules as turning point form.

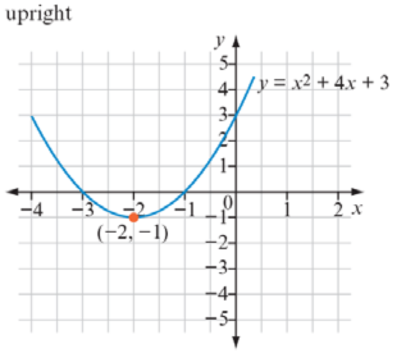
2 a $x = 0$ b $y = 3$ c 3

3 a $y = 0$ b $0 = x^2 + 4x + 3$
c $x = -3$ or $x = -1$ d -3 and -1

4 (0, 3), (−3, 0) and (−1, 0)

5 upright

6



7 Axis of symmetry of a parabola is halfway between x-intercepts. Hence, x-coordinate of turning point is halfway between x values at x-intercepts. y-coordinate of turning point is found by substituting x-coordinate into rule and simplifying.

8 Can sketch quadratic relationships using x- and y-intercepts. x-intercept/s are found by substituting $y = 0$ into rule and solving for x. Equation may need to be factorised first so that Null Factor Law can be used. A parabola can have two, one or no x-intercepts. y-intercept is found by substituting $x = 0$ into rule and simplifying. The x-coordinate of turning point is halfway between x values at x-intercepts. y-coordinate of turning point is found by substituting x-coordinate into rule and simplifying. Orientation of parabola (upright or inverted) can be identified from coefficient of x^2 term: if $a > 0$, parabola is upright and if $a < 0$, parabola is inverted.

Exercise 4E Sketching parabolas using intercepts

1 **a** coordinates of x-intercepts (–3, 0) and (5, 0)
coordinates of y-intercept (0, –15)

c coordinates of x-intercepts (–1, 0) and (1, 0)
coordinates of y-intercepts (0, –1)

2 **a** **i** (0, 0) and (2, 0) **ii** (0, 0)

b **i** (−8, 0) and (0, 0) **ii** (0, 0)

c **i** (−4, 0) and (−2, 0) **ii** (0, 8)

d **i** (2, 0) and (6, 0) **ii** (0, 12)

e **i** (−1, 0) and (5, 0) **ii** (0, −5)

f **i** (−3, 0) and (3, 0) **ii** (0, −9)

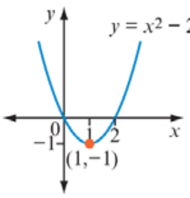
3 **a** coordinates of turning point (1, –16).

b coordinates of turning point (0, –1).

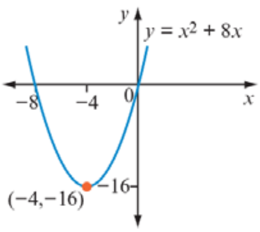
4 **a** (1, −1) **b** (−4, −16) **c** (−3, −1)

d (4, −4) **e** (2, −9) **f** (0, −9)

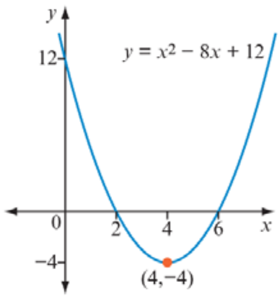
5 **a**



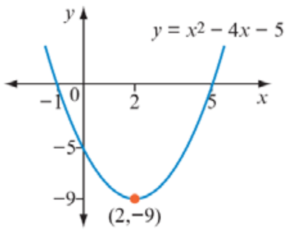
b



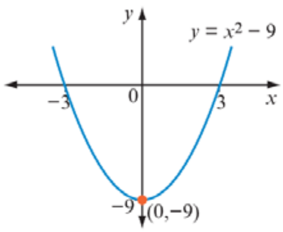
d



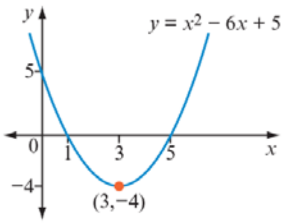
e



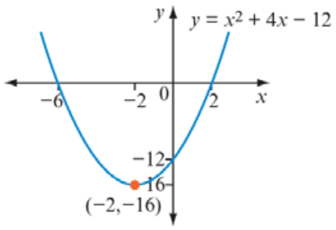
f



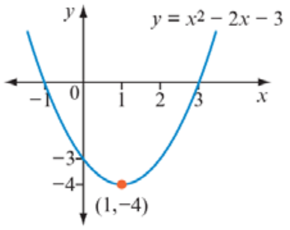
6 **a**



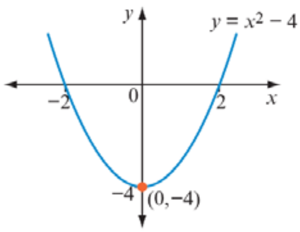
b



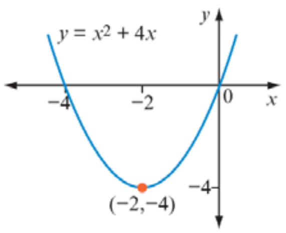
c



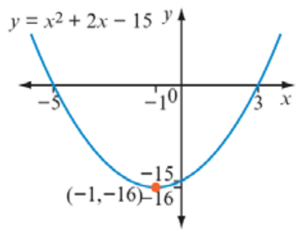
d



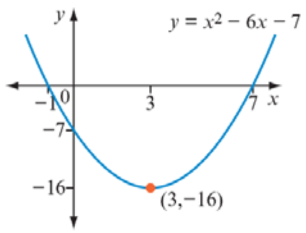
e



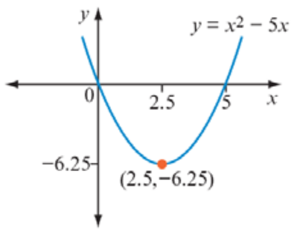
f



g



h



7 **a** C **b** A **c** B

8 Look at coefficient of x^2 term (a). If $a > 0$, parabola is upright and if $a < 0$, parabola is inverted.

9 a Both have x-intercepts of -3 and 2.

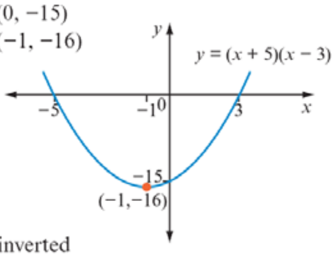
b One graph has negative x^2 coefficient and other has positive x^2 coefficient, so one will be inverted and the other upright.

10 a i upright

ii x-intercepts: $(-5, 0)$ and $(3, 0)$; y-intercept: $(0, -15)$

iii $(-1, -16)$

iv

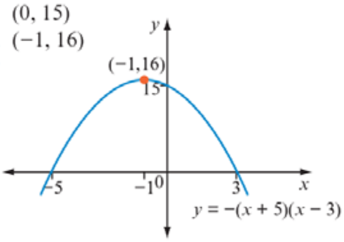


b i inverted

ii x-intercepts: $(-5, 0)$ and $(3, 0)$; y-intercept: $(0, 15)$

iii $(-1, 16)$

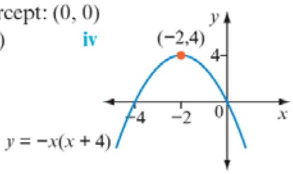
iv



c i inverted

ii x-intercepts: $(-4, 0)$ and $(0, 0)$; y-intercept: $(0, 0)$

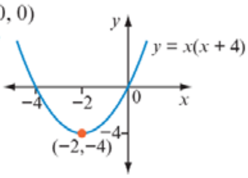
iii $(-2, 4)$ iv



d i upright

ii x-intercepts: $(-4, 0)$ and $(0, 0)$; y-intercept: $(0, 0)$

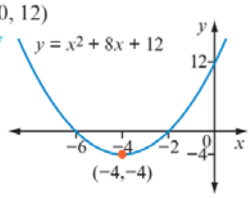
iii $(-2, -4)$ iv



e i upright

ii x-intercepts: $(-6, 0)$ and $(-2, 0)$; y-intercept: $(0, 12)$

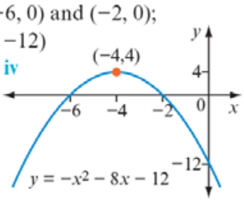
iii $(-4, -4)$ iv



f i inverted

ii x-intercepts: $(-6, 0)$ and $(-2, 0)$; y-intercept: $(0, -12)$

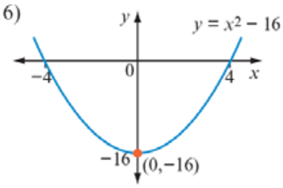
iii $(-4, 4)$ iv



g i upright

ii x-intercepts: $(-4, 0)$ and $(4, 0)$; y-intercept: $(0, -16)$

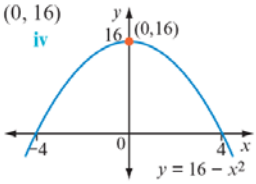
iii $(0, -16)$ iv



h i inverted

ii x-intercepts: $(-4, 0)$ and $(4, 0)$; y-intercept: $(0, 16)$

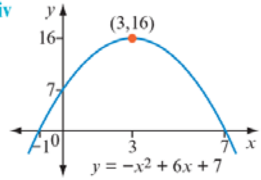
iii $(0, 16)$ iv



i i inverted

ii x-intercepts: $(-1, 0)$ and $(7, 0)$; y-intercept: $(0, 7)$

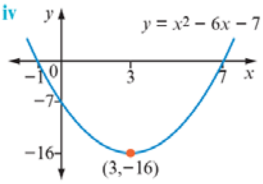
iii $(3, 16)$ iv



j i upright

ii x-intercepts: $(-1, 0)$ and $(7, 0)$; y-intercept: $(0, -7)$

iii $(3, -16)$ iv

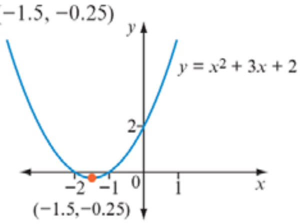


k i upright

ii x-intercepts: $(-2, 0)$ and $(-1, 0)$; y-intercept: $(0, 2)$

iii $(-1.5, -0.25)$

iv

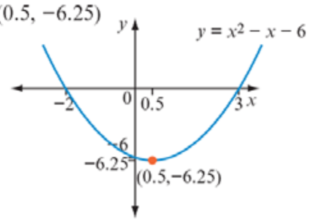


l i upright

ii x-intercepts: $(-2, 0)$ and $(3, 0)$; y-intercept: $(0, -6)$

iii $(0.5, -6.25)$

iv

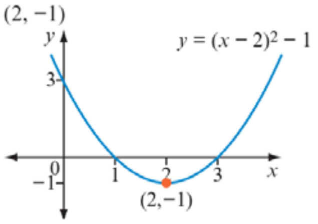


11 a i upright

ii x-intercepts: (1, 0) and (3, 0); y-intercept: (0, 3)

iii (2, −1)

iv

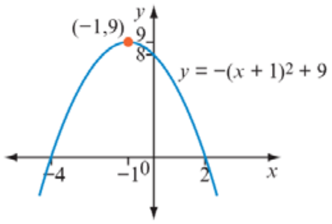


b i inverted

ii x-intercepts: (−4, 0) and (2, 0); y-intercept: (0, 8)

iii (−1, 9)

iv

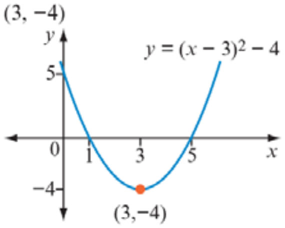


c i upright

ii x-intercepts: (1, 0) and (5, 0); y-intercept: (0, 5)

iii (3, −4)

iv

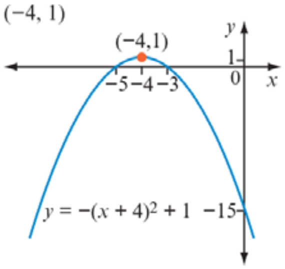


d i inverted

ii x-intercepts: (−5, 0) and (−3, 0); y-intercept: (0, −15)

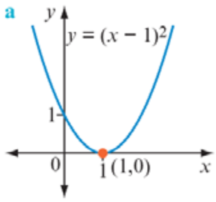
iii (−4, 1)

iv

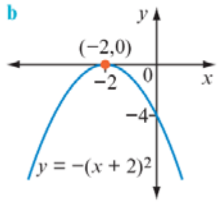


12 a upright **b** one (0, 9) **c** one (3, 0)
d (3, 0)

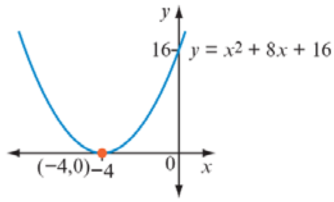
13 a



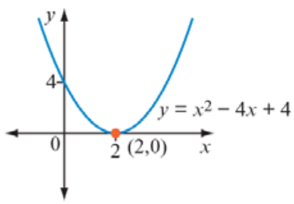
b



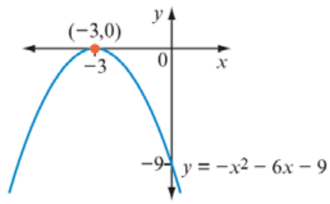
c



d

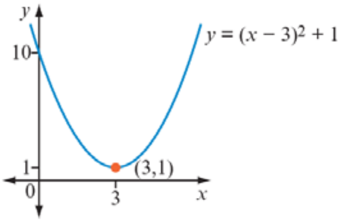


e

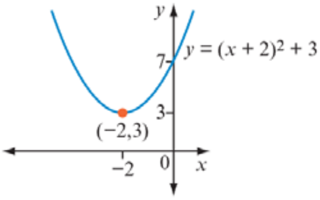


14 a inverted **b** one (0, −1) **c** none
d (0, −1)

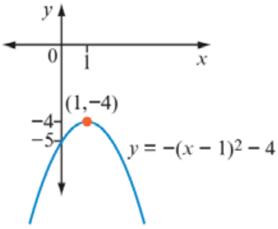
15 **a**



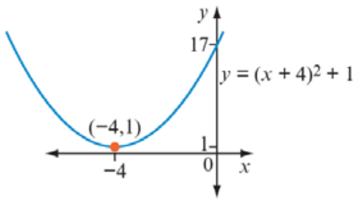
b



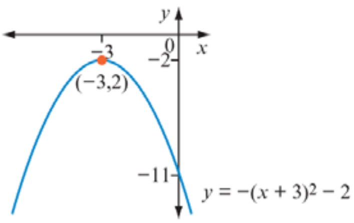
c



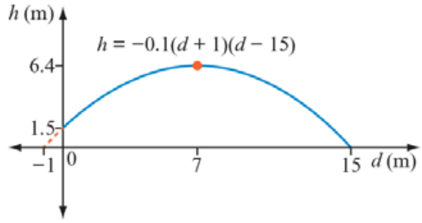
d



e

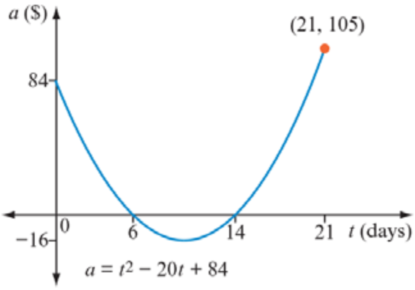


16 **a**



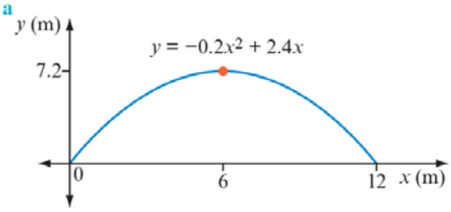
b 6.4 m **c** 1.5 m **d** 15 m

17 **a**



b $84 **c** $48 **d** just after sixth day
e $16 **f** after 14th day
g $105 (at end of the three weeks)

18 **a**



b 7.2 m **c** 6 m **d** 12 m

19, 20 Some possible answers are given.

a $y = (x - 7)(x - 2)$ or $y = -(x - 7)(x - 2)$ or $y = -6(x - 7)(x - 2)$

b $y = x^2 - 8x$ or $y = -x(x - 8)$ or $y = -4x(x - 8)$

c $y = (x + 4)(x - 5)$ or $y = -(x + 4)(x - 5)$ or $y = -3(x + 4)(x - 5)$

Reflect

Possible answer: A parabola may have two, one or no *x*-intercepts but will always have one *y*-intercept.

Resources

SupportSheet

SS 4E-1 Finding *x*- and *y*-intercepts

Focus: To explore where *x*- and *y*-intercepts occur and determine intercepts for linear relationships

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students look at linear graphs and from the graph they observe that the x-intercept occurs when $y = 0$ and the y-intercept occurs when $x = 0$. Students then obtain these values by substitution and evaluation rather than requiring the graph to be drawn.

WorkSheet

WS 4-1 Quadratic relationships

Focus: To solve quadratic equations and apply those skills to sketching quadratic relationships

Resources: 1-cm grid paper (BLM) or graph paper

- This WorkSheet provides a skills review for Exercises 4A–E. Q8–10 relate to Exercise 4E.

Students solve a variety of quadratic equations in preparation for determining x-intercepts when sketching parabolas. They first plot quadratic relationships using a table of values and then describe transformations performed on the graph of $y = x^2$ given relationships shown in the form $y = (x - h)^2 + k$.

Students sketch graphs of quadratic relationships using two methods: by transforming the graph of $y = x^2$ and by finding coordinates of x- and y-intercepts and the turning point.

Investigation

INV 4-2 Fencing the chicken run

Focus: To determine the largest enclosure that can be made with a limited length of fencing

Resources: 1-cm grid paper (BLM) or graph paper, calculator

Students consider the problem of building the fence for a rectangular chicken run. They consider the different dimensions that the chicken run could have, given a fixed length of fencing, with the aim to maximise the area contained in the chicken run.

Students explore this problem for two scenarios:

- all four sides of the chicken run need to be fenced
- three sides of the chicken run need fencing, using an existing structure as one side.

For both scenarios, students will model the problem using a quadratic relationship where the coordinates of the turning point will represent the solution to the problem.

As an extension, students draw a design for the perimeter of the enclosure which includes the positions of fence posts.

BLMs

Cartesian plane grids

1-cm grid paper

Interactives

4E eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4F Circles and other non-linear relationships

Teaching support for pages 184–189

Teaching strategies

Learning focus

To look at other non-linear relationships such as the circle, hyperbola and cubic graph

Start thinking!

The task guides students to:

- recognise the rule for a circle with its centre at the origin
- determine the radius of a circle with its centre at the origin
- consider the translations performed on a circle with its centre at the origin and, in particular, the effect on the coordinates of the centre
- recognise the form of a rule for a circle with the centre not at the origin
- determine the centre and radius of a circle with the centre not at the origin.

Differentiated pathways

Below Level	At Level	Above Level
1–7, 9–13, 15a–c, 16a–c	1, 3–6, 8–12, 14–16	1, 3–6, 8, 9, 11c, d, 12, 14–18
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

At Level

At Level
1, 3–6, 8–12, 14–16

- Demonstrate **4F eTutor** or direct students to do this independently.
- To complete this topic, students will need calculator, ruler, pencil and eraser.
- To assist students with drawing many graphs, provide copies of the BLM **Cartesian plane grid** (see Resources).

- Remind students that relationships can be classified as being linear or non-linear. In a linear relationship both x and y are to a power of 1. Any other relationship is non-linear.
- Students should now recognise that when the highest power of x is 2 the graph will be a parabola.
- Most of this topic is about the graph of a circle. A circle has a rule where both x and y are squared.
- Discuss that the most basic form of a circle is $x^2 + y^2 = r^2$. This graph has:
  - its centre at (0, 0)
  - a radius of r units.
- Explain that a circle with its centre not at (0, 0) is of the form $(x - h)^2 + (y - k)^2 = r^2$. This graph has:
  - its centre at the point with coordinates (h, k)
  - a radius of r units.
- To draw this graph the graph of $x^2 + y^2 = r^2$ is translated h units horizontally and k units vertically.
- Other graphs that students should be able to recognise after completing this topic are:
  - simple cubic relationships of the form $y = x^3 + k$
  - square root relationships of the form $y = \sqrt{x - h}$

    Optional:
  - reciprocal relationships (hyperbolas) of the form $y = \dfrac{1}{x - h}$.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- In Q8, students will need to explain the translation that is made in both the horizontal and vertical directions. With the negative sign in the general form of a circle, the direction of the translation may be counter-intuitive to some students.
- Direct students to **Example 4F-1**. It shows how to sketch a circle from its rule and will help students to complete Q9.
- In Q9, an alternative method for students to find the centre of a circle is to consider the expression in each pair of brackets equal to zero and solve each linear equation to find x and y. This method ensures they have the correct sign.

- You will need to ensure students have access to graphics calculators or graphing software such as *GeoGebra* to complete Q10.
- **Example 4F-2** shows how to write the rule for a circle. This will help students to complete Q11–13.
- In Q15, students plot the graph of $y = x^3$. Discuss with students why this graph has negative values of y but $y = x^2$ does not. Students are guided to see that performing a translation on the curve is best done by translating the point of inflection.
- In Q16, students plot the graph of $y = \sqrt{x}$ and identify its key feature which is the point where y is a minimum. Again, students use their understanding of performing translations to produce sketch graphs of other square root relationships.
- You may like some students to work on part (say, a–c) or all of Q17 which relates to plotting a hyperbola and using transformations to sketch other hyperbolas. This is a more difficult question and introduces the idea of asymptotes.
- For additional practice, students can complete Q1–4 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4F Q1, Q9, Q11 and Q16 are provided. If students have completed Ex 4F Q17 relating to hyperbolas, they can also complete Q5 of WS 4-2. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Below Level

Below Level
1–7, 9–13, 15a–c, 16a–c

- Consistent reminders may be needed for students to take the square root of the constant to find the radius.
- Use the BLM **Cartesian plane grids** (see Resources) to save students time.
- Provide pairs of compasses for students to draw circles.
- In Q1–4, students need to be reminded that all circles with rule of the form $x^2 + y^2 = r^2$ have centre at (0, 0). They only need to concentrate on the relationship between the constant term and the radius.
- In Q5 and Q6, emphasise the connection between the relationship for a circle written in the general form of $(x – h)^2 + (y – k)^2 = r^2$ and the coordinates (h, k) for the centre of this circle.
- You will need to ensure students have access to graphics calculators or graphing software such as *GeoGebra* to complete Q10.

- For Q11, have students write the rule $(x - h)^2 + (y - k)^2 = r^2$ and identify the values for h and k before carefully substituting into the rule.

  e.g. Q11b $(x - h)^2 + (y - k)^2 = r^2$, $h = -2$ and $k = 4$

  $(x - -2)^2 + (y - 4)^2 = 5^2$

  $(x + 2)^2 + (y - 4)^2 = 25$

- For Q12, ensure students write the radius and the coordinates of the centre of the circle first before using the method of Q11.

- For additional practice, students can complete Q1–4 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4F Q1, Q9, Q11 and Q16 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Above Level

Above Level
1, 3–6, 8, 9, 11c, d, 12, 14–18

- In Q8, students will need to explain the translation that is made in both the horizontal and vertical directions. With the negative sign in the general form of a circle, the direction of the translation may be counter-intuitive to some students.

- In Q9, an alternative method for students to find the centre of a circle is to consider the expression in each pair of brackets equal to zero and solve each linear equation to find x and y. This method ensures they have the correct sign.

- You will need to ensure students have access to graphics calculators or graphing software such as *GeoGebra* to complete Q10.

- In Q15, students plot the graph of $y = x^3$. Discuss with students why this graph has negative values of y but $y = x^2$ does not. Students are guided to see that performing a translation on the curve is best done by translating the point of inflection.

- In Q16, students plot the graph of $y = \sqrt{x}$ and identify its key feature which is the point where y is a minimum. Again, students use their understanding of performing translations to produce sketch graphs of other square root relationships. Ask them to compare the graph of this relationship to that of $y^2 = x$. Explain that as the square root only refers to the positive square root, the graph of $y = \sqrt{x}$ is only half of the full ‘sideways’ parabola.

- In Q17, students plot the graph of the hyperbola $y = \dfrac{1}{x}$. Discussion points include:

  – why the graph does not exist at $x = 0$
  – the concept of a limit. To what value does y approach as x gets larger?
  – why the value of y can never equal 0.

- Q18 develops some important concepts that have been established with parabolas but can be applied to all graphs.
- For additional practice, students can complete Q1–5 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4F Q1, Q9, Q11, Q16 and Q17 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Extra activities

1 Quick questions

Calculate each value.

a $\sqrt{64}$ (8)

b $\sqrt{1}$ (1)

c 9^2 (81)

d 12^2 (144)

2 In Q16, you looked at the graph of $y = \sqrt{x}$. This graph could be described as ‘half of a sideways parabola’.

a Why is the graph only half of a parabola? (square root means positive)

b If we consider this to be the top half of a parabola, what rule would create the bottom half of the graph? ($y = -\sqrt{x}$)

3 Consider the circle given by the rule $x^2 + y^2 = 4$.

a Identify the radius and coordinates of the centre of the circle. [radius of 2 units, centre at (0, 0)]

b Pepper’s teacher asks her to make y the subject of this equation. Pepper’s working is shown below.

$x^2 + y^2 = 4$

$y^2 = 4 – x^2$

$y = \sqrt{4 - x^2}$

There is an error (very small) in Pepper's working. What is the error? (last line should read $y = \pm\sqrt{4 - x^2}$)

c Try to draw the graph of $y = \sqrt{4 - x^2}$ and explain how it differs from $x^2 + y^2 = 4$. (semicircle; it is the top half of the graph of $x^2 + y^2 = 4$)

Answers

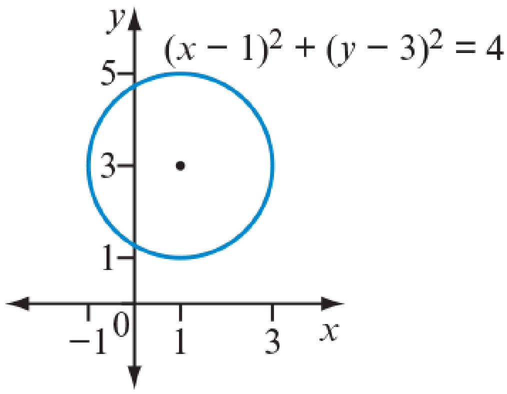
4F Circles and other non-linear relationships

4F Start thinking!

1 a i (0, 0) ii (0, 0) iii (0, 0)
b i 2 units ii 5 units iii 9 units
c Each has centre at (0, 0) and $x^2 + y^2$ written on left side of rule. Radius is equal to square root of number on right side of rule.

2 a i (1, 0) ii (0, 3)
b i translated 1 unit right
ii translated 3 units up
c Each circle has radius 2 units. Values that describe the translations help to determine coordinates of centre of circle.

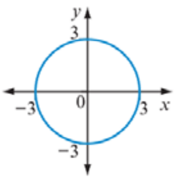
d Graph of $x^2 + y^2 = 4$ translated 1 unit right and 3 units up.



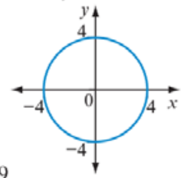
Exercise 4F Circles and other non-linear relationships

1 **a** centre (0, 0), radius = 6 units
b centre (0, 0), radius = 8 units
c centre (0, 0), radius = 1 unit
d centre (0, 0), radius = 10 units

2 **a** (0, 0) **b** 3 units
c centre and points on circle directly above, below, left and right of centre
d, e



3 $x^2 + y^2 = 16$



4 $x^2 + y^2 = 7^2$ or $x^2 + y^2 = 49$

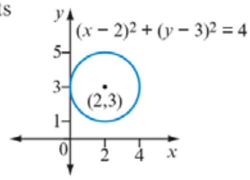
5 **a** $h = 2, k = 5, r = 3$
b (2, 5) **c** 3 units

6 **i** **a** $h = 1, k = 4, r = 4$
b (1, 4) **c** 4 units
ii **a** $h = 6, k = -3, r = 5$
b (6, −3) **c** 5 units
iii **a** $h = -4, k = 2, r = 1$
b (−4, 2) **c** 1 unit
iv **a** $h = -7, k = -2, r = 6$
b (−7, −2) **c** 6 units

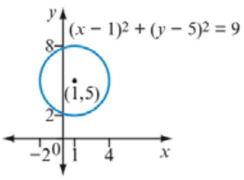
7 **a** (−2, 0) **b** 7 units
c Since $h = -2$ and $k = 0$, centre coordinates (h, k) are (−2, 0). Since $r = 7$, the radius is 7 units.
d translated 2 units left

8 **a** **i** (3, −1) **ii** (−4, −2)
b **i** 5 units **ii** 4 units
c **i** translated 3 units right and 1 unit down
ii translated 4 units left and 2 units down

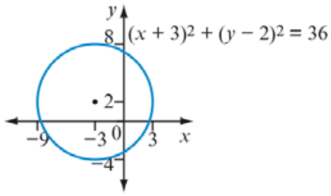
9 **a** centre (2, 3); radius = 2 units



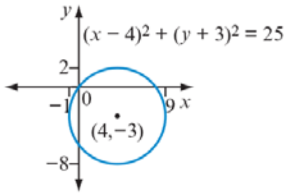
b centre (1, 5); radius = 3 units



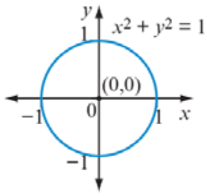
c centre (−3, 2); radius = 6 units



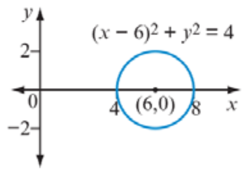
d centre (4, −3); radius = 5 units



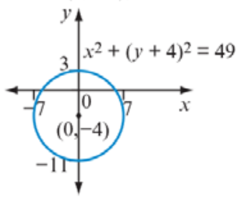
e centre (0, 0); radius = 1 units



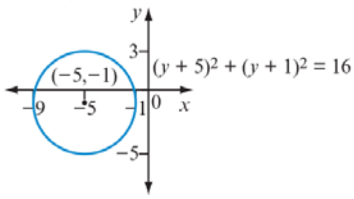
f centre (6, 0); radius = 2 units



g centre (0, −4); radius = 7 units



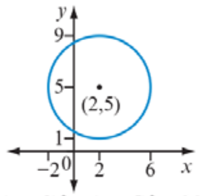
h centre (−5, −1); radius = 4 units



11 **a** $(x-3)^2+(y-5)^2=16$
b $(x+2)^2+(y-4)^2=25$
c $(x+7)^2+(y+6)^2=81$
d $(x-4)^2+(y+8)^2=121$

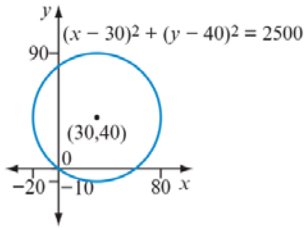
12 **a** centre (4, 2); radius = 2 units
$(x-4)^2+(y-2)^2=4$
b centre (−3, 3); radius = 1 units
$(x+3)^2+(y-3)^2=1$
c centre (−5, −2); radius = 3 units
$(x+5)^2+(y+2)^2=9$

13



$(x-2)^2+(y-5)^2=16$

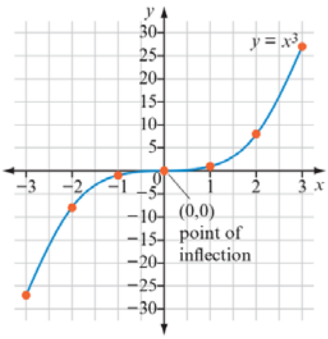
14 **a**



b 314 m **c** 7854 m²

15 a, c

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

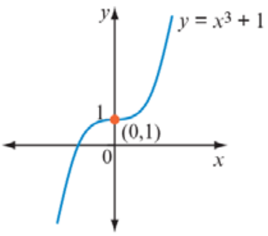


b As x increases, y increases. Graph appears to level out at $x = 0$ before rising more steeply again.

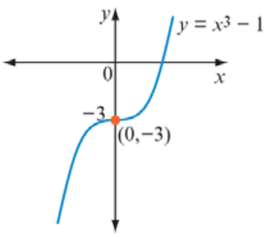
d **i** (0, 2) **ii** translate graph 2 units up

e **i** translate 1 unit up
ii translate 3 units down
iii translate 2 units right
iv translate 4 units left
v translate 1 unit right and 2 units up

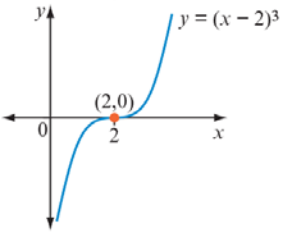
f **i**



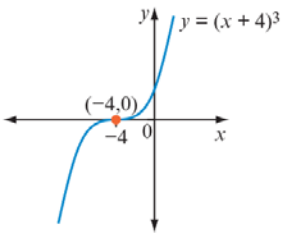
ii



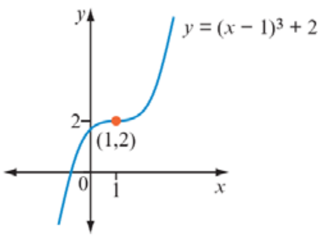
iii



iv

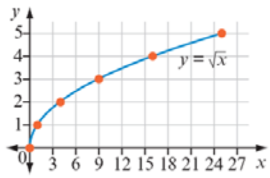


v



16 a, c

x	0	1	4	9	16	25
y	0	1	2	3	4	5

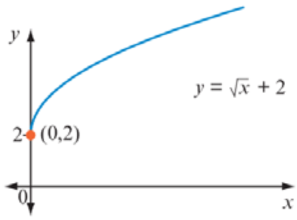


b No; not possible to take square root of negative number.

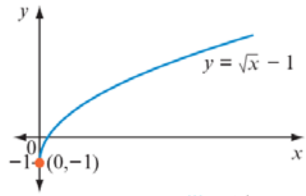
d **i** (3, 0)
ii translate 3 units to right

e **i** translate 2 units up
ii translate 1 unit down
iii translate 1 unit right
iv translate 4 units left
v translate 2 units right and 3 units up

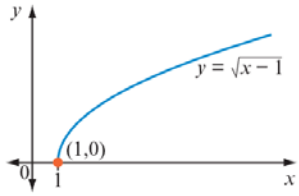
f **i**



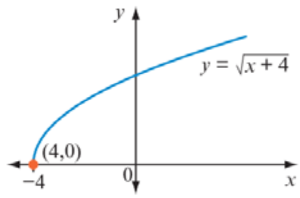
ii



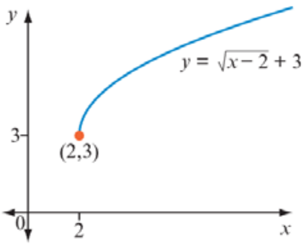
iii



iv

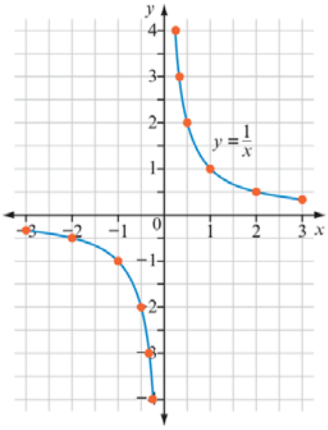


v



17 a

x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	-3	-4	4	3	2	1	$\frac{1}{2}$	$\frac{1}{3}$



b y is undefined when $x = 0$. As x approaches 0 from left, graph approaches $-\infty$ (very small y values) and as x approaches 0 from right, graph approaches $+\infty$ (very large y values).

c **i** $y = 0$ **ii** $x = 0$

d **i** $y = 0$ (horizontal) and $x = 2$ (vertical)

ii translate 2 units right

e **i** translate horizontal asymptote 2 units up

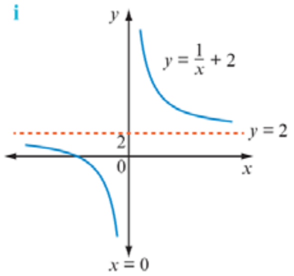
ii translate horizontal asymptote 3 units down

iii translate vertical asymptote 4 units right

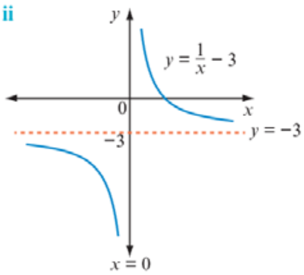
iv translate vertical asymptote 1 unit left

v translate vertical asymptote 3 units right and horizontal asymptote 2 units up

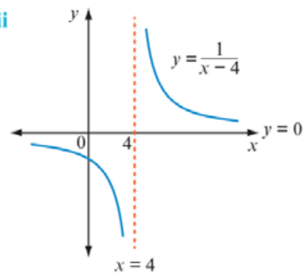
f **i**



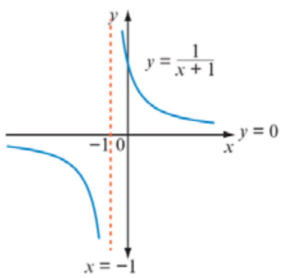
ii



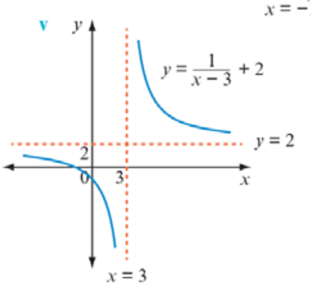
iii



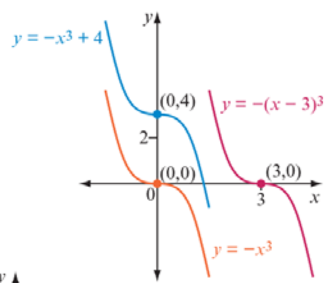
iv



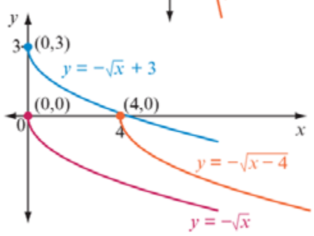
v



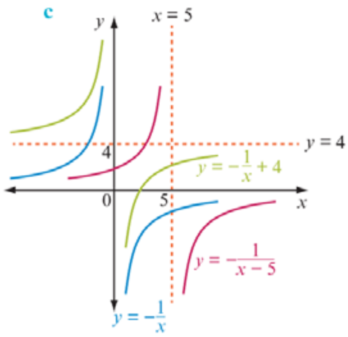
18 a



b



c



Reflect

Possible answer: The radius of a circle is found by taking the square root of the constant term. The centre can be found by setting the expression in each pair of brackets equal to zero and solving the equations to find x and y.

Resources

WorkSheet

WS 4-2 More non-linear relationships

Focus: To understand the key features of circles, square root functions and hyperbolas and to develop rules for linear and non-linear relationships using direct proportion

Resources: ruler, sketching aid (optional), **1-cm grid paper** (BLM) or graph paper (optional)

- This WorkSheet provides a skills review for Exercises 4F–4G. Q1–5 relate to Exercise 4F.

Students work with circles to determine the radius and the coordinates of the centre based on the form of the relationship and hence sketch its graph. They describe and perform transformations on other non-linear relationships including the square root function and the hyperbola and sketch their graphs. They also test for linear and non-linear relationships between two variables involving direct proportion.

BLMs

Cartesian plane grids

1-cm grid paper

Interactives

4F eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4G Relationships and direct proportion

Teaching support for pages 190–195

Teaching strategies

Learning focus

To consider relationships between quantities that vary in proportion to each other

Start thinking!

The task guides students to:

- recognise a direct relationship between two quantities or variables
- see how in a direct relationship, one quantity increases (or decreases) as the other increases (or decreases)
- define and identify the constant of proportionality and see that it is the gradient of the linear graph that represents the relationship between the two quantities
- define the general form of the relationship between x and y as $y = kx$ where y is directly proportional to x and k is the constant of proportionality
- see that $y = kx$ is treated the same way as the linear relationship $y = mx$ where m is the gradient.

Differentiated pathways

Below Level	At Level	Above Level
1–4, 6–8, 9a, b, 10a, b, 11–13	1–6, 7c–e, 8, 9a–d, 10a–d, 11–14	1–6, 8c–f, 9c–f, 10c–f, 11c, 12c, 14, 15
Students complete the **assessment**, **eTutor** and **Guided example** for this topic		

Support strategies for Are you ready? Q8

Focus: To revise linear relationships of the form $y = mx$.

- Direct students to complete **SS 4G-1 Finding the gradient of a linear graph** (see Resources) if they had difficulty with this question or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.

- Students need to recognise that the coefficient of x is the gradient of a linear relationship in the form $y = mx$.

At Level

At Level
1–6, 7c–e, 8, 9a–d, 10a–d, 11–14

- Demonstrate **4G eTutor** or direct students to do this independently.
- To complete this topic, students will need a calculator, ruler, pencil, and eraser.
- Introduce the idea of direct proportion as having the following features.
  - As one quantity increases or decreases, the other does as well.
  - One quantity can be multiplied by a constant to obtain the value of the other. Define this constant as the *constant of proportionality*.
- Explain that price is a simple example of direct proportion (or direct variation). If one can of soft drink costs \$3 then what is the cost of 5 cans? Explain that, in this case:
  - the constant of proportionality is 3
  - the graph will be a linear relationship starting at (0, 0) with a gradient of 3.
- Explain that as students work through the exercise they will meet relationships where one quantity varies with the square or square root of the other.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4G-1**. It shows how to identify whether a relationship shows direct proportion and will help students to complete Q1.
- **Example 4G-2** shows how to find the rule for a linear relationship using direct proportion. This will help students to complete Q3.
- **Example 4G-3** shows how to find the rule for a non-linear relationship using direct proportion. This will help students to complete Q5.
- In Q6, students need to see that only the graphs of straight lines that pass through (0, 0) represent a direct proportion.
- **Example 4G-4** shows how to find the constant of proportionality from given information. This will help students to complete Q8 and Q9.
- In Q9, explain that the symbol α means 'proportional to'. Hence, $y \; \alpha \; x$ can become $y = kx$. Then the method of Q8 can be used to finish the question.

- For additional practice, students can complete Q6–10 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4G Q1, Q3 and Q5 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 How much gold is actually in your gold jewellery?** (see Resources).

Below Level

Below Level
1–4, 6–8, 9a, b, 10a, b, 11–13

- Students will only need a brief exposure to proportions that vary with anything other than x.
- Concentrate on defining and finding the constant of proportionality by solving an equation.
- Explain what is meant by the proportionality statement $y \propto x$ and how it is equivalent to $y = kx$.
- Students may need to complete **SS 4G-1 Finding the gradient of a linear graph** (see Resources).
- In Q1, students will need reminding they are looking for the value of $\frac{y}{x}$ to be the same.
- In Q7, students need to be reminded they are looking for the number by which the independent variable is being multiplied.
- For Q11, some students will need a reminder that gradient = $\frac{\text{rise}}{\text{run}}$ and this is equal to the constant of proportionality. Explain that for Q12, the proportionality sign $\propto$ is replaced by an equals sign together with the constant of proportionality to create the rule for the relationship.
- For students who do not progress past Q5, direct them to Q6–10 of **WS 4-2 More non-linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.

Above Level

Above Level

1–6, 8c–f, 9c–f, 10c–f, 11c, 12c, 14, 15

- In Q6, students need to see that only the graphs of straight lines that pass through (0, 0) represent a direct proportion.
- In Q9, explain that the symbol α means 'proportional to'. Hence, $y \alpha x$ can become $y = kx$. Then the method of Q8 can be used to finish the question.
- For additional practice, students can complete Q6–10 of **WS 4-2 More non-linear relationships** (see Resources). Additional questions similar to Exercise 4G Q1, Q3 and Q5 are provided. This WorkSheet relates to Exercises 4F and 4G, and can be completed progressively or as a skills review of Exercises 4F–G.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 How much gold is actually in your gold jewellery?** (see Resources).

Extra activities

1 Solve each equation.

a $4x = 44$ $(x = 11)$

b $\dfrac{50}{x} = 12.5$ $(x = 4)$

c $9x = 6$ $(x = \dfrac{2}{3})$

d $\dfrac{6}{x} = 4$ $(x = 1.5)$

2 An example where one quantity varies in proportion with the square of another is the area of a circle.

a We can say $A \alpha r^2$ and hence $A = kr^2$. In this case, what is the value of the constant of proportionality? $(k = \pi)$

b Can you think of another example where one quantity varies as the square of another?

c Give an example where one quantity varies as the:

i cube of another quantity (for a cube, $V \propto l^3$; for a sphere, $V \propto r^3$)

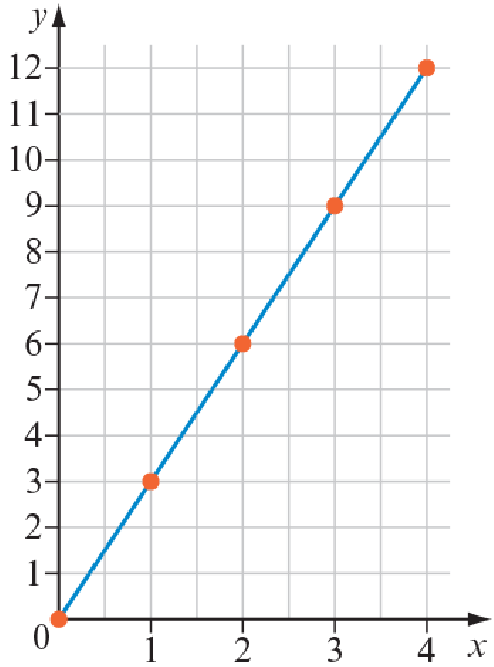
ii square root of another quantity.

Answers

4G Relationships and direct proportion

4G Start thinking!

1 a



b linear

c Increase; as increase is by same amount each time, change is constant.

d $\frac{y}{x} = 3$

e Gradient is 3. Rate of change is same as gradient and is constant for a linear relationship.

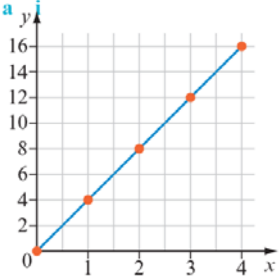
2 Graph is linear and passes through the origin (0, 0).

3 $y = 3x$

Exercise 4G Relationships and direct proportion

1 a and c show direct proportion

2 a i

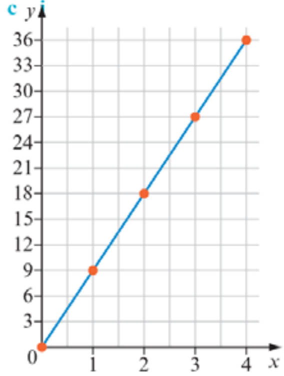


ii 4

iii They are the same.

iv $y = 4x$

c i



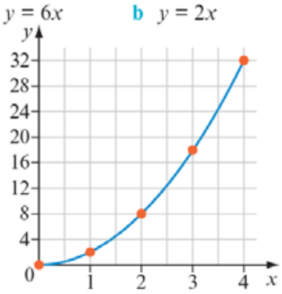
ii 9

iii They are the same.

iv $y = 9x$

3 a $y = 6x$ b $y = 2x$ c $y = \frac{1}{2}x$

4 a



b non-linear

c Increase; as increase is not by same amount each time, change is not constant.

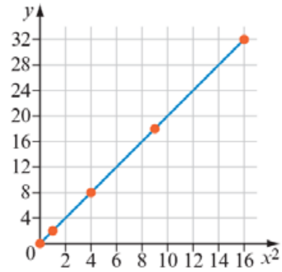
d 2, 4, 6, 8. Value for $\frac{y}{x}$ is different for each set of values.

e Not in direct proportion as $\frac{y}{x}$ is not constant.

f

x^2	0	1	4	9	16
y	0	2	8	18	32

g



h Yes, as it forms a straight line passing through (0, 0).

i–k $y = 2x^2$

5 a $y = 3x^2$ b $y = 7x^2$ c $y = 4x^2$

6 a no direct proportion (not a straight line passing through (0, 0))

b direct proportion (straight line passing through (0, 0))

c no direct proportion (straight line but does not pass through (0, 0))

d direct proportion (straight line passing through (0, 0))

e no direct proportion (not a straight line passing through (0, 0))

f no direct proportion (straight line but does not pass through (0, 0); also, y decreases as x increases)

7 a 4 b 3.5 c 6 d 10 e 2

8 a 10 b 8 c 4 d 7 e 44 f $\frac{1}{2}$

9 a 5 b 1.25 c 2 d 7 e 27 f 2.5

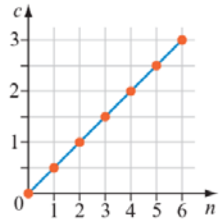
10 a $y = 5x$ b $p = 1.25q^2$ c $d = 2c^3$

d $h = 7\sqrt{g}$ e $a = \dfrac{27}{m}$ f $w = 2.5v$

11 a 3 b 4 c 8

12 a $y = 3x^2$ b $y = 4\sqrt{x}$ c $y = \dfrac{8}{x}$

13 a



b linear

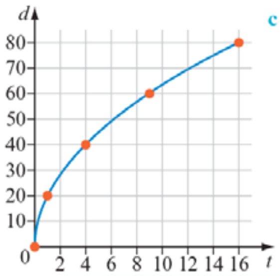
c yes, as forms a straight-line graph that passes through (0, 0)

d $c \propto n$; $c = kn$

e gradient $= \frac{1}{2}$, $c = \frac{1}{2}n$

f \$10

14 a



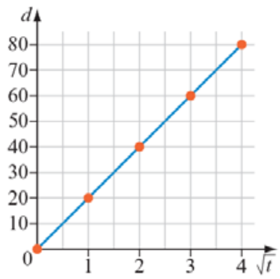
b non-linear

c no, as graph is not a straight line passing through (0, 0)

d

$\sqrt{t}$	0	1	2	3	4
d	0	20	40	60	80

e

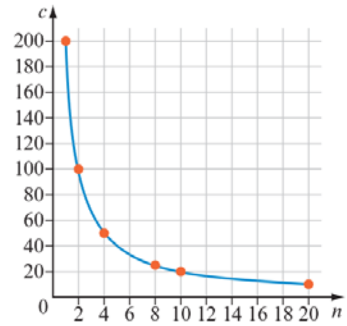


f Yes; it forms a straight-line graph that passes through (0, 0).

g $d \propto \sqrt{t}$; $d = k\sqrt{t}$

h gradient = 20, $d = 20\sqrt{t}$ i 120 km

15 a



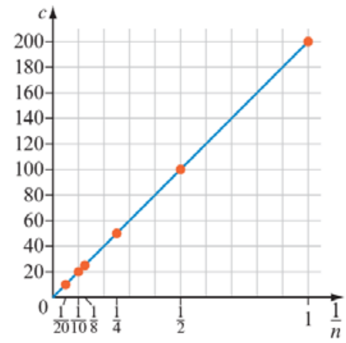
b non-linear

c No; graph is not a straight line passing through (0, 0).

d

$\frac{1}{n}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{20}$
c	200	100	50	25	20	10

e



f Yes; it forms a straight-line graph that passes through (0, 0).

g $c \propto \frac{1}{n}$; $c = \frac{k}{n}$

h gradient = 200, $c = \dfrac{200}{n}$

i \$200 j \$16.67

Reflect

Possible answer: Direct proportion can be used to work out the rule for a non-linear relationship by exploring other powers of the independent variable. For example, compare one quantity with the square, cube or square root of the other.

Resources

SupportSheet

SS 4G-1 Finding the gradient of a linear graph

Focus: To review the method for calculating the gradient of a linear graph

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students look at linear graphs and revise the concept that gradient is the measure of the slope of a line. They consider that a positive gradient slopes upwards to the right while a negative gradient slopes downwards to the right. Students will also calculate the gradient of a linear graph using both gradient = $\frac{\text{rise}}{\text{run}}$ and gradient = $\frac{y_2 - y_1}{x_2 - x_1}$.

WorkSheet

WS 4-2 More non-linear relationships

Focus: To understand the key features of circles, square root functions and hyperbolas and to develop rules for linear and non-linear relationships using direct proportion

Resources: ruler, sketching aid (optional), **1-cm grid paper** (BLM) or graph paper (optional)

- This WorkSheet provides a skills review for Exercises 4F–4G. Q6–10 relate to Exercise 4G.

Students work with circles to determine the radius and the coordinates of the centre based on the form of the relationship and hence sketch its graph. They describe and perform transformations on other non-linear relationships including the square root function and the hyperbola and sketch their graphs. They also test for linear and non-linear relationships between two variables involving direct proportion.

Investigation

INV 4-3 How much gold is actually in your gold jewellery?

Focus: To explore a practical example of direct proportion

Resources: 1-cm grid paper (BLM) or graph paper, calculator

Students consider the measurement of gold purity, the karat. Different grades of gold are looked at and students graph a direct proportion to find the amount of pure gold in any jewellery given the mass of the jewellery and the number of karats of the alloy.

As an extension, students investigate the amount of gold in $1 and $2 coins.

BLM

1-cm grid paper

Interactives

4G eTutor + Guided example

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 196–199

Additional teaching strategies

Multiple choice

1 Answer: C. The highest power of x in the expression $x^2 - 4x$ is 2 (expression has an x^2 term) and the expression does not contain an equals sign.
A: chose $y = x^2 + 2$ which is a quadratic equation as it contains an equals sign.
B: chose $x = 34$ which is a linear relationship.
D: chose $y = 12x$ which is a linear relationship.

2 Answer: A. The highest power of a variable in a quadratic equation is 2 and it contains an equals sign.
B: chose $x = 34$ which is a linear relationship.
C: chose $x^2 - 4x$ which is a quadratic expression not a quadratic equation.
D: chose $y = 12x$ which is a linear relationship.

3 Answer: D. $y = x^2 - 4$ is not a linear relationship as it has a variable to the power of 2.
A: chose a linear relationship where the highest power of x and y is 1.
B: chose a linear relationship where the highest power of x and y is 1.
C: chose a linear relationship where the highest power of x and y is 1.

4 Answer: C. Subtracting 5 from x^2 will move the parabola 5 units down.
A: chose the rule where adding 5 will move the graph 5 units up.
B: chose the rule where multiplying by 5 will make the graph narrower.
D: chose the rule where dividing by 5 will make the graph wider.

5 Answer: D. See below why all other answers are true.
A: chose statement where the factor of 4 dilates the original graph, making it narrower.
B: chose statement where the coefficient of x^2 is negative which means that the original graph is reflected in the x-axis.
C: chose statement where adding 1 translates the original graph vertically.

6 Answer: A. The $(x - 4)$ factor translates the graph of $y = x^2$ and hence its turning point 4 units to the right. So (0, 0) becomes (4, 0).
B: assumed graph is translated 4 units to the left.
C: assumed graph is translated 4 units upwards.
D: assumed graph is translated 4 units downwards.

7 Answer: D. At the x-intercepts, $y = 0$; so $x^2 - 4x - 12 = 0$, $(x - 6)(x + 2) = 0$, $x = 6$ or $x = -2$. The coordinates of the x-intercepts are (6, 0) and (–2, 0).
A: confused signs of the solutions to the quadratic equation and placed them on the y-

axis.
B: found the correct solutions to the quadratic equation but placed them on the y-axis.
C: confused the signs of the solutions to the quadratic equation.

8 Answer: B. At the y-intercept, $x = 0$; so $y = (0 – 5)(0 + 2) = –5 \times 2 = –10$.
A: only considered the expression in the second pair of brackets and solved $x + 2 = 0$.
C: only considered the expression in the first pair of brackets and solved $x – 5 = 0$.
D: assumed value of expression in first pair of brackets is 5 (not –5) so calculated $5 \times 2 = 10$.

9 Answer: B: $\sqrt{9} = 3$ so radius is 3 units.
A: only looked at the first pair of brackets containing the 'x' term.
C: only looked at the second pair of brackets containing the 'y' term.
D: did not take the square root of the constant term.

10 Answer: A. Using the general form of a circle, $h = 2$ and $y = –4$ means that the coordinates of the centre of the circle are (2, –4).
B: has reversed the signs of the x- and y-coordinates or simply used the values directly from each expression in the pairs of brackets.
C: used the values directly from each expression in the pairs of brackets but swapped the order.
D: swapped the order of the x- and y-coordinates.

11 Answer: C. If y is directly proportional to x, the relationship is linear.
A: incorrectly chose a parabola which indicates a squared relationship.
B: incorrectly chose a circle which does not represent a proportional relationship.
D: incorrectly chose a hyperbola which represents an inverse relationship.

Short answer

1 **a** $x^2 – 5x + 6 = 0$, $(x – 3)(x – 2) = 0$, $x = 3$ or $x = 2$

b $x^2 + x – 30 = 0$, $(x – 5)(x + 6) = 0$, $x = 5$ or $x = –6$

c $x^2 + 9 = 0$, $x^2 = –9$ has no solution

d $x^2 – 12x = 0$, $x(x – 12) = 0$, $x = 0$ or $x = 12$

2 **a** **i** upright

ii minimum (0, –4)

iii $x = 1$, $x = –1$, $y = –4$

b **i** inverted

ii maximum (–0.5, 1)

iii $x = -1$, $x = 0$, $y = 0$

c **i** upright

ii minimum (2, 0)

iii $x = 2$, $y = 4$

3 **a** **i** upright

ii narrower

iii minimum

b **i** upright

ii wider

iii minimum

c **i** inverted

ii narrower

iii maximum

d **i** upright

ii same width

iii minimum

e **i** inverted

ii same width

iii maximum

f **i** inverted

ii wider

iii maximum

4 **a** Dilation by a factor of 5.

b Dilation by a factor of $\frac{1}{5}$.

c Dilation by a factor of 5 and reflection in the x-axis

d Translation of 5 units upwards

e Reflection in the x-axis and translation 5 units downwards

f Dilation by a factor of $\frac{1}{5}$ and reflection in the x-axis

5 **a** $y = -5(x + 2)^2 - 1$; parabola is inverted; graph of $y = x^2$ has been dilated by a factor of 5, reflected in the x-axis, translated 2 units left and 1 unit down.

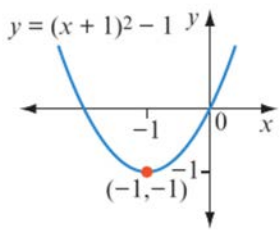
b $y = 5x^2 + 4$; parabola is upright; graph of $y = x^2$ has been dilated by a factor of 5, translated 4 units upwards.

c $y = -\frac{1}{4}(x - 5)^2$; parabola is inverted; graph of $y = x^2$ has been dilated by a factor of $\frac{1}{4}$, reflected in the x-axis, translated 5 units right.

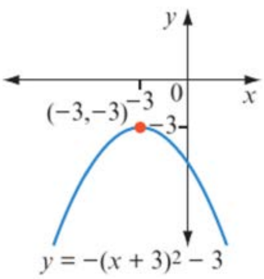
d $y = -3x^2$; parabola is inverted; graph of $y = x^2$ has been dilated by a factor of 3, reflected in the x-axis.

6

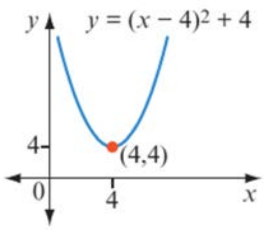
a



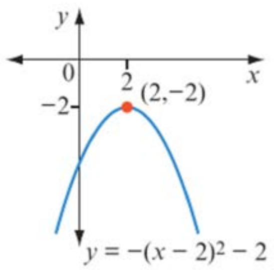
b



c



d



7 **a** At $x = 0$, $y = -0^2 - 4(0) = 0$; coordinates of y-intercept is (0, 0)
At $y = 0$, $-x^2 - 4x = 0$, $-x(x + 4) = 0$, $x = 0$ or $x = -4$; coordinates of x-intercepts are (0, 0) and (–4, 0).

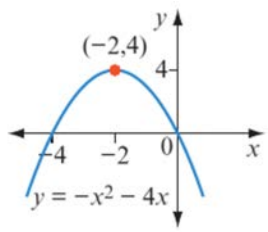
b At $x = 0$, $y = 0^2 - 0 - 12 = -12$; coordinates of y-intercept is (0, –12)
At $y = 0$, $x^2 - x - 12 = 0$, $(x + 3)(x - 4) = 0$, $x = -3$ or $x = 4$; coordinates of x-intercepts are (–3, 0) and (4, 0).

c At $x = 0$, $y = (0 + 5)(0 - 4) = 5 \times -4 = -20$; coordinates of y-intercept is (0, –20)
At $y = 0$, $(x + 5)(x - 4) = 0$, $x = -5$ or $x = 4$; coordinates of x-intercepts are (–5, 0) and (4, 0).

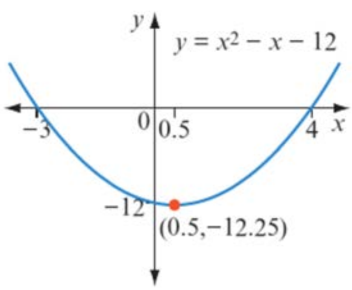
d At $x = 0$, $y = -(0 + 2)(0 + 1) = -2 \times 1 = -2$; coordinates of y-intercept is (0, –2)
At $y = 0$, $-(x + 2)(x + 1) = 0$, $x = -2$ or $x = -1$; coordinates of x-intercepts are (–2, 0) and (–1, 0).

8

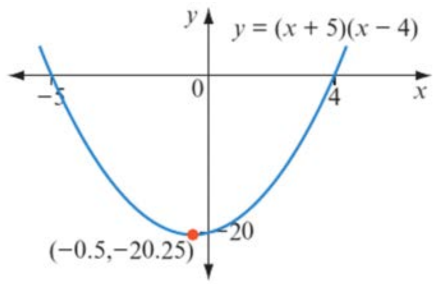
a



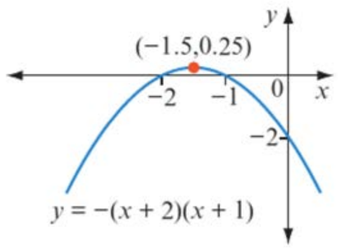
b



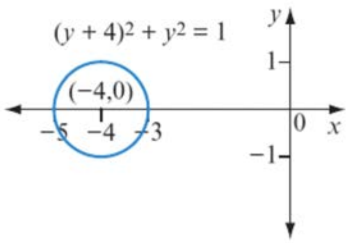
c



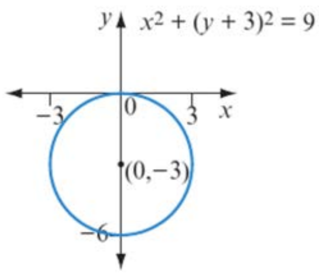
d



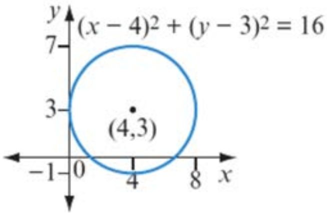
9 **a** centre (–4, 0); radius = 1 unit



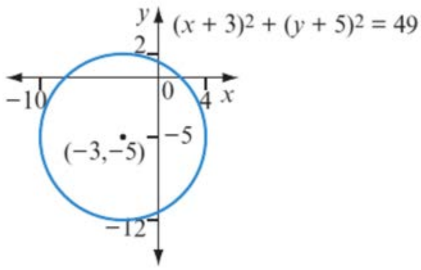
b centre (0, –3); radius = 3 units



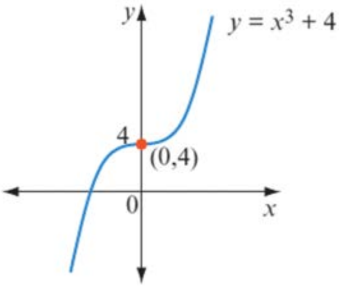
c centre (4, 3); radius = 4 units



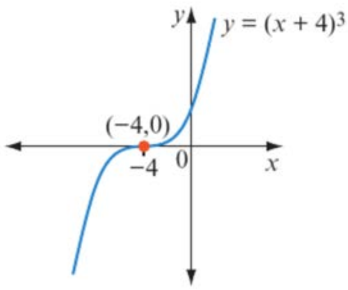
d centre (–3, –5); radius = 7 units



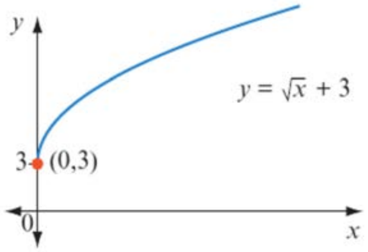
10 **a** $y = x^3$ translated 4 units vertically upwards



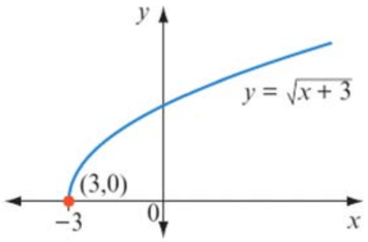
b $y = x^3$ translated 4 units left



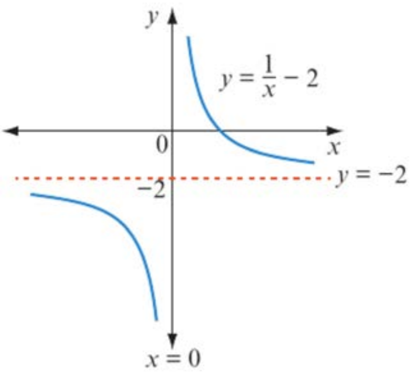
c $y = \sqrt{x}$ translated 3 units vertically upwards



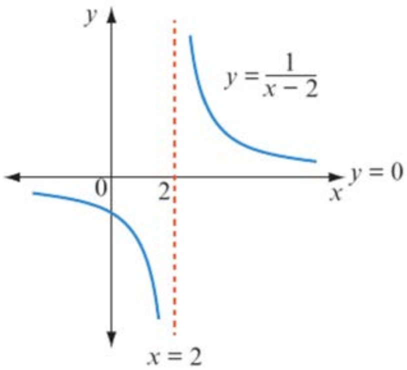
d $y = \sqrt{x}$ translated 3 units left



e $y = \dfrac{1}{x}$ translated 2 units vertically downwards



f $y = \dfrac{1}{x}$ translated 2 units right



11 **a** $y = kx^2$, $4 = k \times 2^2$, $4 = k \times 4$, $k = 1$

b $y = kx^3$, $2 = k \times \left(\frac{1}{2}\right)^3$, $2 = k \times \frac{1}{8}$, $k = 16$

c $y = k\sqrt{x}$, $2 = k \times \sqrt{16}$, $2 = k \times 4$, $k = \dfrac{1}{2}$

d $y = \dfrac{k}{x}$, $8 = \dfrac{k}{\frac{1}{2}}$, $k = 4$

12 **a** Each value of x has been multiplied by 5 to get y, so rule is $y = 5x$.

b There is no constant multiplier for x to get y. Consider x^2.

x^2	0	1	4	9	16
y	0	0.5	2	4.5	8

Each value of x^2 has been multiplied by $\dfrac{1}{2}$ to get y, so rule is $y = \dfrac{1}{2}x^2$.

c There is no constant multiplier for x to get y. Consider x^2.

x^2	0	1	4	9	16
y	0	3	24	81	192

There is no constant multiplier for x^2 to get y. Consider x^3.

x^3	0	1	8	27	64
y	0	3	24	81	192

Each value of x^3 has been multiplied by 3 to get y, so rule is $y = 3x^3$

d There is no constant multiplier for x to get y. Consider $\sqrt{x}$.

$\sqrt{x}$	0	1	2	5	6
y	0	3	6	15	18

Each value of $\sqrt{x}$ has been multiplied by 3 to get y, so rule is $y = 3\sqrt{x}$.

NAPLAN-style practice

Multiple-choice options have been listed as A, B, C and D for ease of reference.

1 Answer: B. $x - 6 = 0$, $x = 6$ or $x + 1 = 0$, $x = -1$
A: incorrectly solved both equations.
C: incorrectly solved the first equation.
D: incorrectly solved the second equation.
Refer to *4A Solving quadratic equations*.

2 Answer B: $(x - 8)(x + 2) = 0$,
$x(x - 8) + 2(x - 8) = 0$
$x^2 - 8x + 2x - 16 = 0$

$x^2 - 6x - 16 = 0$
A: calculated $8x - 2x = 6x$ to get the middle term.
C: calculated $-8x - 2x = -10x$ to get the middle term.
D: calculated $8x + 2x = 10x$ to get the middle term.
Refer to *4A Solving quadratic equations*.

3 $x^2 - 11x + 10 = 0$
$(x - 10)(x - 1) = 0$
$x - 10 = 0$ or $x - 1 = 0$
$x = 10$ or $x = 1$Refer to *4A Solving quadratic equations*.

4 $y = x^2 + 3x - 2$ when $x = -1$
$= (-1)^2 + 3(-1) - 2$
$= 1 - 3 - 2$
$= -4$
Refer to *4B Plotting quadratic relationships*.

5 $y = x^2 + x - 20$
x-intercepts occur when $y = 0$.
$(x + 5)(x - 4) = 0$
$x + 5 = 0$ or $x - 4 = 0$
$x = -5$ or $x = 4$
x intercepts at $(-5, 0)$ and $(4, 0)$.
Refer to *4E Sketching parabolas using intercepts*.

6 Answer: B: x-intercepts: $x^2 + 2x - 15 = 0$, $(x + 5)(x - 3) = 0$, $x = -5, 3$.
Vertical axis of symmetry is halfway between the x-intercepts and has the rule $x = -1$.
A: chose rule for vertical line halfway between –3 and 5.
C: chose rule for horizontal line halfway between –3 and 5.
D: chose rule for horizontal line hallway between –5 and 3.
Refer to *4E Sketching parabolas using intercepts*.

7 $y = x^2 - 8x + 15$
x-intercepts occur when $y = 0$.
$x^2 - 8x + 15 = 0$
$(x - 5)(x - 3) = 0$
$x - 5 = 0$ or $x - 3 = 0$
$x = 5$ or $x =$
So x-intercepts are 3 and 5.
Turning point will be at $x = 4$, $y = 4^2 - 8(4) + 15 = 16 - 32 + 15 = -1$ so coordinates are $(4, -1)$
Refer to *4E Sketching parabolas using intercepts*.

8 Answer D: Adding a constant produces a vertical translation.
A: assumed transformation produces a dilation. This would be $y = 3x^2$.
B: assumed transformation produces a horizontal translation. This would be $y = (x \pm 3)^2$.
C: assumed transformation produces a reflection in the x-axis. This would be $y = -x^2$.
Refer to *4C Parabolas and transformations.*

9 Answer: B: The narrowest parabola will be produced when the size of the coefficient of x^2 is the largest (furthest from 0). The dilation factor is 4.
A: chose relationship with dilation factor of 1 so its graph is the same width as the graph of $y = x^2$.
C: chose relationship with dilation factor of $\frac{1}{2}$ so its graph is wider than the graph of $y = x^2$.
D: chose relationship with dilation factor of 2 so its graph is narrower than the graph of $y = x^2$ but not as narrow as $y = -4x^2 - 5$.

Refer to *4C Parabolas and transformations*.

10 $y = -3(x - 2)^2 - 4$
y-intercept occurs when $x = 0$.
$y = -3(0 - 2)^2 - 4$
$= -3 \times 4 - 4$
$= -16$
Coordinates of y-intercept are (0, –16).
Refer to *4E Sketching parabolas using intercepts.*

11 Using the turning point form of a parabola, $h = 2$ and $k = -4$ so the coordinates of the turning point are (2, –4)
Refer to *4D Sketching parabolas using transformations.*

12 Answer: C. The turning point is located at (–3, 4) so $h = -3$ and $k = 4$. Substituting these values into $y = a(x - h)^2 + k$ gives the correct option of $y = (x + 3)^2 + 4$. When $x = 0$, $y = (0 + 3)^2 + 4 = 13$. This matches the y-intercept shown on the graph.
A: used correct value for k but incorrectly substituted for h.
B: incorrectly substituted for both h and k.
D: incorrectly substituted a value.
Refer to *4D Sketching parabolas using transformations.*

13 Answer B: x-intercepts occur when $y = 0$; $x - 3 = 0$, $x = 3$. There is only one solution so there is only one x-intercept.
A: not recognised the perfect square.
C: assumed all parabolas have two x-intercepts.

D: given the value of the x-intercept not the number of x-intercepts.
Refer to *4E Sketching parabolas using intercepts*.

14 Answer: B: Graph is inverted and $y = -(x^2 - 2x - 3) = -(x - 3)(x + 1)$, x-intercepts at x = 3 and $x = -1$.
A: chose rule for graph that is not inverted.
C: chose rule for graph that is not inverted.
D: chose rule for graph that is inverted but x-intercepts are at $x = -3$ and $x = 1$.
Refer to *4E Sketching parabolas using intercepts*.

15 Answer A: Substituting (2, –3) and $r = 5$ into the general form of a circle:
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 2)^2 + (y + 3)^2 = 25$
B: correctly calculated the value of r^2 but incorrectly substituted values for h and k.
C: correctly substituted values for h and k but value of r has not been squared.
D: incorrectly substituted values for h and k and value of r has not been squared.
Refer to *4F Circles and other non-linear relationships.*

16 Answer B: From general form of a circle $(x - h)^2 + (y - k)^2 = r^2$, $h = 0$, $k = 8$ and $r = \sqrt{4} = 2$. Centre has coordinates (0, 8) and radius is 2 units.
A: incorrectly interpreted the value for k hence y-coordinate of centre is incorrect.
C: did not take square root of constant term.
D: did not take square root of constant term and swapped values for h and k so coordinates for centre are incorrect.
Refer to *4F Circles and other non-linear relationships.*

17 $(x - 2)^2 + (y + 5)^2 = 36$. Radius = 6 units
$A = \pi r^2$, $A = \pi \times 6^2 \approx 113$ square units
Refer to *4F Circles and other non-linear relationships.*

18 Answer: D $y = \sqrt{x}$ is the top half of a sideways parabola.
A: chose rule that produces an upright parabola.
B: chose rule that produces a hyperbola.
C: chose rule that produces a cubic function (upright curve steeper than a parabola).
Refer to *4F Circles and other non-linear relationships.*

19 Answer: A. $y = kx$. At $y = 2$, $x = 10$ so $2 = k \times 10$, $k = 0.2$. Rule is $y = 0.2x$. At $y = 10$, $10 = 0.2x$ so $x = 50$.
B: chose the original y value.
C: divided the two values of y given in the question.
D: swapped the original values for x and y to obtain $k = 5$ and hence solved $10 = 5x$.
Refer to *4G Relationships and direct proportion.*

20 Answer: C. $y = \frac{1}{4}x^2$. At $x = 8$, $y = \frac{1}{4} \times 8^2$, $y = \frac{1}{4} \times 64 = 16$.

A: squared $\frac{1}{4}$ as well as the x value of 8 to obtain 4.
B: incorrectly linked x^2 and y.
D: squared 8 and not accounted for the constant of proportionality.
Refer to *4G Relationships and direct proportion.*

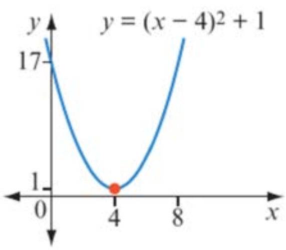
21 At $y = 15$, $x = 25$ so $y = k\sqrt{x}$ becomes $15 = k\sqrt{25}$, $15 = 5k$, $k = 3$.
Refer to *4G Relationships and direct proportion.*

22 Answer D: Direct proportions increase from left to right not decrease.
A: chose correct statement as direct proportions do pass through the origin.
B: chose correct statement as direct proportions are linear.
C: chose correct statement as direct proportions are linear and hence have constant gradient.
Refer to *4G Relationships and direct proportion.*

23 There is a direct relationship between y and x^2 so gradient of line is the constant of proportionality. Gradient is 4 so $k = 4$. Rule is $y = 4x^2$.
Refer to *4G Relationships and direct proportion.*

Analysis

1 a

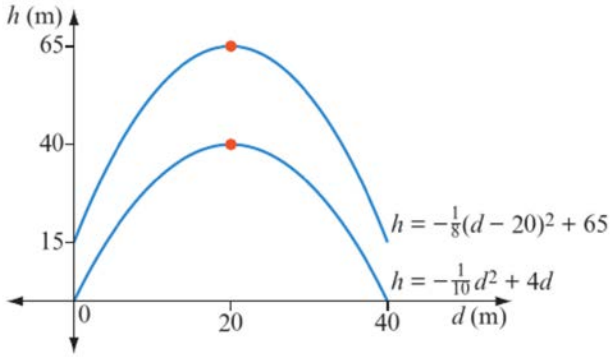


b At $x = 0$, $y = (0 - 4)^2 + 1 = 16 + 1 = 17$
At $y = 17$, $17 = (x - 4)^2 + 1$, $(x - 4)^2 = 16$, $x - 4 = \pm 4$, $x = 0$ or $x = 8$
The two points are (0, 17) and (8, 17).

c Diameter = 8 cm

d At $x = 4$, $y = (4 - 4)^2 + 1 = 1$. The bowl is 1 cm thick.

2 a



b Both have same axis of symmetry (d = 20) and are inverted. Lower arch has maximum turning point at (20, 40) and upper arch has maximum turning point at (20, 65). Lower arch has wider shape than upper arch. End points of upper arch are 15 m above end points of lower arch.

c Bridge ends at d = 0 and d = 40. End points of lower arch at h = 0. For upper arch:

At $d = 0$, $h = -\frac{1}{8}(0-20)^2 + 65 = 15$.

At $d = 40$, $h = -\frac{1}{8}(40-20)^2 + 65 = 15$.

So height of upper arch above the lower arch at the ends of the bridge is 15 m.

d **i** 40 m by looking at the x-intercepts of the lower arch graph

ii For lower arch bridge, $h = -\frac{1}{10}d^2 + 4d$. For d-intercepts, h = 0 so $-\frac{1}{10}d(d-40) = 0$, d = 0 or d = 40. d-intercepts are 0 and 40 so span of bridge is 40 m.

e Max of lower arch at d = 20, $h = -\frac{1}{10} \times 20^2 + 4 \times 20 = 40$ m

Max of upper arch at d = 20, $-\frac{1}{8}(20-20)^2 + 65 = 65$ m

f Difference = 65 – 40 = 25 m

g For lower arch at d = 10, $h = -\frac{1}{10} \times 10^2 + 4 \times 10 = 30$ m

For upper arch at d = 10, h $= -\frac{1}{8}(10-20)^2 + 65 = 52.5$ m

Difference = 52.5 – 30 = 22.5 m

h The two arches are 15 m apart at the start. Approaching the highest points on both arches, the distance between the two arches increases until it is a maximum of 25 m apart at the top of the arch. Approaching the far right side of the bridge, the distance between the two arches decreases until they are a distance of 15 m apart at the end of the bridge.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 4 Chapter test A

Chapter 4 Chapter test B

Summative tests

The three tests, A, B and C, for each chapter accommodate different student ability levels, with one section of overlap in each (the 'Proficient' part). These tests have been carefully mapped against AUSVELS and the Australian Curriculum in order to provide an accurate assessment of each student's level of achievement. When a student's marks are entered into the provided spreadsheet calculator, a letter grade is calculated based upon a weighted average of percentages according to the type of test completed.

Chapter 4 Summative test A: Modified

Aimed at the lower level of student ability.

The top mark a student can achieve in a modified test is a **C**.

Chapter 4 Summative test B: Core

Aimed at the middle level of student ability.

The top mark a student can achieve in a core test is a **B**.

Chapter 4 Summative test C: Extension

Aimed at the upper level of student ability.

The top mark a student can achieve in an extension test is an **A**.

Test answers

Chapter 4 Chapter test answers

Chapter 4 Summative test answers

Summative test spreadsheet calculator

Connect

Teaching support for pages 200-201

Teaching strategies

Path of a soccer ball

Focus: To relate the path of a projectile to a quadratic relationship

- Begin by taking students out onto the football field to kick a soccer ball. Have some of them kick a ball from the ground as well as from a point in the air and others viewing from the sidelines to observe the path of the ball.
- Have students measure the length of kicks and estimate the maximum height of the ball.
- Introduce the two relationships in the task as similar examples.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided for the Connect task.
- An assessment rubric is available (see Resources).

Resources

Assessment rubric

Path of a soccer ball

Australian Curriculum: Mathematics Year 10

The proficiency strands **Understanding**, **Fluency**, **Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Year 10

Number and Algebra

Money and financial mathematics	**Elaborations**	**MyMaths 10+10A**
Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies (ACMNA229)	• working with authentic information, data and interest rates to calculate compound interest and solve related problems	1C Understanding simple interest 1D Working with simple interest 1E Understanding compound interest 1F The compound interest formula 1G Working with compound interest
Patterns and algebra	**Elaborations**	**MyMaths 10+10A**
Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)	• using the distributive law and the index laws to factorise algebraic expressions • understanding the relationship between factorisation and expansion	2D Factorising algebraic expressions
Simplify algebraic products and quotients using index laws (ACMNA231)	• applying knowledge of index laws to algebraic terms, and simplifying algebraic expressions using both positive and negative integral indices	2B Review of index laws

Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)	• expressing the sum and difference of algebraic fractions with a common denominator • using the index laws to simplify products and quotients of algebraic fractions	2F Working with algebraic fractions
Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233)	• exploring the method of completing the square to factorise quadratic expressions and solve quadratic equations • identifying and using common factors, including binomial expressions, to factorise algebraic expressions using the technique of grouping in pairs • using the identities for perfect squares and the difference of squares to factorise quadratic expressions	2C Expanding algebraic expressions 2E Factorising quadratic trinomials of the form $x^2 + bx + c$
Substitute values into formulas to determine an unknown (ACMNA234)	• solving simple equations arising from formulas	4A Solving linear equations
Linear and non-linear relationships	**Elaborations**	**MyMaths 10+10A**
Solve problems involving linear equations, including those derived from formulas (ACMNA235)	• representing word problems with simple linear equations and solving them to answer questions	4A Solving linear equations
Solve linear inequalities and graph their solutions on a number line (ACMNA236)	• representing word problems with simple linear inequalities and solving them to answer questions	4B Solving linear inequalities

Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology (ACMNA237)	• associating the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs	4F Solving linear simultaneous equations graphically 4G Solving linear simultaneous equations algebraically
Solve problems involving parallel and perpendicular lines (ACMNA238)	• solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel • solving problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular	4E Parallel and perpendicular lines
Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)	• sketching graphs of parabolas, and circles • applying translations, reflections and stretches to parabolas and circles • sketching the graphs of exponential functions using transformations	5C Sketching parabolas using intercepts 5D Sketching parabolas using transformations 5E Graphs of circles 5F Graphs of exponential relationships
Solve linear equations involving simple algebraic fractions (ACMNA240)	• solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution • representing word problems, including those involving fractions, as equations and solving them to answer the question	4A Solving linear equations
Solve simple quadratic equations using a range of strategies (ACMNA241)	• using a variety of techniques to solve quadratic equations, including grouping, completing the square, the quadratic formula and choosing two integers with the required product and sum	5A Solving quadratic equations 5B Solving quadratic equations using the quadratic formula

Measurement and Geometry

Using units of measurement	Elaborations	MyMaths 10+10A
Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)	• investigating and determining the volumes and surface areas of composite solids by considering the individual solids from which they are constructed	9C Surface area of prisms and cylinders 9D Volume of prisms and cylinders
Geometric reasoning	**Elaborations**	**MyMaths 10+10A**
Formulate proofs involving congruent triangles and angle properties (ACMMG243)	• applying an understanding of relationships to deduce properties of geometric figures (for example the base angles of an isosceles triangle are equal)	7A Geometry review 7B Congruence 7D Understanding proofs 7E Proofs and triangles
Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)	• distinguishing between a practical demonstration and a proof (for example demonstrating triangles are congruent by placing them on top of each other, as compared to using congruence tests to establish that triangles are congruent) • performing a sequence of steps to determine an unknown angle giving a justification in moving from one step to the next • communicating a proof using a sequence of logically connected statements	7C Similarity 7E Proofs and triangles 7F Proofs and quadrilaterals
Pythagoras and trigonometry	**Elaborations**	**MyMaths 10+10A**
Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245)	• applying Pythagoras' Theorem and trigonometry to problems in surveying and design	8A Finding lengths using Pythagoras’ theorem 8B Finding lengths using trigonometry 8C Finding angles using trigonometry 8D Applications of trigonometry

Statistics and Probability

Chance	**Elaborations**	**MyMaths 10+10A**
Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence (ACMSP246)	• recognising that an event can be dependent on another event and that this will affect the way its probability is calculated	11A Review of theoretical probability 11B Tree diagrams 11C Experiments with and without replacement 11D Independent and dependent events
Use the language of 'if ....then, 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language (ACMSP247)	• using two-way tables and Venn diagrams to understand conditional statements • using arrays and tree diagrams to determine probabilities	11B Tree diagrams 11E Conditional probability with two-way tables and tree diagrams 11F Conditional probability and Venn diagrams
Data representation and interpretation	**Elaborations**	**MyMaths 10+10A**
Determine quartiles and interquartile range (ACMSP248)	• finding the five-number summary (minimum and maximum values, median and upper and lower quartiles) and using its graphical representation, the box plot, as tools for both numerically and visually comparing the centre and spread of data sets	10A Measures of centre 10B Measures of spread 10D Box plots

Construct and interpret box plots and use them to compare data sets (ACMSP249)	• understanding that box plots are an efficient and common way of representing and summarising data and can facilitate comparisons between data sets • using parallel box plots to compare data about the age distribution of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole	10D Box plots
Compare shapes of box plots to corresponding histograms and dot plots (ACMSP250)	• investigating data in different ways to make comparisons and draw conclusions	10D Box plots
Use scatterplots to investigate and comment on relationships between two numerical variables (ACMSP251)	• using authentic data to construct scatterplots, make comparisons and draw conclusions	10E Scatterplots and bivariate data
Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252)	• investigating biodiversity changes in Australia since European occupation • constructing and interpreting data displays representing bivariate data over time	10G Time series 10 Connect task
Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data (ACMSP253)	• investigating the use of statistics in reports regarding the growth of Australia's trade with other countries of the Asia region • evaluating statistical reports comparing the life expectancy of Aboriginal and Torres Strait Islander people with that of the Australian population as a whole	10H Analysing reported statistics 10 Connect task

Year 10 Achievement Standard

By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports.

Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.

Year 10A

Number and Algebra

Real numbers	**Elaborations**	**MyMaths 10+10A**
Define rational and irrational numbers and perform operations with surds and fractional indices (ACMNA264)	• understanding that the real number system includes irrational numbers • extending the index laws to rational number indices • performing the four operations with surds	3A Understanding rational and irrational numbers 3B Multiplying and dividing surds 3C Simplifying surds 3D Adding and subtracting surds 3E Writing surd fractions with a rational denominator 3F Fractional indices
Use the definition of a logarithm to establish and apply the laws of logarithms (ACMNA265)	• investigating the relationship between exponential and logarithmic expressions • simplifying expressions using the logarithm laws	3G Understanding logarithms 3H Working with logarithms
Patterns and algebra	**Elaborations**	**MyMaths 10+10A**
Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems (ACMNA266)	• investigating the relationship between algebraic long division and the factor and remainder theorems	6A Understanding polynomials 6B Division of polynomials 6C Remainder and factor theorems 6D Solving polynomial equations

Linear and non-linear relationships	**Elaborations**	**MyMaths 10+10A**
Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions and their transformations (ACMNA267)	• applying transformations, including translations, reflections in the axes and stretches to help graph parabolas, rectangular hyperbolas, circles and exponential functions	5D Sketching parabolas using transformations 5H Graphs of hyperbolas 5I Sketching non-linear relationships using transformations
Solve simple exponential equations (ACMNA270)	• investigating exponential equations derived from authentic mathematical models based on population growth	5G Solving exponential equations
Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation (ACMNA268)	• investigating the features of graphs of polynomials including axes intercepts and the effect of repeated factors	6E Graphs of polynomial relationships 6F Polynomials and transformations
Factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts (ACMNA269)	• writing quadratic equations that represent practical problems	2G Factorising quadratic trinomials of the form $ax^2 + bx + c$ 5A Solving quadratic equations 5B Solving quadratic equations using the quadratic formula

Measurement and Geometry

Using units of measurement	**Elaborations**	**MyMaths 10+10A**
Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids (ACMMG271)	• using formulas to solve problems • using authentic situations to apply knowledge and understanding of surface area and volume	9E Surface area of pyramids and cones 9F Volume of pyramids and cones 9G Surface area and volume of spheres 9H Surface area and volume of composite shapes
Geometric reasoning	**Elaborations**	**MyMaths 10+10A**
Prove and apply angle and chord properties of circles (ACMMG272)	• performing a sequence of steps to determine an unknown angle or length in a diagram involving a circle, or circles, giving a justification in moving from one step to the next • communicating a proof using a logical sequence of statements • proving results involving chords of circles	7G Circle geometry: circles and angles 7H Circle geometry: chords 7I Circle geometry: tangents and secants
Pythagoras and trigonometry	**Elaborations**	**MyMaths 10+10A**
Establish the sine, cosine and area rules for any triangle and solve related problems (ACMMG273)	• applying knowledge of sine, cosine and area rules to authentic problems such as those involving surveying and design	8F Sine and area rules 8G Cosine rule
Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)	• establishing the symmetrical properties of trigonometric functions • investigating angles of any magnitude • understanding that trigonometric functions are periodic and that this can be used to describe motion	8H Unit circle and trigonometric graphs
Solve simple trigonometric equations (ACMMG275)	• using periodicity and symmetry to solve equations	8I Solving trigonometric equations

Apply Pythagoras' Theorem and trigonometry to solving three-dimensional problems in right-angled triangles (ACMMG276)	• investigating the applications of Pythagoras's Theorem in authentic problems	8E Three-dimensional problems

Statistics and Probability

Chance	**Elaborations**	**MyMaths 10+10A**
Investigate reports of studies in digital media and elsewhere for information on their planning and implementation (ACMSP277)	• evaluating the appropriateness of sampling methods in reports where statements about a population are based on a sample • evaluating whether graphs in a report could mislead, and whether graphs and numerical information support the claims	11G Sampling and reporting
Data representation and interpretation	**Elaborations**	**MyMaths 10+10A**
Calculate and interpret the mean and standard deviation of data and use these to compare data sets (ACMSP278)	• using the standard deviation to describe the spread of a set of data • using the mean and standard deviation to compare numerical data sets	10C Standard deviation

Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation (ACMSP279)	• investigating different techniques for finding a 'line of best fit'	10F Bivariate relationships

Number and Algebra

4 Linear relationships

Teaching support for pages 158–159

Syllabus links

Content descriptions and elaborations

Patterns and algebra

ACMNA234: Substitute values into formulas to determine an unknown

- solving simple equations arising from formulas

Linear and non-linear relationships

ACMNA235: Solve problems involving linear equations, including those derived from formulas

- representing word problems with simple linear equations and solving them to answer questions

ACMNA236: Solve linear inequalities and graph their solutions on a number line

- representing word problems with simple linear equations and solving them to answer questions

ACMNA237: Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology

- associating the solution of simultaneous equations with the coordinates of the intersection of their corresponding graphs

ACMNA238: Solve problems involving parallel and perpendicular lines

- solving problems using the fact that parallel lines have the same gradient and conversely that if two lines have the same gradient then they are parallel
- solving problems using the fact that the product of the gradients of perpendicular lines is –1 and conversely that if the product of the gradients of two lines is –1 then they are perpendicular

ACMNA240: Solve linear equations involving simple algebraic fractions

- solving a wide range of linear equations, including those involving one or two simple algebraic fractions, and checking solutions by substitution
- representing word problems, including those involving fractions, as equations and solving them to answer the question

The proficiency strands **Understanding, Fluency, Problem solving** and **Reasoning** are fully integrated into the content of the chapters.

Teaching strategies

Discussion prompts

- Direct students to examine the opening photo for this chapter.
- Ask students to consider the acceleration of the two cars in the photo. Would the speed of the cars as they accelerate be a linear or non-linear model?

Essential question

The graph of speed versus time will tell you the speed of the car at any time but also the gradient of this graph will tell you the acceleration of the car at any point.

Are you ready?

Prerequisite knowledge and skills can be tested by completing **Are you ready?**. This will give you an indication of the differentiated pathway each student can follow.

Students need to be able to:

- solve simple one-step equations
- substitute into algebraic expressions
- expand a single pair of brackets using the distributive law
- simplify an expression by collecting like terms
- perform the four operations with fractions
- use inequality signs to identify the greater and lesser value
- graph an inequality on a number line
- recognise the x-intercept, y-intercept and gradient on a linear graph
- identify the gradient and y-intercept from a linear rule in the form $y = mx + c$
- find the x- and y-intercepts from a linear rule

- find equivalent equations by multiplying a given equation by an integer.

At the beginning of each topic, there is a suggested differentiated pathway that allows teachers to individualise the learning journey. An evaluation of how each student performed in the **Are you ready?** task can be used to select the best pathway.

Support Strategies and **SupportSheets** to help build student understanding should be attempted before starting the matching topic in the chapter.

Answers

CHAPTER 4 LINEAR RELATIONSHIPS

4 Are you ready?

1 **a** $x = 13$ **b** $x = -8$ **c** $x = 5$ **d** $x = -45$

2 **a** 11 **b** −9 **c** −12 **d** 5

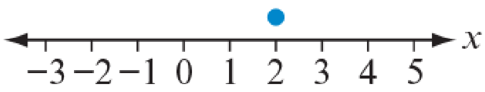
3 **a** $4x + 28$ **b** $-15x + 6$

4 **a** $11x + 8$ **b** $2 - 3x$ **c** $5x - 2$ **d** $3x - 9$

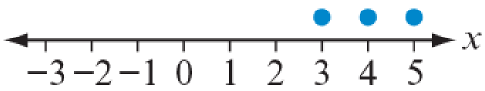
5 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{13}{10}$ or $1\frac{3}{10}$ **d** $\frac{1}{6}$

6 **a** true **b** true **c** true **d** false

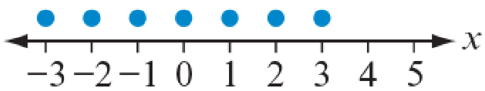
7 **a**



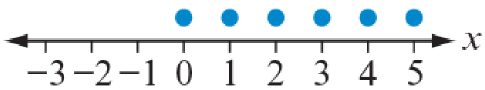
b



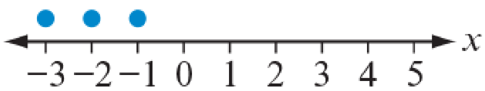
c



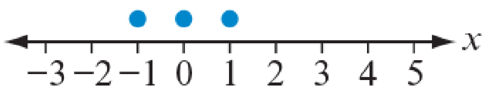
d



e



f



8 **a** 2 **b** −4 **c** 2

9 **a** **i** $c = -4, m = 3$ **ii** $c = 2, m = -1$

iii $c = 0, m = 7$ **iv** $c = -1, m = -\frac{2}{3}$

b **i** rise = 3, run = 1 **ii** rise = −1, run = 1

iii rise = 7, run = 1 **iv** rise = −2, run = 3

10 **a** x-intercept = 1, y-intercept = 4

b x-intercept = 9, y-intercept = 3

c x-intercept = 10, y-intercept = −5

d x-intercept = −3, y-intercept = −6

11 **a** $6x + 3y = 12$ **b** $2x - 6y = 2$

c $-x + 8y = -6$ **d** $-16x + 20y = 0$

Resources

assess: assessments

Each topic of the *MyMaths 10+10A* student text includes auto-marking formative assessment questions. The goal of these assessments is to improve. Assessments are designed to help build students' confidence as they progress through a topic. With feedback on incorrect answers and hints when students get stuck, students can retry any questions they got wrong to improve their score.

assess: testbank

Testbank provides teacher-only access to ready-made chapter tests. It consists of a range of multiple-choice questions (which are auto-graded if completed online).

The testbank can be used to generate tests for end-of-chapter, mid-year or end-of-year tests. Tests can be printed, downloaded, or assigned online.

4A Solving linear equations

Teaching support for pages 160–165

Teaching strategies

Learning focus

To solve a variety of linear equations including those that have the pronumeral on both sides, equations with brackets and equations involving algebraic fractions

Start thinking!

The task guides students to:

- recognise a linear equation
- see how the balance method is used to solve a one-step equation
- consider the order in which the operations need to be performed in solving a two-step equation using the balance method
- follow the step involved in using the balance method to solve a two-step equation involving fractions and negative coefficients.

Differentiated pathways

Below Level	At Level	Above Level
1, 2, 4a–f, 5a–f, 6–8, 10a–f	1–4, 5a–j, 6–8, 9a–f, 10a–l, 11, 12, 15	2, 3, 4e–i, 5e–l, 7, 8d–f, 9d–i, 10g–o, 12–14, 16–19
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q1–5

Focus: To solve simple equations, expand brackets, collect like terms and perform operations with fractions

- Direct students to complete **SS 4A-1 Solving simple linear equations** (see Resources) if they had difficulty with Q1 or require more practice at this skill.
- Direct students to complete **SS 4A-2 Evaluating simple algebraic expressions** (see Resources) if they had difficulty with Q2 or require more practice at this skill.
- Direct students to complete **SS 4A-3 Expanding and simplifying algebraic expressions** (see Resources) if they had difficulty with Q3 and Q4 or require more practice at this skill.

- Direct students to complete **SS 4A-4 Working with fractions** (see Resources) if they had difficulty with Q5 or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to:
  - solve a one-step equation
  - substitute into an expression and evaluate
  - use the distributive law to expand brackets
  - simplify an expression by collecting like terms
  - perform four operations with fractions.

At Level

At Level
1–4, 5a–j, 6–8, 9a–f, 10a–l, 11, 12, 15

- Revise the fact that an equation is an incomplete number sentence and the task is to find the value of the unknown pronumeral.
- An equation is a balance, and in solving the equation whatever is done to one side must also be done to the other in order to maintain the equality. Hence, the balance method is used to solve equations.
- Discuss how in using the balance method the operations that have been used to form the equation must be undone by using the opposite operations in the correct order.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4A-1**. It shows how to solve linear equations and will help students to complete Q1 and Q2.
- **Example 4A-2** shows how to solve an equation when the variable is in the denominator. This will help students to complete Q5.
- **Example 4A-3** shows how to solve an equation with an unknown on both sides. This will help students to complete Q6.
- For Q8, explain that multiplying both sides of the equation by the common denominator produces an equation without algebraic fractions.

- **Example 4A-4** shows how to solve an equation with algebraic fractions on both sides. This will help students to complete Q9.
- For Q10, students need to consider all pronumerals other than *x* as numbers and move them to the other side of the equation using equation-solving techniques.
- For Q11 onwards, where students write an equation from worded information, remind them to define the pronumeral/s used.
- You may like students to use the ‘solve’ function of a calculator or other digital technology to check whether they obtain the same answers.
- For additional practice, students can complete Q1–7 of **WS 4-1 Solving linear equations and inequalities** (see Resources). Additional questions similar to Exercise 4A Q1–7 are provided. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Calendar capers** (see Resources).

Below Level

Below Level
1, 2, 4a–f, 5a–f, 6–8, 10a–f

- To complete this topic, students may need their calculators.
- Students may need to complete **SS 4A-1 Solving simple linear equations** (see Resources).
- Students may need to complete **SS 4A-2 Evaluating simple algebraic expressions** (see Resources).
- Students may need to complete **SS 4A-3 Expanding and simplifying algebraic expressions** (see Resources).
- Students may need to complete **SS 4A-4 Working with fractions** (see Resources).
- Students may need help in some questions to work out the correct order of operations. Have them make a simple flowchart of the expression on the left side of the equation.
- In Q4, remind students to expand expressions to remove brackets and collect like terms before solving each equation.
- In Q6, guide students to remove the pronumeral from the right side of the equation. If this leaves a negative coefficient of *x* on the left side, explain when and how to change the sign on both sides.

- For Q8, explain that multiplying both sides of the equation by the common denominator produces an equation without fractions.
- For students who do not progress past Q7, direct them to Q1–7 of **WS 4-1 Solving linear equations and inequalities** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.
- Students sometimes struggle with what exactly an equation is and what they are doing when they solve an equation. This can be overcome to some extent by writing the equation, replacing the pronumeral with a box. The task is to write the number in the box to make a true statement.
- Once established that the pronumeral takes the place of the box, have students check each of their answers by substitution. Explain that if they do this, they will know immediately if they are correct or incorrect.

Above Level

Above Level
2, 3, 4e–i, 5e–l, 7, 8d–f, 9d–i, 10g–o, 12–14, 16–19

- For Q8, explain that multiplying both sides of the equation by the common denominator produces an equation without algebraic fractions. In Q9, students first need to obtain equivalent fractions with the same denominator before proceeding with the method used in Q8.
- For Q10, students need to consider all pronumerals other than x as numbers and move them to the other side of the equation using equation-solving techniques.
- For Q11 onwards, where students write an equation from worded information, remind them to define the pronumeral/s used.
- For more problem-solving tasks and investigations, direct students to **INV 4-1 Calendar capers** (see Resources).

Extra activities

1 Quick Questions

a Expand $3(2x - 7)$. $(6x - 21)$

b Simplify $2x - 4 + 9 - 11x$. $(5 - 9x)$

c Calculate $\frac{1}{2}+\frac{3}{8}$. $\left(\frac{7}{8}\right)$

d Calculate $\frac{2}{3} \times \frac{4}{5}$. $\left(\frac{8}{15}\right)$

2 A formula is used to calculate a quantity by some given rule. Most formulas are designed to calculate the subject. For example, in the formula $A = l \times w$, A is the subject and is used to find the area of a rectangle. If we are to use the formula to calculate, a different variable and equation will be formed. For example, if the area of a rectangle is 56 and the length is 7, the equation $56 = 7w$ can be used to find the width.

In each of the following, substitute into the given formula and solve the resulting equation.

a $A = \frac{1}{2}bh$ where $A = 72$, $b = 12$ $(h = 12)$

b $E = mc^2$ where $E = 40$, $c = 2$ $(m = 10)$

c $C = \frac{5}{9}(F - 32)$ where $F = 100$ $(C = 37\frac{7}{9})$

Answers

4A Solving linear equations

4A Start thinking!

1 a, c, f and g are linear equations

2 a subtract 9 b $x = 7$

3 a i add 4 ii divide by 3

b You would get an incorrect solution if you did not perform the 'undo' operations in the reverse order to that used to form the equation.

c
$$\begin{aligned} 3x - 4 &= 2 \\ 3x - 4 + 4 &= 2 + 4 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

d
$$\begin{aligned} 3x - 4 &= 2 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

e Substitute the solution into the original equation to check left side equals right side.
LS $= 3 \times 2 - 4 = 2 =$ RS

4 a multiply by 5 then add 2; $x = -18$

b subtract 7 then divide by -4; $x = -2$

Exercise 4A Solving linear equations

1 a $x = 5$ b $x = 1$ c $x = 15$ d $x = -5$
e $x = 8$ f $x = 10$ g $x = 6$ h $x = -4$
i $x = 9$ j $x = -7$ k $x = 3$ l $x = 2$
m $x = 8$ n $x = -1$ o $x = 0$

2 a $x = 10$ b $x = -4$ c $x = 5$ d $x = -8$
e $x = 7$ f $x = 6$ g $x = 8$ h $x = -2$
i $x = 3$

3 a $x = -8\frac{1}{2}$ b $x = 1.7$ c $x = 4$
d $x = -2.464$ e $x = 2.4$
f $x = -9\frac{1}{2}$ or -9.5

4 a $x = -5$ b $x = 3$ c $x = -2$ d $x = 6$
e $x = -\frac{1}{3}$ f $x = 7$ g $x = 3$ h $x = \frac{1}{9}$
i $x = 6$

5 a $x = 2$ b $x = -5$ c $x = -6$ d $x = -4$
e $x = 4$ f $x = 7.5$ g $x = 3\frac{1}{3}$ h $x = \frac{2}{35}$
i $x = 2$ j $x = -1$ k $x = \frac{1}{4}$ l $x = -2\frac{2}{5}$

6 a $x = 2$ b $x = 5$ c $x = -3$ d $x = 1$
e $x = -7$ f $x = -2$ g $x = 3$ h $x = 4$
i $x = \frac{3}{7}$ j $x = -4$ k $x = 2$ l $x = -5$

7 a $x = 5$ b $x = -2$ c $x = 3$ d $x = -1$
e $x = 6$ f $x = 2$ g $x = -4$ h $x = 8$
i $x = 4$ j $x = -1$ k $x = 5$ l $x = 9$

8 a $x = -8$ b $x = 4$ c $x = -2$ d $x = 2$
e $x = -1$ f $x = 4$

9 a $x = -8$ b $x = 31$ c $x = 8$ d $x = 1$
e $x = 17$ f $x = 13$ g $x = -4\frac{2}{3}$ h $x = 6\frac{1}{7}$
i $x = -2\frac{1}{8}$

10 a $x = b - a$ b $x = k + p$ c $x = \frac{d}{c}$
d $x = \frac{h - g}{3}$ e $x = \frac{y - 5}{4}$ f $x = mn - a$
g $x = \frac{5 + 2w}{7}$ h $x = a(b - c)$ i $x = \frac{h}{k}$
j $x = \frac{e + f}{2}$ k $x = \frac{v - 2y}{2}$ l $x = p(w - q)$
m $x = \frac{a}{b + c}$ n $x = \frac{2ny + m}{k}$ o $x = \frac{5c - 3d}{8}$

11 a let n = number of weeks of saving
b $24n + 105 = 219$ c $n = 4.75$
d It will take Tom 5 weeks to save for his new tennis racquet.

12 a \$89.50 b length: 28 m, width: 16 m
c Lisa: 14 goals, Nicole: 9 goals, Tania: 6 goals
d 51 people

13 a $6d + 3.8 = 4d + 9.2$, where d is cost of a dumpling
b \$2.70

14 a $A = 50\ \text{m}^2$ b $h = 7$ mm c $a = 7$ m
d $a = \frac{2A}{h} - b$
e i $a = 52$ cm ii $a = 7.6$ m

15 a $v = 34$ m/s b $t = 5.5$ s

16 15, 17, 19, 21, 23

17 $n - 25 - \frac{n - 25}{3} = 40$, where n is number of cupcakes made; 25 chocolate, 20 lemon, 40 without icing and 85 in total

18 a $x = -6$ b $x = 8$ c $x = -1$ d $x = 13$
e $x = \frac{1}{8}$ f $x = -2\frac{7}{10}$

Reflect

Possible answer: When solving equations, operations are performed in the reverse order to how the expression on the left side was formed.

Resources

SupportSheet

SS 4A-1 Solving simple linear equations

Focus: To use the balance method to solve one-step linear equations

Students review equations and develop an understanding of what solving an equation means. They are guided to solve simple one-step linear equations using the balance method.

SS 4A-2 Evaluating simple algebraic expressions

Focus: To substitute values into a variety of algebraic expressions

Students review substitution into an algebraic expression. They determine the value of expressions by substituting given values into the expressions and evaluating.

SS 4A-3 Expanding and simplifying algebraic expressions

Focus: To expand algebraic expressions by using the distributive law to remove brackets and to simplify expressions by collecting like terms

Students expand algebraic expressions using the distributive law to remove a pair of brackets and simplify expressions by adding and subtracting like terms.

SS 4A-4 Working with fractions

Focus: To perform the four basic operations (addition, subtraction, multiplication and division) on fractions

Students review the requirements for performing the four operations on fractions (addition, subtraction, multiplication and division) in preparation for solving equations involving algebraic fractions.

WorkSheet

WS 4-1 Solving linear equations and inequalities

Focus: To solve a variety of linear equations and inequalities

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4A and 4B. Q1–7 relate to Exercise 4A.

Students solve a range of linear equations involving two-step equations, three step equations, equations with the unknown in the denominator and equations where the unknown appears on both sides. They represent inequalities on a number line and solve linear inequalities.

Investigation

INV 4-1 Calendar capers

Focus: To discover some interesting relationships between dates in any month of a calendar

Resources: calculator

Students look at the calendar and consider the relationships in the sum of the dates in 2 by 2 squares and 3 by 3 squares. Students then form equations to solve that will determine the exact dates in any such squares.

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4B Solving linear inequalities

Teaching support for pages 166–171

Teaching strategies

Learning focus

To solve linear inequalities and graph the solutions on a number line

Start thinking!

The task guides students to:

- use the symbols >, <, ≥ and ≤ to compare numbers
- understand the meaning of a statement about an unknown using one of the above symbols
- consider how an inequality might be represented on a number line
- consider different values of x that will make an inequality statement true
- consider the meaning of an inequality of the form $a < x < b$.

Differentiated pathways

Below Level	At Level	Above Level
1–9, 13, 14	1d–h, 2–10, 11a–c, 12–16	1g, h, 2–4, 6–12, 15–19
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q6 and Q7

Focus: To compare numbers using inequality signs and represent inequalities on a number line

- Direct students to complete **SS 4B-1 Understanding inequality statements** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to use:

– the signs $<$ and $>$ to show a comparison in the size of two numbers

– graphing conventions to show numbers on a number line.

At Level

At Level
1d–h, 2–10, 11a–c, 12–16

- Explain to students that a linear equation has a single solution while an inequality (or inequation) has a range of values that will satisfy it.

- Demonstrate that the balance method is still used in exactly the same way to find the solution to an inequality.

- Explain that the major difference between solving equations and inequalities is that when multiplying or dividing by a negative number, the inequality signs need to be reversed. This can be demonstrated as follows.

  Start with: $4 > 1$

  Adding 10 to both sides: $14 > 11$

  Subtracting 5 from both sides: $9 > 6$

  Multiplying both sides by 2: $18 > 12$

  Dividing both sides by –3: $-6 < -4$

  The inequality remained in the same direction until both sides were divided by a negative number. Similarly show what happens when an inequality like $4 > 1$ is multiplied by a negative number.

- Show students how to graph a solution on a number line. Explain that a circle is put on the number line where the boundary of the solution lies and an arrow is drawn in the direction of the inequality. For $<$ or $>$ an open circle (○) should be used while for $\leq$ or $\geq$ a closed circle (●) should be used.

- Direct students to the **Key ideas**. You may like them to copy this summary.

- Direct students to **Example 4B-1**. It shows how to write inequalities from a number line and will help students to complete Q2 and Q3.

- In Q2, students need to be particularly aware of the closed circle or open circle and may need to have the 'between' solution explained further.

- Have students work through the demonstrations in Q4 carefully as they show the reason why the inequality sign needs to be reversed (to keep statements true) when multiplying or dividing by a negative number.
- **Example 4B-2** shows how to solve inequalities when multiplying or dividing by a positive number. This will help students to complete Q6.
- **Example 4B-3** shows how to solve inequalities when multiplying or dividing by a negative number. This will help students to complete Q7 and Q8.
- In Q8, point out that students must consider the order of operations in the different questions and that in some problems the inequality will need to be reversed and in others it will not.
- **Example 4B-4** shows how to solve inequalities with the unknown on both sides. This will help students to complete Q9 and Q10.
- In Q10, students will need to expand expressions to remove brackets. You can demonstrate that they can avoid using negatives by reversing the direction of an inequality.

  e.g. $6x - 15 < 7x + 4$ can be written as

  $7x + 4 > 6x - 15$ then they can subtract $6x$ from both sides

  $x + 4 > -15$
- Q13 requires students to write an appropriate inequality statement. Some students may need an explanation of what is meant by reserve price. Have them define the pronumeral used for the unknown quantity in each case.
- You may like students to use the 'solve' function of a calculator or other digital technology to check whether they obtain the same answers.
- For additional practice, students can complete Q8–11 of **WS 4-1 Solving linear equations and inequalities** (see Resources). Additional questions similar to Exercise 4B Q2, Q3, Q6 and Q7 are provided. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.

Below Level

Below Level
1–9, 13, 14

- To complete this topic, students may need their calculators.

- Students may need to complete **SS 4B-1 Understanding inequality statements** (see Resources).
- For Q1, ensure that students understand the meaning of each of the four inequality symbols. Specifically, they may need to have explained:
  - the direction in which the arrow of the inequality points. Explain it points to the smaller number
  - the difference between 'less than' and 'less than or equal to'
  - the meaning of a 'between' inequality.
- In Q2, emphasise the meaning of the closed circle or open circle. You may need to explain the 'between' solution further.
- Have students work through the demonstrations in Q4 carefully as they show the reason why the inequality sign needs to be reversed (to keep statements true) when multiplying or dividing by a negative number.
- In Q8, point out that students must consider the order of operations in the different questions and that in some problems the inequality will need to be reversed and in others it will not.
- For students who do not progress past Q7, direct them to Q8–11 of **WS 4-1 Solving linear equations and inequalities** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4A and 4B and can be completed progressively or as a skills review of Exercises 4A–B.
- Below Level students will need some support in remembering the direction of the inequality signs.
- Support them by having them use exactly the same solving methods that they would use for an equation with the different inequality sign used in place of the equals sign.
- They will have the greatest difficulty understanding that the direction of the inequality needs to be turned around when multiply or dividing by a negative number. Demonstrate this several times with examples using numbers to reinforce the concept.

Above Level

Above Level
1g, h, 2–4, 6–12, 15–19

- In Q2, emphasise the meaning of the closed circle or open circle. You may need to explain the 'between' solution further.

- Have students work through the demonstrations in Q4 carefully as they show the reason why the inequality sign needs to be reversed (to keep statements true) when multiplying or dividing by a negative number.
- Q8 contains a mixture of inequalities to solve. Remind students that, in some problems, the inequality sign will need to be reversed and in others it will not.
- In Q17, students need to take care with their algebraic manipulation. Common denominators may need to be applied to whole numbers as well as fractional terms.
- In Q18, the inequalities have three parts. When using the balance method students will need to balance all three parts of the inequality. In part b, when dividing by a negative number, the inequality will also need to be written backwards to maintain the correct ‘between’ concept.

Extra activities

1 Quick Questions

Solve each equation.

a $2x + 6 = 18$ $(x = 6)$

b $3x – 5 = 2x + 11$ $(x = 16)$

c $5(2x – 12) = 80$ $(x = 14)$

d $\frac{x+5}{9} = 11$ $(x = 94)$

2 A surfboard company has found that it is only able to sell surfboards for a minimum price of \$250. They have also found that when surfboards are priced at over \$4325 they will not be able to be sold. Graph the range of suitable prices.

Julie goes running each day but will only do so in a cool temperature. Julie will go before 8.00 am or after 5.00 pm. Graph the suitable times for Julie to go running on a number line. (Consider midnight to be 0 and the number line up to 24.)

Answers

4B Solving linear inequalities

4B Start thinking!

1 < (less than), ≤ (less than or equal to),
> (greater than), ≥ (greater than or equal to)

2 a > b < c < d >

3 a x is greater than 2
b x is less than or equal to 3
c x is greater than or equal to 5
d x is less than −4

4 Each inequality represents a region on the number line and not a single point.

5 Some possible answers are given.
a 7, 30.5, 122 b 4, 0, −5
c −3, −1, 10.6 d −5, −10, −36.4

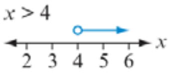
6 x is greater than 2, and less than 9. Some possible values are: 4, 5.7, 8.2.

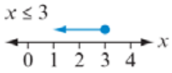
7 −4, −3, −2, −1, 0, 1, 2

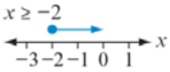
Exercise 4B Solving linear inequalities

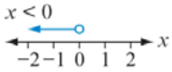
1 **a** 2.4, 3, 7, 8.3, $6\frac{4}{5}$
b $-5, -1.2, 0, -\frac{3}{4}, -10, -4.9$
c 7, 8.3, $6\frac{4}{5}$ **d** $-5, -10, -4.9$
e 2.4, 3, 7, $6\frac{4}{5}$ **f** 2.4, $\frac{1}{2}$, 3, $6\frac{4}{5}$
g $-5, 2.4, -1.2, \frac{1}{2}, 0, -\frac{3}{4}, -4.9$
h $-5, -1.2, -\frac{3}{4}, -4.9$

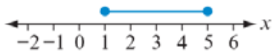
2 **a** $x \le 5$ **b** $x > 1$ **c** $x \ge -4$
d $x < -2$ **e** $3 \le x \le 7$ **f** $-5 < x \le 0$

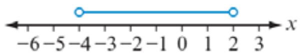
3 **a** $x > 4$


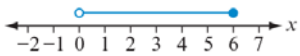
b $x \le 3$


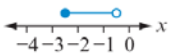
c $x \ge -2$


d $x < 0$


e $1 \le x \le 5$


f $-4 < x < 2$


g $0 < x \le 6$


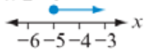
h $-2.5 \le x < -0.5$


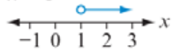
4 **a** **i** $3 < 7$; true **ii** $5 < 9$; true
iii $9 < 13$; true **iv** $-2 < 2$; true
b **i** $1 < 5$; true **ii** $-1 < 3$; true
iii $-5 < -1$; true **iv** $6 < 10$; true
c **i** $4 < 12$; true **ii** $-4 < -12$; not true
iii $6 < 18$; true **iv** $-2 < -6$; not true
d **i** $1 < 3$; true **ii** $-1 < -3$; not true
iii $\frac{2}{3} < 2$; true **iv** $-2 < -6$; not true
e Inequality sign stays the same when adding or subtracting any number. Also stays the same when multiplying or dividing by a positive number. However, direction of inequality sign is reversed when multiplying or dividing by a negative number.
f Some possible answers are given.
i $4 > -8$, $4 + 2 > -8 + 2$ $(6 > -6)$,
$4 - 1 > -8 - 1$ $(3 > -9)$
ii $4 > -8$, $4 \times 2 > -8 \times 2$ $(8 > -16)$,
$4 \div 2 > -8 \div 2$ $(2 > -4)$
iii $4 > -8$, $4 \times -2 < -8 \times -2$ $(-8 < 16)$,
$4 \div -2 < -8 \div -2$ $(-2 < 16)$

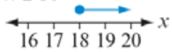
5 **a** $x + 3 > 8$ **b** $-2x \ge 8$ **c** $x \le 10$
d $x < 4$ **e** $-x \ge -9$ **f** $x > -16$
g $x \ge 7$ **h** $-x > 3$ **i** $-6x > -5$

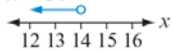
6 **a** $x < 5$ **b** $x \ge 7$ **c** $x \le -6$
d $x > 6$ **e** $x \le -5$ **f** $x < 50$
g $x \ge -20$ **h** $x \le 2$ **i** $x > -5$

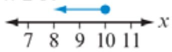
7 **a** $x \ge -6$ **b** $x < -2$ **c** $x \le 3$
d $x > -24$ **e** $x < -7$ **f** $x \ge 1$
g $x > -24$ **h** $x < 7$ **i** $x \le 10$

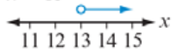
8 **a** $x \ge -5$


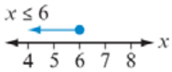
b $x > 1$


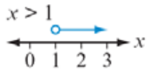
c $x \ge 18$


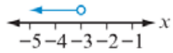
d $x < 14$


e $x \le 10$


f $x > 13$


g $x \le 6$


h $x > 1$


i $x < -3$


9 **a** $x > 4$ **b** $x \le -2$ **c** $x \ge 3$
d $x > 4$ **e** $x < 1$ **f** $x \ge -1$
g $x \le 5$ **h** $x < 2$ **i** $x > \frac{1}{2}$

10 **a** $x > 11$ **b** $x \le 13$ **c** $x \ge -2$
d $x > -19$ **e** $x > -15$ **f** $x \le 1$
g $x \le 3$ **h** $x > -\frac{1}{4}$ **i** $x \ge -6.5$

11 **a** $x \le -7$ **b** $x > 1$ **c** $x \le -3$
d $x < 1$ **e** $x \ge 4$ **f** $x > -5$

12 **a** one
b Linear inequalities can have many solutions within a given range.

13 **a** $p \ge 650\,000$, where p is selling price in \$
b $h > 97$, where h is height of a person in cm
c $s \le 60$, where s is speed in km/h
d $125 \le h \le 196$, where h is height of a person in cm

14 **a** $3p \le 20$, where p is cost of pack of gum in \$; $p \le 6\frac{2}{3}$
b Todd could buy 1, 2, 3, 4, 5 or 6 packs of gum.

15 **a** $m = \frac{1}{2}(20 - x)$, where m is number of watermelons they will each take home.
b $\frac{1}{2}(20 - x) \le 3$; $x \ge 14$
c 14 or more watermelons

16 **a** 25 min **b** 16 min **c** 19 min to 29 min

17 **a** $x \ge -11$ **b** $x < 4$ **c** $x \ge 2$ **d** $x < -8$
e $x \le 2$ **f** $x \le -3$

18 **a** $1 \le x \le 6$ **b** $-4 > x > -7$

Reflect

Possible answer: The method of solving inequalities is much the same as for linear equations; however, care must be taken with the direction of the inequality sign when multiplying or dividing by a negative number.

Resources

SupportSheet

SS 4B-1 Understanding inequality statements

Focus: To compare the size of numbers and to represent inequality statements on a number line

Resources: ruler

Students review the inequality symbols (<, ≤, > and ≥) and use these symbols when comparing numbers. They represent statements involving *x* and these symbols on a number line.

WorkSheet

WS 4-1 Solving linear equations and inequalities

Focus: To solve a variety of linear equations and inequalities

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4A and 4B. Q8–11 relate to Exercise 4B.

Students solve a range of linear equations involving two-step equations, three-step equations, equations with the unknown in the denominator and equations where the unknown appears on both sides. They represent inequalities on a number line and solve linear inequalities.

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4C Sketching linear graphs

Teaching support for pages 172–178

Teaching strategies

Learning focus

To sketch linear graphs using the gradient-intercept method and the *x*- and *y*-intercept method

Start thinking!

The task guides students to:

- describe the graph of a linear relationship
- recognise that two points are needed to graph a linear relationship
- recognise that a linear relationship has a gradient and a *y*-intercept.
- use the gradient and *y*-intercept to sketch the linear relationship.

Differentiated pathways

Below Level	At Level	Above Level
1, 2a–i, 3a–i, 4, 5a, b, 6, 7a, b, 8, 9a–f, 10, 11, 13, 14a, b, 15a–d	1d–i, 2–8, 9d–j, 10, 12–14, 15a–d, 17–22	1f–i, 2f–l, 3f–l, 4, 5b, d, 6, 7c, d, 8, 9g–l, 10, 12–14, 15e–h, 16–24
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q8–10

Focus: To understand the concepts of gradient, *x*-intercept and *y*-intercept as they apply to linear relationships

- Direct students to complete **SS 4C-1 Identifying features of a linear graph** (see Resources) if they had difficulty with these questions or require more practice at this skill.
- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to:
  - identify the gradient, *x*- and *y*-intercepts from a graph
  - identify the gradient and *y*-intercept from a rule in the form $y = mx + c$

- find the x- and y-intercepts from a given rule.

At Level

At Level
1d–i, 2–8, 9d–j, 10, 12–14, 15a–d, 17–22

- To complete this topic, students will need a ruler and pencil, and may like to use grid or graph paper. You may want to provide students with copies of the BLM **1-cm grid paper** (see Resources).
- This topic may need to be split up over 2 or 3 lessons.
- Ensure that students understand the idea of gradient.
- Students may use technology to complete some questions if you need to complete the exercise quickly. Or they may use technology to check their answers.
- Students may be assisted by using the BLM **Cartesian plane grids** (see Resources).
- You may want students to draw several graphs of lines per Cartesian plane as they go through this topic.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4C-1**. It shows how to use the gradient–intercept method to sketch a linear graph and this will help students to complete Q1–4.
- In Q2, students may need to be reminded of their equation-solving methods to rearrange each of the formulas. Encourage them to expand any expressions containing brackets first and then move everything else except y from the left side of the equation.
- Explain that in Q4, any linear rule without a constant term has the origin as both its x- and y-intercept.
- For Q6–8, discuss the form of:
  - a horizontal line that shows all points with the same y value
  - a vertical line that shows all points with the same x value.
- **Example 4C-2** shows how to use the x- and y-intercept method to sketch a linear graph. This will help students to complete Q9–12.
- For Q9–12, emphasise to students that the:
  - x-intercept is the value of x when $y = 0$
  - y-intercept is the value of y when $x = 0$.

- You may like to explain the difference between writing, for example, ‘the *x*-intercept is 3’ and ‘the coordinates of the *x*-intercept are (3, 0)’.
- **Example 4C-3** shows how to sketch horizontal and vertical linear graphs and this will help students to complete Q15.
- For Q18, discuss with students when it is most appropriate to use the gradient-intercept method or the *x*- and *y*-intercept method.
- Students may need some explanation of Q19. They will need to be shown that they would normally draw a linear relationship with arrows on each end indicating that the relationship continues infinitely. In Q19, the line is only drawn for the *x* or *y* values indicated.
- Explain that Q21 and Q22 are examples of how linear relationships are used to model a real life situation and that these models are then used to make predictions.
- For additional practice, students can complete Q1–6 of **WS 4-2 Working with linear relationships** (see Resources). Additional questions similar to Exercise 4C Q1–5, Q9 and Q10 are provided. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.

Below Level

Below Level
1, 2a–i, 3a–i, 4, 5a, b, 6, 7a, b, 8, 9a–f, 10, 11, 13, 14a, b, 15a–d

- Students may need to complete **SS 4C-1 Identifying features of a linear graph** (see Resources).
- Students who are struggling with the gradient-intercept method for Q1 should use the method of finding three points (substitute $x = 0$, $x = 1$, $x = 2$) and then observe the *y*-intercept and gradient.
- In Q2, students may need to be reminded of their equation-solving methods to rearrange each of the formulas. Encourage them to expand any expressions containing brackets first and then move everything else except *y* from the left side of the equation.
- Explain that in Q4, any linear rule without a constant term has the origin as both its *x*- and *y*-intercept.
- For Q6–8, discuss the form of:
  - a horizontal line that shows all points with the same *y* value
  - a vertical line that shows all points with the same *x* value.

- For Q9, emphasise to students that:
  - the *x*-intercept is the value of *x* when $y = 0$. They will need to substitute $y = 0$ into the linear rule and solve for *x*
  - the *y*-intercept is the value of *y* when $x = 0$. They will need to substitute $x = 0$ into the linear rule and solve for *y*.

POTENTIAL DIFFICULTY When the *x*- and *y*-intercepts are the origin, students may struggle to see a second point to use to draw their line. They may need help with the reading of the steps in Q13 to work through the solution to this problem.

- For students who do not progress past Q10, direct them to Q1–6 of **WS 4-2 Working with linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.
- Make use of the BLM **Cartesian plane grids** (see Resources) to save students time with ruling up their own Cartesian planes.
- Where necessary, provide students with the technology to help them with their graphing or to check their answers.

Above Level

Above Level
1f–i, 2f–l, 3f–l, 4, 5b, d, 6, 7c, d, 8, 9g–l, 10, 12–14, 15e–h, 16–24

- Explain that in Q4 any linear rule without a constant term has the origin as both its *x*- and *y*-intercept.
- You may like to explain the difference between writing, for example, ‘the *x*-intercept is 3’ and ‘the coordinates of the *x*-intercept are (3, 0)’.
- In Q16, students must consider that:
  - all vertical and horizontal lines have one axis intercept
  - all other lines will have two axis intercepts although the two axis intercepts can be at the same point for graphs passing through the origin.
- For Q18, discuss with students when it is most appropriate to use the gradient-intercept method or the *x*- and *y*-intercept method.
- Students may need some explanation of Q19. They will need to be shown that they would normally draw a linear relationship with arrows on each end indicating that the

relationship continues infinitely. In Q19, the line is only drawn for the x or y values indicated.

- Explain that Q21 and Q22 are examples of how linear relationships are used to model a real life situation and that these models are then used to make predictions.

Extra activities

1 Quick Questions

Find the value of x when $y = 0$ in the following equations.

a $y = 2x - 6$ (3)

b $2x - 3y = 6$ (3)

Find the value of y when $x = 0$ in the following equations.

c $y = 4 - 2x$ (4)

d $5x - 2y + 10 = 0$ (5)

2 Conversion graphs

When measuring temperature, the convention is that we use degrees Celsius (°C). Until 1972, temperature was measured in degrees Fahrenheit (°F) and this scale is still used in many countries today including the USA.

To convert a temperature from °F to °C, the linear relationship $C = \frac{5}{9}(F - 32)$ can be used.

a By placing F on the horizontal axis and C on the vertical axis draw a graph of this relationship.

b Use your graph to find the value of:

i C when $F = 32$ ($C = 0$)

ii C when $F = 100$ ($C \approx 38$)

iii F when $C = 100$ ($F = 212$)

iv F when $C = 25$. ($F = 77$)

Answers

4C Sketching linear graphs

4C Start thinking!

1 The graph of a linear relationship is a straight line.

2 A straight line connects two points, and this will define the linear relationship.

3 a i $m = 4$, $c = 1$ ii $m = -4$, $c = 1$
iii $m = \frac{1}{4}$, $c = 1$

b i (0, 1) ii (0, 1) iii (0, 1)

c i 4 ii −4 iii $\frac{1}{4}$

d i rise = 4, run = 1 ii rise = −4, run = 1
iii rise = 1, run = 4

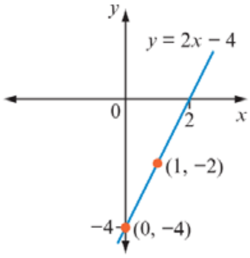
e i $y = 4x + 1$ matches graph C
ii $y = -4x + 1$ matches graph A
iii $y = \frac{1}{4}x + 1$ matches graph B

4 Starting at y-intercept, use rise and run to locate a second point. Ruling a straight line passing through these two points will represent the graph of the linear relationship.

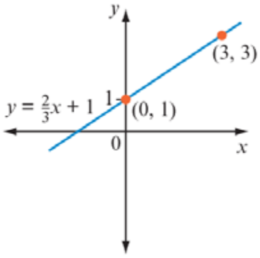
Exercise 4C Sketching linear graphs

ANSWERS

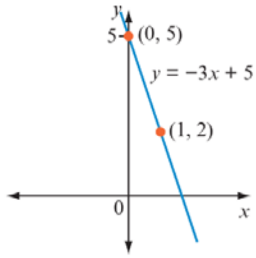
1 a



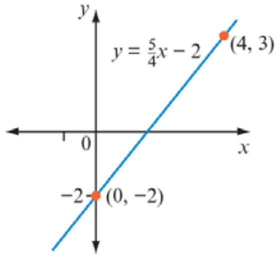
b



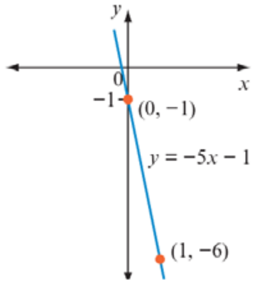
c



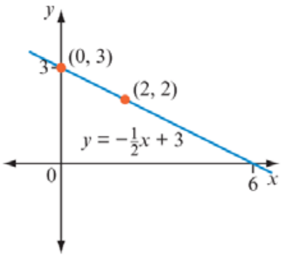
d



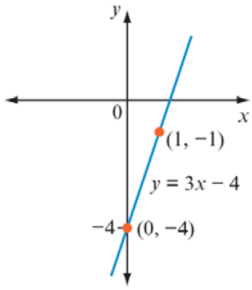
e



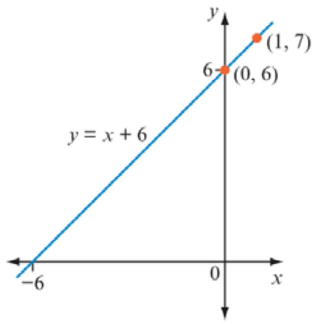
f



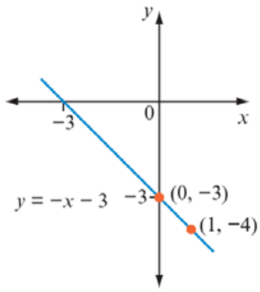
g



h



i



2 a $y = 2x + 1, m = 2, c = 1$

b $y = -3x + 4, m = -3, c = 4$

c $y = -\frac{3}{4}x - 3, m = -\frac{3}{4}, c = -3$

d $y = 3x - 6, m = 3, c = -6$

e $y = -2x + 6, m = -2, c = 6$

f $y = -5x - 2, m = -5, c = -2$

g $y = -3x + 2, m = -3, c = 2$

h $y = 4x - 5, m = 4, c = -5$

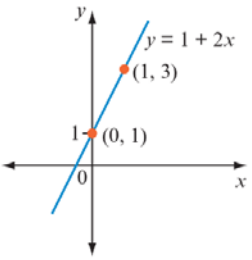
i $y = -2x + 3, m = -2, c = 3$

j $y = \frac{4}{5}x + 3, m = \frac{4}{5}, c = 3$

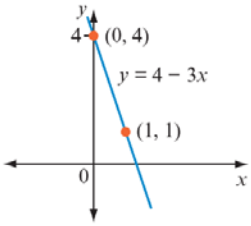
k $y = \frac{3}{2}x + 2, m = \frac{3}{2}, c = 2$

l $y = -\frac{1}{3}x - 2, m = -\frac{1}{3}, c = -2$

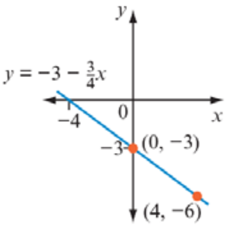
3 a



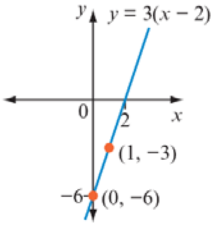
b



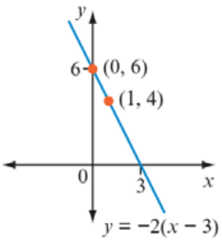
c



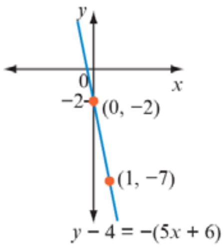
d



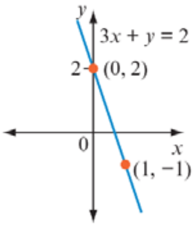
e



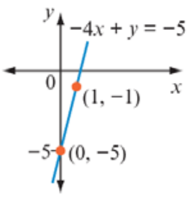
f



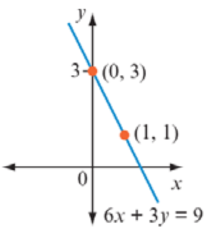
g



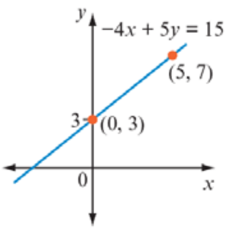
h



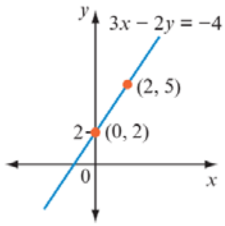
i



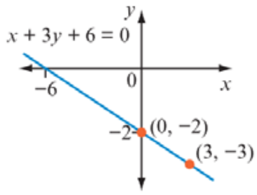
j



k



l

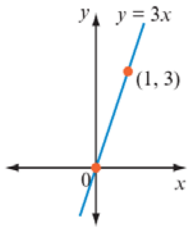


4 a $y = 3x + 0$

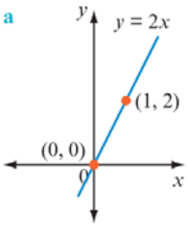
b **i** $m = 3$

ii $c = 0$

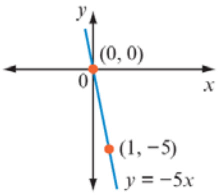
c



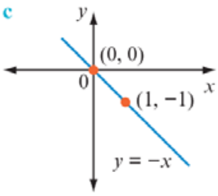
5 a



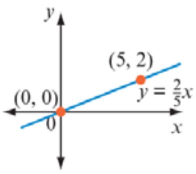
b



c



d

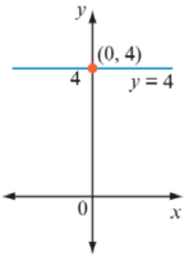


6 a $y = 0x + 4$

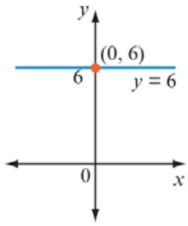
b i $m = 0$

ii $c = 4$

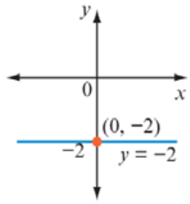
c



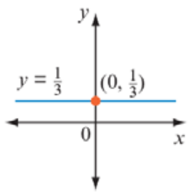
7 a



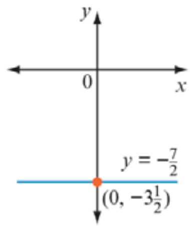
b



c

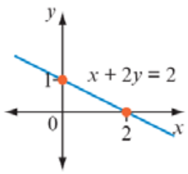


d

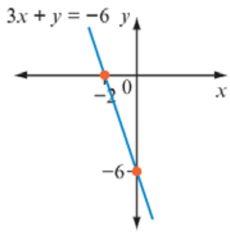


8 The rule $x = 4$ cannot be written in the form $y = mx + c$ since there is no y value specified.

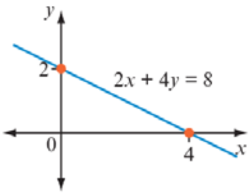
9 **a**



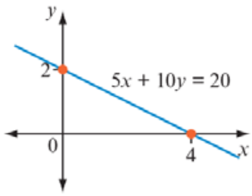
b



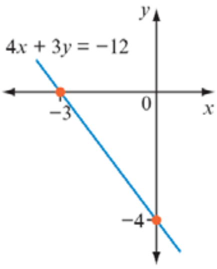
c



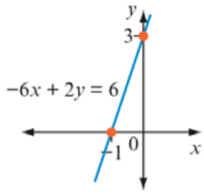
d



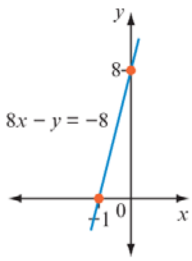
e



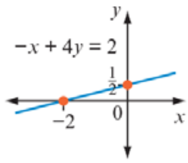
f



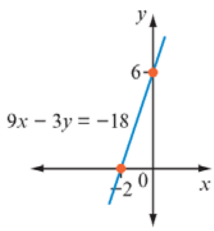
g



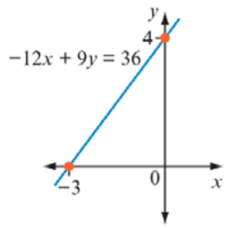
h



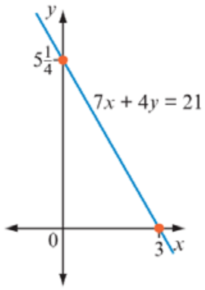
i



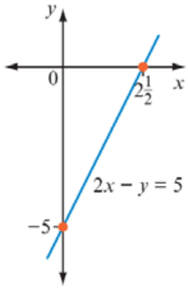
j



k



l



10 **a** x-intercept: $(-5, 0)$, y-intercept: $(0, 5)$

b x-intercept: $(4, 0)$, y-intercept: $(0, -4)$

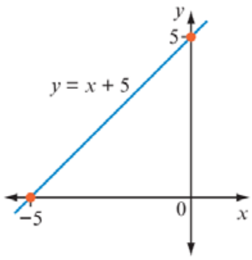
c x-intercept: $(-2, 0)$, y-intercept: $(0, 6)$

d x-intercept: $(4, 0)$, y-intercept: $(0, -8)$

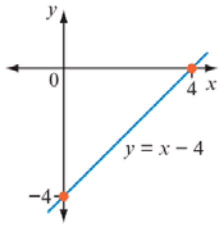
e x-intercept: $(2, 0)$, y-intercept: $(0, 2)$

f x-intercept: $(-\frac{1}{2}, 0)$, y-intercept: $(0, -3)$

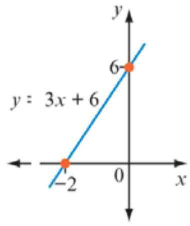
11 a



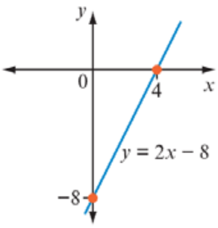
b



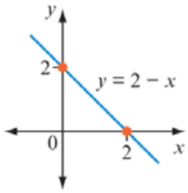
c



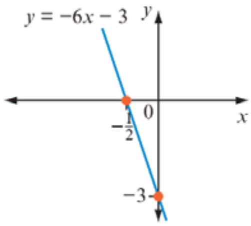
d



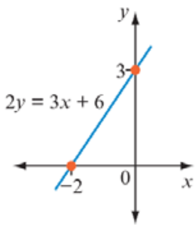
e



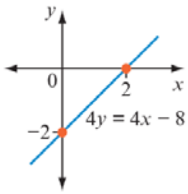
f



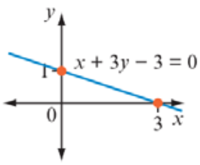
12 a



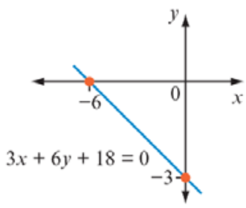
b



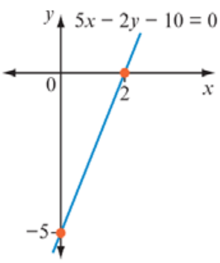
c



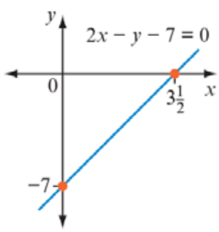
d



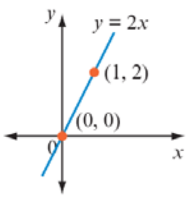
e



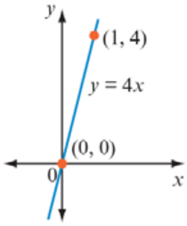
f



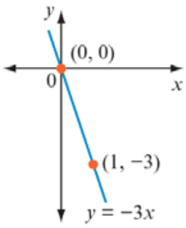
13 a **i** x-intercept: (0, 0)
ii y-intercept: (0, 0)
b no; only have one point
c coordinates of one other point on the line
d $y = 2$
e (0, 0) and (1, 2)
f



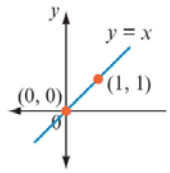
14 a



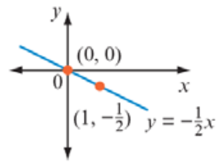
b



c

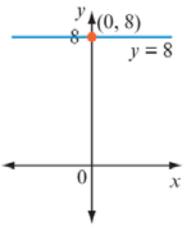


d



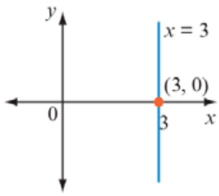
15 a i y-intercept: (0, 8), no x-intercept

ii



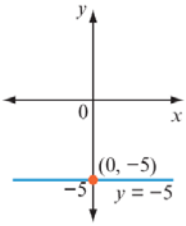
b i x-intercept: (3, 0), no y-intercept

ii



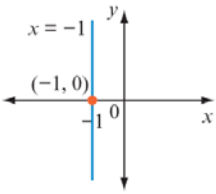
c i y-intercept: (0, −5), no x-intercept

ii



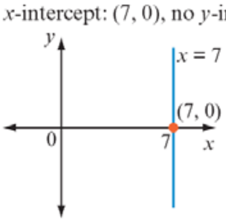
d i x-intercept: (−1, 0), no y-intercept

ii



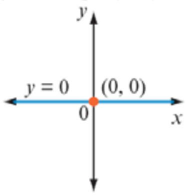
e i x-intercept: (7, 0), no y-intercept

ii



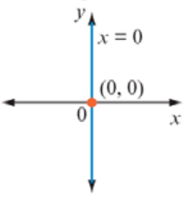
f i y-intercept: (0, 0), no x-intercept

ii



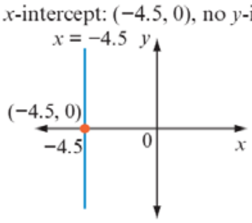
g i x-intercept: (0, 0), no y-intercept

ii



h i x-intercept: (−4.5, 0), no y-intercept

ii



ANSWERS

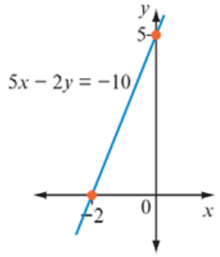
16 A linear graph can have one or two axis intercepts (or perhaps lie along the x-axis or y-axis).

- **a** x-intercept: (2, 0), y-intercept: (0, −4), two intercepts
- **b** no x-intercept, y-intercept: (0, −7), one intercept
- **c** x-intercept: (3, 0), y-intercept: (0, 2), two intercepts
- **d** x-intercept: (0, 0), y-intercept: (0, 0), one intercept
- **e** x-intercept: (10, 0), y-intercept: (0, −50), two intercepts
- **f** x-intercept: (−5, 0), y-intercept: (0, 3), two intercepts
- **g** x-intercept: (28, 0), no y-intercept, one intercept
- **h** x-intercept: (6, 0), y-intercept: (0, −2), two intercepts
- **i** no x-intercept, y-intercept: (0, 5.6), one intercept

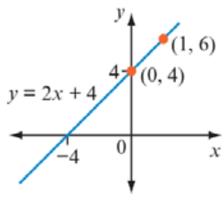
17

- **a** gradient–intercept method
- **b** x- and y-intercept method
- **c** Find the coordinates of another point, and draw the straight line through that point and the origin.
- **d** Locate the y-intercept, and draw a line through that point parallel to the x-axis.
- **e** Locate the x-intercept, and draw a line through that point parallel to the y-axis.

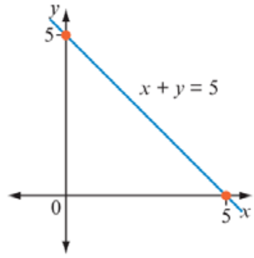
18 **a**



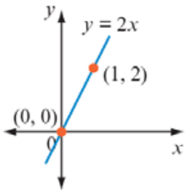
b



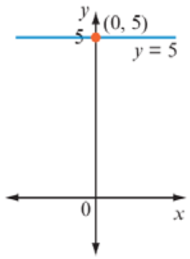
c



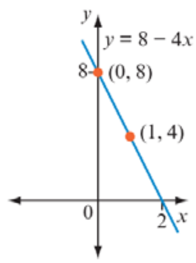
d



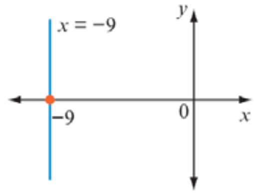
e



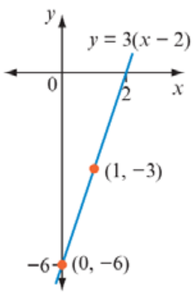
f



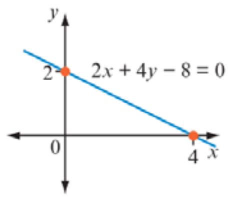
g



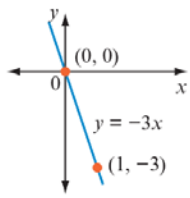
h



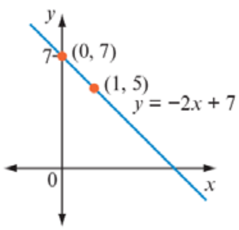
i



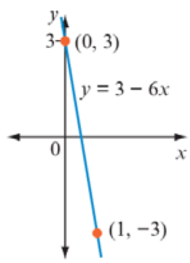
j



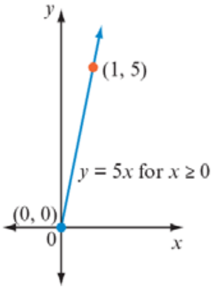
k



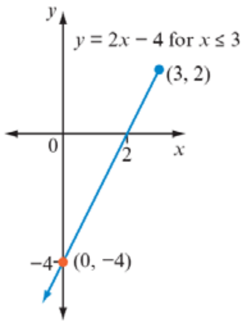
l



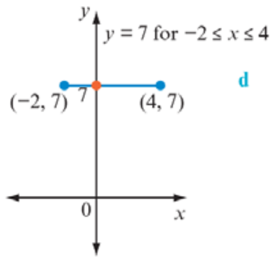
19 a



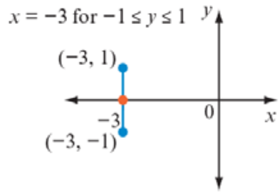
b



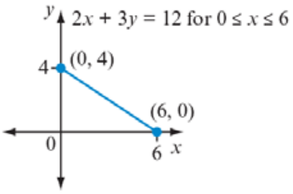
c



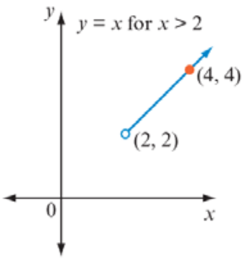
d



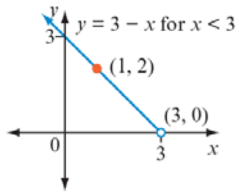
e



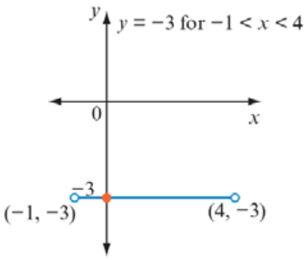
f



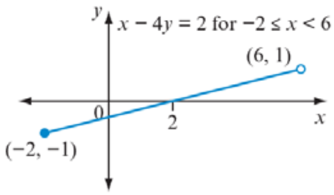
g



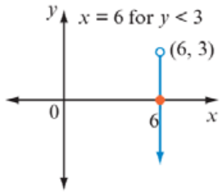
h



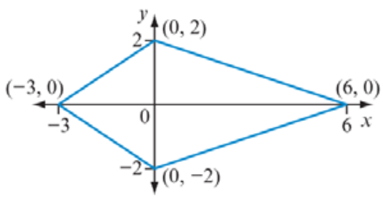
i



j



20 a



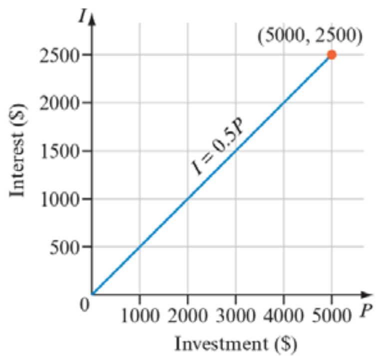
b kite

c **i** 19.9 units **ii** 18 square units

d coordinates of midpoints: (3, 1), (3, −1), (−1.5, −1) and (−1.5, 1)

e perimeter =13 units, area = 9 square units

21 a

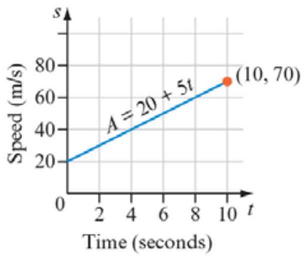


b **i** \$500 **ii** \$1250 **iii** \$2125

c **i** \$1500 **ii** \$2000 **iii** \$4500

d Gradient represents value of RT. This is the rate at which the investment grows over a 10-year period. $RT = 0.05 \times 10 = 0.5$.

22 a



b initial constant speed

c rate of increase in speed

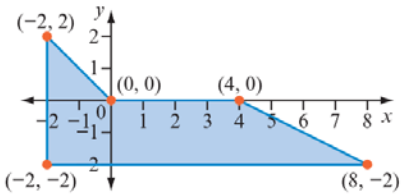
d i 35 m/s ii 45 m/s iii 57.5 m/s

e i 126 km/h ii 162 km/h iii 207 km/h

f i 2 s ii 7 s iii 9 s

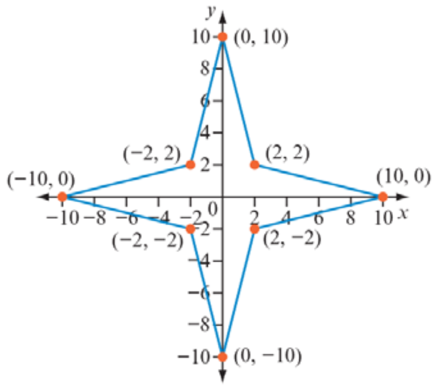
g 8 s **h** 252 km/h reached after 10 s

23 a



b 25.3 units **c** 18 square units

24 a



b 66 units **c** 80 square units

d $y = x$ **e** $y = -x$

Reflect

Possible answer: The common goal of the two methods is to locate two points on a Cartesian plane to enable a line to be ruled through them to represent the linear relationship.

Resources

SupportSheet

SS 4C-1 Identifying features of a linear graph

Focus: To review common features of a linear graph including gradient and the *x*-and *y*-intercepts

Resources: ruler (optional), **1-cm grid paper** (BLM) or graph paper (optional)

Students review the definitions of linear graph, gradient, rise, run, *x*-intercept and *y*-intercept. They work out the vertical rise and horizontal run to calculate the gradient and identify the *x*- and *y*-intercepts from a given linear graph. Students use the general rule $y = mx + c$ and their information for m and c to write the rule for these linear graphs. They also find the gradient and *x*- and *y*-intercepts for linear graphs from given rules.

WorkSheet

WS 4-2 Working with linear relationships

Focus: To sketch linear graphs and to determine the rule for linear relationships given relevant information.

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4C–E. Q1–6 relate to Exercise 4C.

Students sketch linear graphs using the gradient-intercept method and also by using the x- and y-intercept method. They perform calculations to determine the gradient of a line and to find the rule for a linear graph using $y = mx + c$ and $y - y_1 = m(x - x_1)$. Students also work with parallel and perpendicular lines.

BLMs

1-cm grid paper

Cartesian plane grids

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4D Finding the rule for a linear relationship

Teaching support for pages 178–183

Teaching strategies

Learning focus

To determine the rule for a linear relationship

Start thinking!

The task guides students to:

- determine the *y*-intercept and the gradient from a linear graph
- substitute this information into the $y = mx + c$ form of a linear relationship to find the rule
- apply this method to find the rule for more linear graphs given the *y*-intercept and gradient.

Differentiated pathways

Below Level	At Level	Above Level
1, 3–6, 8–11, 15	1–5, 6a–c, 7, 8a–c, 9–18	1–4, 6d–f, 7, 8d–f, 9, 10, 12–21
Students complete the **assessment** for this topic		

At Level

At Level
1–5, 6a–c, 7, 8a–c, 9–18

- Students will need to revise the rule $y = mx + c$ for linear relationships.
- Explain that, to find the rule, students will need to know the gradient and the *y*-intercept.
- Students will be familiar with the formula $m = \frac{\text{rise}}{\text{run}}$ but need to be introduced to the more formal rule $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Direct students to the **Key ideas**. You may like them to copy this summary.

- Direct students to **Example 4D-1**. It shows how to find the gradient of a line through two given points and will help students to complete Q1.
- In Q1, ensure that students set out their working properly, writing the formula at the top of each problem and labelling the values for x_1, y_1, x_2 and y_2.
- In Q3, students are to find the gradient but do not need to use a formula. As the graphs are provided, they should be able to work out the gradient from each line.
- **Example 4D-2** shows how to find the rule given the y-intercept and a point. This will help students to complete Q4 and Q5.
- In Q4, some students may need help in identifying which point is the y-intercept before substituting for c in the rule $y = mx + c$.
- **Example 4D-3** shows how to find the rule given the gradient and a point. This will help students to complete Q6.
- For Q6, explain that, although students have not being given the y-intercept, they can substitute the coordinates of the given point and the gradient into the rule $y = mx + c$ and solve to find c.
- You may like to work through the steps of Q7 as a whole class to produce the point-gradient rule $y - y_1 = m(x - x_1)$.
- **Example 4D-4** shows how to find the rule given two points using $y - y_1 = m(x - x_1)$. This will help students to complete Q9–11.
- In Q10, after first finding the gradient, have some students find the rule using the first point in the pair and others find the rule using the second point. Students will see that the same answer is obtained and so it does not matter which point is used in the formula. This will confirm their findings in Q9.
- In Q13, guide students to write the x- and y-intercepts as pairs of coordinates so they can use the point-gradient formula to find the rule.
- In Q15 and Q16, some students may need to draw the linear relationship to find the rule.
- For additional practice, students can complete Q7–9 of **WS 4-2 Working with linear relationships** (see Resources). Additional questions similar to Exercise 4D Q1, Q3 and Q10 are provided. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.
- For more problem-solving tasks and investigations, direct students to **INV 4-2 The human body** (see Resources).

Below Level

Below Level
1, 3–6, 8–11, 15

- In Q1, encourage students to identify and then write the values for x_1, y_1, x_2 and y_2 to ensure that students substitute the values correctly in the formula for gradient. They should clearly show all working out.
- In Q3, students may need to be guided to see that they can work out the gradient directly from each linear graph.
- In Q4, some students may need help in identifying which point is the *y*-intercept before substituting for c in the rule $y = mx + c$.
- For Q6, explain that, although students have not being given the *y*-intercept, they can substitute the coordinates of the given point and the gradient into the rule $y = mx + c$ and solve to find c.
- You may like to work through the steps of Q7 as a whole class to produce the point-gradient rule $y - y_1 = m(x - x_1)$. Below Level students may find it easier to use the provided formula. However, all the pronumerals will need to be carefully explained.
- When students get to Q9 and Q10, explain that the formula $y - y_1 = m(x - x_1)$ is called the point-gradient formula. In each problem, have students calculate the gradient and label one of the points x_1 and y_1 before substituting.
- In Q10, after first finding the gradient, have some pairs of students find the rule using the first point and other pairs of students find the rule using the second point. Students will see that the same answer is obtained and so it does not matter which point is used in the formula. This will confirm their findings in Q9.
- In Q15, students may benefit from drawing each linear relationship first to assist them in finding the rule.
- For students who do not progress past Q10, direct them to Q7–9 of **WS 4-2 Working with linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.
- Students need to be confident in their understanding that the graph represents all points that satisfy a given rule for a linear relationship.

- They will need to have the two key formulas $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and $y - y_1 = m(x - x_1)$ at their fingertips.
- Where appropriate, allow students to use technology to check their answers.

Above Level

Above Level
1–4, 6d–f, 7, 8d–f, 9, 10, 12–21

- In Q1, an easy mistake to make is to confuse the order of the x and y values in the gradient formula. Encourage students to label the values for x_1, y_1, x_2 and y_2 before substituting into the formula.
- For Q7, explain that the formula $y - y_1 = m(x - x_1)$ is called the point-gradient formula.
- In Q10, after first finding the gradient, have some students find the rule using the first point in the pair and others find the rule using the second point. Students will see that the same answer is obtained and so it does not matter which point is used in the formula. This will confirm their findings in Q9.
- In Q13, guide students to write the x- and y-intercepts as pairs of coordinates so they can use the point-gradient formula to find the rule.
- In Q19, students will obtain a fractional answer for the gradient. Explain that the best way to express the rule without fractions is to multiply both sides of the equation by the denominator before expanding as shown below for part a.

  $$y - y_1 = m(x - x_1)$$

  $$y - 8 = \tfrac{1}{2}(x - 20)$$

  $$2y - 16 = 1(x - 20)$$

  $$2y - 16 = x - 20$$

  $$2y = x - 4$$

- For more problem-solving tasks and investigations, direct students to **INV 4-2 The human body** (see Resources).

Extra activities

1 Quick Questions

Solve each equation.

a $7 = 2 \times 3 + c$ $(c = 1)$

b $–4 = 3 \times 4 + c$ $(c = –16)$

c $9 = 5 \times (–4) + c$ $(c = 29)$

d $10 = c – 4 \times 3$ $(c = 22)$

2 Jaimee is organising a school disco. It will cost $480 to stage the disco and she is planning to charge $12 per ticket.

a Write a rule that will represent the profit (or loss) that Jaimee will make.
($P = 12n – 480$ where P is the profit and n is the number of tickets sold)

b The number of people that will attend can be found using the rule $N = 950 – 12c$ where c is the cost of attending in dollars. Find the number of people who will attend the disco and Jaimee's profit or loss.
($N = 950 – 12 \times 12 = 806$; $P = 12 \times 806 – 480 = \$9192$)

Answers

4D Finding the rule for a linear relationship

4D Start thinking!

1 a Graph shows line drawn through two points on the Cartesian plane.
  b Rule can be calculated from gradient and y-intercept.
  c 3 d (0, 2) e $m = 3$, $c = 2$
  f $y = 3x + 2$

2 c −2 d (0, 3) e $m = -2$, $c = 3$
  f $y = -2x + 3$

3 General rule for straight line is $y = mx + c$. Gradient (m) is coefficient of x, and y-coordinate of y-intercept represents value of c.

4 a $y = 3x - 2$
  b $y = -\frac{5}{3}x + 1$ or $5x + 3y = 3$

Exercise 4D Finding the rule for a linear relationship

1 **a** 4 **b** −2 **c** $\frac{1}{2}$ **d** −1 **e** 5 **f** −3

2 No. Consider the two points (2, 6) and (1, 4).
Taking (2, 6) as (x_1, y_1) and (1, 4) as (x_2, y_2),
$m = \frac{4-6}{1-2} = \frac{-2}{-1} = 2$.
Taking (1, 4) as (x_1, y_1) and (2, 6) as (x_2, y_2),
$m = \frac{6-4}{2-1} = \frac{2}{1} = 2$.

3 **a** $y = 2x + 1$ **b** $y = 4x - 3$ **c** $y = -3x + 5$
d $y = -x - 1$ **e** $y = \frac{3}{5}x$ **f** $y = -\frac{1}{2}x + 2$

4 **a** $y = 4x + 2$ **b** $y = x - 7$ **c** $y = \frac{1}{2}x + 5$
d $y = 10x + 6$ **e** $y = -3x - 5$ **f** $y = -\frac{1}{4}x + 4$

5 **a** $y = 5x - 3$ **b** $y = -2x + 4$ **c** $y = \frac{4}{5}x + 2$

6 **a** $y = 3x + 1$ **b** $y = 2x + 3$ **c** $y = -4x - 2$
d $y = -x + 9$ **e** $y = 5x + 9$ **f** $y = -3x - 1$

7 **a** $c = y_1 - mx_1$ **b** $y = mx + y_1 - mx_1$
c $y - y_1 = mx - mx_1$, $y - y_1 = m(x - x_1)$

8 **a** $y = 3x + 1$ **b** $y = 2x + 3$ **c** $y = -4x - 2$
d $y = -x + 9$ **e** $y = 5x + 9$ **f** $y = -3x - 1$

9 The answer is the same. Either of the two points can be used.

10 **a** $y = 2x + 1$ **b** $y = 6x - 28$ **c** $y = -3x + 10$
d $y = -2x + 4$ **e** $y = x - 1$ **f** $y = -x - 6$

11 **a** $y = 3x - 11$ **b** $y = -2x + 6$ **c** $y = x + 2$

12 $m = \frac{7-3}{3-1} = 2$ so $y = 2x + c$
Substituting the point (1, 3) gives $3 = 2 + c$ or $c = 1$.
So the rule is $y = 2x + 1$.

13 **a** $y = -2x + 4$ **b** $y = x + 3$ **c** $y = 2x - 6$
d $y = -5x + 5$

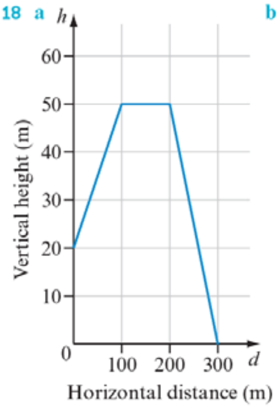
14 **a** $y = -3x + 5$ **b** $y = 2x + 4$ **c** $y = 8$
d $y = \frac{1}{3}x$ **e** $x = -\frac{5}{2}$ **f** $y = 2x - 1$

15 **a** $y = \frac{1}{200}x + 50$ **b** $y = -\frac{1}{15}x + 6$ **c** $y = \frac{1}{5}x$

16 **a** $a = 20n + 80$ **b** 21 weeks

17 **a** $w = -195n + 1560$ **b** \$975
c 8 months

18 **a**

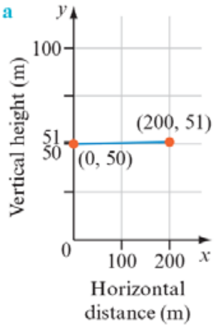


b In order of the flight:
$h = 0.3d + 20$, $h = 50$,
$h = -0.5d + 150$

c **i** 29 m **ii** 44 m **iii** 50 m **iv** 30 m
d $66\frac{2}{3}$ m and 220 m

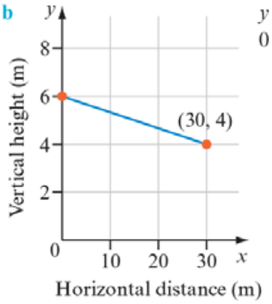
19 **a** $x - 2y = 4$ **b** $y = -23$
c $2x + 3y + 20 = 0$ **d** $12x - 17y = 0$

20 **a**



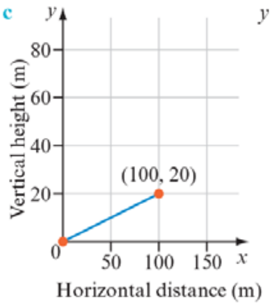
$y = \frac{1}{200}x + 50$ for $0 \le x \le 200$

b



$y = -\frac{1}{15}x + 6$ for $0 \le x \le 30$

c



$y = \frac{1}{5}x$ for $0 \le x \le 100$

21 In order of the flight:
$h = 0.3d + 20$ for $0 \le d \le 100$
$h = 50$ for $100 \le d \le 200$
$h = -0.5d + 150$ for $200 \le d \le 300$

Reflect

Possible answer: The rule for a linear graph can be found by identifying the gradient (m) and y-intercept (c) and substituting this into the rule $y = mx + c$. Alternatively, the coordinates of a point on the graph and the gradient can be substituted into the rule $y - y_1 = m(x - x_1)$.

Resources

WorkSheet

WS 4-2 Working with linear relationships

Focus: To sketch linear graphs and to determine the rule for linear relationships given relevant information

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4C–E. Q8 and Q9 relate to Exercise 4D.

Students sketch linear graphs using the gradient-intercept method and also by using the *x*- and *y*-intercept method. They perform calculations to determine the gradient of a line and to find the rule for a linear graph using $y = mx + c$ and $y - y_1 = m(x - x_1)$. Students also work with parallel and perpendicular lines.

Investigation

INV 4-2 The human body

Focus: To discover a relationship between measurements of various parts of the human body

Resources: calculator, measuring tape

Students look at measurements of various parts of the body; for example, height, waist, head length and reach. These measurements are compared and an approximate ratio found enabling a linear relationship to be written to compare different measurements.

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4E Parallel and perpendicular lines

Teaching support for pages 184–189

Teaching strategies

Learning focus

To understand the relationship between the gradients of parallel and perpendicular lines and write rules for these lines

Start thinking!

The task guides students to:

- look at the graphs of three linear relationships that are either parallel or perpendicular
- find the gradient and *y*-intercept of each linear graph
- recognise that parallel lines have equal gradients
- recognise that the product of the gradients of two perpendicular lines is –1.

Differentiated pathways

Below Level	At Level	Above Level
1–10, 12, 13a, b, 16, 18	2, 3, 5–9, 11–17, 19–22	2, 3, 5, 7–9, 11–13, 15, 17, 19–27
Students complete the **assessment** for this topic		

At Level

At Level
2, 3, 5–9, 11–17, 19–22

- You may like to provide students with copies of the BLM **Cartesian plane grids** (see Resources).
- Ensure that students are familiar with the key ideas from the previous topic.
  - $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
  - $y - y_1 = m(x - x_1)$

- Discuss the definition of gradient as being the slope of a line. Therefore, as parallel lines have the same slope, the gradients must be equal.
- Discuss the definition of perpendicular lines as being at right angles to each other. Demonstrate that if two lines are perpendicular the signs of the two gradients must be opposite. Students should also be able to see that if one gradient is steep the other must be slight. Introduce the idea of the gradients being negative reciprocals.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- In Q2, students need to read the gradient from the rule $y = mx + c$ and recognise that parallel lines have equal gradients.
- Direct students to **Example 4E-1**. It shows how to write the rule for a parallel line using the gradient and y-intercept and will help students to complete Q3 and Q4.
- For Q3, students again use the rule $y = mx + c$ to find the gradient and then use this same rule with a new value of c to write the required rule of the parallel line. Students should progress to the point where they can see they only need to change the value of c using the new y-intercept.
- **Example 4E-2** shows how to write the rule for a parallel line using the gradient and a point. This will help students to complete Q5.
- **Example 4E-3** shows how to write the rule for a perpendicular line using the gradient and y-intercept. This will help students to complete Q9.
- For Q9, students need to understand that unlike parallel lines, the coefficients of x and y will change. Therefore, it is not as simple as changing the constant term.
- **Example 4E-4** shows how to write the rule for a perpendicular line using the gradient and a point. This will help students to complete Q12.
- In Q13, guide students to write both rules in the form $y = mx + c$ and examine if the gradients are equal or negative reciprocals.
- In Q14, there will be many possible answers. Have students compare their answers and recognise that they are all of the same form, $2y - 10x = k$ (or $y = 5x + k$).
- In Q15, there will be many possible answers. Again, have students compare their answers and see they are all of the form $8x - y - k = 0$. Discuss the similarities and differences to the given line $x + 8y - 4 = 0$.
- In Q16, students use the gradient formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between two points and compare this to the gradient of the graph of $y = 4x - 7$.

- In Q20, students consider the gradients of opposite sides of a quadrilateral. Have students draw a conclusion about what type of quadrilateral ABCD would be. Extend this conversation to discuss what we would know about the gradients of sides and diagonals of common quadrilaterals.

- For Q21, guide students to see that two of the sides must be perpendicular to form a right-angled triangle and hence they need to calculate the gradients of the line segments. To show that the triangle is also isosceles, they need to think about which two sides to consider and find their lengths using the distance formula. Finding the length of the third side will enable them to calculate the perimeter of the triangle.

- In Q22, students need to remember that a rhombus is a quadrilateral with four equal side lengths and that the opposite sides of a parallelogram are equal in length.

- For additional practice, students can complete Q10–12 of **WS 4-2 Working with linear relationships** (see Resources). Additional questions similar to Exercise 4E Q2, Q3 and Q8 are provided. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.

Below Level

Below Level
1–10, 12, 13a, b, 16, 18

- For Q1, ensure that students draw their Cartesian plane quite large to enable them to see the key features of the graphs.

- In Q2, students need to read the gradient from the rule $y = mx + c$ and recognise that parallel lines have equal gradients.

- For Q3, students again use the rule $y = mx + c$ to find the gradient and then use this same rule with a new value of c to find the required rule of the parallel line.

- In Q4, students should reach the point where they see that the rules for all parallel lines only differ by the constant term.

POTENTIAL DIFFICULTY
Students need to be familiar with the key language of the topic. Define words such as perpendicular, product and reciprocal so students can progress to Q6.

- After completing Q6, students should be able to see that the gradient of a perpendicular line can be found by taking the negative reciprocal of the first gradient.

- For Q9, students need to understand that unlike parallel lines, the coefficients of x and y will change. Therefore, it is not as simple as changing the constant term.

- For students who do not progress past Q8, direct them to Q10–12 of **WS 4-2 Working with linear relationships** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4C, 4D and 4E and can be completed progressively or as a skills review of Exercises 4C–E.

- Have students look at families of parallel lines and see that the coefficients of x and y remain the same and that only the constant term changes. This should include looking at equations in the form
  - $y = mx + c$
  - $ax + by = c$
  - $ax + by + c = 0$
- Repeat this task for perpendicular line pairs and have students look at the change in coefficients.

Above Level

Above Level
2, 3, 5, 7–9, 11–13, 15, 17, 19–27

- For Q3, students should observe that they only need to change the value of c using the new y-intercept.
- For Q9, students need to understand that unlike parallel lines, the coefficients of x and y will change. Therefore, it is not as simple as changing the constant term.
- In Q13, guide students to write both rules in the form $y = mx + c$ and examine if the gradients are equal or negative reciprocals.
- In Q17, students use the gradient formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between two points and compare this to the gradient of the graph of $y = –2x + 5$.
- In Q20, students consider the gradients of opposite sides of a quadrilateral. Have students draw a conclusion about what type of quadrilateral ABCD would be. Extend this conversation to discuss what we would know about the gradients of sides and diagonals of common quadrilaterals.
- For Q21, guide students to see that two of the sides must be perpendicular to form a right-angled triangle and hence they need to calculate the gradients of the line segments. To show that the triangle is also isosceles, they need to think about which two sides to consider and find their lengths using the distance formula. Finding the length of the third side will enable them to calculate the perimeter of the triangle.

- In Q22, students need to remember that a rhombus is a quadrilateral with four equal side lengths and that the opposite sides of a parallelogram are equal in length.
- For Q25, students may need the term ‘perpendicular bisector’ defined as a line that cuts another line exactly in half at right angles.
- For Q26, students need to know that the diagonals of a kite are perpendicular.

Extra activities

1 Quick Questions

Write each equation in the form $y = mx + c$.

a $2y = 4x + 8$ $(y = 2x + 4)$

b $2x + 5y = 10$ $(y = -\frac{2}{5}x + 2)$

c $5x - 2y = 10$ $(y = \frac{5}{2}x - 5)$

d $6x + 4y - 5 = 0$ $(y = -\frac{3}{2}x + \frac{5}{4})$

Answers

ANSWERS

4E Parallel and perpendicular lines

4E Start thinking!

1 A gradient: 2, y-intercept: 2
B gradient: 2, y-intercept: -2
C gradient: $-\frac{1}{2}$, y-intercept: 3

2 A $y = 2x + 2$ B $y = 2x - 2$ C $y = -\frac{1}{2}x + 3$

3 A and B are parallel.

4 gradient

5 90°; perpendicular

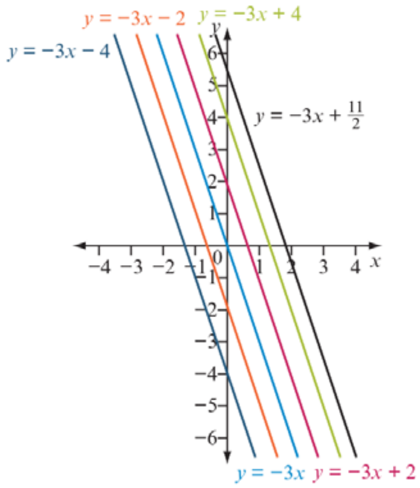
6 -1

7 -1

8 Parallel lines have the same gradient. The product of the gradients of perpendicular lines is -1.

Exercise 4E Parallel and perpendicular lines

1 a



b All lines are parallel and have same gradient. They have different y-intercepts.
c All have same gradient (-3).
d **i** gradient = 3, so not parallel
ii gradient = -3, so parallel
iii gradient = 1, so not parallel

2 a 7 **b** -1 **c** $-\frac{3}{2}$ **d** 0 **e** -4 **f** -2

3 a $y = -2x + 3$ **b** $y = 6x - 4$
c $y = x$ **d** $y = -5x + 2$

4 a $y = -3x - 5$
b One possible answer is: $y = -3x + 20$.

5 a $y = 4x - 13$ **b** $y = -3x - 6$
c $y = -x - 2$ **d** $y = \frac{1}{2}x - 3$

6 a **i** gradients: 3 and $-\frac{1}{3}$
ii gradients: $\frac{3}{4}$ and $-\frac{4}{3}$
b Both products are -1.
c The product of gradients of perpendicular lines is always -1.
d Take negative reciprocal of known gradient.

7 a $-\frac{1}{5}$ **b** $\frac{1}{8}$ **c** 1 **d** -6 **e** $-\frac{5}{4}$ **f** $\frac{2}{3}$

8 a $-\frac{1}{2}$ **b** $\frac{1}{3}$ **c** -7 **d** $\frac{5}{2}$ **e** $\frac{1}{9}$ **f** -4

9 a $y = \frac{1}{2}x + 4$ **b** $y = -\frac{1}{5}x - 1$
c $y = -x$ **d** $y = \frac{2}{7}x + 3$

10 a $y = -\frac{1}{2}x + 3$ **b** $y = x + 3$
c $y = -\frac{2}{3}x + 3$

11 a **i** $y = 2x - 4$ **ii** $y = -\frac{1}{2}x - 4$
b **i** $y = -x - 4$ **ii** $y = x - 4$
c **i** $y = \frac{3}{2}x - 4$ **ii** $y = -\frac{2}{3}x - 4$

12 a $y = -\frac{1}{5}x - 7$ **b** $y = \frac{1}{7}x + 3$
c $y = -x - 3$ **d** $y = \frac{3}{2}x + 10$

13 a parallel; same gradient of 4
b perpendicular; gradients of $\frac{2}{3}$ and $-\frac{3}{2}$
c perpendicular; gradients of $\frac{3}{8}$ and $-\frac{8}{3}$
d neither; gradients of -2 and 2
e neither; gradients of -5 and $-\frac{1}{5}$
f parallel; same gradient of $\frac{2}{7}$

14 Some possible answers are:
$y - 5x = 6$ and $4y - 20x + 1 = 0$.

15 Some possible answers are:
$y = 8x + 2$ and $8x - y = 12$.

16 Gradient of line joining $(2, -3)$ and $(4, 5)$ is 4.
Gradient of $y = 4x - 7$ is 4.
So lines are parallel.

17 Gradient of line joining $(-11, -7)$ and $(-1, -2)$ is $\frac{1}{2}$.
Gradient of $y = -2x + 5$ is -2.
Product of gradients is -1, so lines are perpendicular.

18 a $\frac{2}{3}$ **b** **i** $y = \frac{2}{3}x - 5$ **ii** $y = -\frac{3}{2}x + 2$

19 a $y = -3x$ **b** $y = \frac{1}{3}x + 9$ **c** $y = \frac{1}{3}x - 1$

20 a Yes, both have gradient of $\frac{3}{7}$.
b No. AC has gradient of $\frac{5}{4}$, while BD has gradient of $-\frac{2}{3}$.

21 a gradient of AB: 3, gradient of BC: $-\frac{1}{3}$. So line segments AB and BC are perpendicular. This makes triangle ABC right-angled.
b length of AB: 9.5 units, length of BC: 6.3 units. So triangle is not isosceles right-angled.
c 27.2 units **d** 29.9 square units

22 a AD ∥ BC, each with gradients of $\frac{1}{2}$, and AB ∥ DC, each with gradients of 2.
length of AD = length of BC =
length of AB = length of DC = 2.2 units
b EF ∥ HG, each with gradients of $\frac{3}{7}$ and HE ∥ GF, each with gradients of 2.
length of EF = length of HG = 7.6 units,
length of HE = length of GF = 8.9 units

23 gradient of KN = gradient of LM = $-\frac{1}{2}$, gradient of MN = gradient of LK = 2
So opposite sides are parallel, and adjacent sides are at right angles.
length of KL = length of LM = length of MN = length of NK = 4.5 units
perimeter: 17.9 units, area: 20.3 square units

24 gradient of PS = gradient of QR = 1, gradient of PQ = gradient of SR = −1
So opposite sides are parallel, and adjacent sides are at right angles.
length of PS = length of QR = 4.2 units,
length of PQ = length of SR = 5.7 units
perimeter: 19.8 units, area: 23.9 square units

25 $y = x - 5$

26 length of YX = length of YZ = 7.1 units and length of WX = length of WZ = 20.6 units
gradient of XZ = $-\frac{3}{4}$ and gradient of WY = $\frac{4}{3}$, so diagonals are at right angles.
midpoint of XZ is (3, 5). This point also lies on the diagonal WY, which has equation $y = \frac{4}{3}x + 1$.

27 Gradients of the line segments are: AB = $\frac{1}{2}$, BC = −3, CD = $\frac{1}{2}$, DA = $\frac{6}{5}$, AC = 0, BD = $\frac{9}{11}$.
This indicates quadrilateral is a trapezium (one pair of parallel sides).

Reflect

Possible answer: Parallel lines have the same gradient while the gradients of two perpendicular lines multiply to make –1.

Resources

WorkSheet

WS 4-2 Working with linear relationships

Focus: To sketch linear graphs and to determine the rule for linear relationships given relevant information

Resources: ruler

- This WorkSheet provides a skills review of Exercises 4C–E. Q10–12 relate to Exercise 4E.

Students sketch linear graphs using the gradient-intercept method and also by using the *x*- and *y*-intercept method. They perform calculations to determine the gradient of a line and to find the rule for a linear graph using $y = mx + c$ and $y - y_1 = m(x - x_1)$. Students also work with parallel and perpendicular lines.

BLM

Cartesian plane grids

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4F Solving simultaneous linear equations graphically

Teaching support for pages 190–195

Teaching strategies

Learning focus

To find the point of intersection of two linear graphs and recognise that this point is a solution to both equations

Start thinking!

The task guides students to:

- look at two linear graphs and identify the coordinates of points on each graph to complete a table of values
- use the table to identify which *x*-value has a *y*-value that is the same for both linear graphs
- recognise a coordinate pair that lies on both equations and see that this is the point of intersection
- find the rule or equation for each line
- see that the coordinates of the point of intersection satisfies both equations simultaneously.

Differentiated pathways

Below Level	At Level	Above Level
1–6, 9	1–3, 4e–l, 5e–l, 6–14	1e, f, 2, 4i–l, 5i–l, 6–8, 10–17
Students complete the **assessment** for this topic		

At Level

At Level
1–3, 4e–l, 5e–l, 6–14

- You may like students to use the BLM **Cartesian plane grids** (see Resources) to save them ruling up their own grids.

- Students need to understand that a single linear equation with one unknown has a single (unique) solution.
- If there are two unknowns in a single linear equation, there are an infinite number of solutions and these solutions are represented by the graph of that equation.
- If there are two linear equations and two unknowns, the point of intersection of those two graphs will be a solution of both equations.
- The values of x and y are said to be solution to the pair of simultaneous linear equations.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4F-1**. It shows how to check solutions to simultaneous linear equations and will help students to complete Q1.
- For Q1, ensure that students understand that the solution needs to satisfy both equations. Guide them to substitute the x and y values separately into each side of the equation and show that LS = RS, as demonstrated in the example.
- **Example 4F-2** shows how to identify the solution to simultaneous linear equations from graphs. This will help students to complete Q2.
- In Q2, ensure that students correctly identify the x value and the y value and don't reverse them. This will be important for their substitution in Q3.
- **Example 4F-3** shows how to solve simultaneous linear equations graphically. This will help students to complete Q4, Q6 and Q7.
- For Q4, Q6 and Q7, it may be useful to provide the BLM **Cartesian plane grids** (see Resources). Emphasise to students that graphs need to be drawn to an accurate scale if the solution is to be read from the graphs.
- In Q6, some students may need a reminder about the graphs of horizontal and vertical lines.
- In Q8, if students cannot see the reason there is no solution, have them graph both equations to recognise that they are parallel.
- Q9 is an example that can be used to demonstrate marketing concepts larger organisations use on a bigger scale. Ensure that students correctly assign n on the horizontal axis and a on the vertical axis.
- Q10 requires students to develop their own equations. For those who are unable to do this, have them use the diagram to derive $x + y = 20$ and $y = x + 4$.

- In Q11 and Q12, it will be important that students define the variables at the beginning of the question. For example in Q11, begin with ‘Let Lachlan’s age be *x* and Tia’s age be *y*.’
- Q13 will require students to draw a diagram and label the length and the width.
- For additional practice, students can complete Q1 and Q2 of **WS 4-3 Solving simultaneous equations** (see Resources). Additional questions similar to Exercise 4F Q2 and Q4 are provided. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.

Below Level

Below Level
1–6, 9

- For Q1, ensure that students understand that the solution needs to satisfy both equations. Guide them to substitute the *x* and *y* values separately into each side of the equation and show that LS = RS, as demonstrated in the example.
- In Q2, ensure that students correctly identify the *x* value and the *y* value and don’t reverse them. This will be important for their substitution in Q3.
- In Q4 and Q6, it may be useful to provide the BLM **Cartesian plane grids** (see Resources). Emphasise to students that graphs need to be drawn to an accurate scale if the solution is to be read from the graphs.
- In Q6, some students may need a reminder about the graphs of horizontal and vertical lines.
- Q9 is an example that can be used to demonstrate marketing concepts larger organisations use on a bigger scale. Ensure that students correctly assign *n* on the horizontal axis and *a* on the vertical axis.
- For students who do not progress past Q4, direct them to Q1 and Q2 of **WS 4-3 Solving simultaneous equations** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.
- Ensure that students are properly equipped to draw accurate diagrams, as many Below Level students will lack the fine motor skills to do this unaided.
- If this becomes too difficult, provide them with technology that will assist them to complete this exercise.

Above Level

Above Level
1e, f, 2, 4i–l, 5i–l, 6–8, 10–17

- It may be useful to provide the BLM **Cartesian plane grids** (see Resources). Emphasise to students that graphs need to be drawn to an accurate scale if the solution is to be read from the graphs.
- In Q8, if students cannot see the reason there is no solution, have them graph both equations to recognise that they are parallel.
- For Q11–14, it will be important that students define the variables at the beginning of the problem.
- In Q15, students need to draw the graphs by:
  - plotting the points (0, –5) and (4, –6) and ruling a line though them to obtain the first graph
  - using the gradient of –3 from the point (4, –6) to draw the second graph.
- In Q17, students will need to see that the two equations are in fact the same equation.

Extra activities

1 **Quick Questions**

Solve each equation.

a $2x + 9 = 31$ ($x = 11$)

b $9 - 3y = -15$ ($y = 8$)

c $\dfrac{z+4}{5} = 6$ ($z = 26$)

d $\dfrac{9-2a}{4} = -1$ ($a = 6.5$)

2 Give an example of a simultaneous equation pair that has:

a no solution

b one solution

c infinite solutions.

Answers

4F Solving linear simultaneous equations graphically

4F Start thinking!

1

x	−1	0	1	2	3	4	5
y_A	−6	−4	−2	0	2	4	6
y_B	6	5	4	3	2	1	0

2 $x = 3$ **3** (3, 2) **4** (3, 2)

5 This point lies on both lines, and satisfies both equations simultaneously.

6 **a** $y = 2x - 4$ **b** $y = -x + 5$

7 Line A: RS = 2 × 3 − 4 = 2 = LS
Line B: RS = −3 + 5 = 2 = LS

8 $x = 4$ **9** $x = 3, y = 2$

10 Identify coordinates of point of intersection.

Exercise 4F Solving linear simultaneous equations graphically

1 **a** solution **b** solution
c not a solution **d** solution
e not a solution **f** not a solution

2 **a** **i** (4, 3) **ii** $x = 4, y = 3$
b **i** (2, −4) **ii** $x = 2, y = -4$
c **i** (−5, −2) **ii** $x = -5, y = -2$
d **i** (−2, 3) **ii** $x = -2, y = 3$

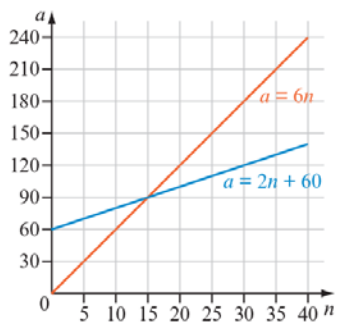
4 **a** $x = 5, y = 4$ **b** $x = 2, y = 6$
c $x = 6, y = 4$ **d** $x = -2, y = 5$
e $x = -1, y = 3$ **f** $x = -3, y = 1$
g $x = 3, y = 2$ **h** $x = 4, y = -2$
i $x = -2, y = -3$ **j** $x = -5, y = 6$
k $x = 2, y = -2$ **l** $x = 3, y = 5$

6 **a** $x = 3, y = -4$ **b** $x = 0, y = 0$
c $x = 2, y = 1$

7 **a** $x = \frac{1}{2}, y = \frac{1}{2}$ **b** $x = -3, y = -2\frac{1}{2}$
c $x = 1\frac{1}{2}, y = -4\frac{1}{2}$

8 Both graphs have gradient of 2. Lines are parallel so there is no point of intersection.

9 **a** To make n cards, cost of plain white cards is $2 \times n$ or $2n$. So, amount of money spent (in \$) is cost of plain white cards plus start-up cost of \$60.
b For selling n cards, the amount of money received (in \$) is $6 \times n$ or $6n$.
c



d (15, 90); this shows number of cards to be sold (15) so that amount of money received is same as amount of money spent (\$90).
e 15 cards **f** 16 to 40 cards

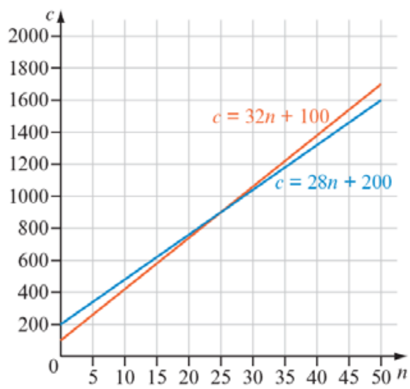
10 **a** $2x + 2y = 40$ or $x + y = 20$
b $y = x + 4$
c $x = 8, y = 12$; postcard has length of 12 cm and width of 8 cm.

11 **a** For x = Lachlan's age, y = Tia's age, equations are $y = 2x$ and $x + y = 51$.
b $x = 17, y = 34$
c Tia is 34 years old, Lachlan is 17 years old.

12 **a** For d = cost of drink, c = cost of choc top, equations are $d = c + 2$ and $5d + 7c = 70$
b Drink costs \$7, choc top costs \$5.

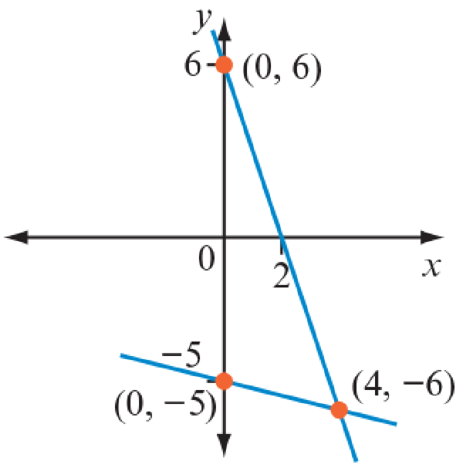
13 For l = length of land, w = width of land, equations are $l = w + 20$ and $2l + w = 124$. Land is 48 m long and 28 m wide.

14 Let n = number of people and c = catering cost.
a $c = 28n + 200$ **b** $c = 32n + 100$
c



d Cool Food Club; at $n = 18$, the line for Cool Food Club is lower than the line for Angie's Catering. This represents a lower cost for 18 people.
e Angie's Catering **f** 25 people at cost of \$900
g When catering for less than 25 people, Cool Food Club is cheaper. When catering for more than 25 people, Angie's Catering is cheaper. Cost is same when catering for 25 people.

15 a



b $y = -\frac{1}{4}x - 5$, $y = -3x + 6$

c $x = 4$, $y = -6$

d For $x = 4$ and $y = -6$,

For $y = -\frac{1}{4}x - 5$, RS $= -\frac{1}{4} \times 4 - 5 = -6 =$ LS.

For $y = -3x + 6$, RS $= -3 \times 4 + 6 = -6 =$ LS.

16 One possible answer is: $y = 2x - 18$ and $y = 3 - x$.

17 They are equivalent equations. When graphed, the position of the lines is identical; that is, one line lies on top of the other. There are an infinite number of points where the lines intersect so there are infinite solutions.

Reflect

Possible answer: The point of intersection shows you the only values of x and y that satisfy both of the given equations.

Resources

WorkSheet

WS 4-3 Solving simultaneous equations

Focus: To solve simultaneous equations graphically and using algebraic methods

Resources: 1-cm grid paper (BLM) or graph paper, ruler

- This WorkSheet provides a skills review of Exercises 4F and 4G. Q1 and Q2 relate to Exercise 4F.

Students solve simultaneous equations graphically and algebraically. The algebraic methods involve using substitution and elimination.

BLMs

Cartesian plane grids

1-cm grid paper

assess

Students are encouraged to complete the review questions in the assessment for this topic.

4G Solving simultaneous linear equations algebraically

Teaching support for pages 196–201

Teaching strategies

Learning focus

To explore methods of solving a pair of simultaneous equations algebraically, making it unnecessary to draw a graph to solve the equations

Start thinking!

The task guides students to:

- consider the solution of a simultaneous equation pair presented graphically
- see that if *y* is the subject of one equation, this can be substituted into the other so a single equation with one unknown is formed
- solve the single linear equation for *x* and substitute this value into an equation to find the value of *y*
- recognise the algebraic solution is the same as the point of intersection found from the graphs
- consider the elimination method of solving a simultaneous equation pair by adding equations.

Differentiated pathways

Below Level	At Level	Above Level
1–8, 10a, d, g, j, 11	1–4, 5d–f, 6–9, 10a–l, 11–15	1g–l, 2f–i, 3, 4, 5e, f, 6d–f, 7–9, 10j–o, 12–18
Students complete the **assessment** for this topic		

Support strategies for Are you ready? Q11

Focus: To multiply an equation by an integer value to create an equivalent equation

- Direct students to complete **SS 4G-1 Forming equivalent linear equations** (see Resources) if they had difficulty with this question or require more practice at this skill.

- You may need to undertake some explicit teaching so that students understand these skills.
- Students need to be able to multiply each term in the equation by an integer value.

At Level

At Level
1–4, 5d–f, 6–9, 10a–l, 11–15

- Students need to understand there are two different methods of solving simultaneous equations algebraically:
  - the substitution method
  - the elimination method.
- Explain that the substitution method is best suited to when at least one of the equations has a variable written as the subject. This enables one equation to be substituted into the other.
- Explain that the elimination method is best suited to when equations are able to be added or subtracted in such a way that one of the variables is eliminated.
- Direct students to the **Key ideas**. You may like them to copy this summary.
- Direct students to **Example 4G-1**. It shows how to solve simultaneous linear equations using substitution and will help students to complete Q1 and Q2.
- **Example 4G-2** shows how to solve simultaneous linear equations using elimination. This will help students to complete Q3 and Q4.
- In Q3, each equation pair has one variable eliminated by adding the equations together. Guide students to understand that adding will eliminate a variable if the variable in each equation has the same coefficient with opposite sign.
- In Q4, each equation pair has one variable eliminated by subtracting the equations. Guide students to understand that subtracting will eliminate a variable if the variable in each equation has the same coefficient and the same sign.
- Q5 provides practice in forming an equivalent equation with a desired term. This prepares students for Q6.
- **Example 4G-3** shows how to solve simultaneous linear equations using elimination where one of the equations is multiplied by an integer to produce an equivalent equation. This will help students to complete Q6.

- After completing Q8, discuss with students how there are two ways in which they can multiply both equations to use the elimination method. One pair of equivalent equations will enable x to be eliminated while the other pair will allow y to be eliminated.
- In Q9, students identify when each method is most appropriate to be used. This is to be applied in Q10. Remind students that if:
  - one linear equation is of the form $y = mx + c$ or $x = my + c$, substitution is more appropriate
  - if the linear equations are of the form $ax + by = c$, then elimination is simpler.
- For Q11–15, students need to write their own equations. Ensure that students begin each problem by defining their variables and give a worded answer. Remind students that they need to write two different linear equations that link the two unknowns.
- For additional practice, students can complete Q3–10 of **WS 4-3 Solving simultaneous equations** (see Resources). Additional questions similar to Exercise 4G Q1–4, Q6 and Q8 are provided. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 Drawing a circumcircle around a triangle** (see Resources).

Below Level

Below Level
1–8, 10a, d, g, j, 11

- In Q3, each equation pair has one variable eliminated by adding the equations together. Guide students to understand that adding will eliminate a variable if the variable in each equation has the same coefficient with opposite sign.
- In Q4, each equation pair has one variable eliminated by subtracting the equations. Guide students to understand that subtracting will eliminate a variable if the variable in each equation has the same coefficient and the same sign.
- Q5 provides practice in forming an equivalent equation with a desired term. This prepares students for Q6. Students may need to complete **SS 4G-1 Forming equivalent linear equations** (see Resources).
- In Q10, students identify which method is most appropriate to be used. Remind students that if:
  - one linear equation is of the form $y = mx + c$ or $x = my + c$, substitution is more appropriate

- if the linear equations are of the form $ax + by = c$, then elimination is simpler.

- For Q11, students need to write their own equations. Ensure that students begin by defining the two variables and give a worded answer. Remind students that they need to write two different linear equations that link the two unknowns.

- For students who do not progress past Q8, direct them to Q3–10 of **WS 4-3 Solving simultaneous equations** (see Resources) for additional skill practice. This WorkSheet relates to Exercises 4F and 4G and can be completed progressively or as a skills review of Exercises 4F–G.

- Below Level students may lack the algebraic skills to manipulate equations sufficiently well. If this is the case, have them concentrate on Q1 and Q2 (substitution) and Q3 and Q4 (elimination with no multiplication of equations required)

- If students continue to struggle, have them go back to solving linear equation pairs graphically, with the help of technology if required.

Above Level

Above Level
1g–l, 2f–i, 3, 4, 5e, f, 6d–f, 7–9, 10j–o, 12–18

- For Q3 and Q4, encourage students to recognise when to add and when to subtract a pair of linear equations to eliminate a variable.

- After completing Q8, discuss with students how there are two ways in which they can multiply both equations to use the elimination method. One pair of equivalent equations will enable x to be eliminated while the other pair will allow y to be eliminated.

- In Q9, students identify when each method is most appropriate to be used. This is to be applied in Q10. Remind students that if:

  - one linear equation is of the form $y = mx + c$ or $x = my + c$, substitution is more appropriate

  - if the linear equations are of the form $ax + by = c$, then elimination is simpler.

- For Q11–15, students need to write their own equations. Ensure that students begin each problem by defining their variables and give a worded answer. Remind students that they need to write two different linear equations that link the two unknowns.

- In Q17, students should use the substitution method and will obtain the statement $11 = 5$. Explain that this is a contradiction and can never be true and so the two linear equations have no solution. This result can be checked by having students graph the two linear relationships. They should see that the lines are parallel and hence never intersect.

- In Q18, students should use the substitution method and will obtain the statement 6 = 6. Explain that this is always true and so the two linear equations will be equal for every value of *x* and *y*, hence there are infinite solutions. This result can be checked by having students graph the two linear relationships. They should see that the same line is produced for both linear relationships and hence there are infinite points where the lines intersect.
- For more problem-solving tasks and investigations, direct students to **INV 4-3 Drawing a circumcircle around a triangle** (see Resources).

Extra activities

1 Quick Questions

Make *y* the subject of each equation.

a $x + y = 4$ $(y = 4 - x)$

b $2x - y + 7 = 0$ $(y = 2x + 7)$

c $3x + 2y - 10 = 0$ $\left(y = \dfrac{10 - 3x}{2}\right)$

d $x - 5y + 10 = 0$ $\left(y = \dfrac{x + 10}{5}\right)$

2 Consider the linear relationship $y = 2x - 8$ and the parabola $y = x^2 - 5x + 4$.

a Use substitution to create a single quadratic equation involving *x*.
$(x^2 - 7x + 12 = 0)$

b Solve the quadratic equation to find two values of *x*. $(x = 3, x = 4)$

c Substitute these values of *x* to find two corresponding values of *y*. $(y = -2, y = 0)$

d Explain what this means in terms of the original two graphs. [The line cuts the parabola twice, at (3, –2) and (4, 0)]

e Leighton says that a straight line will always cut a parabola twice. Is Leighton correct? Explain your answer. (Leighton is wrong. The line might cut the graph twice but it may also just touch the parabola at one point or not intersect at all.)

Answers

4G Solving linear simultaneous equations algebraically

4G Start thinking!

1 $x = 3, y = 2$

2 a $x + x - 1 = 5, 2x - 1 = 5$ b $x = 3$

c $x = 3, y = 2$

3 a $2x$ b 6 c y has been eliminated.

d $x = 3$; when $x = 3$, $x + y = 5$ becomes $3 + y = 5$ so $y = 2$

e $x = 3, y = 2$

f New equation is $2y = 4$; solution is the same.

Exercise 4G Solving linear simultaneous equations algebraically

1 a $x = 2, y = 7$ b $x = 4, y = 1$ c $x = -1, y = 3$ d $x = 6, y = 5$ e $x = -4, y = -2$ f $x = 2, y = 12$ g $x = 1, y = 3$ h $x = -3, y = 11$ i $x = 5, y = -5$ j $x = 5, y = 3$ k $x = -4, y = -1$ l $x = 5, y = 6$

2 a $x = 1, y = 8$ b $x = 6, y = 4$ c $x = -3, y = -5$ d $x = 5, y = -1$ e $x = -1, y = 4$ f $x = 4, y = -7$ g $x = 2, y = 1$ h $x = -2, y = 6$ i $x = -3, y = 19$

3 a $x = 2, y = 5$ b $x = -3, y = 1$ c $x = 1, y = -5$ d $x = -4, y = -2$ e $x = 5, y = 3$ f $x = -1, y = 2$

4 a $x = 5, y = 3$ b $x = -2, y = 9$ c $x = 3, y = -7$ d $x = -1, y = 5$ e $x = 6, y = 2$ f $x = -3, y = -4$

5 a $3x + 6y = 15$ b $-6x + 2y = 2$ c $-5x - 20y = 10$ d $20x + 8y = 12$ e $-4x + 6y = 14$ f $-12x + 6y = 9$

6 a $x = 3, y = 2$ b $x = 2, y = -1$ c $x = -4, y = 7$ d $x = 5, y = 3$ e $x = -3, y = -4$ f $x = -2, y = 6$

7 a No; ① + ② gives $11x + 5y = -7$ and ① − ② gives $3x - y = -9$.
b $21x + 6y = -24$ c $8x + 6y = 2$
d y; subtract the equations e $x = -2, y = 3$

8 a $x = 1, y = 4$ b $x = -3, y = 1$ c $x = 2, y = -1$ d $x = -4, y = 2$ e $x = 5, y = 5$ f $x = -3, y = -2$

9 Graphical method best when information given to draw graphs; substitution method best when one or both of the equations have a variable term as subject of the equation; elimination method best when equations are in the form $ax + by = d$.

10 a $x = 2, y = 0$ b $x = \frac{1}{3}, y = -3$
c $x = \frac{3}{2}, y = \frac{2}{5}$ or $x = 1.5, y = 0.4$
d $x = -9, y = -7$ e $x = 4, y = 2$ f $x = -2, y = -3$ g $x = 3, y = 10$ h $x = 5, y = -2$ i $x = 2, y = -1$ j $x = -2, y = 2$ k $x = 7, y = 4$ l $x = -4, y = 1$ m $x = -0.8, y = 2.3$
n $x = -\frac{1}{4}, y = 5$ or $x = -0.25, y = 5$
o $x = 28, y = 41$

11 a For two numbers x and y, equations are $x + y = 245$, $x - y = 91$. Solution is $x = 168$, $y = 77$.
b Two numbers are 77 and 168.

12 a $2l + 2w = 810$ or $l + w = 405$ b $l = 2w$
c 270 cm long and 135 cm wide

13 For x = width of pillow case, y = length of pillow case, equations are $y = 2x - 18$ and $2x + 2y = 240$ (or $x + y = 120$). Solution is $x = 46$, $y = 74$ so pillow case is 74 cm long and 46 cm wide.

14 For g = number of points for a goal, b = number of points for a behind, equations are $15g + 6b = 132$ and $13g + 12b = 128$. Solution is $g = 8$, $b = 2$ so number of points for a goal is 8 and number of points for a behind is 2.

15 For p = cost of a pie, r = cost of a sausage roll, equations are $7p + 6r = 63$ and $5p + 5r = 48$. Solution is $p = 5.4$, $r = 4.2$ so cost of a pie is \$5.40 and cost of a sausage roll is \$4.20.

16 a Strategy A: equation ③ is $10x - 6y = -2$.
① − ③: $-3x = -12$, $x = 4$
Substituting $x = 4$ in ②: $20 - 3y = -1$, $y = 7$
Solution is $x = 4$, $y = 7$.
Strategy B: equation ③ is $-10x + 6y = 2$.
① + ③: $-3x = -12$, $x = 4$
Substituting $x = 4$ in ②: $20 - 3y = -1$, $y = 7$
Solution is $x = 4$, $y = 7$.
Strategy C: equation ③ is $35x - 30y = -70$ and equation ④ is $35x - 21y = -7$.
③ − ④: $-9y = -63$, $y = 7$
Substituting $y = 7$ in ②: $5x - 21 = -1$, $x = 4$
Solution is $x = 4$, $y = 7$.

b Some possible answers are:
i Multiply equation ① by 5 to form equation ③ and multiply equation ② by −7 for form equation ④. Add equations ③ and ④.
ii Graph equations ① and ② and identify the coordinates of the point of intersection.
iii Rearrange equation ② to obtain $3y = 5x - 1$ and substitute into equation ①. That is, substitute $5x - 1$ for $3y$ in $7x - 2(3y) = -14$.

17 Substituting $y = 11 - 4x$ into $8x + 2y = 5$ gives:
$8x + 2(11 - 4x) = 5$
$8x + 22 - 8x = 5$
There is no solution as there is no x value that makes this equation true.

18 Substituting $y = 2x - 3$ into $4x - 2y = 6$ gives:
$4x - 2(2x - 3) = 6$
$4x - 4x + 6 = 6$
There are an infinite number of solutions as all values of x make this equation true.

Reflect

Possible answer: The aim of the first step in solving a pair of simultaneous linear equations is to reduce the two linear equations with two unknowns to a single linear equation with one unknown.

Resources

SupportSheet

SS 4G-1 Forming equivalent linear equations

Focus: To explore how equivalent linear relationships are formed

Resources: ruler, **1-cm grid paper** (BLM) or graph paper (optional)

Students develop an understanding of equivalent linear equations by sketching their graphs. They form equivalent linear equations by multiplying the entire equation by an integer. This skill is needed for the elimination method when solving linear simultaneous equations.

WorkSheet

WS 4-3 Solving simultaneous equations

Focus: To solve simultaneous equations graphically and using algebraic methods

Resources: 1-cm grid paper (BLM) or graph paper, ruler

- This WorkSheet provides a skills review of Exercises 4F and 4G. Q3–10 relate to Exercise 4G.

Students solve simultaneous equations graphically and algebraically. The algebraic methods involve using substitution and elimination.

Investigation

INV 4-3 Drawing a circumcircle around a triangle

Focus: To find the circumcentre of a triangle algebraically, then draw the circumcircle

Resources: calculator, **1-cm grid paper** (BLM) or graph paper, pair of compasses

Students find the midpoints and gradients of the three sides of a triangle. They calculate the gradient of the perpendicular line through each midpoint and hence write the equation for each of the three perpendicular bisectors. Students solve these linear equations simultaneously to find the coordinates of the circumcentre of the triangle. By finding the distance of each vertex from the circumcentre, students find the radius of the circumcircle enabling them to use a pair of compasses to draw the circumcircle.

BLM

1-cm grid paper

assess

Students are encouraged to complete the review questions in the assessment for this topic.

Chapter review

Teaching support for pages 202–205

Additional teaching strategies

Multiple choice

1 Answer: A. $\frac{x-4}{2} = -3$, $x - 4 = -6$, $x = -2$
B: calculated 6 – 4 instead of –6 + 4 to obtain 2.
C: calculated –6 – 4 instead of –6 + 4 to obtain –10.
D: calculated 6 + 4 instead of –6 + 4 to obtain 10.

2 Answer: C. LS = $\frac{12}{-5} = -2.4$ = RS
A: LS = 3(–5) – 7 = –15 – 7 = –22 $\neq$ RS. x = –5 is not a solution.
B: LS = $\frac{2(-5)+3}{13} = \frac{-10+3}{13} = \frac{-7}{13} \neq$ RS. x = –5 is not a solution.
D: LS = $\frac{3}{2-(-5)} = \frac{3}{7} \neq$ RS. x = –5 is not a solution.

3 Answer: C. –3.5 > –4.7, so statement of –3.5 < –4.7 is false.
A: chose a correct statement as –7 < –4 is true.
B: chose a correct statement as $\frac{2}{3} < \frac{3}{4}$ is true.
D: chose a correct statement as $\frac{8}{9} < \frac{9}{8}$ is true.

4 Answer: B. –6.3 < –6.2, so –6.3 could not be a value for x.
A: chose a value that could be x as –4.9 > –6.2.
C: chose a value that could be x as 6.2 > –6.2.
D: chose a value that could be x as 0 > –6.2.

5 Answer: C. $12x + 4y = 8$, when $x = 0$, $12(0) + 4y = 8$, $y = 2$
A: chose the constant term of 8.
B: considered $12x + 4y - 8 = 0$ and chose the constant term of –8.
D: confused the use of the negative sign and chose –2.

6 Answer: D. $12x + 4y = 8$; $4y = -12x + 8$; $y = -3x + 2$; $m = -3$, $c = 2$
A: chose the coefficient of x in the original rule for the gradient.
B: considered $4y = -12x + 8$ and taken the gradient to be the coefficient of x.
C: divided by 4 but not put the rule in the form $y = mx + c$.

7 Answer: A. $y = \frac{3}{4}x + \frac{5}{8}$, 8$y$ = 6x + 5; 6x – 8y + 5 = 0
B: considered the wrong sign for the constant term.
C: reversed the coefficients of x and y.
D: reversed the coefficients of x and y.

8 Answer: C. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6-8}{-3-1} = \dfrac{-14}{-4} = \dfrac{7}{2}$
A: added 8 and –6 in the numerator to obtain 2 and incorrectly subtracted 1 and –3 in the denominator to obtain 2.
B: added the y values in the numerator and added the x values in the denominator.
D: reversed the numerator and the denominator in the formula.

9 Answer: D. 3x + 2y = 6; 2y = –3x + 6; $y = -\frac{3}{2}x + 3$; $m = -\frac{3}{2}$
A: used the rule for perpendicular lines
B: made 2y the subject and used the coefficient of x.
C: divided the original rule by 2 and used the coefficient of x.

10 Answer: B. Gradient of perpendicular line, $m_2 = -\dfrac{1}{-\frac{3}{4}} = \dfrac{4}{3}$
A: taken the reciprocal but not changed the sign.
C: changed the sign but not taken the reciprocal.
D: used the same gradient.

11 Answer: C. x + 3 = –2x + 6; 3x + 3 = 6; 3x = 3, x = 1; y = 1 + 3 = 4; point is (1, 4)
A: subtracted 2x from x at the first step of the solution to find x.
B: added 3 to 6 instead of subtracting 3 from 6 when solving for x.
D: miscalculated y by ignoring the negative sign in the coefficient of x when substituting x = 1 into y = –2x + 6.

12 Answer: D. Graphing these two equations produces the same line so there are many solutions.
A: assumed the two lines have no point of intersection and hence the two equations have no simultaneous solutions.
B: assumed the two lines have one point of intersection and hence the two equations have one simultaneous solution.
C: assumed the two lines have two points of intersection and hence the two equations have two simultaneous solutions.

13 Answer: A. x – 2y = 3; x – 2(3x – 4) = 3; x – 6x + 8 = 3; –5x + 8 = 3
B: incorrectly multiplied –2 by –4 to obtain –8 when expanding the pair of brackets.
C: incorrectly added x and –6x to obtain 5x.
D: incorrectly added x and –6x to produce 5x and incorrectly multiplied –2 by –4 to obtain –8 when expanding the pair of brackets.

14 Answer: A. $(2x - y + 3) - (4x - y + 5) = 0$; $-2x - 2 = 0$
B: incorrectly subtracted 5 from 3 to obtain –8.
C: subtracted equation 1 from equation 2.
D: subtracted equation 1 from equation 2 and incorrectly subtracted 3 from 5 to obtain 8.

Short answer

1 **a** $3 - 2(4 - 5x) = -20$; $3 - 8 + 10x = -20$; $-5 + 10x = -20$; $10x = -15$, $x = -1.5$

b $4(3x + 5) = 3(5x - 3)$; $12x + 20 = 15x - 9$; $-3x + 20 = -9$; $-3x = -29$; $x = 9\frac{2}{3}$

2 **a** $\frac{4+x}{3} = \frac{2x-5}{3}$; $4 + x = 2x - 5$; $-x = -9$; $x = 9$

b $\frac{5-3x}{2} = \frac{2x+5}{3}$; $3(5 - 3x) = 2(2x + 5)$; $15 - 9x = 4x + 10$; $5 = 13x$; $x = \frac{5}{13}$

3 **a** $A = \frac{h(a+b)}{2}$; $2A = ha + hb$; $hb = 2A - ha$; $b = \frac{2A-ha}{h}$ or $b = \frac{2A}{h} - a$

b $A = 2[(l+w)h]$; $A = 2(l + w) \times h$; $h = \frac{A}{2(l+w)}$

4 **a** For $x > -1$, x can be $1\frac{5}{8}$, 2.6, 8.5 or $\frac{3}{4}$.

b For $x \le 0.75$, x can be –3.9, –8.5 or $\frac{3}{4}$.

c For $x \ge 2.6$, x can be 2.6 or 8.5.

5 **a** $x \ge -3$

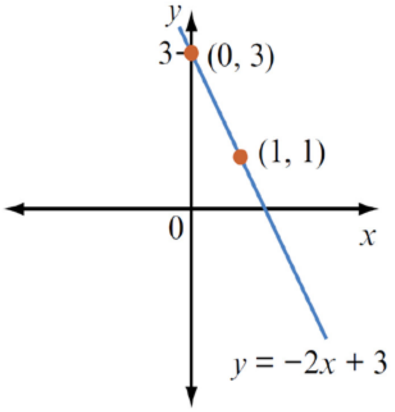
b $x < -10$

c $x \le 2.5$

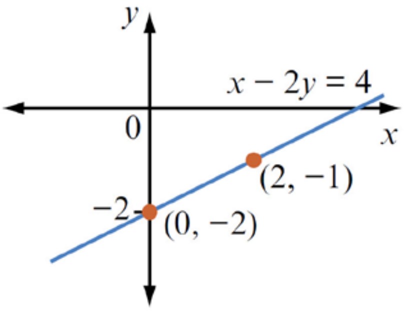
6 **a** $5 - 2x < 7$; $-2x < 2$; $x > -1$

b $\frac{2-3x}{5} > 2$; $2 - 3x > 10$; $-3x > 8$; $x < -\frac{8}{3}$ or $x < -2\frac{2}{3}$

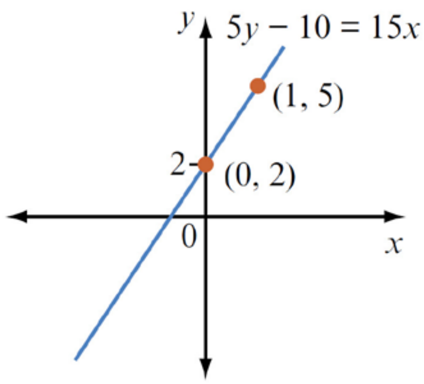
7 **a** $y = -2x + 3$; $m = -2$, $c = 3$



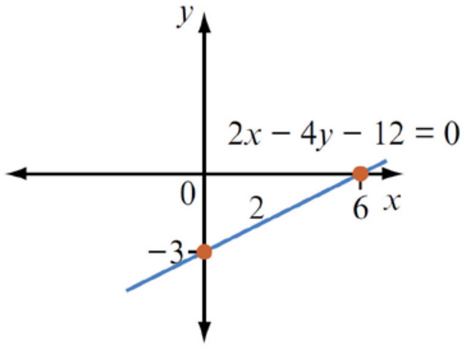
b $x - 2y = 4$; $-2y = -x + 4$; $y = \frac{1}{2}x - 2$; $m = \frac{1}{2}$, $c = -2$



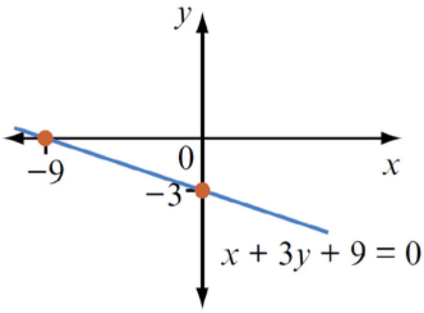
c $5y - 10 = 15x$; $5y = 15x + 10$; $y = 3x + 2$; $m = 3$, $c = 2$



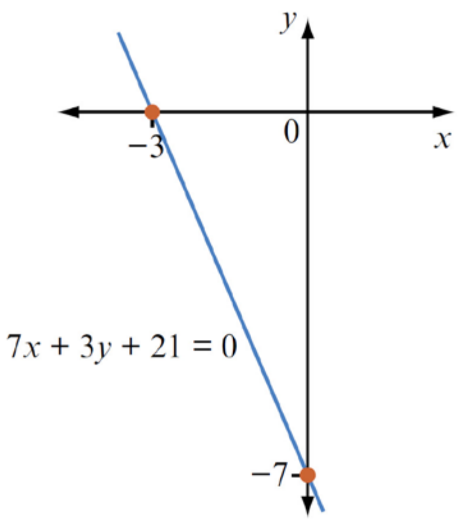
8 **a** $2x - 4y - 12 = 0$; when $x = 0$, $4y = -12$, $y = -3$; when $y = 0$, $2x = 12$; $x = 6$; intercepts are $(0, -3)$ and $(6, 0)$



b $x + 3y + 9 = 0$; when $x = 0$, $3y = –9$, $y = –3$; when $y = 0$, $x + 9 = 0$; $x = –9$; intercepts are (0, –3) and (–9, 0)



c $7x + 3y + 21 = 0$; when $x = 0$, $3y = –21$, $y = –7$; when $y = 0$, $7x = –21$; $x = –3$; intercepts are (0, –7) and (–3, 0)



9 **a** $m = \dfrac{6-(-4)}{5-0} = 2$; $y – (–4) = 2(x – 0)$; $y + 4 = 2x$; $y = 2x – 4$

b $m = \dfrac{6-6}{-2-3} = 0$; horizontal line; $y – 6 = 0(x – 3)$; $y – 6 = 0$; $y = 6$

c $m = \dfrac{-3-5}{-3-(-3)}$ = undefined; vertical line; $x = -3$

10 **a** $y - 5 = 2(x - 1)$; $y - 5 = 2x - 2$; $y = 2x + 3$

b $y - 5 = -2(x - 1)$; $y - 5 = -2x + 2$; $y = -2x + 7$

11 **a** $m = \dfrac{4-3}{2-1} = 1$; $y - 3 = 1(x - 1)$; $y - 3 = x - 1$; $y = x + 2$

b $m = \dfrac{-7-5}{3+1} = \dfrac{-12}{4} = -3$; $y - 5 = -3(x + 1)$; $y - 5 = -3x - 3$; $y = -3x + 2$

c $m = \dfrac{5+4}{5+4} = 1$; $y - 5 = 1(x - 5)$; $y - 5 = x - 5$; $y = x$

12 **a** $m = -4$, $c = -3$; $y = -4x - 3$

b $m = -½$, $c = -4$; $y = -½x - 4$

13 **a** $m = -\frac{1}{3}$, (3, 2); $y - 2 = -\frac{1}{3}(x - 3)$; $3y - 6 = -x + 3$; $3y = -x + 9$; $y = -\frac{1}{3}x + 3$

b $m = 3$, (–3, –2); $y + 2 = 3(x + 3)$; $y + 2 = 3x + 9$; $y = 3x + 7$

14 **a** Point of intersection on graph is (1, 2); solution is $x = 1$ and $y = 2$.

b Point of intersection on graph is (6, 17); solution is $x = 6$ and $y = 17$.

15 **a** $3x + 4y = -1$ [1]
$y = x - 2$ [2]
Substituting [2] into [1]: $3x + 4(x - 2) = -1$, $3x + 4x - 8 = -1$, $7x = 7$, $x = 1$
Substituting $x = 1$ into [2]: $y = 1 - 2 = -1$
Solution is $x = 1$, $y = -1$.

b $y = 4x + 3$ [1]
$x - 2y = 8$ [2]
Substituting [1] into [2]: $x - 2(4x + 3) = 8$, $x - 8x - 6 = 8$, $-7x = 14$, $x = -2$
Substituting $x = -2$ into [1]: $y = 4(-2) + 3 = -5$
Solution is $x = -2$, $y = -5$.

c $7x - y = 4$ [1]
$y = 4x + 2$ [2]
Substituting [2] into [1]: $7x - (4x + 2) = 4$; $7x - 4x - 2 = 4$, $3x = 6$, $x = 2$
Substituting $x = 2$ into [2]: $y = 4(2) + 2 = 10$
Solution is $x = 2$, $y = 10$.

16 **a** $2x + y = 8$ [1]

$4x - y = 4$ [2]

[1] + [2]: $6x = 12$, $x = 2$

Substituting $x = 2$ into [1]: $2(2) + y = 8$, $4 + y = 8$, $y = 4$

Solution is $x = 2$, $y = 4$.

b $x + 5y = 13$ [1]

$x + 2y = 4$ [2]

[1] – [2]: $3y = 9$, $y = 3$

Substituting $y = 3$ into [1]: $x + 5(3) = 13$, $x + 15 = 13$, $x = -2$

Solution is $x = -2$, $y = 3$.

c $2x - y = 7$ [1]

$3x + 5y = 4$ [2]

[1] × 5: $10x - 5y = 35$ [3]

[2] + [3]: $13x = 39$, $x = 3$

Substituting $x = 3$ into [1]: $2(3) - y = 7$, $6 - y = 7$, $-y = 1$, $y = -1$

Solution is $x = 3$, $y = -1$.

Mixed practice

1 $m = \dfrac{1+4}{-4-1} = -1$

a $m = -1$, $(4, 4)$; $y - 4 = -1(x - 4)$; $y - 4 = -x + 4$; $x + y - 8 = 0$ or $y = -x + 8$

b $m = 1$, $(1, 1)$; $y - 1 = 1(x - 1)$; $y - 1 = x - 1$; $y = x$

Refer to *4E Parallel and perpendicular lines*

2 Answer: C. $2 - \dfrac{x}{4} < 1$; $-\dfrac{x}{4} < -1$; $\dfrac{x}{4} > 1$, $x > 4$

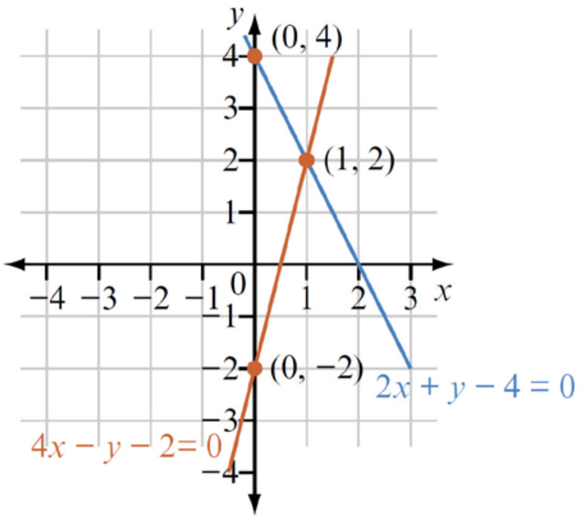
A: did not reverse the inequality after dividing both sides by –1 (or after multiplying by –4).

B: did not reverse the inequality and incorrectly divided by –1 (or incorrectly multiplied by –4).

D: incorrectly divided by –1 (or incorrectly multiplied by –4).

Refer to *4B Solving linear inequalities*

3 **a**



b Point of intersection is (1, 2).

Refer to *4F Solving simultaneous equations graphically*

4 **a** $x + 2y + 10 = 0$; $2y = -x - 10$; $y = -\frac{1}{2}x - 5$; $m_1 = -\frac{1}{2}$
$2x - y - 3 = 0$; $y = 2x - 3$; $m_2 = 2$
$m_1 \times m_2 = -\frac{1}{2} \times 2 = -1$ $\therefore$ Lines are perpendicular.

b $x + y = 3$; $y = 3 - x$; $m_1 = -1$
$y = x$; $m_2 = 1$;
$m_1 \times m_2 = -1 \times 1 = -1$ $\therefore$ Lines are perpendicular.

c $2x + 3y = 4$; $3y = 4 - 2x$; $y = \frac{4}{3} - \frac{2}{3}x$; $m_1 = -\frac{2}{3}$
$3x + 2y = 4$; $2y = 4 - 3x$; $y = 2 - \frac{3}{2}x$; $m_2 = -\frac{3}{2}$
$m_1 \times m_2 = -\frac{2}{3} \times -\frac{3}{2} = 1$ $\therefore$ Lines are neither perpendicular or parallel.
Refer to *4E Parallel and perpendicular lines*

5 $l = w + 28$; $l = 5w$; $5w = w + 28$; $4w = 28$; $w = 7$; $l = 7 + 28 = 35$
Rectangle has length of 35 m and width of 7 m.
Refer to *4G Solving linear simultaneous equations algebraically.*

6 Answer: D. A gradient of 0 means that the line is horizontal.
A: chose a line that has a positive gradient.
B: chose a line that has a negative gradient.
C: chose a line that is vertical and has an undefined gradient.
Refer to *4C Sketching linear graphs*

7 Point of intersection of the lines has coordinates (1, –1) so the solution to the simultaneous equations $y = 2x - 3$ and $y = -5x + 4$ is $x = 1$, $y = -1$.
Refer to *4F Solving linear simultaneous equations graphically.*

8 Answer: A. rise = 9 – (–1) = 10, run = 5 – (–2) = 7
B: confused the rise and the run, and so found the difference in the x values to obtain the rise and found the difference in the y values to obtain the run.
C: mistakenly added the y values to obtain the rise and added the x values to obtain the run.
D: confused the rise and the run, and mistakenly added the x values to obtain the rise and added the y values to obtain the run.
Refer to *4C Sketching linear graphs.*

9 **a** $V = \dfrac{x+y+z}{3}$; $3V = x + y + z$; $z = 3V - x - y$

b **i** $z = 3(12) - 7 - 19 = 36 - 7 - 19 = 10$

ii $z = 3(42) - 5 - 12 = 126 - 5 - 12 = 109$

Refer to *4A Solving linear equations.*

10 Answer: B. At $y = 0$, $x = -2$ so coordinates of the x-intercept are $(-2, 0)$; at $x = 0$, $y = 2$ so coordinates of the y-intercept are $(0, 2)$.
A: confused the order of the x- and y-intercepts.
C: confused the order of the x- and y-intercepts and reversed the signs.
D: mistakenly calculated the signs.
Refer to *4C Sketching linear graphs.*

11 Coordinates of the intersection points are $(1, -3)$, $(1, 5)$, $(-4, -3)$ and $(-4, 5)$.
Refer to *4F Solving linear simultaneous equations graphically.*

12 **a** $y + 3x = 8$ [1]
$y = 3x + 2$ [2]
Substituting [2] into [1]: $3x + 2 + 3x = 8$, $6x + 2 = 8$, $6x = 6$, $x = 1$
Substituting $x = 1$ into [2]: $y = 3(1) + 2$, $y = 5$
Solution is $x = 1$, $y = 5$.

b $3x + 2y = 1$ [1]
$3x + 7y = 11$ [2]
[2] – [1]: $5y = 10$, $y = 2$
Substituting $y = 2$ into [1]: $3x + 2(2) = 1$, $3x + 4 = 1$, $3x = -3$, $x = -1$
Solution is $x = -1$, $y = 2$.

c $4x - 5y = 3$ [1]
$6x - 11y = 1$ [2]
[1] × 3: $12x - 15y = 9$ [3]
[2] × 2: $12x - 22y = 2$ [4]
[3] – [4]: $7y = 7$, $y = 1$

Substituting $y = 1$ into [1]: $4x – 5(1) = 3$, $4x – 5 = 5$, $4x = 8$, $x = 2$
Solution is $x = 2$, $y = 1$.

Refer to *4G Solving linear simultaneous equations algebraically.*

13 Let h = cost of one hamburger ($), d = cost of one drink ($)
$3h + 2d = 32$ [1]
$h + 4d = 29$ [2]
[1] × 2: $6h + 4d = 64$ [3]
[3] – [2]: $5h = 35$, $h = 7$
Substituting $h = 7$ into [1]: $3(7) + 2d = 32$, $21 + 2d = 32$, $2d = 11$, $d = 5.5$
A hamburger costs $7 and a drink costs $5.50.
Refer to *4G Solving linear simultaneous equations algebraically.*

14 **a** $5 – 4x \geq 2x – 1$; $5 \geq 6x – 1$; $6x – 1 \leq 5$; $6x \leq 6$; $x \leq 1$

b $2x + 3 < 5x + 9$; $3 < 3x + 9$; $3x + 9 > 3$; $3x > –6$; $x > –2$

c $5(2 – 3x) \leq 3(5 – 2x)$; $10 – 15x \leq 15 – 6x$; $10 \leq 15 + 9x$; $15 + 9x \geq 10$; $9x \geq –5$; $x \geq -\frac{5}{9}$

d $–2(x – 4) > 5(3 – 2x)$; $–2x + 8 > 15 – 10x$; $8x + 8 > 15$; $8x > 7$; $x > \frac{7}{8}$

Refer to *4B Solving linear inequalities.*

15 **a** Coordinates of midpoint = $\left(\frac{5+(-7)}{2}, \frac{-6+2}{2}\right) = \left(\frac{-2}{2}, \frac{-4}{2}\right) = (-1,-2)$

b Coordinates of midpoint = $\left(\frac{-3+9}{2}, \frac{-1+(-3)}{2}\right) = \left(\frac{6}{2}, \frac{-4}{2}\right) = (3,-2)$

Refer to *4C Sketching linear graphs.*

16 **a** $d = \sqrt{(-4-5)^2 + (3--2)^2} = \sqrt{(-9)^2 + 5^2} = \sqrt{81+25} = \sqrt{106} \approx 10.3$ units

b $d = \sqrt{(1--7)^2 + (-4-3)^2} = \sqrt{8^2 + (-7)^2} = \sqrt{64+49} = \sqrt{113} \approx 10.6$ units

c $d = \sqrt{(-3-4)^2 + (-4--3)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} \approx 7.1$ units

Refer to *4C Sketching linear graphs.*

17 **a** True; $y = –2$ represents a horizontal line.

b True; (0, 0) satisfies the equation $y = 2x$.

c False; the x-axis has equation $y = 0$.

Refer to *4C Sketching linear graphs*.

18 $m = \dfrac{-2-(-4)}{4-3} = 2$

$y + 4 = 2(x - 3)$; $y + 4 = 2x - 6$; $y = 2x - 10$

a $m = 2$

b $c = -10$

Refer to *4C Sketching linear graphs*

19 Answer: D. $\dfrac{2x}{7} = 8$; $2x = 56$; $x = 28$

A: forgotten to divide by 2 as the final step.
B: multiplied 7 × 8 incorrectly to obtain 48.
C: multiplied 7 × 8 incorrectly to obtain 48 and not divided by 2 as the final step.
Refer to *4A Solving linear equations.*

20 Answer: B. $\dfrac{8-x}{3} = 4$

A: mistakenly subtracted 8 from x.
C: only divided x by three and not the result of subtracting x from 8.
D: mistakenly divided the result (4) by 3.
Refer to *4A Solving linear equations*.

21 Let x = unknown number.

a $2x - 3 = 17$, $2x = 20$, $x = 10$; so unknown number is 10

b $7(x - 1) = 21$; $7x - 7 = 21$; $7x = 28$; $x = 4$; so unknown number is 4

c $5(3x - 12) = 15$; $15x - 60 = 15$; $15x = 75$; $x = 5$; so unknown number is 5

Refer to *4A Solving linear equations.*

22 **a** $\dfrac{1-2x}{4} < \dfrac{x+2}{3}$; $3(1 - 2x) < 4(x + 2)$; $3 - 6x < 4x + 8$; $3 < 10x + 8$; $10x + 8 > 3$;

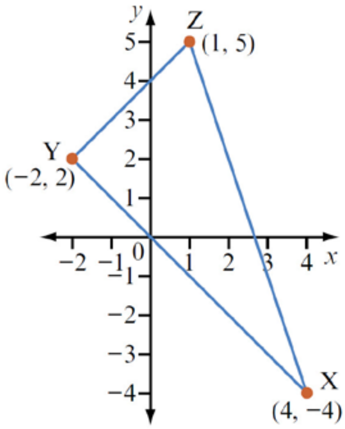
$10x > -5$; $x > -0.5$

b $\dfrac{x+1}{6} \geq \dfrac{x-1}{8}$; $8(x + 1) \geq 6(x - 1)$; $8x + 8 \geq 6x - 6$; $2x + 8 \geq -6$; $2x \geq -14$; $x \geq -7$

Refer to *4B Solving linear inequalities.*

Analysis

a



b For XY: $m = \dfrac{2--4}{-2-4} = \dfrac{6}{-6} = -1$; $y + 4 = -1(x - 4)$; $y + 4 = -x + 4$; $y = -x$

For YZ: $m = \dfrac{5-2}{1--2} = \dfrac{3}{3} = 1$; $y - 5 = 1(x - 1)$; $y - 5 = x - 1$; $y = x + 4$

For ZX: $m = \dfrac{5--4}{1-4} = \dfrac{9}{-3} = -3$; $y - 5 = -3(x - 1)$; $y - 5 = -3x + 3$; $y = -3x + 8$

c Gradient of XY = $m_1 = -1$, gradient of YZ = $m_2 = 1$.

$m_1 \times m_2 = -1$ therefore XY and YZ are perpendicular and triangle XYZ is right-angled with right angle at vertex Y.

d $x = \sqrt{(-2-1)^2 + (2-5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} \approx 4.24$ units

$y = \sqrt{(4-1)^2 + (-4-5)^2} = \sqrt{(3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} \approx 9.49$ units

$z = \sqrt{(4--2)^2 + (-4-2)^2} = \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} \approx 8.49$ units

e $c^2 = \left(\sqrt{90}\right)^2 = 90$; $a^2 + b^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{72}\right)^2 = 18 + 72 = 90$

f Midpoint of ZX, M = $\left(\dfrac{4+1}{2}, \dfrac{-4+5}{2}\right)$ = (2.5, 0.5)

g $\text{MY} = \sqrt{(2.5--2)^2 + (0.5-2)^2} = \sqrt{(4.5)^2 + (-1.5)^2} = \sqrt{20.25+2.25} = \sqrt{22.5} \approx 4.74$ units

h The two smaller triangles are MYZ and MYX. As MY is half the length of XZ, it means that MY = MZ = MX and the two triangles are isosceles triangles. All angles are acute, making the triangles acute-angled isosceles triangles.

i Gradient of XZ = –3 so gradient of YH = $\frac{1}{3}$

Rule for YH: $y - 2 = \frac{1}{3}(x + 2)$; $y - 2 = \frac{1}{3}x + \frac{2}{3}$; $y = \frac{1}{3}x + 2\frac{2}{3}$

j Rule for XZ: $y = -3x + 8$ [1]

Rule for YH: $y = \frac{1}{3}x + 2\frac{2}{3}$ [2]

Substituting [1] into [2]: $-3x + 8 = \frac{1}{3}x + 2\frac{2}{3}$, $-9x + 24 = x + 8$; $10x = 16$, $x = 1\frac{3}{5}$

Substituting $x = 1\frac{3}{5}$ into [1]: $y = -3(1\frac{3}{5}) + 8 = 3\frac{1}{5}$

Coordinates of H are $(1\frac{3}{5}, 3\frac{1}{5})$ or (1.6, 3.2).

k $\text{YH} = \sqrt{(1.6-(-2))^2 + (3.2-2)^2} = \sqrt{(3.6)^2 + (1.2)^2} = \sqrt{14.4} \approx 3.79$ units

l Using YX as the base and YZ as the perpendicular height:
area = ½ × 8.49 × 4.24 = 18.0 square units
Using XZ as the base and YH as the perpendicular height:
area = ½ × 9.49 × 3.79 = 18.0 square units

m $\text{YH} = \dfrac{4.24 \times 8.49}{9.49} = 3.79$ units

n With centre M, and radius MX = MY = MZ, a circle can be drawn to touch the vertices of the right-angled triangle. This is one of the geometry facts which states that the angle in a semi-circle is a right angle.

Resources

Chapter tests

There are two parallel chapter tests (Test A and B) available.

Chapter 4 Chapter test A

Chapter 4 Chapter test B

Summative tests

The three tests, A, B and C, for each chapter accommodate different student ability levels, with one section of overlap in each (the 'Proficient' part). These tests have been carefully mapped against AUSVELS and the Australian Curriculum in order to provide an accurate assessment of each student's level of achievement. When a student's marks are entered into the provided spreadsheet calculator, a letter grade is calculated based upon a weighted average of percentages according to the type of test completed.

Chapter 4 Summative test A: Modified

Aimed at the lower level of student ability.

The top mark a student can achieve in a modified test is a **C**.

Chapter 4 Summative test B: Core

Aimed at the middle level of student ability.

The top mark a student can achieve in a core test is a **B**.

Chapter 4 Summative test C: Extension

Aimed at the upper level of student ability.

The top mark a student can achieve in an extension test is an **A**.

Test answers

Chapter 4 Chapter test answers

Chapter 4 Summative test answers

Summative test spreadsheet calculator

Connect

Teaching support for pages 206–207

Teaching strategies

Comparing taxi charges

Focus: To use linear relationships to model a real-life application

- Students may need to have the concept of flagfall and distance rate defined. Explain how the tariff is affected by the time and day of the week.
- They need to be guided to see that in terms of a linear relationship, the flagfall is shown by the vertical (y) intercept and that the distance rate is shown by the gradient.
- Students should then be able write a linear rule for the taxi price in each city.
- To complete the task, students will need to apply the following skills from the chapter:
  - writing a linear rule
  - substituting into an equation
  - solving linear equations
  - solving and writing linear inequalities
  - graphing a linear relationship
  - solving simultaneous linear equations.
- Encourage students to be creative in presenting their report but stress that correct calculations with appropriate reasoning should be shown. They need to justify their findings and include any assumptions they have made.
- Sample answers are provided for the Connect task.
- An assessment rubric is available (see Resources).

Resources

Assessment rubrics

Comparing taxi changes

Do you need to contact

Oxford University Press?

Customer Service

Email: cs.au@oup.com
Phone: 1300 650 616
Fax: 1300 813 602

Digital Support

Email: digitalsupport.au@oup.com
Phone: 1300 650 616

Mailing Address

GPO Box 2784
Melbourne Victoria 3001

Operating Hours

8:30am – 4:45pm
Monday to Friday AEST
(Excluding public holidays)

We look forward to hearing from you.

OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND